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Derivation and Measurement of the Velocity Parameters of Hydrodynamics Oscillating System

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Preliminary note

An analytical derivation for the velocity parameters of elements of a hydrodynamic acoustic flow oscillating system, namely the liquid flow velocities, the mean values of liquid oscillation velocities and the phase velocity of acoustic wave were presented in this paper. There are also presented results of indirect measurements. The stated hydrodynamic oscillating system generates an amplify modulated high-velocity liquid flow to divide materials by water jet technology.

Izvod i mjerenje brzine hidrauličkog oscilatornog sustava

Prethodno priopćenje

Ovaj rad prikazuje analitički izvod parametara brzine za elemente hidrodinamičkog akustičnog oscilatornog sustava strujanja, to jest brzine strujanja tekućine, srednje vrijednosti brzina titranja tekućine i faze brzine akustičnog vala. Isto tako predstavljeni su rezultati indirektnih mjerenja. Analizirani hidrodinamički oscilatorni sustav generira povećan usklađeni visoko-brzinski tok tekućine koji omogućava razdvajanje materijala pri korištenju tehnologije rezanja mlazom vode.

1. Introduction

Processes inside a hydrodynamic oscillating system (associated with hydrodynamics and flow modulations) and also processes outside this system (associated with dividing of various materials) are closely connected with an analysis of flow velocities, mean oscillation velocities and phase velocity of acoustic wave. Many technical experts are concerned with problems of water jet dividing of materials and they emulate to increase the quality of process of material dividing as well as the created topography of surface [1], [2], [3], [4]. The stated hydrodynamic oscillating system can be defined as a oscillating liquid medium which by means of an implanted resonator strengthens the acoustic oscillations with the frequency from units Hz to tens kHz. This system consists of the following parts, i.e. elements: cylindrical chambers (Figures 1, ad 2, 3, 5) and cylindrical tubes (Figures 1, ad 1, 6, 7). Water is supplied to the system by a piston pump (exerts the pressure from 5 MPa to

Symbols/Oznake

v_1	- flow velocity in *i*-th element, m/s - brzina toka *i*- tog elementa
ρ	- liquid density in *i*-th element, kg/m^3 - gustoća tekućine *i* – tog elementa
ρ_0	- liquid density at the exit of nozzle, kg/m^3 - gustoća tekućine na izlazu iz sapnice
r_i	- radius of *i*-th element, m - polumjer *i* – tog elementa
r_0	- radius of exit tube, m - polumjer na izlazu iz cijevi
v_0	- velocity at the exit of system, m/s - brzina toka na izlazu iz sustava
u_i	- oscillation velocity in the *i*-th element, m/s - brzina titranja *i* – tog elementa
p	- liquid pressure, MPa - tlak tekućine
ω	- angular velocity, rad/s - kutna brzina
Δp	- pressure loss owing to the turbulent friction, MPa - gubitak tlaka zbog trenja turbulencije
Re	- Reynolds number - Reynoldsov broj
A_{0i}	- oscillation amplitudes, m - amplitude titranja
S_i	- cross-section of *i*-th element of system, m^2 - površina presjeka *i*-tog elementa sustava
f_{MZ}	- a measured fundamental frequency, Hz - mjerena osnovna frekvencija
m_{ai}	- acoustic mass, kg - masa akustičnog titranja
b_{ai}	- coefficient of acoustic absorbing - faktor akustične apsorpcije
v_φ	- phase velocity of wave, m/s - fazna brzina vala
γ	- liquid compressibility, m2/N - stišljivost tekućine

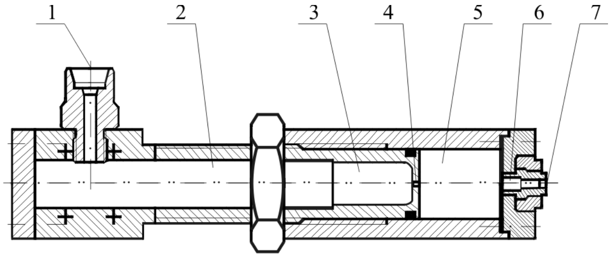

Figure 1. A schematic diagram of hydrodynamic oscillating system

Slika 1. Shematski prikaz hidrodinamičkog oscilatornog sustava

25 MPa), a supply lateral tube 1, goes through an entry (calming) chamber composed of the upper 2 and the lower 3 part; through an exit tube, i.e. nozzle 6, 7 this water abandons the system in a form of a cutting flexibly tool, i.e. a liquid jet. Into the system we implant (Figure 1) a resonant chamber 5 with an entry tube 4, by means of a socket.

Theoretically, the physical elements are represented by characteristic acoustic parameters. The tubes are represented by acoustic mass, the chambers then by acoustic compliance [6]. The determination of characteristic acoustic parameters is necessary for the analytical evaluation of fundamental frequency, amplitude, pressure and energy of liquid oscillations. The directly measured or exactly adjusted input parameters are primary the geometrical dimensions of elements (their length and radius) and the liquid pressure; secondary then the liquid density inside and at the exit of system (differences in the liquid density of elements are insignificant), liquid viscosity, liquid compressibility and others. The indirectly measured velocity parameters have relatively high informative value for dynamics of the process inside hydrodynamic oscillating system.

2. Derivation and measurement of flow velocity in the elements

The analysis of measurement results has shown that cardinal parameters describing the properties of liquid flow are a liquid pressure, a flow velocity, a liquid density and a liquid compressibility. By evaluation of Reynold's number we survey that the liquid flow is highly turbulent (Table 1). Requirements of technical practice tend towards the simplification of description of turbulent field. We solve in this way only the mean values of flow velocity in the axis of symmetry of elements and no problems of turbulence in a partial elementary locality of system elements. We calculate the flow velocity v_i in *i*-th element of system by the continuity flow equation:

$$v_i = \frac{r_0^2\, v_0\, \rho_0}{r_i^2\, \rho}\ ,\ i = 1,2,...,6, \tag{1}$$

where ρ is the liquid density in *i*-th element (simplified, inside the whole system the liquid density can be taken as constant); ρ_0 is the liquid density at the exit of nozzle; r_i is the radius of *i*-th element; r_0 is the radius of exit tube, i.e. the radius of nozzle; v_0 is the flow velocity at the exit of system.

Table 1. An example of comparison of flow velocities in the elements and multiples of critical values of Reynolds number

Tablica 1. Primjer usporedbe brzine toka u elementima i višestrukih kritičnih vrijednosti Reynoldsovog broja

p MPa	v_1 $\mathrm{m{\cdot}s^{-1}}$	$v_{2,3}$ $\mathrm{m{\cdot}s^{-1}}$	v_4 $\mathrm{m{\cdot}s^{-1}}$	v_5 $\mathrm{m{\cdot}s^{-1}}$	v_6 $\mathrm{m{\cdot}s^{-1}}$
5	9,986	0,799	22,469	0,399	89,875
10	14,091	1,128	31,706	0,564	126,822
15	17,221	1,378	38,748	0,689	154,992
20	19,844	1,588	44,648	0,794	178,593
25	22,140	1,772	49,815	0,886	199,261

p MPa	$n_1=\dfrac{Re_1}{2320}$	$n_{2,3}=\dfrac{Re_{2,3}}{2320}$	$n_4=\dfrac{Re_4}{2320}$	$n_5=\dfrac{Re_5}{2320}$	$n_6=\dfrac{Re_6}{2320}$
5	12,9	3,4	19,3	2,6	38,7
10	18,2	4,9	27,4	3,7	54,7
15	22,3	6,0	33,5	4,5	67,0
20	25,8	6,9	38,7	5,2	77,4
25	28,8	7,7	43,2	5,8	86,5

The changes of flow velocities (Table 1) can be commented in this way: with the increasing value of liquid pressure (see columns of Table 1) the flow velocities increase in all elements of system. At setting the certain liquid pressure the values of flow velocities in the elements change according to the changes of cross-sections of system elements (see rows of Table 1). Through the chambers the liquid flows with velocities of a order smaller than in the tubes. In the resonant chamber this liquid flows relatively slowly than in the entry chamber. As well, in the nozzle this liquid flows of a order faster than in the connecting socket and in the supply tube. The exit velocity v_0 is determined by equation (2), where the conversion coefficient 0.9 is given empirically [5]

$$v_0 = 0.9\sqrt{\frac{2p}{\rho_0}}\,. \tag{2}$$

3. Derivation and measurement of oscillation velocities in elements

In accordance with experience, the liquid pressure p is taken (by the Bernoulli's equation) as corresponding only to the dynamic components of liquid flow and oscillation pressure, because the study system is not a classical hydrodynamic "pipeline". At the liquid pressures from 5 MPa to 25 MPa, at the total very small length of given system (approximately 0.21 m) and at the very small diameters of elements (the tubes have the radius of a order in millimeters, the chambers in centimetres), we disregard of an influence of static pressure component and also pressure component corresponding to the non-horizontal position of system in the gravitational field. The dissipations of liquid pressure and energy as a consequence of hydrodynamic local wastes are nonmeasurable, only the pressure dissipations by turbulent friction are significant.

The mean value of oscillation velocity u_i in the i-th element is a function of liquid pressure p, pressure loss Δp owing to the turbulent friction, the radius r_i of i-th element, the liquid density ρ in the elements, the liquid density ρ_0 at the exit of system, the exit flow velocity v_0 and the radius r_0 of nozzle. The value (4) we obtain by substitution of the equation (1) to the equation (3)

$$p = \Delta p + \frac{1}{2}\rho u_i^2 + \frac{1}{2}\rho v_i^2\,,\ i = 1,2,\ldots,6; \tag{3}$$

$$u_i = \sqrt{\frac{2}{\rho}\left(p - \Delta p - \frac{r_0^4\, v_0^2\, \rho_0^2}{2\, r_i^4\, \rho}\right)}\,. \tag{4}$$

Table 2. An example of oscillation velocities in the elements of system

Tablica 2. Primjer promjene brzina u elementima sustava

p MPa	u_1	$u_{\mathrm{mean}\ 2,3}$	u_4	u_5	u_6
	$\mathrm{m{\cdot}s^{-1}}$				
5	89,42	89,97	87,13	89,98	4,30
10	126,32	127,10	123,09	127,11	8,51
15	154,55	155,50	150,60	155,51	12,65
20	178,27	179,37	173,73	179,37	16,74
25	199,11	200,33	194,05	200,34	20,77

The changes of mean values of oscillation velocities (Table 2) can be commented as follows: with the increasing value of liquid pressure (see columns of Table 2) the mean values of oscillation velocities increase also in all elements of system. At setting the certain liquid pressure the mean values of oscillation velocities in the elements of system do not change significantly with the exception of nozzle (see rows of Table 2). We can see that in the nozzle the liquid flows of an order faster, but oscillates of an order smaller than in the other elements of system.

Note: The mean values of oscillation velocities we can not directly measure, these we evaluate indirectly, i.e. by calculation from input directly measured parameters. Their evaluations serve consecutive to the calculations of energy budget, i.e. oscillation amplitudes A_{0i}, liquid pressure and liquid energy of oscillations. The derivation of equation (5) is analogical to the theory of acoustic circuits and the derivation of equation (6) to the theory of mechanical oscillations. The calculations of oscillation amplitudes double in a way (5), (6) give comparable results (for example the oscillation amplitudes are approximately 0,003m).

$$A_{0i} = \frac{(\,p - \Delta p\,)\,S_i}{m_a\sqrt{\left(\omega_0^2 - \omega^2\right)^2 + 4\,b_a{}^2\omega^2}} \Leftrightarrow \omega = \omega_0\,;$$

$$A_{0i} = \frac{(\,p - \Delta p\,)S_i}{4\pi\, f_{ZM}\, m_{ai} b_{ai}}\,, \tag{5}$$

$$u_i = 2\pi f_{ZM} A_{0i} \Rightarrow A_{0\,i} = \frac{1}{2\pi\, f_{ZM}}\sqrt{\frac{2\left(p - \Delta p\right)}{\rho} - v_i^2}\,, \tag{6}$$

S_i is a cross-section of *i*-th element of system, f_{ZM} a measured fundamental frequency, m_{ai} an acoustic mass, b_{ai} a coefficient of acoustic absorbing [7], [8], [9].

4. Derivation and measurement of phase velocity of acoustic wave

In aerodynamic oscillating systems is evaluated only the phase velocity of sound; the air flow velocity is omitted with respect to this velocity. According to the acoustic theory, it is supposed that oscillations of gas particles are subject to the Poisson´s law. At relatively fast oscillations the temperatures of compressed and diluted particles of real gas do not equalize conformable with an approximately adiabatic process.

Contrariwise, in a hydrodynamic oscillating system it´s possible to consider a wave length insofar big that the temperatures of compressed and diluted particles of real liquid are equalizing and thus the indirect dependence of liquid pressure and liquid volume can be supposed. By partial differentiation to liquid volume and liquid pressure we are able to derive the interdependence of elementary changes of liquid volume and liquid pressure. A knowledge of wave length is very important: the condition of functionality of an acoustic circuits is, that the lengths of elements are smaller than the undivided odd multiples of quarter-wave and the diameters of elements are smaller than the undivided multiples of half-wave.

By the Laplace formula of longitudinal wave propagation through liquids the phase velocity v_φ of wave is generally a function of material characteristics, i.e. the liquid compressibility γ and liquid density ρ (7), specifically a function of water pressure p (8). Results of equations (7), (8) given an comparable values.

$$v_\varphi = \sqrt{\frac{1}{\gamma\,\rho}}\,; \tag{7}$$

$$v_\varphi = 2{\cdot}10^{-6}(p - \Delta p) + 1432\,. \tag{8}$$

Table 3. An example of phase velocities

Tablica 3. Primjer brzinskih faza

***p*, MPa**	**v_φ, m·s^{-1}**
5	1440,1
10	1448,2
15	1456,3
20	1464,4
25	1472,5

By means of the phase velocity v_φ of acoustic wave and the measured fundamental frequency f_{ZM} we evaluate the lengths of acoustic waves in the elements of systems. A necessary condition of functionality of the given acoustic element in a series of other elements is the non-existence of stationary wave in it. The stationary waves would cause either the interruption of elements connected in series, or the whole system could function as an open “pipe” and generate the undesirable fundamental frequency. Owing to the small geometrical parameters of elements this situation did not arise.

5. Conclusion

In this contribution, the evaluation of velocity parameters in the elements of the hydrodynamic oscillating system was carried out and simultaneously an example from the database of indirect measure results (by way approximately of 200 direct input values) was presented. The parameters of flow velocities, oscillation velocities and phase velocity were derived by analogy with the theory of acoustic air circuits and in accordance with experimental results were verified.

The results of measurement were produced, verified and presented either by directly measured or accurately adjusted input parameters (primarily the liquid pressure, the geometrical dimensions of all elements of system and fundamental frequency) obtained at the Institute of Geonics of Academy of Sciences of the Czech Republic in Ostrava. Because conformity between theoretically predicted results and laboratory measure results of all parameters does not exceed 6%, for the needs of technical practice the accuracy of relative uncertainties and relative differences of measurement results can be considered to be satisfactory.

The analysis of velocity parameters is necessary, because the calculation of flow velocity and oscillation velocity substitute to the calculations of acoustic characteristic parameters, i.e. an acoustic mass and an acoustic compliance, and then those results to the calculations of final parameters, i.e. the fundamental frequency, amplitude, pressure and energy of oscillations.

The hydrodynamic resonators are applied in technical practice, but a simulation their parameters and also a regulation are missing.

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