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EVALUATION OF BENFORD'S LOW APPLICATION IN STOCK PRICES AND STOCK TURNOVER

PROCJENA MOGUĆNOSTI PRIMJENE BENFORDOVOG ZAKONA U ANALIZI CIJENA I PROMETA DIONICA

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Abstract

Application of Benford's Law in the field of financial analysis is very rarely covered. In this paper it is researched possibility of usage of this law in analysis of stock prices and stock turnover in Zagreb stock exchange. On the basis of online available and public data, sets of input numbers are prepared. These sets are checked against Benford's Law. Results show that sets partially fit to this law. Stock turnover data conforms to Benford's Law, while daily closing stock prices do not. Probably, psychological factors significantly influence daily closing stock prices, so these values do not conform to Benford's distribution.

Sažetak

O primjeni Benfordovog zakona u području financijskih analiza u literaturi postoji malo izvora. U ovom radu se istražuje mogućnost primjene ovog zakona u analizi kretanja cijena i prometa dionica na burzi, i to na primjeru Zagrebačke burze. Temeljem online raspoloživih podataka, oblikovani su skupovi ulaznih nizova na koje je primijenjen Benfordov zakon. Dobiveni rezultati ukazuju da je ovaj zakon djelomično primjenljiv i za takvu problematiku. Potpuno je primjenljiv za analize i procjene ukupnog dnevnog promete dionica, ali nije primjenljiv za analize i procjene zadnjih dnevnih cijena dionica. Vjerojatno na ove cijene na tržištu značajno utječu psihološki čimbenici, pa stoga ova pojava ne slijedi Benfordovu distribuciju.

1. Introduction

Reason for writing this paper is researcher's curiosity in examining possibility of use of modern computer technology in analysis of change in stock prices and stock turnover on specific stock exchange but avoiding only ordinary statistical approach. In this paper publicly available data on stock prices and stock turnovers from Zagreb Stock Exchange are used as input values, and such set of values is analysed by use of Benford's Law. After research it was possible to make conclusion about deviations in stock prices and stock turnover changes on before mentioned stock exchange.

Doubtlessly, stock markets have very important influence on world economy, and consequently on Croatian economy. Changes in economy are often described and explained by changes in stock prices and/or stock turnover. Not bearing in mind considerable dependency of national and world economies on these parameters, it is especially interesting to answer the question if it is possible to note whether there are some regularities in creation of stock prices and stock turnover changes. Some interesting questions arise: are stock prices uniformly distributed, do stock prices follow certain rules that are applicable for some number sets and are psychological and some other market aspects important in stock prices creation. In this paper, it is examined if number sets of stock prices and stock turnovers on Zagreb Stock Exchange agree with Benford's Law.

Examination of conformity with Benford's law according to first digit i.e. examination of frequencies of certain first digits in daily closing stock prices and daily stock turnovers is included. Intuitively, it may be assumed that stock prices and stock turnovers are uniformly distributed which means that appearance of each digit on leading (most important, most valuable) position in stock prices and stock turnovers is equal.

All stock exchanges in the world use so called HLOC (high-low-open-close) prices in order to show range of daily stock prices changes. The greatest achieved price for certain stock is marked *high*, while the smallest achieved price for certain stock during working day hours is marked low. Open and close stock price mark first and last agreed price for certain stock. Sometimes HLOC values show prices for week, month, year or specific period of time.

Analysis in this paper is based on closing (*close*) stock price and daily turnover. With regard to available data, it is possible to conduct analysis against lowest (*low*) and highest (*high*) daily stock prices and against volume i.e. total number of stocks traded during the day. Also, it would be interesting to perform analysis according to volatility of stock prices with respect to closing price on previous day and closing price of observed day (that is, comparison between closing prices in two consecutive days). Unfortunately, at this very moment, there are no available historical data on opening stock prices, so it is not possible to analyse data according to stock prices intraday volatility (volatility during specific day), i.e. according to change between last and first stock price during a day.

Data source for historical stock prices and stock turnovers was web site www.zse.hr.

2. Research Method

Benford's Law was used as a basis of research in above setting of a problem. This Law defines expected digit frequencies in certain number sets. Also, it is often called „First Digit Law", „First Digit Phenomenon" and „Leading Digit Phenomenon". American astronomer Simon Newcomb was first who found out that numbers more frequently begin with smaller digits than with greater digits. Newcomb noticed that pages in logarithm tables were more dirty at the start, i. e. more used, and progressively cleaner as approaching to the end. He concluded that numbers more often begin with digit 1 than with any other digit, and in addition, that probability of each following digit (up to 9) at the most significant position in number progressively decreases.

Frank Benford gathered more than 20.000 observations from different sources (geographical area population, river areas, physical constants etc.). He analysed frequencies of first digits for each number set. After he summarized all individual analyses he concluded that probability of first digit being 1 is 0,30103 which equals $\log_{10}2$, probability of first digit being 2 is 0,17609 which equals $\log_{10}3/2$ etc. Benford's Law of first digit i.e. probability V of appearance of digit z on leftmost position in number can be expressed by following formula:

$$V(z_1)=\log_{10}(1+1/z_1),\ z_1 \in \{1,2,...,9\}$$

Probabilities of each digit on the most significant position in number is shown in table 1.

Table 1. Probabilities of each digit on the first position in number according to Benford's Law

digit (z_1)	1	2	3	4	5	6	7	8	9
probability $V(z_1)$	0,30103	0,17609	0.12494	0.09691	0.07918	0.06695	0.05799	0.05115	0.04576

Source: [Nigrini & Mittermaier], page 54.

It was explained that Benford's Law also applies to other significant digits in numbers. Furthermore, if second digit is observed, probability V of appearance of digit z on 2nd position in number is:

$$V(z_2)=\log_{10}(1+1/1z_2),\ z_2 \in \{1,2,...,9\}$$

Also, Benford's Law defines following rules of appearance of first two ($V(z_1z_2)$) and first three ($V(z_1z_2z_3)$) digits in number:

$$V(z_1z_2)=\log_{10}(1+1/z_1z_2),\ z_1z_2 \in \{10,11,...,99\}$$

$$V(z_1z_2z_3)=\log_{10}(1+1/z_1z_2z_3),\ z_1z_2z_3 \in \{100,101,...,999\}$$

This law starts from assumption that number set sorted ascending forms geometric series. Intuitive explanation of Benford's Law is pretty clear. If town with population of 10.000 is observed, first digit is 1. Digit 1 will stay on first position of population number until population rise for 100%, which is 20.000 inhabitants. After this, only rise of 50% is needed in order to change first digit from 2 to digit 3. It is clear that town will have digit 1 the most of the time because the most time is needed to change first digit from 1 to 2.

In Nigrini /**1**/ and Nigrini & Mittermaier /**2**/ criteria are set for number series to conform to Benford's Law:

1. Number series must describe values of same or similar phenomenon. E.g. lake area, heights of mountains, total yearly revenue of companies, total daily turnover on stock exchange.
2. Number series should not have defined minimal and maximal values. If minimal commission on foreign currency exchange in exchange office is 3 kunas, then set of commission values will not fit to Benford's Law, because large number of commission values will have digit 3 as a first digit. Digit 0 is allowed minimum.
3. Number series should not comprise of so called assigned numbers. These numbers are assigned to various phenomena instead of description, and their important attribute is that there is no sense to perform mathematical operations on these numbers. Examples are citizens identification numbers, bank account numbers, telephone numbers, numbers on car registration plates etc.
4. This law does not apply to numbers which creation is influenced by psychological factors, like prices in supermarket or ATM cash withdrawals.

Very important feature of Benford's Law is invariance. If certain number set fits Benford's Law, then the set will follow the law independently on measurement unit in which it is expressed. Consequently, if all numbers in a set that conforms to Benford's Law are multiplied by a constant, then new set will also conform to the law. For example, if the law is followed by set of total yearly companies turnover, then the law will be followed independently on currency in which turnovers are expressed. Invariance rule also holds for reciprocal number sets. For example, if the law is followed by set of prices in kunas per stock, it will hold for numbers of stocks per kuna.

3. Status of literature about the problem

There is very extensive literature on various fields of usage of Benford's Law. But, we did not manage to find any literature which will indicate that research on usage of Benford's Law in analysis of daily stock prices changes and their daily turnover was carried out.

Some authors intuitively claim that frequencies of stock prices in certain period conform to Benford's Law, while others claim stock prices can not fit Benford's Law. In Hill /**3**/, on page 1, it is stated that certain digits in stock-market prices should occur more often than others. Unfortunately, author does not give detailed explanation of this assertion or proves its truth. Reason for inherently non conformance of stock prices with Benford's Law some authors find in fact that stock prices are to significant extent formed on the basis of market psychology /**4**/ or under the influence of financially powerful groups, when some other rules are valid.

Although we tried to find as much as possible papers on this subject, we did not manage to find any. Consequently, we think former research of Benford's Law is restricted on analysis of stock market indices or their conformance to Benford's Law according to first, first two and first three digits. On the basis of investigations of certain digit frequencies on first, first two and first three positions in daily market indices, some authors make conclusion about existence of psychological barriers in stock markets. In Ley /**5**/ it is shown that number sets of one-day returns on the Dow-Jones Industrial Average Index (DJIA) and the Standard and Poor's Index (S&P) agrees with Benford's Law.

Research carried out in Doucouliagos /**6**/ shows that stock prices on Australian stock market do not follow uniform distribution. Stock indices are also analysed and it is stated that for Nasdaq psychological barrier was positioned on value 5.000 in march of 2000. Dow Jones Industrial Average Index had important psychological limit on value 10.000, as Nikkei on value 25.000. In DeCeuster & Dhaene & Schatteman /**7**/ it is explained that research of eventual psychological barriers in change of stock indices should not be based on comparison with uniform distribution, but on comparison with Benford's Law distribution. Starting from such changed hypothesis, authors shown that there are no reasonable arguments on existence of psychological barriers for stock indices. Particularly, authors assume that stock indices must fit to Benford's Law, with deviation from uniform distribution as a consequence. In order to proof if psychological barriers in stock indices values exist, it is necessary to check existence of relevant deviation of indices number set from Benford's Law distribution. Authors researched stock indices FTSE 100, DJIA and Nikkei and concluded that there are no reasonable proofs on existence of psychological levels of certain indices if Benford's Law is defined as a basis of comparison. According to previous research, it can be concluded that values of stock indices are not behaving in conformance with uniform distribution but are close to Benford's distribution.

If significant deviations against uniform distribution of stock indices exist i.e. indices are following Benford's Law, with regard to invariance of number sets for which Benford's Law holds, we may assume that stock prices are diverging from uniform distribution. Consequently, set of stock prices that is large enough should fit characteristics of Benford's Law.

On the basis of this, it may be supposed that number set of closing daily stock prices conform to Benford's Law.

Taking into account that total daily turnover of stocks is calculated on the basis of volume (quantity) and prices of single stock, it should be assumed that this parameter conforms to Benford's Law.

4. Results

According to problem explanations we gave in previous chapters, we set following hypotheses:

1. If number set of closing daily stock prices, for leading (most significant) digit, conforms to Benford's Law.
2. If number set of total daily stock turnovers, for leading digit, conforms to Benford's Law

Our research included operations on Zagreb Stock Exchange in period between January 1st 1998 and February 26th 2008., and involved closing daily stock prices and total daily stock turnovers. On web sites of Zagreb Stock Exchange, following historical data are available for each stock traded during a day:

— date
— highest daily price (HIGH)
— lowest daily price (LOW)
— closing daily price (CLOSE)
— percentage of change according to previous closing daily price
— volume, i.e. total number of stocks traded during a day
— turnover, i.e. total amount traded for stock during a day

Total number of data rows of stock prices and turnovers in observed period was 82.134.

We used Chi-square (χ^2) test in order to evaluate conformance of stock prices and turnovers with Benford's Law. With that test we wanted to show if eventual deviation of frequencies of observed phenomena (closing daily stock prices and total daily stock turnovers) from Benford's Law frequencies, are incidental or not. We set two null hypotheses and tested significance level on 5%.

Firstly we conducted analysis of closing daily stock prices in previously defined period. The results are shown in table 2.

Table 2. Application of Benford's Law on changes in closing daily stock prices

digit	BZ	fo	fs	fs-fo	Po	χ2
1	0,30103	24.724,40	24402	-322,40	-1,3039	4,2040
2	0,17609	14.462,43	13996	-466,43	-3,2251	15,0429
3	0,12494	10.261,97	11609	1.347,03	13,1264	176,8169
4	0,09691	7.959,80	8608	648,20	8,1434	52,7857
5	0,07918	6.503,63	7056	552,37	8,4932	46,9142
6	0,06695	5.498,74	5213	-285,74	-5,1964	14,8484
7	0,05799	4.763,23	4363	-400,23	-8,4024	33,6293
8	0,05115	4.201,46	4149	-52,46	-1,2486	0,6550
9	0,04576	3.758,34	2738	-1.020,34	-27,1486	277,0089
		Σ	82134	POP	8,4764	621,9053

Explanation of symbols used:
BZ – probability of digit according to Benford's Law
fo – expected frequency – number of observations expected according to Benford's Law
fs – actual frequency
fs-fo – difference between actual and expected frequency
Po – percentage of deviation of actual from expected frequency
POP – average deviation from percentages of deviation (sum of absolute values Po divided by number of frequency categories i.e. 9)

Average deviation from percentages of deviation (POP) is used in Nigrini & Mittermaier - page 59 for intuitive explanation if certain number set conforms to Benford's Law. This measure does not have defined limit values i.e. range in which it can be stated whether mutual deviation of value sets is significant or not. Furthermore, by means of average deviation from percentages of deviation it can not be judged if set of closing daily stock prices conforms to Benford's Law.

According to 8 degrees of freedom and testing on significance level of 5%, in order to confirm first null hypothesis, value of χ^2 should be less than 15,507. Since it is not the case, we have to reject assumption that number set of closing daily stock values for 10 year period conforms to Benford's Law.

We performed additional analyses, in order to determine in which relationships with Benford's Law are certain subsets of closing stock prices (with regards to periods, closing prices amounts). By separate analysis of first and second 5 years, we verified that both subsets do not fit Benford's Law. Namely, for first 5 years (total of 13670 stock prices) χ^2 amounts to 627,0497 which is far greater than cut-off value for null hypothesis acceptance (15,507). Value of χ^2 for second 5 years (total of 68464 stock prices) is 799,196 which is also much greater value than cut-off value.

Data on stock prices were analysed in separate periods of nearly 2 years. In each 2 years period significant deviations from Benford's Law were registered, what is shown in table 3.

Table 3. Deviation of closing stock prices from Benford's Law depending on observation periods

period in years (date of beginning – end)	N	χ^2
0-2 (1.1.1998.-12.1.2000.)	3721	746,4312
2-4 (13.1.2000.-24.1.2002.)	5906	1182,085
4-6 (25.1.2002.-5.2.2004.)	9192	526,4596
6-8 (6.2.2004.-17.2.2006.)	21651	900,7057
8-10 (18.2.2006.-26.2.2008.)	41664	634,971

In order to accept null hypothesis on conformance of number set from certain two year period with Benford's Law, χ^2 should be less than 15,507.

We carried out stratification of closing stock prices on intervals <0,100>, [100, 1000> and [1000,∞> kunas, and analysed conformance of these strata with Benford's Law. according to given results shown in table 4., we may make conclusion that no subset fits Benford's Law.

Table 4. Deviation of closing stock prices from Benford's Law depending on price intervals

interval	N	χ^2
<0,100>	13552	9473,7188
[100, 1000>	46060	1513,77
[1000,∞>	22522	3204,7751

If we analyse prices from 100 to 1.000 kunas more closely, which is the interval with biggest number of observations, specifically prices starting with digit 9 (which is interval [900, 1000>), significant deviation from Benford's distribution can be noticed. Number of instances belonging to this interval is 48% less than it should be according to Benford's distribution and even 78% less than uniform distribution. Also, all prices starting with digit 6 and greater have significantly lesser number of actual observations than expected. Number of prices starting with digit 6 is 12% less than expected by Benford's distribution and 47% less than uniform distribution. For digit 7, number of observations is 41% less than Benford's distribution and 69% less than if compared with uniform distribution. Digit 8 is on first position in 22% less cases than it should be according to Benford's Law, and 64% less than prices would fit uniform distribution. It can be concluded that traders who aim at prices between 100 and 1.000 kunas (probably in this group there is great number of „small" stockholders, i.e. persons) reluctantly buy more expensive stocks (greater than 600 kunas), and are especially reluctant to trade stocks more expensive than 900 kunas. In addition, for stock prices between 100 and 1.000 kunas it is noticeable that only prices which have digit 1 on first position conform to Benford's Law. Digit 1 appears as first positioned digit in prices with probability of 0,30454, which deviates from Benford's distribution for only 1% with χ^2 1,8825.

When analysing stock prices greater than 1.000 kunas, it is noticed considerable excess of prices which start with digit 1 (46% more than Benford's distribution), and considerable lack of all prices starting with digits 4 to 9 (from 34% to 64% when compared to distribution according to Benford's Law). Interestingly, deficiencies get bigger as approaching to digit 9. So, digit 4 appears on first position in 34% less cases than in Benford's and 42% less than in uniform distribution. For digit 5, deviation is 42% from Benford's and 58% from uniform distribution. Especially significant deviation is identified for digit 8 (53% less than Benford's and 78% less than uniform distribution). As with interval of prices between 100 and 1.000 kunas, the biggest deviation is with digit 9: frequency of this digit on most important position in numbers is 64% less than Benford's and 85% less than uniform distribution.

On the basis of this result, it can be concluded that buyers aiming at stocks more expensive than 1.000 kunas (probably in this group there is more „big" stockholders) are not ready to pay for stocks more expensive than 4.000 kunas per stock, and especially not prices greater than 8.000 kunas. Also, it can be stated that buyers are more often ready to buy stocks in interval 1.000 to 4.000 kunas, and especially stocks with prices from 1.000 to 2.000 kunas, probably because they expect further rise of these stock prices.

On the basis of last two examples of first digit analysis it can be stated that there are certain psychological barriers in process of forming stock prices on Zagreb Stock Exchange. By analysis of first two and first three digits frequencies and their relations to corresponding Benford's Law frequencies, probably it could be concluded that certain rules exist in creation of significant (psychological) barriers in stock prices.

Further, in table 5 we show analysis of Benford's Law application on total daily stock turnovers on Zagreb Stock Exchange in interval January 1st 1998 and February 26th 2008.

Table 5. Application of Benford's Law on total daily stock turnovers

digit	BZ	fo	fs	fs-fo	Po	χ2
1	0,30103	24.725,40	24600	123,40	-0,5072	0,6360
2	0,17609	14.463,43	14469	-5,57	0,0385	0,0021
3	0,12494	10.261,97	10234	27,97	-0,2726	0,0762
4	0,09691	7.959,80	7954	5,80	-0,0729	0,0042
5	0,07918	6.503,63	6465	38,63	-0,5940	0,2295
6	0,06695	5.498,74	5490	8,74	-0,1590	0,0139
7	0,05799	4.763,23	4935	-171,77	3,6062	6,1943
8	0,05115	4.201,46	4154	47,46	-1,1296	0,5361
9	0,04576	3.758,34	3833	-74,66	1,9865	1,4831
		∑	82134	POP	0,9295	9,1754

In order to confirm second null hypothesis, concerning 8 degrees of freedom and testing on 5% significance level, value of χ^2 should be less than 15,507. Taking into account that this value is 9,1754, it results that set of turnovers for mentioned 10 year period conforms to Benford's Law.

We conducted additional analyses, in order to identify relationship between Benford's Law and correspondent subsets of daily stock turnovers in observed period. Separate analyses of first and second five-year periods showed that both subsets conform to Benford's Law. Particularly, for first five-year period χ^2 amounts to 3,8397 which is less than cut-off value for null hypothesis acceptance (15,507). χ^2 value for second five-year period (68464 stock prices) is 8,5629 which is also less than cut-off value.

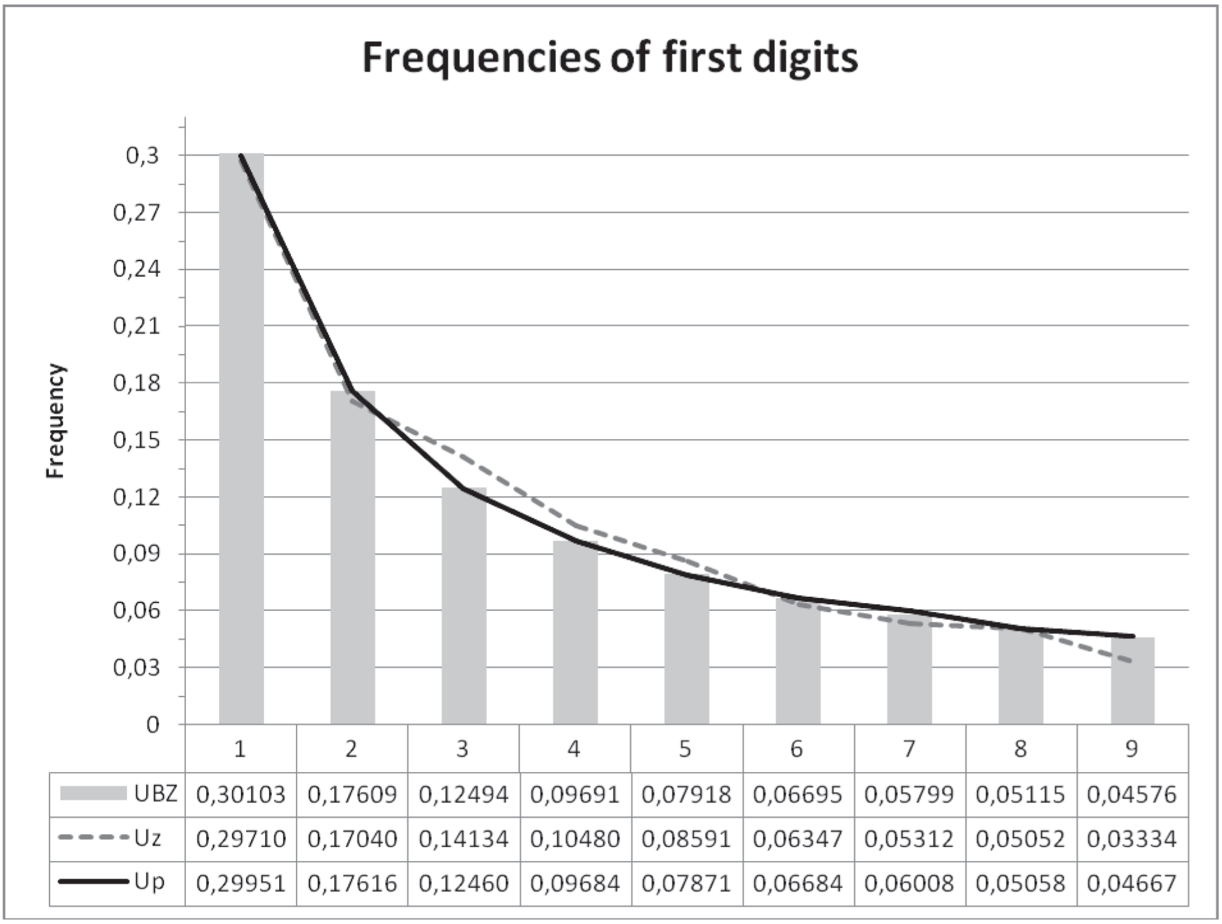
Data are analysed in nearly two-year intervals. In each two-year period insignificant deviation from Benford's distribution was recorded (Table 6).

Table 6. Deviations of total daily stock turnovers from Benford's Law according to observation periods

period in years (date of beginning – end)	N	χ^2
0-2 (1.1.1998.-12.1.2000.)	3721	6,3036
2-4 (13.1.2000.-24.1.2002.)	5906	7,9421
4-6 (25.1.2002.-5.2.2004.)	9192	14,0826
6-8 (6.2.2004.-17.2.2006.)	21651	5,1697
8-10 (18.2.2006.-26.2.2008.)	41664	6,9666

On picture 1 frequencies of certain digits (from 1 to 9) on first position in numbers (significantly most important) are shown. Abbreviation UBZ (presented by column) stands for frequencies according to Benford's Law, Uz shows frequencies of digits in daily closing stock prices, while Up stands for digit frequencies of daily stock turnovers.

It can be concluded that daily stock turnovers conform to Benford's Law for all observed periods. Psychological factors obviously does not have influence on frequencies of first digits appearance in stock turnovers, as it is the case with closing daily stock prices where it can be stated that psychological barriers exist.



	1	2	3	4	5	6	7	8	9
UBZ	0,30103	0,17609	0,12494	0,09691	0,07918	0,06695	0,05799	0,05115	0,04576
Uz	0,29710	0,17040	0,14134	0,10480	0,08591	0,06347	0,05312	0,05052	0,03334
Up	0,29951	0,17616	0,12460	0,09684	0,07871	0,06684	0,06008	0,05058	0,04667

Picture 1. Frequencies of digits on the most significant position

5. Conclusion

Basic objective of this paper was to examine, by virtue of modern ICT technology, whether Benford's Law applies to changes of daily closing stock prices and daily stock turnovers. Consequently, these parameters were observed on Zagreb stock exchange in period January 1st 1998 to February 26th 2008. Observation included 82.134 data rows.

Results we reached show that closing daily stock prices in observed ten-year period do not conform to Benford's Law. The same conclusion is valid for more detailed observation of these data, performed on two-year periods. The same is valid for stratification of stock prices (less than 100 kunas, 100 – 1000 kunas, over 1000 kunas). However, total daily stock turnovers on Zagreb Stock Exchange, observed in ten-year period, completely fit to Benford's Law. The same conclusion is valid for stratification of this data on five-year and two-year periods.

Results confirm assumptions of some other authors who claim that Benford's Law is not conformed by number sets on which psychology has important influence. We confirmed that on the example of closing daily stock prices. However, we confirmed general expectation that number sets on which creation psychological factors does not have direct influence fit to Benford's Law. This assumption is proved in example of daily stock turnovers analysis. Furthermore, it is noted that certain stock prices appear far seldom than it could be expected according to Benford's, and especially according to uniform distribution. It can be concluded that certain psychological barriers exist, i.e. (un)willingness of investors to buy stocks over certain prices.

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