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Discriminant function analysis of Upper Miocene and Pliocene sands from the southwestern part of the Pannonian Basin System, Croatia

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ABSTRACT

Multiple discriminant analysis (MDA) of Upper Miocene and Pliocene sands of the SW portion of the Pannonian Basin System was performed in order to determine the optimum criteria for separating the previously defined informal lithostratigraphic units – Ozalj, Andraševec, Hum Zabočki, Cernik and Pluska. The heavy mineral association is employed as independent variables, or descriptors, in the process of discrimination and results of analysis established a clear and strong bond between characteristic heavy mineral assemblages in the Upper Miocene and Pliocene sands and their affinity to particular lithostratigraphic units. In this respect, the Pluska unit is especially highlighted, followed by the Ozalj and Hum Zabočki units. Close heavy mineral/lithostratigraphic unit relationships were very useful in unravelling the basic sedimentological meaning hidden beneath specific discriminant functions in the mathematical model. The most prominent is DF1 accounting for almost 3/4 of the total variance of the investigated sedimentary system. In accordance with its 'key' mineral it is labeled kyanite function which indicates the (metamorphic) provenance of the siliciclastic mineral detritus. The second most important, DF2, is labeled garnet-zircon function and, being bipolar, it points at both provenance of siliciclastic material (older sedimentary rocks) and hydraulic conditions during the transport and sedimentation of the detritus. Finally, DF3 is of minor importance indicating the stability of the heavy mineral association in sandy sediments. Three lithostratigraphic units, namely Ozalj, Pluska and Hum Zabočki are clearly separated by the functions labeled after their distinctive ('key') heavy minerals.

Keywords: Discriminant function analysis, heavy mineral association, lithostratigraphic units, Upper Miocene, Pannonian Basin System

1. INTRODUCTION

The Pannonian Basin System (PBS) is a system of back-arc basins the evolution of which started in the Early Miocene epoch as a response to continental collision and subduction of the European Platform below the Apulian Platform (ROYDEN, 1988; HORVÁTH, 1995; KOVÁČ et al., 1998). Surrounded by the Alps, Carpathians and Dinarides (Fig. 1), it included a number of different sized, deep, depressions and basins separated by a comparatively shallow complex of basement rocks (HORVÁTH & ROYDEN, 1981; ROYDEN, 1988). The first phase of basin development was characterized by tectonic thinning of the crust and isostatic subsidence (syn-rift), while the second phase was marked by the cessation of rifting and subsidence caused by cooling of the lithosphere (post-rift) (HORVÁTH & ROYDEN, 1981; ROYDEN et al., 1983; ROYDEN, 1988). In the south western part of the PBS the syn-rift phase lasted from the Ottnangian to the Middle Badennian, while the post-rift phase extended from the Middle Badennian to the end of the Pontian (PAVELIĆ, 2001). Palaeogeographically, the PBS extended over the major part of Central Paratethys – a sedimentary basin having passed through a succession of stages of isolation and reconnection to the open oceans (Indopacific and Mediterranean) during its evolution. The final isolation at the end of the Middle Miocene resulted in development of Lake Pannon that existed as a separate sedimentary basin during the Late Miocene (RÖGL & STEININGER, 1983; RÖGL, 1998; 1999; MAGYAR et al., 1999).

From the Upper Miocene to the Lower Pliocene, great quantities of clastic detritus were transported to the southwestern part of the PBS from both the surrounding mountain ranges and the uplifted areas within the basin itself (ŠĆAVNIĆAR, 1979; ŠIMUNIĆ & ŠIMUNIĆ, 1987; KOVAČIĆ et al., 2004; KOVAČIĆ & GRIZELJ, 2006; GRIZELJ et al., 2007), triggering the submersion that marked the onset of compressional tectonics in basin development (JAMIČIĆ, 1995; TOMLJENOVIĆ & CSONTOS, 2001; MARTON et al., 2002), and the subsequent gradual and diachronous infilling of Lake Pannon (MAGYAR et al., 1999; KOVAČIĆ et al., 2004).

In earlier research work, the Upper Miocene and Pliocene sedimentary rocks of the southwestern part of the PBS were typically divided according to their endemic fossil assemblages (e.g. ŠIKIĆ et al., 1979; BASCH, 1983). However, according to the recent geological investigations related to the Basic Geological Map of the Republic of Croatia 1:50000, a number of informal lithostratigraphic units were distinguished among the investigated sedimentary rocks, based principally on the field observations of their lithological characteristics. A considerable body of data was thus created including the modal composition of sands, which represent the most important lithological member of the newly established units. Such a large amount of numerical data is quite a propitious material for a statistical procedure which employs various multivariate methods and techniques. One of the most often exploited tools in geosciences is discriminant function analysis which is used to compare a number of groups for which there already exists a sound (geological) basis for separation. For instance, here, lithostratigraphic units can serve as *á priori* established geological groups to which the collected geological objects (samples) can be assigned using some independent geological criterion such as geological mapping (ROCK, 1988). A key problem in the study can be summarized in a simple question: can such predefined groups be distinguished effectively utilizing the selected suite of known attributes in each sample such as, for example, the heavy mineral association (HMA), or subtle differences in the mineral composition that can disturb the homogeneity of groups acknowledged on 'coarser' geological criteria derived from geological mapping, and cause them to overlap significantly? It may also be important to allocate each new 'unknown' object (sample) to one of the previously defined groups following the principle of least distance (greatest similarity). This being so, the main scope of this work is defined as follows: a) to determine the strength of the relationship between the composition of the HMA of the studied sands and their lithostratigraphic affiliation after the method of multiple (multi-group) discriminant analysis (MDA) has been applied; b) to decide which minerals contribute most to discrimination between the lithostratigraphic units, and; c) to answer to what extent discrimination between individual groups facilitates interpretation of geological events.

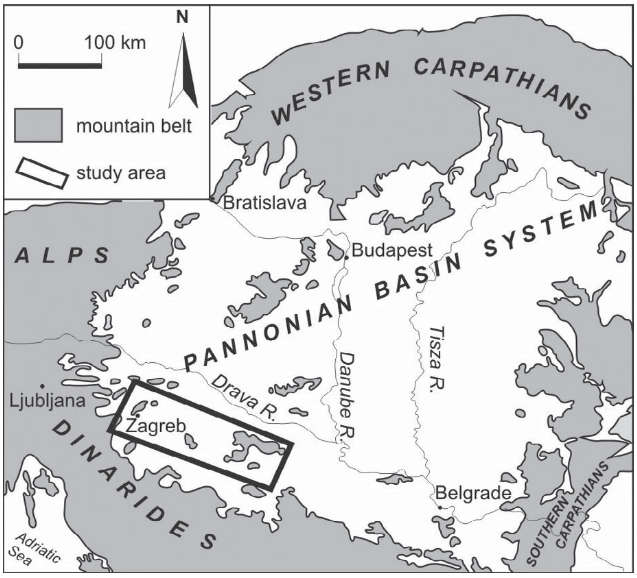

Figure 1: The Pannonian Basin System and its surroundings (after ROYDEN, 1988). The area delineated is in the South-western part of the Pannonian Basin System, shown in Fig. 2.

2. LITHOSTRATIGRAPHIC UNITS, FACIES, AND SEDIMENTARY ENVIRONMENTS

The investigated area is situated in the central and eastern parts of the Republic of Croatia. It consists of three different and geographically separated regions over about 200 km distance – Hrvatsko Zagorje with the Medvednica Mt., Žumberak Mt., and Slavonian Mts. (Fig. 2).

During the systematic fieldwork that included surface geological mapping, subject to the requirements of the Basic Geological Map of Croatia 1:50000, the Upper Miocene and Pliocene sediments were divided into six informal lithostratigraphic units, namely the Croatica (Cro), Medvedski Breg (MeB), Ozalj (Oza), Andraševec (And), Hum Zabočki (HZb), Pluska (Plu), and Cernik (Cer) units. In the Slavonian Mts., the MeB unit was, however, referred to as the Pavlovci unit while simultaneously the designation Nova Gradiška unit is applied to the HZb unit by some researchers (KOVAČIĆ et al., 2005). Their stratigraphic position and inter-correlation is displayed in Fig. 3. and their description is accepted from the works of VRSALJKO (2003), KOVAČIĆ (2004) and KOVAČIĆ & GRIZELJ (2006). In contrast to the widely accepted lithostratigraphic nomenclature in this part of the PBS, which is defined basically according to the results of subsurface geological investigations (ŠIMON, 1973, 1980;

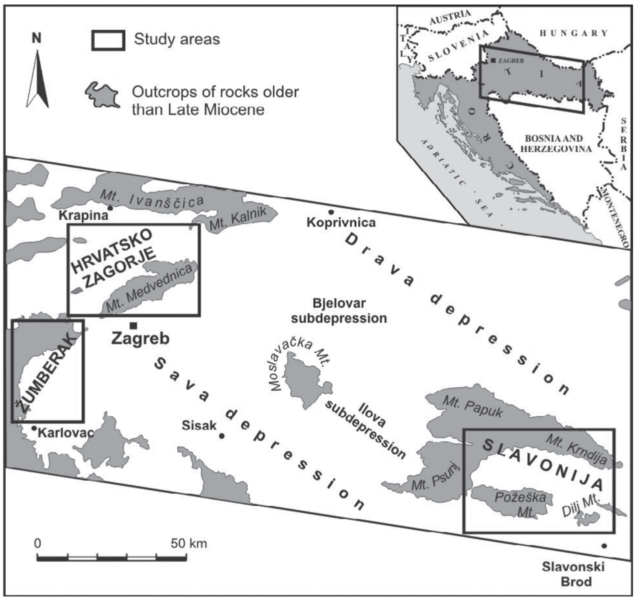

Figure 2: The South-western part of Pannonian Basin System with location of the study-area (Hrvatsko Zagorje, Mt. Medvednica, Mt. Žumberak and Slavonian Mts.).

VELIĆ, 1980), these units are informal in character. However, some recent investigations could easily assign them a rank of formation (KOVAČIĆ, 2004).

The Cro unit (Lower Pannonian) occurs in the region of Hrvatsko Zagorje and the Slavonian Mts. The sharp boundary with the underlying Sarmatian beds is conformable, while the lateral-vertical transition to the younger MeB and Oza units is gradual (Fig. 3). It is composed of thin-bedded clayey limestones and marls with occasional intercalations of calcareous sands deposited in a shallow water lacustrine environment of low salinity.

The Oza unit (Lower–Middle Pannonian) is widespread on the Žumberak and Medvednica Mountains, unconformably overlying the Middle Miocene and older deposits, or occurring as the lateral equivalent of the Cro unit and the lower part of the MeB unit. It consists of medium-grained clastic sedimentary material deposited in coastal lacustrine, fluvial, or distributary channel environments. The variability of locally derived clastic detritus clearly indicates the diverse composition of the parent rocks in the area.

The MeB unit (Lower Pannonian–Upper Pontian) is present over the entire investigated area. It conformably overlies the Cro and Oza units or Sarmatian deposits consisting mostly of marls deposited in the deep-water brackish environment (Fig. 3). Sands and gravels are interbedded within the lowermost parts of the unit, along the contact with the Oza unit, the composition and origin of which correspond closely to the underlying Oza clastics.

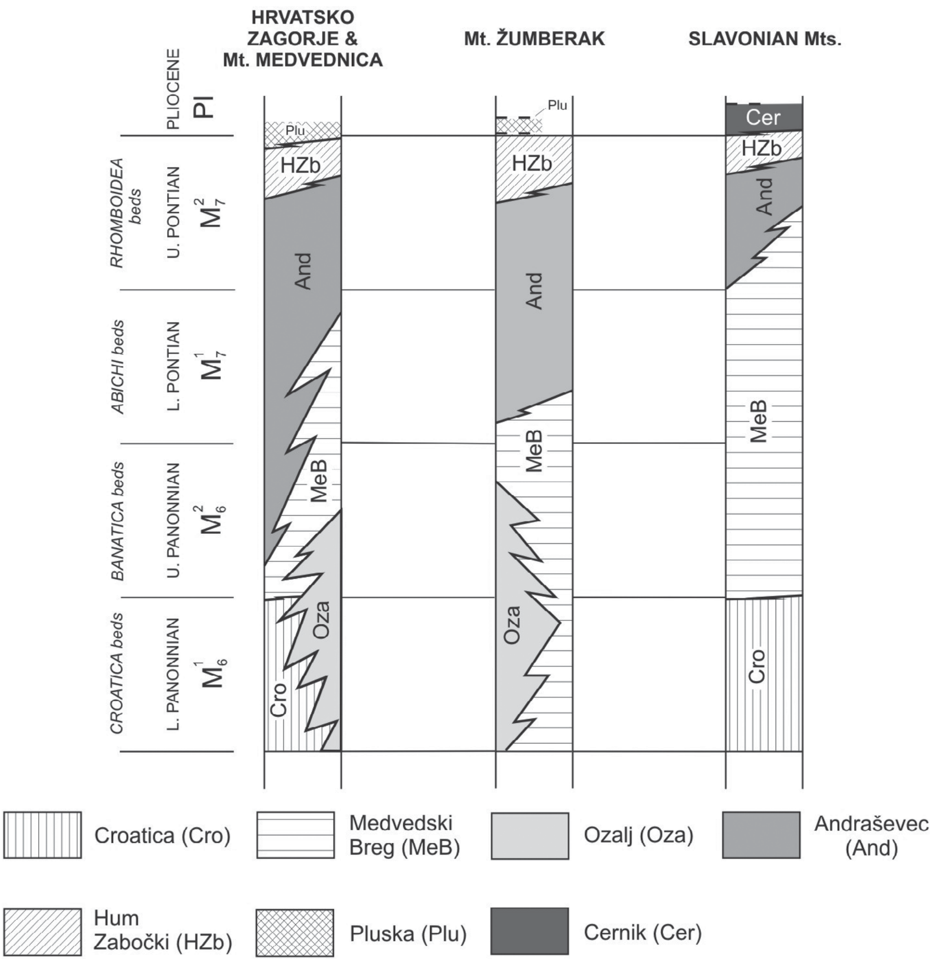

Figure 3: Schematic geological sections of the Upper Miocene deposits with informal lithostratigraphic units of Hrvatsko Zagorje, Mt. Medvednica, Mt. Žumberak and the Slavonian Mts. Stratigraphic position of the litostratigraphic units is based on changes of endemic fauna, so it is highly speculative.

The And unit (Lower Pannonian–Upper Pontian) covers the entire study area. This unit conformably overlies the MeB unit and is also conformably overlain by the HZb unit. It is composed of alternating layers of sands, silts and marls deposited in the prodelta-deltaic slope lake environment. Siliciclastic detritus is mineralogically and structurally relatively mature. Its composition is homogeneous over the entire area and derives its origin mostly from metamorphic and older sedimentary rocks outcropping in the source area relatively far to the north.

The HZb unit (Upper Pontian) is widespread in the whole studied area, overlying conformably the older, And unit, and passing conformably upward into the overlying Cer and Plu units (Fig. 3). It is characterized by alternation of sand and silt beds deposited in the delta front in the shallow brackish lacustrine environment. According to both composition and origin of its detritus it is not significantly different from the And unit.

The Plu unit (Upper Pontian–Pliocene) crops out in Hrvatsko Zagorje and in the Žumberak Mts. overlying conformably the HZb unit, while its upper boundary remains undetermined. It consists of clays, silts and sands with lenses of gravel and coal deposited in a river or distributary channels, alluvial plains, coastal lagoons and swamps. The clastic detritus is comparatively mature in a mineralogical and structural sense, and is similar in composition to the detritus of the And and HZb units. The latter two differ in their HMA, indicating some variations in the composition of the parent rocks.

The Cer unit (Lower Pliocene) is detected only in the Slavonian Mts. region, overlying the HZb unit (Fig. 3). It consists of clays, silts and sands with lenses of gravel deposited in a sedimentary environment similar to that of the Plu unit. The composition and origin of detritus is close to the And and HZb units.

3. METHODS

3.1. Sampling, sample preparation and analysis

A total of 101 samples were collected during the field investigations that included geological mapping and construction of the geological columns of unbound sand-silt sedimentary material. Sampling was designed to cover the entire study area, comprising all relevant time periods and investigated informal lithostratigraphic units.

Samples were sieved to the 0.09–0.16 mm size fraction, followed by subsequent dissolution of calcite. The heavy mineral association (HMA) was separated using bromoform ($\delta(CHBr_3) = 2.84\ gcm^{-3}$). Qualitative and quantitative composition of the HMA was established after the determination of 300–400 grains applying *the ribbon counting method* (MENGE & MAURER, 1992). Results are presented in Table 1 (accepted and modified from KOVAČIĆ & GRIZELJ, 2006).

3.2. Data processing

Multiple (multi-group) discriminant analysis (MDA) is a powerful multivariate technique which is recently widely applied to many problems in geology, particularly in cases when a large amount of data is collected from different lithological or lithostratigraphical units, in an attempt to define their boundaries. Its principal purpose is to establish the major sources of difference between the *a priori* defined groups allowing the minimum misclassification error rates for their members. Generally, this is achieved in such a way that the variance between the original groups is maximized in relation to the variance within each particular group (DILLON & GOLDSTEIN, 1984). In the process, a hypothesis is tested that all observed groups have the same multivariate mean, against the alternative that at least one mean is different (ROCK, 1988). If the alternative hypothesis is not rejected (which does not mean that the separation of all groups is necessarily optimal), then discriminant scores can be computed from the original data set allocating each object (single sample) along one or more mutually independent (perpendicular) vectors – linear discriminant functions (e.g., KRUMBEIN & GREYBILL, 1965; DOORNKAMP & KING, 1971; DAVIS, 1986). In this way the multivariate problem is simply and parsimoniously cut down to the least dimensional solution depending on the number of groups (K–1), or variables (p) if the latter is greater ($K<p$).

Investigation is focused on separation between lithostratigraphic units using the data on the HMA from the sampled siliciclastic material of the Upper Miocene and Lower Pliocene sands in the SW part of the PBS. Five groups described above as Oza, And, HZb, Plu and Cer (lithostratigraphic units Cro and MeB are not defined as specific groups) including more than a hundred samples in total were subject to investigation. Descriptor variables are represented by 10 minerals of HMA including chlorite (Chl), dolomite (Dol), tourmaline (Tur), zircon (Zrn), rutile (Rt), amphibole (Amph), garnet (Grt), kyanite (Ky), staurolite (St) and minerals from the group of epidote-zoisite-clinozoisite (EpZoCzo). Regardless of the positive skewed frequency distributions of the analyzed variables, a bias usually ascribed to complex nonlinear responses within the natural system (siliciclastic material of the sands) represented by HMA, the data were not subject to transformation. Common transformation procedures were omitted following the idea that data processed by various multivariate methods should be left in their original form since transformed variables can develop their own patterns of behavior (SIZE, 1987; MANN, 1987; AGRAWAL, 1995). A single variable exhibiting distribution close to normal was EpZoCzo.

4. RESULTS AND DISCUSSION

Discriminant analysis of the investigated lithostratigraphic units represents a multiple-group discrimination involving five groups ($K=5$). In this case discriminant model is built of four discriminant functions (K–1) completely explaining the differences between the respective groups. Before that, the overall significance of their discrimination is tested by appropriate multivariate tests (Table 2a) disclosing the vanishingly low associated probability ($p<0.000$), a rationale required to safely proceed with computing discriminant functions (DFs). With respect to the statistical significance of the variation between the observed groups (p-level, Table 2b) and

Table 1: Composition of the HMA of Upper Miocene and Pliocene sands from the southwestern part of the Pannonian Basin System. Chl – chlorite, Dol – dolomite, Op – opaque mineral, Bi – biotite, THM – Translucent heavy minerals, Tur – tourmaline, Zrn – zircon, Rt – rutile, Amph – amphibole, Grt – garnet, Ky – kyanite, St – staurolite, EpZoCzo – epidote-zoisite-clinozoisite, oth – other minerals, + – less than 1%.

Sample	Unit	Heavy minerals (%)					Translucent heavy minerals (%)								
		Chl	Dol	Op	Bi	THM	Tur	Zrn	Rt	Amph	Grt	Ky	St	EpZoCzo	Oth
Bol 1/2		+		34		66	2		+		1	+	4	87	5
5/4				40		60	2		2		+		5	88	2
Dbr-I 2/1				23		77		+	+	+			4	91	3
4/1				29		71	1	+			1		2	92	3
12/1				16		84				+			1	95	3
Bor 1/1	Oza			23		77	+	+		3	2		1	87	6
Oza-I 3/2			2	92		6	11	48	38			+	2	+	
6/1			1	93		6	14	53	27			2	2	+	1
Sla 3/4			70	27		3	25	6	4		40	2	9	9	5
Top-II 3/1			2	92	+	6	18	27	35			1	12	3	4
6/4			17	56	10	7	13	43	28	1	10		3		2
Kra-I 13/1		1	91	6		2	46	5	13		8	2	6	15	5
Mal-I 5/2		1	70	11		18	10	3	7		24	2	15	31	8
7/1		17	41	11		31	8	1	4	1	28	3	14	34	7
KrT-II 16/1		1	61	8		30	4	+	7	+	49	3	11	20	5
8/1		5	34	13		48	6		4		25	5	21	34	5
11/1		5	11	18		66	5		3		41	2	19	26	4
16/1		3		12		85	4		3	4	42	4	12	27	4
Pač-I 1/1		7	11	13		69	3		5		65	1	7	13	6
20/1		2	5	11		82	2		5		72	3	5	11	2
39/1		18	11	14		57	8	+	3		53	2	16	14	3
Pož-I 1/2		1	5	8		86	6	1	5	5	62	1	7	11	2
6/1		21	10	13		56	4	+	3	4	36	1	13	34	4
Mat-I 1/1		4		10	2	84	6		4	+	57	3	3	14	12
Bob-I 10/1		41	27	9		23	6		2		28	7	12	37	8
25/1		35	31	7		27	5		2	2	28	5	18	35	5
Pož-I 23/1		15	24	14		47	3	+	2	4	32	3	17	35	3
And-I 2/1		+		14	+	85	1	+	2		63	1	2	26	4
29/1		19		32	1	48	5	1	1		54	+	7	21	10
Mat-I 8/1		+		10		89	4		3		74	1	4	8	6
14/1		1		11		88	5		2		71	2	4	10	6
Mir-I 2/1		13		21	1	65	8		9	+	37	6	6	26	7
6/1	And	+		8		92	2		3	+	57	4	3	28	2
6/3				16		84	2		4		57	2	2	31	2
7/4		8		17	+	75	7		2		26	5	8	38	14
12/1		2		13		85	3		3		39	3	7	39	6
Maj-I 1/2		29	31	7		33	4	+	1	2	36	2	12	40	2
3/1		15	16	12		57	6		1	3	33	3	17	34	3
Tol –II 8/1		21	39	4		36	5		5	16	20	4	16	27	7
Bek-I 2/3		3		9	1	87	2		1	4	31	1	3	46	12
MMa-I 5/1				8		92	5		2	3	7	2	12	57	12
11/1		1		6		93	2		2	3	26	1	10	44	12
Bek-II 3/1		21		8	15	56	2		5	1	17	+	3	59	
11/1		39		12		49	5		1	13	12	1	5	51	12
Sel-I 1/1		7		10		83	3	+	1	20	28	2	7	28	10
Sem-I 12/1		3		18	+	78	5		2	4	22	4	10	39	14
22/1		1		12	1	86	4		1	3	15	2	9	51	15
BDI-I 5/1		2		15		83	2		3	9	41	2	6	32	5
6/3		1		11		88	2		2	7	40	2	4	32	11
Mil-I 1/3		17	5	15		63	6		4	3	19	2	20	40	6
3/1		6	11	13		70	3		4	1	20	1	27	40	4
PoP 1/1		2	2	7		89	1	1	4	3	35	7	19	25	5
5/4		7	10	6		77	3		1	1	23	3	33	33	3
Nac-III 6/1				21		79	3	+	4		2	3	21	57	9

Table 1: continued.

Sample	Unit	Heavy minerals (%)					Translucent heavy minerals (%)								
		Chl	Dol	Op	Bi	THM	Tur	Zrn	Rt	Amph	Grt	Ky	St	EpZoCzo	Oth
Tol-II 14/1		24	37	6		33	3		2	8	28	3	19	29	5
Sel-I 30/1		47		18		35	4	1	4	9	18		3	58	3
38/1		2		11	+	87	2		2	7	27	3	6	43	10
VV 1		+		14		85	4	1	6	2	19	3	13	41	11
Plu-I 1/1		33	32	6	+	28	2		4	21	23	6	12	20	12
8/1		27	39	3		31	6	+	1	26	18	6	17	15	10
11/1		7	4	10		79	4	+	3	14	28	3	15	23	9
19/1				14		86	4	+	2	22	+	6	14	42	9
Plu-III 1/1		2	+	11		86	3		4	6	38	6	7	28	8
Mal-I 12/2		9	11	18		62	2		2	6	40	4	12	26	8
PG-2	HZb	2	8	14		76	2	+	1	7	48	4	5	28	4
D-1			+	7		93	3		2	11	46	4	10	19	5
D-2				8		92	3		2		2	6	19	64	4
Nac-III 6/3				9		91	33		14			23	24	2	4
Plu-II 1/1				21		79	9	2	5	4		5	27	40	8
4/1				14		86	5	1	2	3		5	29	52	3
Plu-III 2/1		+		16		86	5		4	5	1	7	15	56	7
3/1				22		78	6		8		+	7	22	52	4
4/1				34		66	7	+	8	+	+	13	28	39	3
PB 1				32		68	8	+	17			11	18	43	2
Mal-I 14/2				29		71	9	+	7			5	24	47	7
Plu-II 6/2				25		75	11	+	11	+		10	21	43	3
6/1			+	17		82	7		8	+	2	11	38	29	4
Vbo-I 17/1		12		7		81	23		1	3	1	10	26	33	3
Sel-I 45/1				24		76	2	1	6	3	32	5	5	38	8
Dzg-I 3/1				47		53	33		14			23	24	2	4
10/1				48		52	25	1	18			26	23	3	4
20/2			+	46		54	18	+	12	+		25	27	10	7
21/2	Plu		+	63		36	20	2	18			21	30	4	5
Plu-II 7/2				39		61	13	2	14	1		17	33	17	3
VTv-I 1/1				53		47	12	3	26			22	32	2	3
6/1				51		49	15		23			24	35	+	2
9/1				55		45	29	2	13		+	29	24		2
17/1				48		52	18	1	15			26	37	+	2
Mal K/1				49		51	33	2	12			22	27	2	2
K/2				51		49	29	1	15			21	29	3	2
Nac-III 12/2				11		89	25	1	18			26	23	3	4
Nac-IV 1/1				17		83	18	+	12	+		25	27	10	7
5/1				29		71	20	2	18			21	30	4	5
7/1				20		80	13	2	14	1		17	33	17	3
Vrb 7/1		2		29		69	12	3	26			22	32	2	3
13/1	Cer	2	32	21		45	15		23			24	35	+	2
17/1			6	26		68	29	2	13		+	29	24		2
22/1			9	21		70	18	1	15			26	37	+	2
27/1		1	13	14		72	33	2	12			22	27	2	2
39/1		1		25		74	29	1	15			21	29	3	2

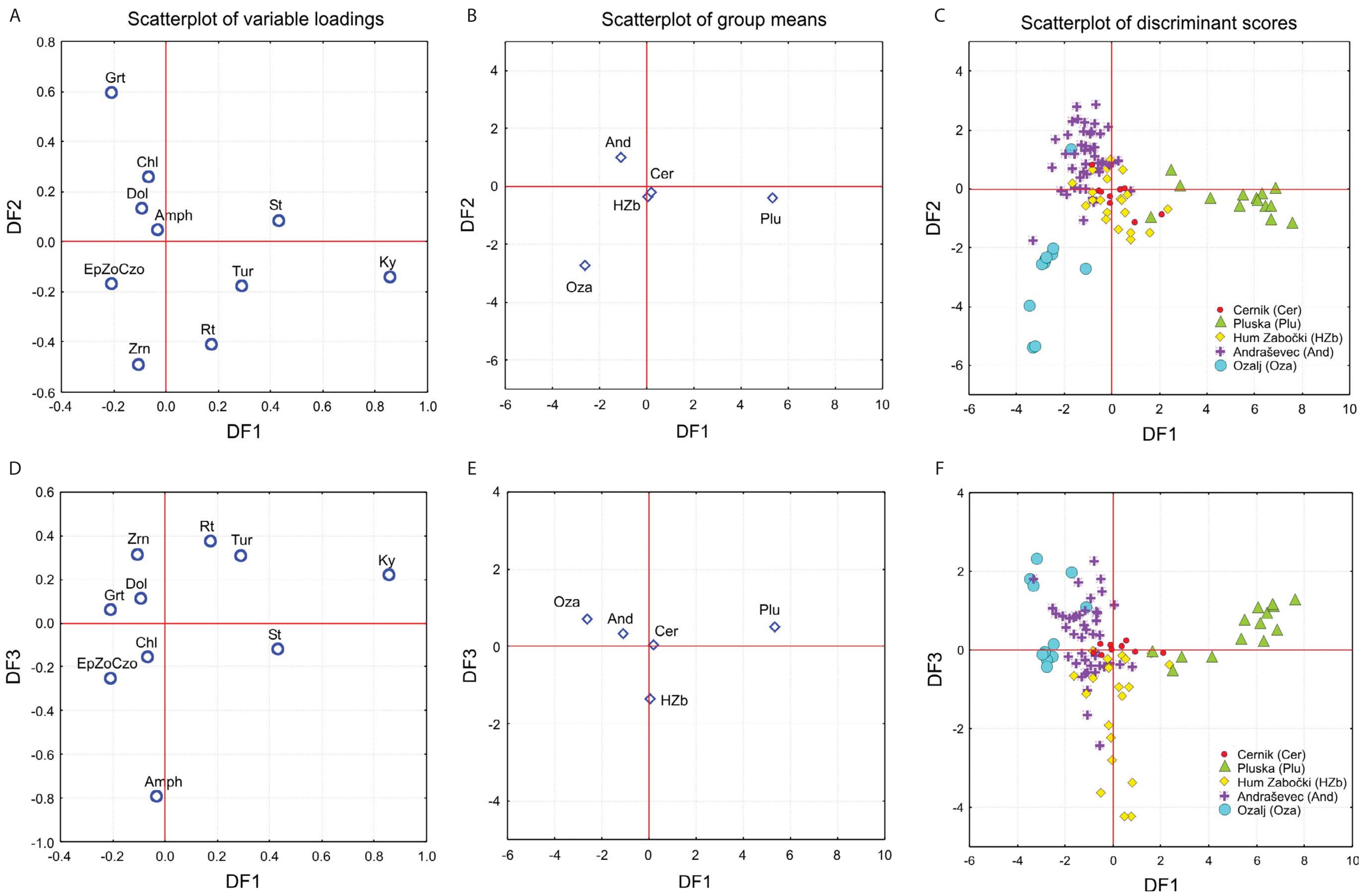

Figure 4: Scatterplot of variable loadings (A,D), group means (B,E) and discriminant scores (C,F) in reduced discriminant space (DF1/DF2 and DF1/DF3).

Table 2: Tests of significance. a) Multivariate test of the overall significance of discrimination. b) Tests of residual roots (discriminant functions).

a)	Number of variables in the model	10
	Wilks' λ	0.038
	Approximate F ratio	11.387
	Degrees of freedom	40; 331
	p-level	p<0.000

b)	DF	Eigen-value	Eigen (%)	Canon. R	Wilks' λ	Chi-Sqr.	df	p-level
	1	5.533	73.06	0.920	0.038	303.1	40	0.000
	2	1.390	18.35	0.763	0.247	129.5	27	0.000
	3	0.568	7.50	0.602	0.589	48.9	16	0.000
	4*	0.082	1.08	0.275	0.924	7.3	7	0.402

Note: DF marked* is statistically insignificant

accompanying eigenvalues, only the first three DFs which explain almost all the variability of the system (98.9%) were selected to define dimensionality of the discriminant space.

4.1. Building a model

To construct a functional model around the computed DFs is vital to MDA since it opens the door to the hidden relationship between groups not detected by direct observations and measurements. In this way the natural processes can be inferred underlying the mathematical structure of analyzed data. Individual assessment of each descriptor (HMA) is needed for this purpose, and discriminant loadings (structure coefficients), which represent simple correlations of variables with respective DFs, are used here to assist the overall discrimination between the groups. Inspection in a model is particularly helpful via the variable diagrams which provide the easiest and most informative insight into the structure of discriminant space, both indicating which descriptors should be kept in the geological interpretation of discriminant functions and which descriptors most clearly separate the *a priori* defined groups. In this respect, a number of variables with minor contributions to discrimination are excluded from the interpretation of the model. Among these are Dol, Chl and EpZoCzo which crowd together around the main centroid owing to their small discriminant loadings (Fig. 4A, D). Furthermore, the relationship between variables and related groups can be likewise displayed geometrically, viewing DFs as mutually perpendicular vectors (axes) in the reduced discriminant space. However, the variable and group diagrams cannot be contrasted directly since different scales are employed in each case. The scatterplot of variable loadings (Fig. 4A, D) is drawn with the axes shown as normalized vectors, while scatterplots of group means (Fig. 4B, E) and discriminant scores (Fig. 4C, F) are constructed using discriminant score vectors. As a result, interdependence between variables and related groups can be elucidated regarding their shared position along the respective discriminant axis. In both diagrams, contribution to the overall discrimination of points placed close to the axes intersection is negligible. Typically, a reciprocal relationship between the group means (centroids) and variables along the respective discriminant axis will determine the geological nature of the computed mathematical model. Nevertheless, however revealing these diagrams may be about the real geological processes generating the character of mutual group relationships, the best way to evaluate how much the groups really differ is to take a closer look into the distance matrix, (matrix of squared Mahalanobis distances; Table 3). Based on these values, the Plu unit falls farthest away from all other units, namely the Oza (69.38), And (43.46), HZb (31.33) and Cer (27.82). However, there is some overlap between the Cer, HZb and And units. The closest together are Cer and HZb (2.96), then Cer and And (4.13), and finally And and HZb (5.98). Judging from the calculated Mahalanobis distances, the Oza unit is not very different from the Cer (15.91), And (16.61) and HZb (17.45) units, lying somewhere in the middle of the first two cases. The rationale for similarities or disparities between the investigated groups (lithostratigraphic units) can be found in specific geologic processes which control the spatial distribution of the descriptor variables (HMA) during evolution of the tectonic and sedimentological history of the study area.

Table 3: Squared Mahalanobis distances.

	Oza	And	HZb	Plu	Cer
Oza	0.00	16.62	17.45	69.38	15.91
And	16.62	0.00	5.98	43.46	4.13
HZb	17.45	5.98	0.00	31.33	2.96
Plu	69.38	43.46	31.33	0.00	27.82
Cer	15.91	4.13	2.96	27.82	0.00

4.2. Labeling the discriminant functions

Labeling discriminant functions is a starting point in interpreting the mathematically computed (structural) model in terms of processes. In this manner, the structural model is transformed into a process with each DF contributing to a single independent (geologic) process in the overall separation of the groups. Variables participate in this structure-process conversion as the building blocks assuming the role of process descriptors.

In the computed mathematical model, the first discriminant function DF1 makes the greatest contribution to discrimination between the groups, accounting for more than 73% of the total variability. It is essentially monopolar, being marked by the high positive loadings of dominant translucent heavy minerals, kyanite (Ky) and, to a lesser degree, staurolite (St), while the major part of the HMA suite is concentrated around the main centroid (Fig. 4A, D). If compared to the scatterplot of group means, it illustrates the separation of the Plu unit from the other four groups (particularly Oza) based on enrichment/depletion of kyanite in their modal composition. While the Plu unit is characteristic for its higher content of kyanite, other lithostratigraphic units in the SW part of PBS are rather indifferent to it (with the exception of the Oza unit showing a slightly decreased content of Ky). In accordance with the relative prominence of this mineral, DF1 can be readily identified as the kyanite function. Geologically, the significant abundance of Ky and St in the detritus of the Plu unit sands serves as a good indicator of the weathering of rocks of medium grade regional metamorphism in the source area. Due to the marked dominance of DF1 in the discriminant model it unmistakably highlights the central geologic process in the development of the Upper Miocene and Pliocene sands in the SW part of the PBS – a process controlling the distribution of Ky as the 'key' mineral from the incipient stage of parent rock weathering through transport to sedimentation and possible re-deposition of sedimentary material in the later stages of the development of the sedimentary basin or its particular portions.

The second discriminant function DF2 accounts for only 18% of the total variance between the groups. In contrast to DF1 it is characteristically bipolar and can be interpreted as reflecting the inverse relationship between garnet on the one side and zircon with (to a less extent) rutile on the other, after which it can be properly labeled the garnet-zircon function. From visual inspection of the related variable and group scatterplots (Fig. 4A, B), it is evident that after acknowledging this variable pattern, DF2 further separates the Oza unit from the remaining groups. This partition is based principally on the reduced content of garnet and seconded by the higher content of zircon and rutile in the Oza unit sand composition. The content of garnet is slightly higher with part of the samples in the And unit (Fig. 4C) while the remaining three groups (Cer, HZb and Plu units) remain indifferent in this relationship sticking to the middle position with the least mean values. This situation highlights their average HMA composition with respect to the DF2 labeling minerals. In a geological (sedimentological) sense, DF2 must be observed through the bond of the two minerals, Zrn and Rt, which are amongst the ultra stable detrital minerals in the sedimentary systems, against Grt which is considered to be the indicator of high energy conditions during transport (ROTHWELL, 1989). Being a garnet-zircon function, DF2 in a functional sense (in a sedimentary system) can either reflect variations in the parent rock mineral composition via its mineral content in the detrital material. In other words, Zrn and Rt can be found in the composition of older sedimentary rocks (most probably derived from more acid igneous rocks), or signal the prevailing hydraulic conditions closely associated with sedimentary environment. This amounts to saying that heavy minerals (having great density) are concentrated under high energy conditions (Grt) while others, such as Zrn and Rt prefer environments of low stream power. It could be noted in passing that a similar pattern of sedimentary process regime was already deduced in a quite different environment where the Quaternary sediments were divided into several genetic groups (PEH et al., 1998). Although results can be inconclusive in this respect due to the still limited volume of similar investigations needed for reliable cross-comparison, it is interesting to observe that source rocks seem to play the dominant role in distribution of HMA in the basin systems, at least at a regional scale, while the hydraulic conditions controlling their physical and chemical stability during transport and deposition are secondary.

The third discriminant function DF3 (explaining barely 7.5% of the system variance) is also bipolar but dominantly loaded with only one variable – amphibole (Amph) – by which the HZb unit is weakly separated from the others (Fig. 4D, E). Amphibole is negatively associated with the set of variables weakly loaded on the positive pole of DF3 among which some minerals, very resistant to weathering, such as Rt, Zrn and Tur, are highlighted. Since amphibole is a highly unstable mineral component this discriminant function can be interpreted as the amphibole function representing, in sedimentological sense, a function of the chemical stability of HMA in the investigated arenaceous (sandy) deposits. Obviously, the HZb unit must have been deposited under conditions of prolonged stability or amphibole would not otherwise be significantly present to serve as the key mineral in its discrimination. Similarly, with respect to environmental quiescence and stability, it could be said that all other groups are more or less well separated from HZb towards the Zrn(Rt) quarter of the diagram, indicating the more turbulent conditions of their deposition.

4.3. Classification results

Classification efficiency is often proved to be of greater use geologically in the process of discrimination than various statistical tests (ROCK, 1988). It is particularly evident in cases when MDA is applied to a large volume of data scattered across various spatial boundaries such as geological (lithostratigraphical) units, depth intervals, or profile lines, being composed of the same suite of observed or measured attributes (e.g. physical or chemical properties) (e.g. SAFTIĆ et al., 2001; PEH et al., 2008; KOVAČIĆ, 2004; GRIZELJ, 2008). It is also evident in applying statistical analysis in order to test fro group differences based on lithostratigraphical units properties (e.g. MALVIĆ et al., 2005). In this context, classification rates in original or *a priori* defined lithostratigraphic units can be readily inspected by contrasting the mathematically 'predicted' (computed) and original ('natural') group membership of individual samples (Table 4).

From Table 4 it is apparent that the discriminant model is organized with relatively high efficiency which exceeds 80%. However, for certain groups, namely Cer and HZb, overlap is considerable which is evident from the low Mahalanobis distances (Table 3), and can be visually scanned from respective sample diagrams (Fig. 4C, D). For instance,

Table 4: Classification matrix.

Observed group memberships	Predicted group memberships					
	Oza	And	HZb	Plu	Cer	% correct
Oza	10	1	0	0	0	90.91
And	1	40	3	0	1	88.89
HZb	0	5	14	0	3	63.64
Plu	0	0	2	12	0	85.71
Cer	0	4	0	0	5	55.56
total	11	50	19	12	9	80.20

only five samples out of nine of the Cer unit are assigned to that particular group while the rest are more similar to the And unit, which has a classification rate barely in excess of 55%. In the case of the HZb unit, the success rate is somewhat greater and amounts to over 63%. Here, eight samples out of 22 are closer to other groups, including the And (5) and Cer (3) units. This information indicates the significant inhomogeneity of the Cer unit where more than 44% of allocations are unequivocally misclassified (And unit), while at the same time, an equal number of samples are *a posteriori* allocated to the former at the expense of other litostratigraphic units: HZb (3) and And (1).

The Oza, Plu and And lithostratigraphic units are distinguished by relatively high classification efficiency. Among these, Oza unit is recognized for its homogeneity (Table 4). Its modal composition is so unique that one can hardly find similar sand grade sedimentary material accounting for the fact that only a single sample of this unit is incorrectly classified changing its group affiliation for And, and vice versa. One of the samples originally ascribed to And was later classified as a member of the Oza unit (Table 4). Such an almost optimal separation can be explained by a different source for the clastic material in its detrital composition relative to other units. Namely, the clastic detritus of the Oza unit is of local provenance (after KOVAČIĆ & GRIZELJ, 2006), and is derived from the weathering of older sedimentary rocks while the material forming the other units must have been transported from more distant areas, originating form the weathering of a diverse suite of metamorphic, sedimentary and igneous rocks. A cross-exchange of two single group assignments between the Oza and And units can be interpreted in the knowledge that both samples were collected in the Žumberak Mt. where clastic detritus from both sources combines.

A high classification rate is also characteristic for the Plu unit (Table 4). This is expected in as much as, (after KOVAČIĆ & GRIZELJ, 2006), a significant proportion of its detritus is derived from the alumina-rich metamorphic rocks as opposed to other units. Misclassification of two Plu samples into the HZb unit can be easily explained by the fact that the former conformably overlies the latter while their boundary is not clearly defined.

Also, according to KOVAČIĆ & GRIZELJ (2006), the same petrologic province responsible for supplying sediment for the And, HZb and Cer units, produces great similarity of their clastic material which is the main reason why a considerable number of samples show reciprocally incorrect affiliations. However, almost 90% of the And samples are *a priori* correctly classified, strongly confirming its homogeneous composition (Table 4). In this connection, with the And unit being the 'heaviest', some samples were assimilated from the other two units, a typical effect where one group, being larger than the other, draws the members of a smaller group closer to its own centroid. As a result, the post hoc assignments of smaller groups inevitably undergo low classification rates (slightly over 50%) but a larger group better conveys the mean modal composition.

5. CONCLUSION

Multiple discriminant analysis applied to the Lower Miocene and Pliocene sands from the SW part of the Pannonian Basin System showed that specific mineral associations characteristically relate to particular lithostratigraphic units outcropping in the area. These relationships allow some definite conclusions to be drawn about specific stages of the sedimentary cycle, and conditions prevailing during weathering, transport and deposition of the studied sedimentary material. Important results of MDA can be summarized as follows:

(1) Three discriminant functions account for almost all variance between the groups (99%) and can be easily interpreted in geological terms as: a) DF1 – kyanite (Ky) function indicating source rock mineralogy (alumina-rich, metamorphic rocks); b) DF2 – garnet-zircon/rutile (Grt-Zrn/Rt) function explaining the relationship between source rock mineralogy (older clastic sediments and felsic igneous rocks – Zrn/Rt) and hydraulic conditions controlling transport and deposition of detritus (Grt) and; c) DF3 – amphibolite function accounting for the processes of chemical and physical stability in the sedimentary cycle.

(2) The Kyanite function is by far the most important part of the discriminant model accounting for almost 3/4 of all the processes in the investigated area, placing great importance on the area of alumina-rich metamorphic rocks as the main provenance of the clastic material.

(3) Oza, Plu and And units are set apart by the highest classification efficiency. The most discrete (most different) among these is the Plu unit (by DF1). However, the Ozalj unit is recognized for its homogeneity (the greatest classification rate). The Andraševec unit is the most massive, prone to scavenging the members of smaller groups. At the other end of the scale, the Cer and HZb units represent the mean composition of HMA with respect of all the investigated groups.

Finally, it must be stated that interpretation of changing affiliations assessed by discriminant analysis can greatly improve geological mapping results because otherwise very similar sand units, previously differentiated into predefined lithostratigraphic and chronostratigraphic compartments during field work, can now be reconsidered. Inhomogeneity of the HMA within particular units can be used as an indicator tool for visualizing the dynamics of a sedimentary system and of its development, particularly concerning the processes of palaeotransport, sedimentation and subsequent re-deposition. Some widespread lithostratigraphic such as the Andraševec (Hrvatsko Zagorje, Žumberak Mt. and Slavonian Mts.) may indicate specific variations in lithology, texture, facies, sedimentary environments, sedimentary control on depositional sequences, and even age. In order to specify which geologic process had the dominant role in the sedimentary cycle resulting in the final differentiation of lithostratigraphic units, it is very important in discriminant analysis to associate individual groups with respective variable descriptors (HMA). Thus, one can single out the process which is involved, and with what priority, in the sedimentary regime. As previously mentioned, the most valid implication emerging from the discriminant model is the complete dominance of DF1 since it explains the greatest proportion of the total variability. For this reason it unequivocally (in the process sense) highlights the most important issues in the genesis of the Upper Miocene sand deposits in the WS portion of PBS; the provenance of the detritus indicating the alumina-rich metamorphic source rocks, typical for the Plu unit.

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