FORMAL MODEL FOR ASSESSING THE SUITABILITY OF A COMPETITION SYSTEM IN BASKETBALL

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Abstract:

An appropriate competition system should be in the function of the development of a particular sport, that is, it should ensure in the best possible and most humane way the integral sport development of individuals and teams. The purpose of this study was to analyse the different competition systems so that their nature can be objectively determined on the basis of the predicted deviation of ranking of the competitors in relation to their actual quality. A mathematical model was designed to achieve the above-mentioned goal. With regard to the results obtained it is possible to recommend the round-robin system, in which each team plays against each other team once, because this kind of a competition system enables competition results which are in accordance with the predicted actual quality of the competitors.

Key words: *mathematical model, evaluation, competition system, round-robin competition system, cup competition system, team sports, basketball, actual quality of a player*

FORMALES MODELL FÜR DIE EINSCHÄTZUNG DER ANGEMESSENHEIT EINES WETTBEWERBSSYSTEMS IN BASKETBALL

Zusammenfassung:

Ein angemessenes Wettbewerbssystem soll seine Rolle in der Entwicklung einer bestimmten Sportart haben, d.h. es soll in der besten Art und Weise eine umfangreiche sportbezogene Entwicklung von Individuen und von den Teams sichern. Das Ziel dieser Forschung war, verschiedene Wettbewerbssysteme zu analysieren, so dass ihre Beschaffenheit objektiv bestimmt werden kann aufgrund der vorausgesagten Abweichung von den Plazierungen der Teilnehmer im Zusammenhang mit deren eigentlichen Qualität. Ein mathematisches Modell wurde gemacht, um dieses Ziel zu realisieren. Die Ergebnisse ermöglichen, den Wettbewerb, in dem jeder einmal gegen jeden spielt (Round-Robin), zu empfehlen, da ein solcher Wettbewerbssystem die Ergebnisse ermöglicht, die der vorausgesagten eigentlichen Qualität von Teilnehmern entsprechen.

Schlüsselwörter: *mathematisches Modell, Einschätzung, Wettbewerbssystem, Round-Robin-Wettbewerb, Pokalwettbewerbssystem, Mannschaftssport, Basketball, eigentliche Qualität von Spielern*

Introduction

One of the biggest current issues of sport science or kinesiology in the field of applied kinesiology in sport is rational management of the process of sport preparation and of selection of athletes, as well as the role of a competition in it. It is generally accepted that a competition system plays a significant part since top-level (professional) athletes do prepare for a dozen and more years to be the best at competitions (Trninić, 1996). So, success at a competition (final ranking) is the goal of every sport training process; and vice versa, it is a measure of the qualitative level of players and teams (Dežman & Tkalčić, 2002). A competition is an opportunity for overall potential and actual quality of athletes to be manifested and, more importantly, to be developed. From that point of view, a suitable competition system should be in the function of the development of a particular sport, that is, it should ensure, in the best possible and most humane way, the integral sport development of both individuals and teams. At the moment, the number of various competitions and their systems in operation is becoming ever bigger in professional sport, which seriously jeopardizes the developmental programmes of sport preparation. Large, long-lasting competitions (with two or even three matches per week during a season) have become counterproductive – there is an evident lack of space and time for the development of individual players, teams and the game itself. The consequence is that in a couple of decades basketball has lost its skillful, multifaceted (intellectual) nature; nowadays it is primarily a physical game. This fact provokes many questions at all levels of top-level competition sport. Practitioners of coaching know that competition in sport preparation is of the outmost importance for the improvement of the actual quality of players and teams. Therefore, the issue of the appropriateness of a competition system should also be investigated systematically and solved scientifically, as is already the case with the issues of sport preparation, of regeneration and recovery, and of the evaluation of the potential and actual quality of players (Figure 1).

It would be desirable to have competition systems shaped in such a way as to provide an integral sport improvement and as many opportunities as possible for achieving competitive results congruent with players' and team's actual quality. When regarded as a component of the structure of sport preparation, a competition system is the crucial component of the system of sport which allows the structure of the actual quality of an athlete in a particular sport to be manifested and recognised (Trninić, Perica, & Dizdar, 1999). On the other hand, it itself produces and shapes the actual quality by offering competition experience. Hence, to what extent a particular sport has developed and the sport preparation is efficient enough (especially in the fields of sport fitness and performance) can be qualitatively recognised through games played within a competition system. Therefore, the appropriate competition system should provide that sport success (final ranking) is, to the greatest possible extent, a consequence of the actual quality of

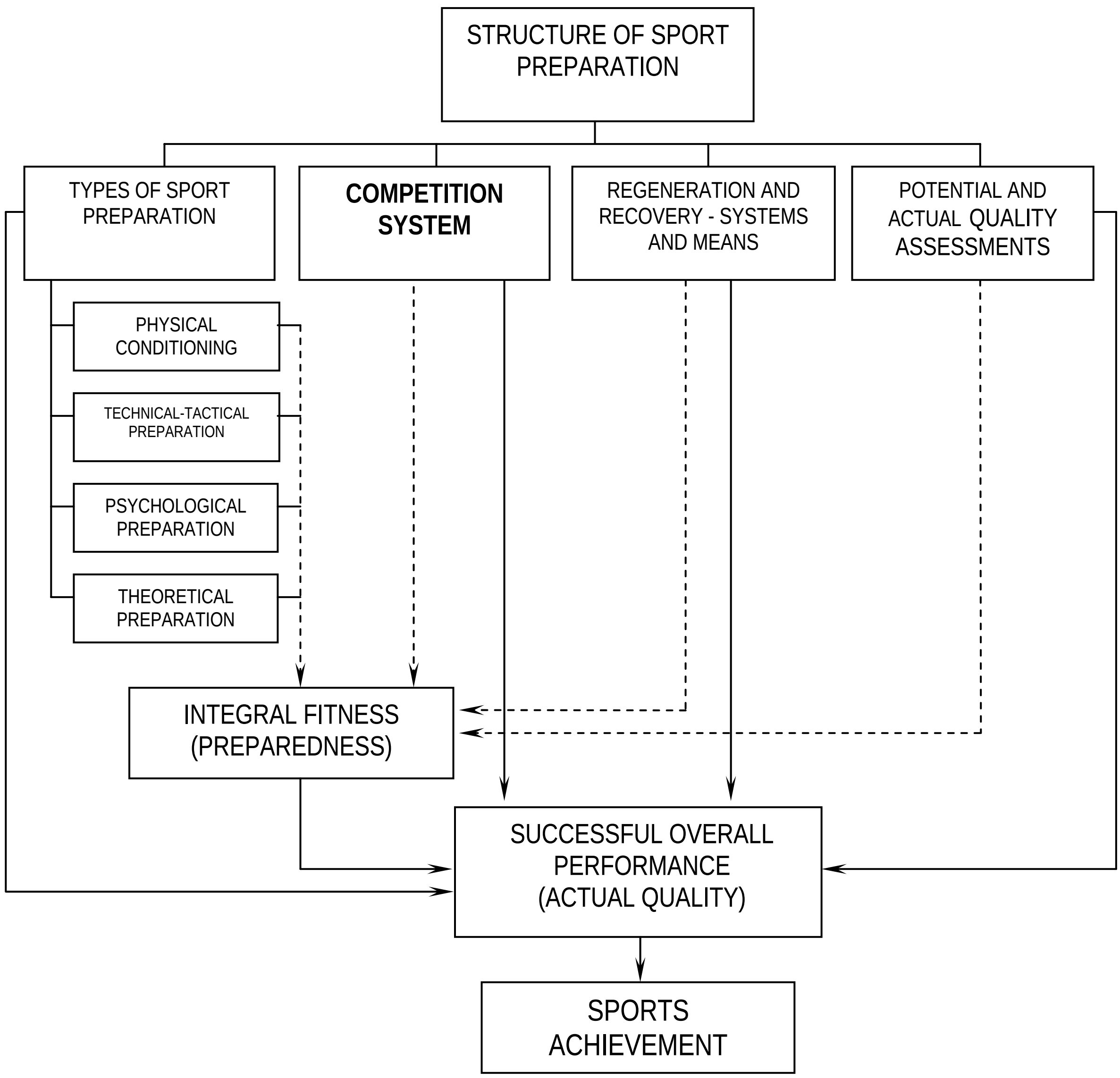

Figure 1. The structure of integrated sport preparation.

players and teams, and not of other factors, even a coincidence. The appropriateness of competitions should be determined on the basis of scientific findings and inferences, as well as on the basis of knowledge and experience of experts.

Back in the year 1978 Thiess and associates (according to Milanović, 2004:14-1) described a competition system as a social phenomenon in which individuals and groups (teams) match, compare their knowledge, skills and sport condition under strict conditions defined by the rules and norms. As such, a competition system must contribute to the development of a particular sport. Competitions can be classified into various competition systems: league, cup, tournament; and divided according to competition format in individual, pair, team, and national team competitions.

The only element of the system of sport preparation (Figure 1) which allows insights into the improvements of the *structure* of actual quality is undoubtedly the suitable system of competition. In this study different competition systems are analysed from the aspect of sport development and actual quality manifestation. The analysis is based on the predicted deviation of ranking of a competitor in relation to his/her team's actual quality as assessed (hypothetically) by experts. The competition system, whose deviation from the predicted ranking on the basis of the assessed actual quality of the contestants is smaller, will be considered as more appropriate. A mathematical model has been designed to achieve the previously mentioned goal.

Development of mathematical models is a very important process in every scientific discipline. Here are a few examples. Most of modern physical theories are developed as mathematical models (Cottingham & Greenwood, 1998). Graphs are used to model molecules in chemistry and topological indices (Todeschini & Consonni, 2000) calculated from these graphs are used to predict the properties of molecules. Such research studies evolved even in the particular branch of mathematics and chemistry called mathematical chemistry (Trinajstić, 1992; Gutman & Polanski, 1986). Mathematical models are becoming increasingly popular in social sciences, too. Let us just mention the widely popular doctoral thesis of Nash Bridges which was awarded the Nobel Prize (Bridges, 1950) for economy. In kinesiology, there is also a lot of mathematical modelling some of which is based on its close connection to game theory (Trninić & Dizdar, 2000; Dežman, Trninić, & Dizdar, 2001; Trninić, Dizdar, & Dežman, 2002). In the present study, round-robin tournaments, cup systems and mixed systems are analysed. Tournaments have been also studied in discrete mathematics (as complete directed graphs) (Harrary & Moser, 1966) and their connection with the management of companies was established (Lasear & Rosen, 1981; Green & Stokey, 1983). An especially interesting problem is the problem of the connection of rewards with the performance of the contesters (Rosen, 1986). Also, the problems of competition models are closely related with sorting out the problems. The research started about fifty years ago (see e.g. Ford & Johnson, 1959, and references within). Then, it flourished in many directions (a lot of sorting algorithms are known today: bubble-sort, quick-sort, merge-sort, max-sort and so on). Finally, competition models go far beyond just sport (e.g. there are competitions in the area of research and development; Taylor, 1995).

Methods

The mathematical model developed here is based on probability (Alon, Spencer, & Erdós, 1992; Sarapa, 1992) and algorithmic theory (Gibbons, 1985). All programs are implemented in the programming language Visual C++ (Horton, 1998) as the console applications.

The mathematical model is based on the following presumptions:

1) There are 8 contestants in a competition. The number 8 has been chosen because it is the largest potention of the number for which these kinds of analyses are possible with present daycomputers.
2) The actual quality (Trninić, Perica, & Dizdar, 1999; Trninić, Dizdar, & Dežman, 2003) of each competitor is hypothetically expressed by one of the numbers 1,…,8 (each number is assigned to precisely one contestant), as if it had been ranked after expert evaluation. The smaller the number, the better the contestant, i.e. the better the actual quality assessed. For the sake of simplicity, the contestant with the assigned number k will be called contestant k and will be denoted by N_k.
3) Each game is played by two contestants and it always finishes with one of the two alternative outcomes: either one or other contestant is the winner. This rule is applicable to several team and individual sports, such as tennis or volleyball. Here we impose this rule to make our calculations simpler, because if we observe 28 matches with only two outcomes, we have 2^{28} = 268,435,456 possible outcomes, which is not too complicated for the computer analysis. However, if we observe three possible outcomes (which includes also a draw), we have $3^{28} \approx$ 22, 876,792,454,961, which is a too large number to be processed.
4) It is considered that the probability of the victory of the contestant over the contestant is given by:

$$\frac{b + 1 - \min\{a,b\}}{(a + 1 - \min\{a,b\}) + (b + 1 - \min\{a,b\})}$$

For example, in the match between contestants 3 and 5, the probability of the victory of the contestant 3 (the favorite in this match) equals ¾. However, two alternative assumptions are also possible.

4’) It is considered that the probability of the victory of the contestant a over the contestant b is given by:

$$\frac{b}{a+b}.$$

For example, in the match between contestants 3 and 5, the probability of the victory of the contestant 3 (the favorite in this match) equals ⅝ (this assumption is in accordance with Zipf’s code, which assumes that quality of the contestant x is $^1/_x$ (note that:

$$\frac{\frac{1}{a}}{\frac{1}{a}+\frac{1}{b}} = \frac{b}{a+b}\ ))$$

4”) It is considered that the probability of the victory of the contestant a over the contestant b is given by:

$$\frac{\sqrt{b}}{\sqrt{a}+\sqrt{b}}.$$

For example, in the match between contestants 3 and 5, the probability of the victory of the contestant 3 (the favorite in this match) equals

$$\frac{\sqrt{5}}{\sqrt{3}+\sqrt{5}}$$

(this assumption is a modification of Zipf’s code, which assumes that the quality of the contestant is

$$\frac{1}{\sqrt{x}}$$

(note that:

$$\frac{\frac{1}{\sqrt{a}}}{\frac{1}{\sqrt{a}}+\frac{1}{\sqrt{b}}} = \frac{\sqrt{b}}{\sqrt{a}+\sqrt{b}}\ ))$$

Evaluation of competition systems

Round-robin system

Round-robin system is a competition in which each team plays against each other team once. There are 7 rounds and in each of them there are 4 matches, which gives a total of 28 matches, hence the number of the possible outcomes is $2^{28} =$ 268,435,456. One of the possible outcomes is presented by the following table:

Table 1. One of the possible outcomes of the round-robin system

	N_1	N_2	N_3	N_4	N_5	N_6	N_7	N_8
N_1		1	0	1	1	1	1	1
N_2			1	1	0	1	1	1
N_3				1	0	1	1	1
N_4					1	1	1	1
N_5						1	0	1
N_6							1	0
N_7								1
N_8								

In this case, the final score can be presented by the following table:

Table 2. Ranking of the contestants

Rank	Contestant	Points
1	N_1	6
2-3	N_2	5
2-3	N_3	5
4-5	N_4	4
4-5	N_5	4
6	N_6	2
7-8	N_7	1
7-8	N_8	1

In order to consider the occurrences of the shared ranks, we display Table 2 in the following form:

Table 3. Ranking of the contestants (where shared ranks are taken under consideration)

Rank	Contestant	Points
1	N_1	6
2.5	N_2	5
2.5	N_3	5
4.5	N_4	4
4.5	N_5	4
6	N_6	2
7.5	N_7	1
7.5	N_8	1

The objective standing (by their assessed actual quality) of the contestant N_i is i, hence we are able to calculate the aberration of this ranking and the ranking given in the table above. We obtain:

$|1-1|+|2-2.5|+|3-2.5|+|4-4.4|+|5-4.5|+|6-7.5|+|7-6|+|8-7.5| = 5.$

Hence, the average aberration of the rank is ⅝ = 0.625.

Our aim was to find the average aberration of the ranks obtained by the table of the competition and the assessed actual quality of the contestants. Denote by *AO* the set of all possible outcomes of the competition. Let $x \in$ AO. Denote by $PROB(x)$, the probability that x is the outcome of the competition and by $D(x)$ the average aberration of the competition ranking and the objective ranking, i.e. $D(x) = \sum_{i=1}^{8}|x_I - i|$, where x_I is the rank of the contestant i in the outcome x. Our task is to calculate:

$$\sum_{x \in AS} PROB(x) \cdot D(x)$$

Using the computer (the pseudocode of the algorithm is presented in the next section), we obtain (for the round-robin system):

$$\sum_{x \in AS} PROB(x) \cdot D(x) = 1.047428).$$

Cup system (elimination system; the pre-arranged draw)

The competition is played according to the cup system (the elimination system - a failure to win results in an elimination). There are four quarter-final matches. The winners proceed to play for the first four places, whereas the losers play for the places 5^{th} – 8^{th}. There are three rounds and in each round there are four matches, which gives a total of 12 matches.

The draw is pre-arranged and it is presented by the following figure:

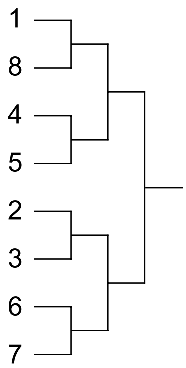

Figure 2. The graphical presentation of the pre-arranged draw.

Hence, after 8 matches all contestants are ranked from the 1^{st} to the 8^{th} place. Analogously, as in the previous section, we are able to determine the aberration of the results of ranking each outcome and the ranking according to the assessed actual quality of the contestants. The total number of possible outcomes is 2^{12} = 4,096. Denote the set of all possible outcomes (for the selected draw t) by $AS(t)$. Let $PROB(x)$ and $D(x)$ be defined as in the last section. We need to calculate:

$$\sum_{x \in AS(t)} PROB(x) \cdot D(x).$$

Using the computer (the algorithm is presented below in the pseudocode), we obtain:

$$\sum_{x \in AS(t)} PROB(x) \cdot D(x) = 1.152927$$

Cup system (a random, not pre-arranged draw)

The system of the competition is the same as the one described in the previous section, except for the fact that the draw is not pre-arranged. We have

$$\frac{8!}{2^{8-1}} = 315$$

different draws. Denote by $AS(q)$the set of all possible outcomes of the tournament with draw q, by $PROB_q(x)$the probability of outcome x when draw q is used, let $D(x)$be defined as in the previous section and let T be the set of all possible 315 draws. We have:

$$\sum_{q \in T} \frac{\sum_{x \in AS(q)} PROB(x) \cdot D(x)}{315} = 1.528560$$

Combined system (the pre-arranged draw)

The contestants are divided in two groups, each consisting of 4 contestants. Group A consists of contestants 1, 4, 5 and 8, whereas group B consists of contestants 2, 3, 6 and 7. The group competition is played as the round-robin system. If two contestants are tied (have the same number of points), then we give the better rank to the one that has won the game between these two contestants. Three contestants can have the same number of points only in the following two possible cases:

1) The winner of the group has 2 points, whereas the others have 1 point
2) The last in the group has 0 points, whereas the others have 2 points.

It can be easily seen that in both cases the principle of the number of points won in the games between the contestants that are tied does not discriminate the contestants with the same number of points, so the matter should be resolved by a draw. All four contestants can never have the same number of points. The first two contestants from each group compete for the 1^{st} to 4^{th} place (the cup system: semi-finals A1-B2 and A2-B1), the last two contestants from every group compete for the 5th to 8th place (the cup system: semi-finals A3-B4 and A4-B3). Finally, we are able to rank all the contestants and compare the ranks obtained in this way with their ranks according to their assessed actual, objective quality. The average aberration is 1.238890. The program that calculates this is given below in the pseudocode.

Combined system (the random draw)

All the rules are the same as in the section above, the difference being that the competition scheme of competitors in groups A and B is not pre-arranged; it is determined by a draw. We have

$$\binom{8}{4} = 70$$

possible draws and after an average aberration of each of the draws is calculated, we sum them up and divide by 70. The obtained result is 1.314522.

According to the results obtained in this study, the authors recommend the round-robin competition system, in which each team plays against each other team once, because the competition results achieved in such a kind of the competition displayed the highest level of congruence with the assessed actual quality of the competitors, that is, sport achievements are least uncertain from the aspect of actual quality. Unlike the round-robin system, the cup (elimination) system can produce the least reliable and the least expected competition results. The combined competition system is, from the aspect of reality of competition results, positioned between these two extremes. If someone organizes a cup competition system or combined competition system, the pre-arranged draw is recommended since it gives better results than the random draw. This model, proven by solid mathematical equation, confirms the findings from certain previous research studies (Dežman & Tkalčić, 2002), as well as the intuitive standpoints of sports experts on the appropriate competition systems which give competition results that are in accordance with the actual quality of the contestants.

However, it is well known that the cup system is extremely economic or efficient, because it has a very small number of matches. One may define the efficiency of the competition model as the product of the number of matches and the square of average aberration (because, we would wish for both of these numbers to be small). Values of this product are given in Table 4.

Table 4. Efficiency of competition models

COMPETITION SYSTEM	PRODUCT OF THE NUMBER OF MATCHES AND AVERAGE ABERRATION
Round-robin system	$1.047428^2 \cdot 28 \approx 30.719$
Cup system (prearranged draw)	$1.152927^2 \cdot 12 \approx 15.951$
Cup system	$1.528560^2 \cdot 12 \approx 28.038$
Combined system (prearranged draw)	$1.238890^2 \cdot 20 \approx 30.697$
Combined system	$1.314522^2 \cdot 20 \approx 34.559$

Hence, one can easily see that the most efficient competition system is the cup system with a prearranged draw, although it is least congruent with the actual quality of the individuals and teams. This is also in accordance with the intuition of the sport experts.

Appendix

Here, we compare the models with assumption 4), 4’) and 4’’). One can easily see that all the results and the conclusions are similar:

COMPETITION SYSTEM	AVERAGE ABERRATION WITH THE ASSUMPTION 4)	AVERAGE ABERRATION WITH THE ASSUMPTION 4’)	AVERAGE ABERRATION WITH THE ASSUMPTION 4’’)
Round-robin system	1.047428	1.411005	1.894108
Cup system (prearranged draw)	1.152927	1.503702	2.010783
Cup system	1.528560	1.834310	2.194800
Combined system (prearranged draw)	1.238890	1.626241	2.061255
Combined system	1.314522	1.683133	2.101372

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FORMALNI MODEL ZA PROCJENU PRIMJERENOSTI SUSTAVA NATJECANJA U KOŠARCI

Sažetak

Uvod

Odgovarajući sustav natjecanja treba na najbolji način osigurati integralno sportsko usavršavanje pojedinaca i timova te mora biti u funkciji razvoja pojedine sportske grane. Cilj ovog rada je analizirati različite sustave natjecanja kako bi se objektivno utvrdila njhova priroda u odnosu na mogućnosti koje pružaju natjecateljima, i to na temelju očekivanog odstupanja plasmana natjecatelja u odnosu na njihovu stvarnu kvalitetu.

Metode

Za ostvarenje navedenog cilja oblikovan je matematički model. Tim modelom je promatrano više natjecateljskih sustava i izračunato je očekivano odstupanje plasmana natjecatelja u odnosu na njihovu stvarnu kvalitetu.

Rezultati i rasprava

Dobivene su sljedeće vrijednosti odstupanja: 0.625 za jednokružni sustav natjecanja “svak sa svakim”; 1.152927 za kup-sustav s dirigiranim ždrijebom; 1.528560 za kup-sustav s nedirigiranim ždrijebom; 1.23890 za mješoviti sustav s dirigiranim ždrijebom te 1.314522 za mješoviti sustav s nedirigiranim ždrijebom.

Zaključak

U skladu s dobivenim rezultatima moguće je predložiti jednokružni sustav natjecanja “svak sa svakim” jer takav sustav natjecanja omogućava postizanje natjecateljskih rezultata koji su najviše u skladu sa stvarnom kvalitetom natjecatelja i očekivanim rezultatima.