Velocity profile simulation for natural gas flow underneath waterbody following a full-bore rupture of an offshore pipeline

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PROFESSIONAL PAPER

This work develops a model based on principle of conservation of momentum to predict natural gas flow pattern in waterbody following an accidental release through a full bore rupture (FBR) from a submerged pipeline. The model was discretized using Finite Difference Method; Crank-Nicholson numerical technique was applied to simulate it while MATLAB 7 was used to simulate the resulting algorithm. Solutions to the model are generated at various mesh points in the computational domain to show the flow pattern at various points within the waterbody both vertically and horizontally. This model gives a good representation of the flow pattern when compared with the existing similar models, thus, the model is useful for the Accident Response Planning Unit (ARPU) in case of such disaster.

Key words: natural gas, pipeline, velocity profile, full bore rupture, ARPU

INTRODUCTION

Natural gas is a naturally occurring mixture of simple hydrocarbons and non-hydrocarbons that exists as a gas at ordinary temperature and pressure (Maddox and Cannon, 1998).[9] The gas consists principally of methane (CH_4) and ethane (C_2H_6) with functional amounts of propane (C_3H_8) and butane (C_4H_{10}). In addition to hydrocarbon components, raw natural gas contains a varying amount of non-hydrocarbon contaminants or diluents, such as hydrogen sulfide (H_2S), carbon dioxide (CO_2), nitrogen (N_2) and helium (He). Its composition varies with origin, type genesis and location of the deposit, geological structure of the region and other factors.[18]

The gas is used as domestic and industrial fuel, raw material for the synthesis of methanols, formaldehyde and other chemical compounds[5] and for thermal generation purpose.[12] It is also used in air conditioning systems for domestic cooling; it is an important base ingredient for plastic, fertilizer and anti-freeze. It is used in metal preheating (mostly for iron and steel), drying and dehumidification, glass melting and food processing.

Natural gas is continuously transported through a complex network of pipelines designed to quickly and efficiently transport the gas from its origin to areas of high demand under high pressure and temperature of about 113 K.[11] For offshore continuous transportation of the gas, pipes are laid in trenches dug on the floor of the water body after which the pipe is fitted with a concrete casing to ensure that it stays at the bottom of the water. Another form of offshore continuous transportation of the gas is by allowing water to suspend the pipe-length through buoyancy.[14] However, these pipes are subject to rupture through corrosion, assembly errors, manufacturing defects, improper maintenance, fastener failure, design errors, improper material and improper heat treatment, casting discontinuities, fluctuation in operating conditions and inadequate environmental protection and control.[13,10,15] Following this failure, the conveyed fluid escapes into the waterbody to disturb its composition and biomass. Release of pollutants such as hydrocarbons and hydrogen sulfide to waterbody leads to mass mortality of many organisms, including fish and benthic molluscs[18], renders the water unfit for human consumption[21], hydrate formation and release of Volatile Organic Compounds (VOCs) to the atmosphere.

To minimize these problems, numerous models have been developed to study the velocity profile of the gas from the point of discharge in order to accurately represent the real world situation by means of iterative procedure.[2,23,15] Obanijesu and Mosobalaje[15] developed a similar model on concentration profile. This work develops a model to predict the velocity profile of the gas inside waterbody following such an accident based on the principle of conservation of momentum. The developed model was discretized using Finite Difference Method (FDM) while Crank-Nicholson numerical technique was applied to simulate the resulting algorithm.

Simulating such scenario is important in understanding the behavior of such fluid in a stratified or unstratified waterbody. It also makes the prediction of the extent of pollution possible in terms of concentration and area size. Also, the direction of flow of the polluting fluid in the receiving waterbody could be easily predicted. Also when new elements are introduced into a system, this study can be used to anticipate bottlenecks or other problems that may arise in the behavior of such system. It can be used to experiment new situations about which we have a little or no information so as to prepare for the aftermath.

METHODOLOGY

Model Development

According to Govier and Aziz[6], the 2-dimensional model equation for conservation of momentum for fluid flowing through a pipe is given as

$$\frac{\partial(U_{pi})}{\partial t}+\frac{\partial(\rho Upip_j)}{\partial xp_j}=-\frac{\partial p}{\partial x_{pi}}+\frac{\partial D_{ij}}{\partial x_{pj}}+\rho g_{pi}+F_{pi} \qquad (1)$$

where

U_{pi} and U_{pj} are the velocities of the gas phase in i and j – direction respectively,

ρ is the density of the gas phase,

x_i and x_j are the positions of the gas in i- and j- direction respectively,

p is the pressure,

g_{pi} is the gravity force in the i-direction,

F_{pi} is the source term in the i-direction (contribution from discharge point),

D_{ij} is the stress tensor (a second rank tensor whose components are stresses exerted across surface perpendicular to the coordinate direction).

However, gas constant relationship is given by Smith et al [19] as

$$pV=nRT \qquad (2)$$

Re-arrangement of equation (2) gives

$$p=\rho RT \qquad (3)$$

where

$$\rho=\frac{n}{V} \qquad (4)$$

$$r=\frac{R}{W} \qquad (5)$$

R is the gas constant

W is the air molecular weight

T is the temperature

D_{ij} is expressed by Abou-Arab[1] as

$$D_{ij}=\frac{\mu\delta U_{pi}}{\delta x_{p,j}}+\frac{\delta U_{pj}}{\delta x_{p,i}}-\frac{2\mu\delta U_{p,i}}{3\delta x_{p,i}}\delta_{ij} \qquad (6)$$

where

μ is the viscosity of the gas and

δ_{ij} is the Kroneeker delta

According to Kumar[8]

$$\delta_{ij} \text{ for } i=j \text{ and } \delta_{ij}=0 \text{ for } i\neq j \qquad (7)$$

According to Chung[4] however,

$i\neq j$ in the flow path except at the point of discharge where $i=j$ $(t=o)$ (8)

Substituting (7) and (8) into (6) gives

$$D_{ij}=\frac{\mu\delta U_{p,i}}{\delta x_{p,j}}+\frac{\delta U_{pj}}{\delta x_{p,i}} \qquad (9)$$

Substituting equation (9) into equation (1) gives

$$\frac{\delta(\rho U_{pi})}{\delta t}+\frac{\delta(\rho U_{pi},U_{pj})}{\delta x_{pj}}=\frac{-\delta p}{\delta x_{pi}}+\frac{\delta}{\delta x_{pj}}\left[\frac{\mu\delta U_{p,i}}{\delta x_{pj}}-\frac{\delta Uj}{\delta x_{Pl}}\right]+\rho g_{pj}-F_{p,i} \qquad (10)$$

Further expansion of equation (10) yields

$$\frac{\delta(\rho U_{pi})}{\delta t}+\frac{\delta(\rho U_{pi}U_{pj})}{\delta x_{pj}}=\frac{-\delta p}{\delta x_{pi}}+\frac{\mu\delta^2 U_{pi}}{\delta x^2{}_{pj}}+\frac{\delta^2 U_{pj}}{\delta x_{pj}\delta x_{pi}}+\rho g_{pj}+F_{p,i} \qquad (11)$$

Further re-arrangement of equation (11) gives the model equation as

$$\frac{\delta(\rho U_{pi})}{\delta t}+\delta U_{pi}\frac{\delta(U_{pj})}{\delta x_{pj}}=\frac{-\delta p}{\delta x_{pi}}+\frac{\mu\delta^2 U_{pi}}{\delta x^2{}_{pj}}+\frac{\delta^2 U_{pj}}{\delta x_{pj}\delta x_{pi}}+\rho g_{pj}+F_{p,i} \qquad (12)$$

Model Simulation

To simulate the model, numerical technique is developed using FDM (Figure 1). The partial derivatives in the partial differential equation are written in partial differential form as given by Kreyzig.[7] The computational domain was divided into a system of regular meshes and an approximation of the differential equation is then found at the point of intersection of these lines. The approximation was done replacing the derivatives of the equation by a finite difference approximation.

Applying Finite Difference Method to each component of equation (12) gives

$$\frac{\delta(\rho U_{pi})}{\delta t}=\rho\frac{(U_{i,j}+1-U_{ij})}{K} \text{ (Forward)} \qquad (13)$$

$$\frac{\rho U_{pi}\delta(U_{pi})}{\delta x_{p,j}}=\frac{\rho U_{pi}(U_{ij}-U_{i-1j})}{xj} \text{ (Backward)} \qquad (14)$$

$$\frac{\mu\delta^2 U_{pi}}{\delta x^2_{pj}}=\frac{\mu(U_{i+1j}-2U_{ij}+U_{i-1j})}{x_j^2} \text{ (Central)} \qquad (15)$$

$$\frac{\delta U_{pj}}{\delta x_{pj}}=\frac{U_{i+1j}-U_{i-1j}}{2x_j} \text{ (Central)} \qquad (16)$$

$$\frac{\delta U_{pj}}{\delta x_{pi}}=\frac{U_{i,j+1}-U_{i,j-1}}{2x_i} \text{ (Central)} \qquad (17)$$

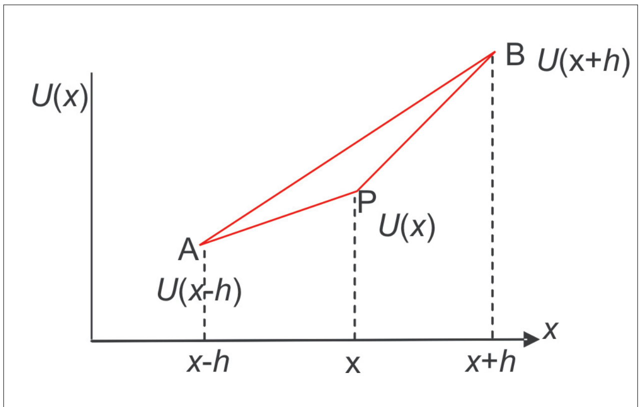

Fig. 1. Geometrical representation of finite difference method
Sl. 1. Geometrijski prikaz metode konačnih razlika

Inserting equations (13) - (17) into equation (12) gives the model in the descritized form as

$$\frac{\rho\left(U_{i,j+1}-U_{i-1j}\right)}{K}+\rho U_{pi}\frac{\left(U_{ij}-U_{i-1j}\right)}{x_j}=\frac{\mu\left(U_{i+1j}-2U_{ij}+U_{i-1j}\right)}{x_j^2}+ \\ +\left[\frac{\left(U_{i,j+1}-U_{ij-1}\right)}{2x_j}\times\left(\frac{U_{i,j+1}-U_{ij-1}}{2x_i}\right)\right]+\frac{p}{x_{pi}^2}+\rho g_{pj}+F_{pi} \tag{18}$$

But from vector multiplication according to Kreysizg[7]

$$iX_i=0 \tag{19}$$

Hence, equation (16) becomes

$$\frac{\rho\left(U_{i,j+1}-U_{ij}\right)}{K}+\frac{\rho U_{pi}\left(U_{ij}-U_{i-1j}\right)}{x_j}= \\ =\frac{\mu\left(U_{i+1j}-2U_{ij}+U_{i-1j}\right)}{x_j^2}+\frac{p}{x_{pi}^2}+\rho g_{pj}+F_{pi} \tag{20}$$

Re-arranging equation (20) gives

$$\frac{\rho\left(U_{i,j+1}-U_{ij}\right)}{K}=\frac{\mu\left(U_{i+1j}-2U_{ij}+U_{i-1j}\right)}{x_{pj}^2}-\frac{\rho U_{pi}\left(U_{ij}-U_{i-1j}\right)}{x_j}+ \\ +\frac{p}{x_{pi}^2}+\rho g_{pj}+F_{pi} \tag{21}$$

Taking L.C.M of equation (21) gives

$$\frac{\rho\left(U_{i,j+1}-U_{ij}\right)}{K}= \\ =\frac{\mu x_{pi}^2\left(U_{i+1,j}-2U_{ij}+U_{i-1,j}\right)-\rho U_{pi}x_{pj}x_{pi}^2\left(U_{ij}-U_{i-1,j}\right)+x_{pj}^2p+x_{pj}^2x_{pi}^2\rho g_{pj}+x_{pj}^2x_{pi}^2F_{pi}}{\left(x_{pj}^2\right)\left(x_{pi}^2\right)} \tag{22}$$

For simplification, let

$$\begin{aligned} &\mu x_{pi}^2=A\\ &\rho U_{pi}x_{pj}x_{pi}^2=B\\ &x_{pj}^2p=C\\ &x_{pj}^2,x_{pi}^2\rho g_{pj}=D\\ &x_{pj}^2,x_{pi}^2F_{pi}=E\\ &\left(x_{pj}^2\right)\left(x_{pi}^2\right)=h^2 \end{aligned} \tag{23}$$

Substituting equation (23) into equation (22) gives

$$\frac{\rho\left(U_{i,j+1}-U_{ij}\right)}{K}= \\ =\frac{A\left(U_{i+1j}-2U_{ij}+U_{i-1j}\right)-B\left(U_{ij}-U_{i-1j}\right)+C+D+E}{h^2} \tag{24}$$

Further simplification and re-arrangements gives equation (24) as

$$\frac{\rho\left(U_{i,j+1}-U_{ij}\right)}{K}= \\ =\frac{AU_{i+1j}-(2A+B)U_{ij}+AU_{i-1j}+BU_{i-1j}+C+D+E}{h^2} \tag{25}$$

Applying Crank-Nicholson Method to equation (25) by replacing the right side of the equation above by ½ times the sum of two such difference quotients at two time rows of j and $j+1$ (Kreyszig)[7] gives

$$\frac{\rho\left(U_{i,j+1}-U_{ij}\right)}{K}= \\ =\frac{1}{2h^2}\left(AU_{i+1j}-(2A+B)U_{ij}+AU_{i-1j}+BU_{i-1j}+C+D+E\right)+ \\ +\frac{1}{2h^2}\left(AU_{i+1j+1}-(2A+B)U_{ij+1}+AU_{i-1j+1}+BU_{i-1j+1}+C+D+E\right) \tag{26}$$

Letting $r=\frac{K}{h^2}$ in equation (26), followed by multiplication of both sides by $2K$ gives

$$2\rho\left(U_{i,j+1}-U_{ij}\right)= \\ =r\left(AU_{i+1,j}-(2A+B)U_{ij}+AU_{i-1,j}+BUi-1,j+C+D+E\right)+ \\ +r\left(AU_{i+1,j+1}-(2A-B)U_{i,j+1}+AU_{i-1,j-1}-BU_{i-1,j-1}-C-D-E\right) \tag{27}$$

Expansion followed by further re-arrangement gives equation 27 as

$$U_{i,j+1}(2\rho+r(2A+B))-r\left[U_{i-1,j+1}(A+B)+AU_{i+1,j+1}\right]= \\ =U_{ij}(2\rho-r(2A+B))+r\left[AU_{i+1,j}+U_{i-1,j}(A+B)+2(C+D+E)\right] \tag{28}$$

The three terms on the left hand side of equation (28) i.e. i-1,i,i+1 are unknown whereas those on the right hand side are known.

To solve equation (28), appropriate initial and boundary conditions are applied.

At the initial condition i.e before the rupture,

$$U(x,0)=0.0 \tag{29}$$

This means that the velocity of the solute (natural gas) in the waterbody before the discharge is zero.

After the rupture, the upstream and downstream boundary conditions are applicable. The upstream region is that region very close to the point of discharge and is governed by

$$U(0,t)=U_i(t) \tag{30}$$

Whereas, downstream region is that region far away from the point of release and is governed by

$$\frac{\partial U}{\partial x_i}=0.0 \tag{31}$$

Matlab program was developed to solve the model algorithm and the results are displayed as Figure 3.

Data Generation

The data used to test the model are generated based on the composition of a Nigerian gas field and the individual properties of the components (Table 1) while the flow rate and the internal pipe diameter are based on an offshore segment of the presently on-going West African Gas Pipeline (WAGP) project (Figure 2 and Table 2). Full bore rupture was assumed at the point of failure.

The gas viscosity, temperature and density were individually calculated using

$$\mu_{NG}=\sum C_i\mu_i \tag{32}$$

$$T_{NG}=\sum C_iT_i \tag{33}$$

$$\rho_{NG}=\sum C_i\rho_i \tag{34}$$

Table 1. Temperature, viscosity and density of typical gas pipeline

Source * Sonibare and Akeredolu[20]
** Smith et al[19]

Molecular formula	Composition* (%)	T** (K)	μ** (Pa·s)	ρ** (kg/m^3)
CH_4	69	90.69	1.33×10^{-5}	28.18
C_2H_6	9	90.36	1.29×10^{-5}	21.64
C_3H_8	7.6	85.47	1.01×10^{-5}	16.583
C_4H_{10}	3.4	134.86	6.90×10^{-6}	12.62
CO_2	2.6	216.58	1.85×10^{-5}	26.828
N_2	4	63.15	2.15×10^{-5}	31.063
H_2S	3.8	187.68	1.65×10^{-5}	29.13
He	0.6	2.20	2.25×10^{-5}	37.115

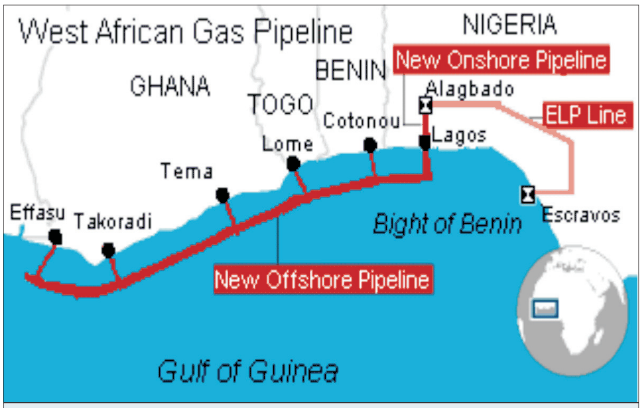

Fig. 2. West African Gas Pipeline (WAGP) Route
Sl. 2. Trasa zapadnoafričkog plinovoda (WAGP)

Table 2. Operating Condition of a typical pipeline project in Nigeria

Source: Obanijesu and Macaulay[17]

Flow rate (m^3/d)	4.24×10^9
Internal pipes diameter (in)	30

Where C_i is the concentration of component *i*

The point of rupture is taken to be circular and this gives the area (*A*) as

$$A = \frac{\pi d^2}{4} \tag{35}$$

where *d* is the pipe's internal diameter

Mass flow rate (*M*) is given as

$$M = \rho V \tag{36}$$

where *V* is the volumetric flow rate, while the source term F_i is given as

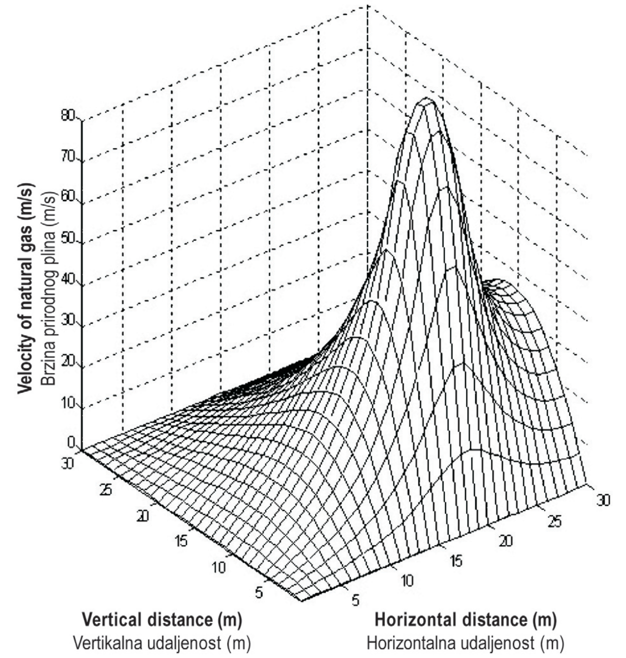

Fig. 3. Velocity profile in the horizontal and vertical directions for the gas from rupture point
Sl. 3. Profil brzine plina iz točke puknuća u horizontalnom i u vertikalnom smjeru

Table 3. Generated Data used to test the model initial conditions

Pressure	733.46 Pa
Viscosity	1.344×10^{-3} (Pa·s)
Density	26.350 kg/m^3
Temperature	97.092 K
Gas constant	8.3143 /J·mol^{-1} ·K^{-1}
Air molecular weight	29 kg/kmol
Gravity force	6.672×10^{-11}Nm^2/kg^2
Source term (Fi)	287 995 9.342 kg/m^{2}s

$$F_i = \frac{M}{A} \tag{37}$$

RESULTS AND DISCUSSION

A computer code written in MATLAB 7 was developed to solve the model and the result displayed in two-dimensional form as shown in Figure 3. From the curve, it is shown that at time zero, nothing was released to the waterbody and the velocity is zero along horizontal and vertical planes (distances) but as the time increases, velocity begins to increase with distance. Due to difference in the densities and viscosities of the components in the two-phase flow (the gas and water i.e. 26.35 kg/m^3 and 1000 kg/m^3 respectively), the less dense phase tends to flow at higher velocity than the other. This led to the unsteady state situation initially observed in the curve (Figure 3) for both the horizontal and vertical flows. This situation is easily explainable since velocity is determined through the relative velocity (slip velocity), which is the velocity of the dispersed phase relative to that of the continuous phase.

At some distance away from the point of discharge, velocity of the escaped gas (both vertical and horizontal) starts reducing till it equals the velocity of the ambient current. This is so since the driving pressure of the escaped gas reduces with distance (Cirpka, 2008).[3] The decrease will continue till the gas flows in undulating motion along the water stream and finally gets released to the atmosphere probably through water inversion.

Conclusions

The model has successfully predicted natural gas flow pattern in waterbody following an accidental release from a submerged pipeline and gives a good representation of the flow pattern when compared with the existing similar models. Hence, in the process of offshore transportation of this energy source, it is necessary to properly prevent leakage or rupture along the pipe-length in order to avoid the resulting consequences. This could be achieved through regular pigging of the pipes to prevent scaling, immediate replacement of any fractured pipe and development of other management schemes.

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