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POBOLJŠANJE KVALITETE PRIKAZA SIGNALA PRIMJENOM MATEMATIČKIH OPERACIJA NA VREMENSKO-FREKVENCIJSKIM DISTRIBUCIJAMA
SIGNAL REPRESENTATION QUALITY ENHANCEMENT BY APPLYING MATHEMATICAL OPERATIONS TO TIME-FREQUENCY DISTRIBUTIONS

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Sažetak: *Vremensko-frekvencijske distribucije efikasan su alat u analizi i obradi nestacionarnih signala. Same karakteristike nestacionarnih signala (frekvencijska modulacija, svojstvo višekomponentnosti) dovode do odstupanja od idealnog vremensko-frekvencijskog prikaza, tj. prikaza bez unutarnjih i vanjskih interferencija. U ovom je radu opisan učinak određenih matematičkih operacija na vremensko-frekvencijske distribucije u smislu poboljšanja kvalitete prikaza. Operacijama na spektrogramu teži se poboljšanju koncentracije energije oko trenutne frekvencije signala, dok se množenjem Wigner-Villove distribucije pseudo Wigner-Villovom distribucijom teži uklanjanju interferencije, što je i uspoređeno s već poznatim maskiranjem Wigner-Villove distribucije spektrogramom.*

Ključne riječi:
- spektrogram
- Wigner-Villova distribucija
- pseudo Wigner-Villova distribucija
- višekomponentni signali

Abstract*: Time-frequency distributions represent an efficient tool in the analyzing and processing of nonstationary signals. Some characteristics of nonstationary signals (such as frequency modulation or the property of multi-components) result in the appearance of undesirable interference terms (cross-terms) which do not exist in an ideal time-frequency representation. In this paper, several mathematical operations are used in order to enhance the quality of time-frequency distributions representation. Mathematical operations applied to the spectrogram have led to an improvement in the concentration of the signal energy about its instantaneous frequency. The multiplication of the Wigner-Ville distribution by the Pseudo Wigner-Ville distribution, on the other hand, is proposed as a method for cross-terms suppression, and its performance is numerically compared to that of the Wigner-Ville distribution masked by the spectrogram.*

Key words:
- spectrogram
- Wigner-Ville Distribution
- pseudo Wigner-Ville Distribution
- multi-component signals

1. UVOD

Dva klasična prikaza signala su prikaz u vremenskoj domeni, $s(t)$, i prikaz u frekvencijskoj domeni, $S(f)$ [1]. U tim obama prikazima varijable t i f međusobno su isključive: da bi se dobila ovisnost signala o jednoj varijabli, druga mora biti zanemarena [1]. Vremensko-frekvencijska distribucija (VFD), $\rho(t, f)$, je distribucija energije signala u odnosu na vrijeme i frekvenciju, tj. energija signala u potpunosti je očuvana u (t, f) ravnini, tako da vrijedi [2]:

1. INTRODUCTION

Two common representations of a signal are the representation in the time domain, $s(t)$, and in the frequency domain, $S(f)$ [1]. In both, the variables t and f are mutually exclusive; the signal's dependency on one variable excludes the other. In time-frequency distributions (TFD), $\rho(t, f)$, the time and frequency variables are not exclusive, but are used concurrently, such that the signal's energy in the (t, f) plane is fully preserved, that is [2]:

$$E = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \rho(t,f)\, dt\, df \tag{1}$$

Vremensko-frekvencijska funkcija *ρ(t, f)* je idealno oštar brid koji opisuje trenutnu frekvenciju (TF) signala u vremensko-frekvencijskoj ravnini [1], tj. vrh vremenskog presjeka VFD jednokomponentnog frekvencijski moduliranog (FM) signala u odnosu na frekvenciju odražava TF signala [2]:

Ideally, the time-frequency (TF) function *ρ(t, f)* should take the form of a sharp edge, describing the signal's instantaneous frequency (IF) law over the time-frequency plane [1], i.e. the peaks of the TFD with respect to frequency of a mono-component frequency modulated (FM) signal should reflect the IF of the signal [2]:

$$\left.\frac{\partial \rho(t,f)}{\partial f}\right|_{f=f_i(t)} = 0 \tag{2}$$

Isto vrijedi i za pojedine komponente višekomponentnih signala [2].

Prednost je koju VFD nude, u odnosu na uobičajene prikaze u vremenu i frekvenciji, u dostupnosti glavnih karakteristika signala (trenutna amplituda, trenutna frekvencija i frekvencijski opseg) iz samo jednog prikaza. VFD su korisne i kod višekomponentnih signala jer jasno izdvajaju njihove komponente, što nije lako kod klasičnih prikaza [1].

U ovom je radu promatran utjecaj matematičkih operacija na VFD (spektrogram, Wigner-Villova distribucija, pseudo Wigner-Villova distribucija) testnih signala različitih FM, u svrhu uklanjanja nepoželjnih pojava iz vremensko-frekvencijskih prikaza, kao što su loša koncentracija energije oko TF signala ili različite interferencije.

The same applies to the individual components of multi-component signals [2].

The major advantage of the TF approach is the availability of the signal's characteristics (such as instantaneous amplitude, instantaneous frequency and bandwidth) from only one representation. TFDs are useful in dealing with multi-component signals since they allow for the separation of the components from each other, which is not an easy task within the classic representations [1].

This paper presents the effects of mathematical operations on the TFDs (spectrogram, Wigner-Ville distribution, Pseudo Wigner-Ville distribution) of different FM test signals, in order to suppress undesirable effects such as the energy dispersion about the signal's IF or various interference terms.

2. OPERACIJE NA SPEKTROGRAMU

Prvi korak pri definiranju spektrograma je uvođenje vremenskog elementa u Fourierovu transformaciju signala. To se postiže množenjem promatranog signala *s(τ)* realnim vremenskim otvorom, *w(t-τ)* centriranim oko trenutka *t*. Računajući Fourierovu transformaciju toga umnoška, za svaki trenutak *t* dobiva se Fourierova transformacija na vremenskom otvoru (engl. *Short-time Fourier transform* (STFT)) signala *s(t)* definirana kao $F_s(t,f;w)$.

Spektrogram signala *s(t)* dobiven je kvadriranjem apsolutne vrijednosti STFT istog signala [4]:

2. OPERATIONS ON THE SPECTROGRAM

The first step in defining the spectrogram is the introduction of time-dependency in the definition of the Fourier transformation of a signal. The solution is given by multiplying the signal *s(τ)* by a real window *w(t-τ)* centered at time *t*. By calculating the Fourier transformation of this product, and repeating it for each time instant *t*, we obtain a transformation known as the *Short-time Fourier transform* (STFT), denoted by $F_s(t,f;w)$.

The squared magnitude of the STFT is the signal's spectrogram, which is defined for the signal *s(t)* as follows [4]:

$$S(t,f) = \left|F_s(t,f;w)\right|^2 = \left|\int_{-\infty}^{\infty} s(\tau)\, w(t-\tau)\, e^{-j2\pi f\tau}\, d\tau\right|^2 . \tag{3}$$

Optimalno trajanje vremenskog otvora, odnosno ono koje minimizira širinu brida VFD na polovici amplitude opisuje izraz [1]

The optimal window duration, which minimizes the half-power bandwidth of the resulting ridge in the *(t ,f)* plane, is given by the expression [1]

$$\Delta = \sqrt{2}\left|\frac{df_i(t)}{dt}\right|^{-\frac{1}{2}} \tag{4}$$

Kod višekomponentnih signala pojavit će se interferencija između komponenata samo u dijelovima VF ravnine gdje se spektrogrami pojedinih komponenata dodiruju ili preklapaju.

U svrhu približavanja idealnim karakteristikama VFD, tj. poboljšanja koncentracije energije oko TF signala i time smanjenja eventualne loše rezolucije komponenata, djeluje se na spektrogram određenim matematičkim operacijama.

Matematičke operacije koje daju najbolje rezultate su operacije eksponenciranja spektrograma. Eksponenciranjem se smanjuje varijanca energije oko TF signala tako što se povećava razlika između energije vrha spektrograma i njegovih marginalnih dijelova. Međutim takvim će postupkom oslabjeti i dijelovi signala koji su slabije zastupljeni u vremenu, prije nego spektrogram dostigne konačnu amplitudu.

Kao testni signal u ovom je članku korišten linearno frekvencijski modulirani (LFM) signal definiran kao [1]:

The cross-terms will appear in the resulting spectrogram of multi-component signals only in those areas of the *(t, f)* plane where the spectrograms of each component are either touching or overlapping.

In order to match the performance of the spectrogram to that of the TFD's ideal characteristics (in terms of concentration and resolution) a modification of the original spectrogram by the application of various mathematical operations is proposed.

The best results are obtained by taking the spectrogram to different powers. The obtained spectrogram has a reduced energy variance along the signal's IF, as it better emphasizes differences in the energy level between the peaks of the spectrogram and its marginal parts. Consequently, the parts of the spectrogram located in time before the spectrogram reaches its final amplitude, decrease. The test signal used in this paper has a linear frequency modulation (LFM) law, and it is defined as [1]:

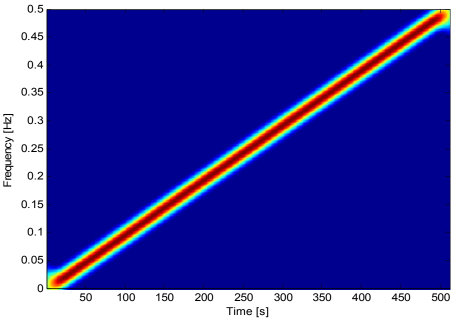
$$s(t) = \cos(2\pi\,(f_0 t + \frac{\alpha}{2}t^2)), \qquad 0 \le t \le T \tag{5}$$

gdje je T=512 s, $f_0 = f_{\min} = 0$, $f_1 = f_{\max} = 0.5$, odnosno gradijent frekvencije $\alpha = 9.765 \cdot 10^{-4}$.

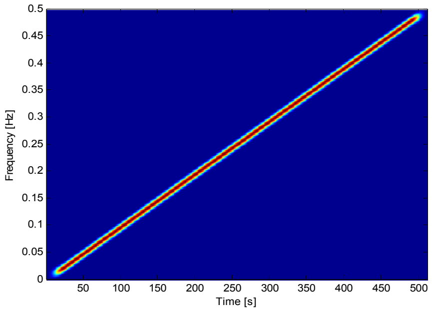
Spektrogram signala (5) uz optimalni vremenski otvor Hammingova oblika ($\Delta = 45$) i isti spektrogram uzdignut na šestu potenciju (normiran na energiju početnog spektrograma) prikazani su na slikama 1 i 2. Usporede li se slike 1 i 2, vidljivo je da je koncentracija energije oko TF signala bolja kod potenciranog spektrograma (slika 2), no spektrogram gubi intenzitet amplitude u vremenski rubnim područjima. Kako kriterij kvalitete prikaza ne bi bio samo vizualni, uvode se numerički pokazatelji kvalitete VFD: apsolutna pogreška TF, $\varepsilon_{f_i(t)}$ (razlika zadane analitičke frekvencije signala i one estimirane iz VFD u trenutku *t*), frekvencijski opseg na polovici maksimalne amplitude $B_i(t)$ te vrijeme rasta spektrograma t_r (vrijeme potrebno da spektrogram dostigne 90 % vrijednosti konačne amplitude). Na slici 3 usporedno su prikazani normirani vremenski presjeci za središnji trenutak *(t*=256*)* spektrograma sa slika 1 i 2.

where T=512 s, $f_0 = f_{\min} = 0$, $f_1 = f_{\max} = 0.5$, and the frequency gradient is $\alpha = 9.765 \cdot 10^{-4}$.

The spectrogram of the signal (5) with the Hamming window of the optimal length ($\Delta = 45$), and the same spectrogram to the power of six (normalized to the original spectrogram's energy), are shown in Figures 1 and 2, respectively. By comparing Figures 1 and 2, it can be observed that after the exponentiation of the spectrogram, an improvement in the energy concentration around the IF is achieved, with a lower energy intensity in time marginal areas. Since the representation quality criteria should not be only visual, the following numerical indicators of the TFD quality are introduced: the IF bias, $\varepsilon_{f_i(t)}$, (mismatch between the analytical IF and the IF estimated from the representation at the time instant *t*), the half-power bandwidth of the resulting ridge, $B_i(t)$, and the spectrogram rise time, t_r (the time in which the spectrogram rises to 90 % of its maximum value). Time slices of the spectrograms from Figures 1 and 2 at the central time moment *(t*=256*)* are shown in Figure 3.



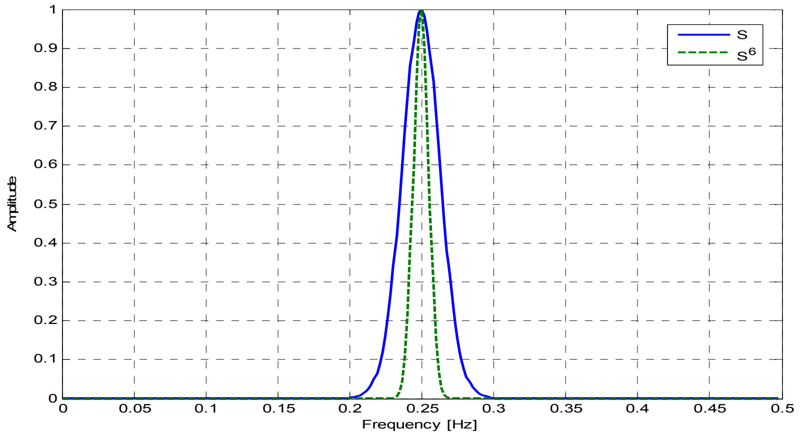
Slika 1. Spektrogram LFM signala s(t)
Figure 1. The spectrogram of the signal s(t)



Slika 2. Šesta potencija spektrograma signala s(t)
Figure 2. The spectrogram to the power of six of the signal s(t)

Slika 3 potvrđuje da je za spektrogram LFM signala odstupanje estimirane od analitičke frekvencije nula [3], stoga nema odstupanja ni nakon operacije eksponenciranja spektrograma.

Figure 3 confirms that the bias of the spectrogram peak-based IF estimator is zero for LFM signals [3], and hence also remains zero for the spectrogram after the exponentiation operation.



Slika 3. Usporedni presjek originalnog spektrograma i šeste potencije spektrograma signala s(t) u trenutku t=256
Figure 3. Time slices of the original spectrogram and the spectrogram to the power of six of the signal s(t) at time t=256

U tablici 1 navedeni su podaci koji će se koristiti kao kriteriji kvalitete prikaza. Eksponenciranjem se postiglo smanjenje frekvencijskog opsega gotovo 2.5 puta na polovici amplitude u odnosu na najuži mogući frekvencijski opseg dobiven optimiranjem vremenskog otvora. Vrijeme porasta spektrograma povećalo se za svega 5 sekundi.

Table 1 presents the numerical indicators which are used as the representation quality criteria. The exponentiation has led to a spectrogram with an approximately 2.5 times narrower half-power bandwidth, when compared to the bandwidth of the original spectrogram for the optimal window width. The rise time of the resulting spectrogram has increased for 5 seconds only.

Tablica 1. Kriteriji kvalitete prikaza za vremenske presjeke sa slike 3
Table 1. Representation quality indicators for the time slices in Figure 3

	$\varepsilon_{f_i} = \left\| f_i(256) - \hat{f}_i(256) \right\|$	$B_i(256)$	t_r
$S(t,f)$	$\|0.25 - 0.25\|$	$\approx 0.265 - 0.234 = 0.031$	14
$S^6(t,f)$	$\|0.25 - 0.25\|$	$\approx 0.256 - 0.243 = 0.013$	19

Dakle, eksponenciranjem spektrograma postiže se bolja koncentracija energije oko TF signala uz minimalan gubitak podataka o njegovoj dinamici.
Bolja koncentracija energije kod višekomponentnih signala povlači i bolju rezoluciju, pa je potenciranje korisno u minimiziranju interferencije. Kod višekomponentnih signala interferencija među komponentama može djelovati i na njih same, uzrokujući varijacije razine energije. Potenciranjem se varijacije naglašavaju, što može dovesti do krivog tumačenja signala. Kako bi se to izbjeglo, uputno je usrednjiti nekoliko spektrograma istog signala, koji se međusobno razlikuju po odabiru vremenskog otvora, na način da je za jedan spektrogram odabran optimalni otvor, a ostali su naizmjence usmjereni boljoj vremenskoj, odnosno frekvencijskoj rezoluciji.

Na slici 4 prikazan je spektrogram uz optimalno trajanje vremenskog otvora Hammingova oblika dvokomponentnog LFM signala koji je definiran kao

Therefore, it can be concluded that the spectrogram exponentiation gives better energy concentration around the IF, with a minimal loss in the dynamic features of the signal.
Good energy concentration in the multi-component signals gives better resolution, thus the exponentiation is also useful in interference minimization. In multi-component signals, the cross-terms located between the components can cause energy variations in the components themselves. Through exponentiation, these variations will be higher, which can be misleading. The distortions could be avoided by performing the exponentiation of the spectrogram obtained by averaging several spectrograms of the analyzed signal. The spectrograms differ from each other in terms of the window width selection, such that one spectrogram has the optimal window width, while the other two have the window for a better time resolution and a better frequency resolution, respectively.
Figure 4 shows the spectrogram with the optimal Hamming window of a two-component LFM signal defined as

$$s_{12}(t) = s_1(t) + s_2(t) = \cos(2\pi\,(f_1 t + \frac{\alpha}{2}t^2)) + (2\pi\,(f_2 t + \frac{\alpha}{2}t^2)), \qquad 0 \le t \le T \tag{6}$$

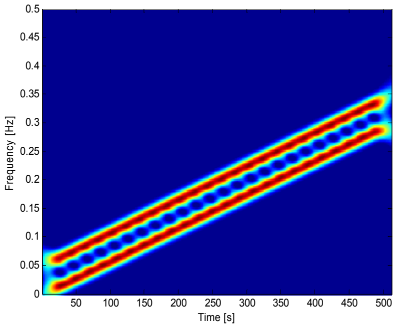
gdje je T=512 s, $f_1 = 0$, $f_2 = 0.05$, odnosno gradijent frekvencije $\alpha = 5.86\cdot 10^{-4}$.
Slika 5 prikazuje spektrogram dobiven kubiranjem sume spektrograma $S_1(t,f)$ signala iz (6) za optimalni vremenski otvor, $\Delta = 59$, te dvaju spektrograma čiji se vremenski otvori odabiru tako da jedan odgovara boljoj vremenskoj ($\Delta = 51$) $S_2(t,f)$, a drugi boljoj frekvencijskoj ($\Delta = 69$) rezoluciji $S_3(t,f)$. Rezultantni spektrogram $S_R(t,f)$ je:

where T=512 s, $f_1 = 0$, $f_2 = 0.05$ and the frequency gradient is $\alpha = 5.86\cdot 10^{-4}$.
Figure 5 shows the spectrogram of the signal (6), obtained by cubing the sum of three spectrograms of the signal (6); $S_1(t,f)$ is the spectrogram with the optimal window length $\Delta = 59$, while the windows for the spectrograms $S_2(t,f)$ ($\Delta = 51$) and $S_3(t,f)$ ($\Delta = 69$) are chosen to insure better time and frequency resolution, respectively. The resulting spectrogram $S_R(t,f)$ is:

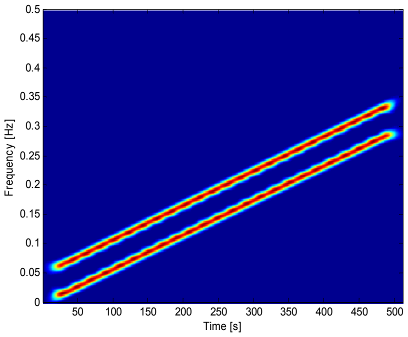
$$S_R(t,f) = \left[S_1(t,f) + S_2(t,f) + S_3(t,f)\right]^3 \tag{7}$$

Rezultantni spektrogram na slici 5 normiran je na energiju spektrograma sa slike 4.

The resulting spectrogram in Figure 5 is normalized to the energy of the original spectrogram (Figure 4).



Slika 4. Spektrogram signala (6)
Figure 4. Spectrogram of the signal (6)



Slika 5. Spektrogram signala (6) dobiven pomoću (7)
Figure 5. Spectrogram of the signal (6) obtained by (7)

U tablici 2 prikazani su već uvedeni numerički kriteriji za svaku komponentu signala te dodatni kriterij $A_{\text{int}}(256)$ koji označava amplitudu interferencije u trenutku t=256 s na srednjoj vrijednosti trenutnih frekvencija komponenata.

Table 2 shows the representation quality indicators for each of the signal's two components. An additional indicator, $A_{\text{int}}(256)$, is introduced as a measurement of the interference amplitude at time t=256 s, measured at the average value of the instantaneous frequencies of the components.

Tablica 2. Kriteriji kvalitete prikaza za vremenski presjek spektrograma signala $s_{12}(t)$ *i spektrograma istog signala dobivenog pomoću (7) za t=256 s*

Table 2. Representation quality indicators for the time slices of the spectrogram of the signal $s_{12}(t)$ *and the spectrogram obtained by (7) at time t=256 s*

	$\varepsilon_{f_i} = \left\lvert f_i(256) - \hat{f}_i(256) \right\rvert$		$B_i(256)$		t_r		$A_{\text{int}}(256)$
	$s_1(t)$	$s_2(t)$	$s_1(t)$	$s_2(t)$	$s_1(t)$	$s_2(t)$	
$S_1(t,f)$	$1.6\cdot10^{-3}$	$8\cdot10^{-4}$	0.024	0.024	22	21	0.206
$S_R(t,f)$	$1.6\cdot10^{-3}$	$8\cdot10^{-4}$	0.013	0.013	24	35	0.01

Eksponenciranje sume spektrograma popravilo je rezoluciju komponenata, ali i dovelo do nešto kasnijeg ulaska spektrograma u konačnu amplitudu.

The exponentiation of the sum of spectrograms has led to a better resolution of components, but has resulted in a longer rise time.

3. OPERACIJE NA WIGNER-VILLOVOJ DISTRIBUCIJI

3. OPERATIONS ON THE WIGNER-VILLE DISTRIBUTION

Wigner-Villova distribucija (WVD) teži udovoljavanju zahtjeva da signal bude prikazan oštrim bridom (matematički predstavljenim nizom delta funkcija) koji prati TF signala [1]. Međutim, taj je zahtjev zadovoljen samo kod jednokomponentnog LFM signala, jer kod WVD dvije točke interferiraju bez obzira na njihovu udaljenost u VF ravnini i doprinose trećoj točki koja se nalazi na geometrijskoj sredini između njih [4]. WVD analitičkog ekvivalenta signala *s(t)* , *z(t)*, je [1]:

The aim of the Wigner-Ville distribution (WVD) is to obtain a signal representation whose form in the *(t, f)* plane shows a sharp ridge (mathematically, an array of delta functions) describing the IF law of the signal [1]. Such a request is fulfilled only for a mono-component LFM signal because in the WVD, two points in the *(t, f)* plane will always interfere, regardless of their separation, to create a third point which is located at their midpoint [4]. The WVD of a signal *s(t)* , whose analytic equivalent is *z(t)*, is [1]:

$$W_z(t,f) = \int_{-\infty}^{\infty} z\left(t+\frac{\tau}{2}\right) z^*\left(t-\frac{\tau}{2}\right) e^{-j2\pi f\tau} d\tau \ . \tag{8}$$

Modifikacija WVD je pseudo WVD (PWVD) kod koje se zbog praktične nemogućnosti poznavanja umnoška $q_z(t,\tau) = z\left(t+\frac{\tau}{2}\right) z^*\left(t-\frac{\tau}{2}\right)$ u intervalu od $-\infty$ do $+\infty$, taj umnožak nadomješta vremenski ograničenom verzijom, što vodi do definicije PWVD [4]:

The WVD requires a knowledge of the product $q_z(t,\tau) = z\left(t+\frac{\tau}{2}\right) z^*\left(t-\frac{\tau}{2}\right)$ from $-\infty$ to $+\infty$, which can be a problem in practice. The Pseudo WVD (PWVD) is a modification of the WVD in which $q_z(t,\tau)$ is replaced by a windowed version of it, leading to the definition of the PWVD as [4]:

$$PW_z(t,f) = \int_{-\infty}^{\infty} h(\tau)\, z\left(t+\frac{\tau}{2}\right) z^*\left(t-\frac{\tau}{2}\right) e^{-j2\pi f\tau} d\tau \ . \tag{9}$$

pri čemu je $h(\tau)$ odabrani vremenski otvor.

where $h(\tau)$ is the chosen analysis window.

Množenje signala s vremenskim otvorom ekvivalentno je gađenju WVD u frekvenciji [4]. Posljedica je toga smanjenje komponente interferencije u smjeru vremenske osi, ali i gubitak frekvencijske rezolucije.

Kao testni signal odabran je parabolično frekvencijski modulirani (PFM) signal kod čije se WVD pojavljuju interferencije unutar parabole (slika 6), što je kod PWVD (slika 7) frekvencijskim glađenjem izbjegnuto, ali uz gubitak koncentracije energije oko TF signala.

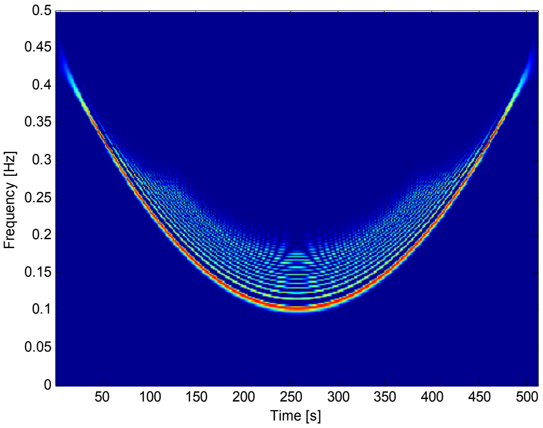
Kako bi se objedinile dobre karakteristike WVD (dobra koncentracija energije oko TF) i PWVD (izostanak interferencije), nameće se zamisao o njihovu množenju, odnosno maskiranje WVD s PWVD-om. Cilj je i pokazati kako je maskiranjem WVD s PWVD-om moguće postići bolje rezultate, u smislu koncentracije energije i uklanjanja interferencije, u odnosu na maskiranje WVD spektrogramom [1].

Upravo zato što odabir vremenskog otvora $h(\tau)$ kod PWVD omogućava postupno glađenje WVD u frekvenciji, cilj je pronaći vremenski otvor s kojim će se prolazak kroz nulu glavne latice PWVD poklopiti s minimumom prve sporedne latice WVD; na taj će način prva sporedna latica njihova umnoška imati minimalnu amplitudu. Vremenski otvor koji zadovoljava taj uvjet za analizirani signal ima trajanje 81 *s*. Slika 8 prikazuje distribuciju dobivenu umnoškom distribucija sa slika 6 i 7.

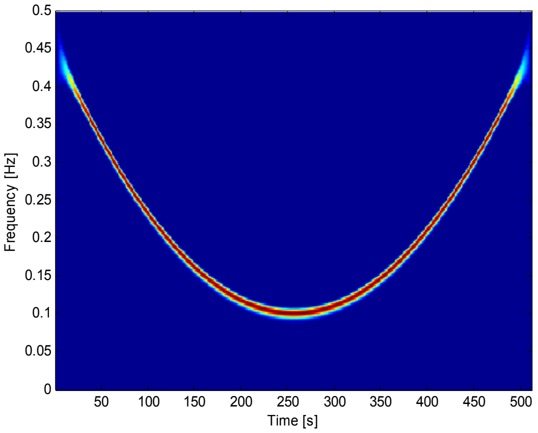
The multiplication of the signal and the window is equivalent to the smoothing of the WVD in the frequency direction [4]. The result of it is a reduction of the interferences in the direction of the time axis, but also a loss of the frequency resolution.

As a test signal, a parabolic frequency modulated signal (PFM) is used to show how the inner-artifacts in the WVD (Figure 6) can be reduced by frequency smoothing performed by the PWVD (Figure 7), with an inevitable loss in the energy concentration around the IF law.

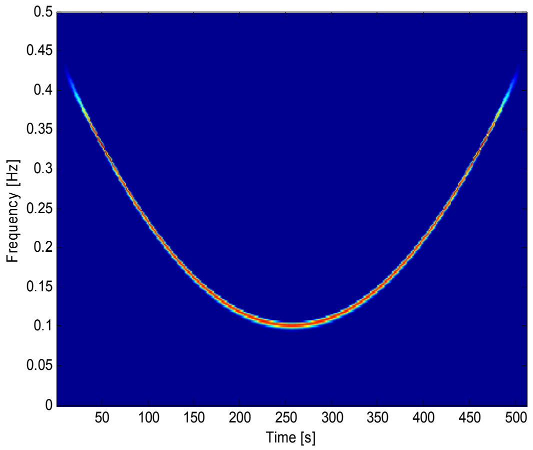
To take advantage of the good characteristics of the WVD (good energy concentration around the IF) and of the PWVD (suppression of the interference), a new distribution is proposed, obtained by multiplication of the WVD by the PWVD, i.e. this is the WVD masked by the PWVD. The purpose of this modification is to show that by masking the WVD with the PWVD, better results are obtained in terms of energy concentration and interference suppression than by masking the WVD with the spectrogram [1]. Since the window choice in the PWVD allows gradual frequency smoothing, the aim is to find a window by which the zero-crossing point of the main-lobe of the PWVD and the minimum of the first side-lobe of the WVD will overlap, which ensures the minimal amplitude of the first side-lobe of their product. The window duration that fulfills this criterion for the analyzed signal is 81 *s*. Figure 8 shows the product of the distributions in Figures 6 and 7.



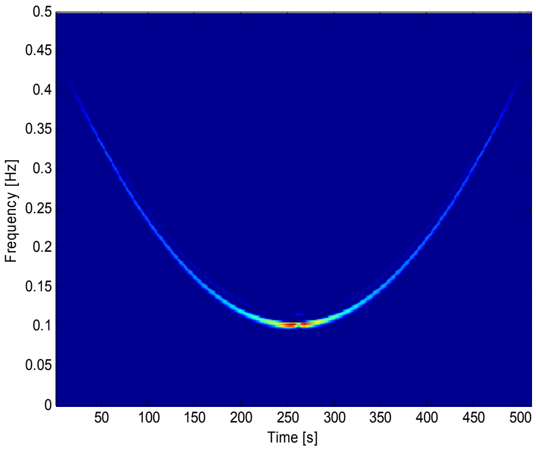
Slika 6. WVD paraboličnog FM signala
Figure 6. WVD of a parabolic FM signal



Slika 7. PWVD paraboličnog FM signala
Figure 7. PWVD of a parabolic FM signal



Slika 8. WVD paraboličnog FM signala maskirana PWVD-om
Figure 8. WVD of a parabolic FM signal masked by the PWVD



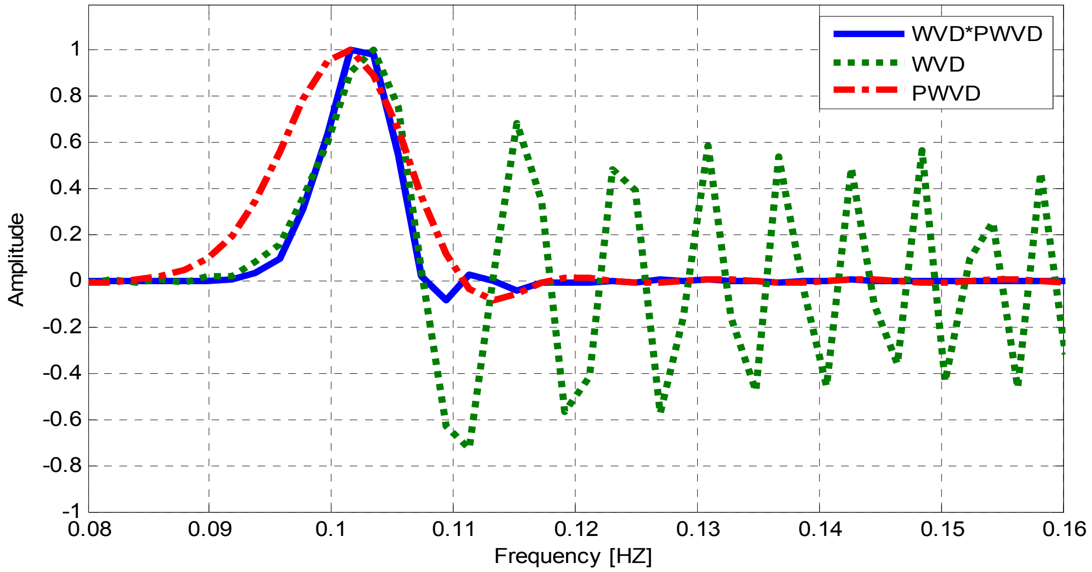
Slika 9. WVD paraboličnog FM signala maskirana spektrogramom
Figure 9. WVD of a parabolic FM signal masked by the spectrogram

Da bi se dobio najuži mogući frekvencijski opseg spektrograma (koji će kod maskiranja WVD izdvojiti samo glavnu laticu signala), koristi se izraz (4). Međutim WVD maskirana tako dobivenim spektrogramom (slika 9) ima veće varijacije energije signala u odnosu na distribuciju sa slike 8.

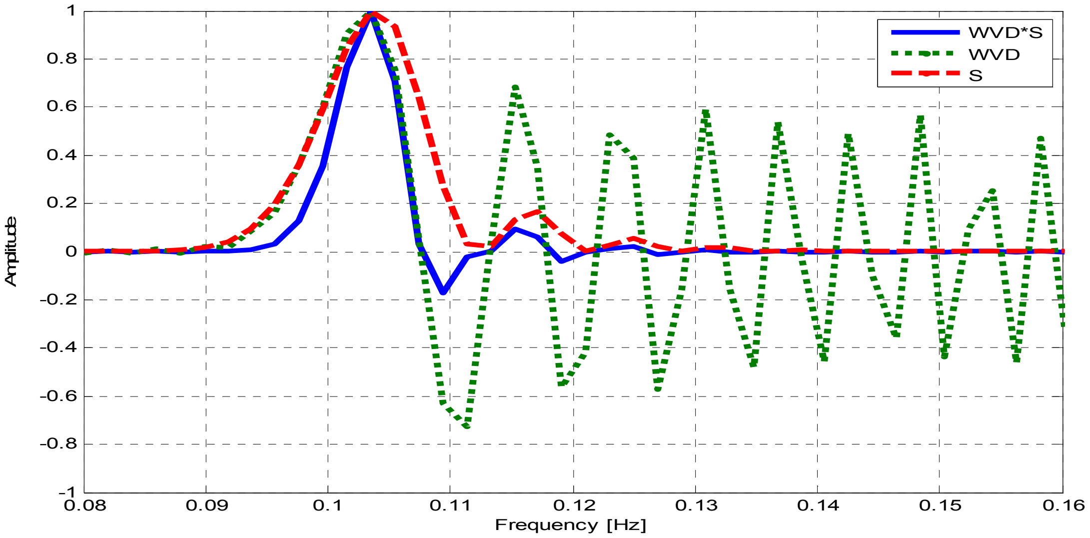
Na slikama 10 i 11 prikazani su vremenski presjeci za t=256 s WVD maskirane PWVD-om i WVD maskirane spektrogramom.

In order to extract only the main-lobe of the WVD after the masking, expression (4) will be used to obtain the narrowest spectrogram bandwidth. However, the resulting masked WVD (Figure 9) has large energy variations when compared to the distribution in Figure 8.

Figures 10 and 11 show the slices of the WVD masked by the PWVD and by the spectrogram at time t=256 s.



Slika 10. Presjeci WVD, PWVD i njihova umnoška paraboličnog FM signala u trenutku t=256
Figure 10. Slices of the WVD, the PWVD and their product for a parabolic FM signal at time t=256



Slika 11. Presjeci WVD, spektrograma i njihova umnoška paraboličnog FM signala u trenutku t=256
Figure 11. Slices of the WVD, the spectrogram and their product for a parabolic FM signal at time t=256

Numerički kriteriji kvalitete prikaza (apsolutna pogreška trenutne frekvencije ε_{f_i}, frekvencijski opseg na polovici maksimalne amplitude B_i, te maksimalna amplituda prve bočne latice izražena u postocima normirane glavne latice), dobiveni na temelju slika 10 i 11, dani su u tablici 3.

The numerical representation quality indicators (the IF bias ε_{f_i}, the signal half-power bandwidth B_i and the maximum amplitude of the first side-lobe expressed as a percentage of the normalized main-lobe) estimated from Figures 10 and 11 are presented in Table 3.

Tablica 3. Kriteriji kvalitete prikaza za presjek WVD, PWVD, spektrograma, umnoška WVD i PWVD te umnoška WVD i spektrograma u trenutku t=256
Table 3. Representation quality indicators for the slices of the WVD, the PWVD, the spectrogram, the product of the WVD and the PWVD, and the product of the WVD and the spectrogram at time t=256

	$\varepsilon_{f_i} = \left\| f_i(256) - \hat{f}_i(256) \right\|$	$B_i(256)$	$\|A_{\max}\|$
$WVD(256,f)$	$\|0.1-0.1035\| = 0.0035$	$\approx 0.1063 - 0.0989 = 0.0074$	72%
$PWVD(256,f)$	$\|0.1-0.1016\| = 0.0016$	$\approx 0.1065 - 0.095 = 0.0115$	8%
$WVD(256,f)\cdot PWVD(256,f)$	$\|0.1-0.1016\| = 0.0016$	$\approx 0.1056 - 0.0989 = 0.0067$	8%
$S(256,f)$	$\|0.1-0.1035\| = 0.0035$	$\approx 0.1082 - 0.0989 = 0.0093$	17%
$WVD(256,f)\cdot S(256,f)$	$\|0.1-0.1035\| = 0.0035$	$\approx 0.106 - 0.1003 = 0.0057$	17%

Iz podataka u tablici 3 vidi se da PWVD preciznije prati TF signala, dok WVD ima uži frekvencijski opseg. Glađenje PWVD, koje uzrokuje širenje frekvencijskog opsega, izglađuje interferenciju, pa njezin iznos pada sa

The results from Table 3 confirm that the PWVD describes the IF of the analyzed signal more precisely than the WVD, but it gives a wider bandwidth. The frequency smoothing, that causes the bandwidth

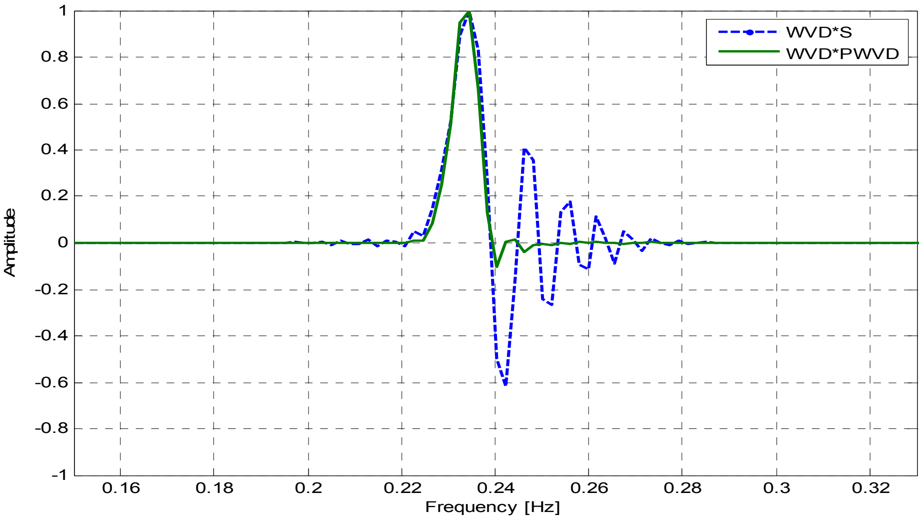
72 % vrijednosti normirane amplitude glavne latice kod WVD na 8 % kod PWVD. Distribucija dobivena umnoškom WVD i PWVD ima pogrešku TF jednaku onoj PWVD, a frekvencijski opseg uži od WVD. Amplituda prve bočne latice (koja predstavlja unutarnju interferenciju) iznosi 8 % kao i kod PWVD. Dobivena je distribucija u kojoj su objedinjeni mala pogreška u TF, uzak frekvencijski opseg i zanemariva razina interferencije.

Iz podataka o spektrogramu vidljivo je da je optimalni iznos vremenskog otvora, koji u trenutku t=256 za paraboličan FM signal ima beskonačno trajanje, uzrokovao pogrešku u TF. Ograničenost spektrograma na pozitivne vrijednosti energije ne dopušta preklapanje maksimalne amplitude prve bočne latice WVD i prolaska kroz nulu glavne latice spektrograma. Posljedica je toga slabije prigušenje interferencije u njihovu umnošku (17 % vrijednosti normirane amplitude glavne latice umnoška). Osim toga optimiranjem vremenskog otvora spektrograma (izraz 4) dobiva se spektrogram koji ima uži frekvencijski opseg u središnjem dijelu parabole u odnosu na dijelove parabole koji imaju veću promjenu TF. Zbog toga će množenje WVD spektrogramom imati veću interferenciju u dijelovima parabole s derivacijom različitom od nule. Na slici 12 prikazani su presjeci WVD maskirane PWVD-om odnosno spektrogramom u trenutku t=100 *s*.

Dakle, kad je riječ o otklanjanju interferencije iz WVD paraboličnog FM signala, bolji se rezultati postižu maskiranjem WVD s PWVD-om nego maskiranjem spektrogramom.

widening, reduces the interference intensity from 72 % of the normalized main-lobe amplitude in the WVD to 8 % in the PWVD. The distribution obtained as the product of the WVD and the PWVD gives a narrower bandwidth than the WVD and the IF bias that is equal to the one of the PWVD. The amplitude of the first side-lobe (that represents the inner-artifact) is 8 %, the same as in the PWVD. Thus, the resulting distribution has a smaller IF bias, a narrow bandwidth, and insignificant interferences.

The results for the spectrogram and the spectrogram-masked WVD show that the optimal window duration for the spectrogram, which for a PFM signal at the central time t=256 goes to infinity, causes the IF bias. The spectrogram limitation to positive energy values does not allow the overlapping of the first side-lobe of the WVD and the zero-crossing point of the main-lobe of the spectrogram. A consequence of this is a significant decrease in the interference-suppression ability of the method (the first side-lobe of their product is 17 % of the normalized main-lobe amplitude). Furthermore, the result of the window optimization is a spectrogram with a narrower bandwidth in the central part of the parabola in comparison to the parts with larger changes in the IF. This is why the product of the WVD and the spectrogram exhibits significant interference levels in the parts of the parabola whose derivative is different from zero. Figure 12 shows the time slices of the WVD masked both by the PWVD and the spectrogram at time t=100 *s*. Finally, better results in interference suppression are obtained by masking the WVD with the PWVD, than with the spectrogram.



Slika 12. Presjek WVD maskirane PWVD-om i WVD maskirane spektrogramom paraboličnog FM signala u trenutku t=100

Figure 12. Slices of the PWVD-masked WVD and of the spectrogram-masked WVD for a parabolic FM signal at time t=100

4. ZAKLJUČAK

U ovom je članku promatran efekt određenih matematičkih operacija na kvalitetu prikaza kvadratnih VFD nekoliko testnih signala.

Eksponenciranje spektrograma ima znatan utjecaj na koncentraciju energije kod jednokomponentnih signala (smanjenje frekvencijskog opsega za 2.5 puta kod LFM signala), što dovodi do poboljšanja rezolucije i u slučaju višekompnentnih signala.

Potenciranje je primjenjivo na svaku VFD koju odlikuju slabija koncentracija energije oko TF i niža razina interferencije u odnosu na amplitude komponenata signala.Primijećeno je da maskiranje WVD sa PWVD-om daje bolje uklanjanje interferencije od maskiranja spektrogramom. Niža razina interferencije u rezultantnoj distribuciji posljedica je sličnih karakteristika u distribuciji energije WVD i PWVD (oscilacije između pozitivnih i negativnih vrijednosti) i preciznog prilagođavanja PWVD (odabirom trajanja vremenskog otvora) željenom rezultatu, tj. distribuciji koja morfološki zadržava karakteristike WVD ali ne sadrži interferencijske pojave.

Maskiranje je primjenjivo na svaku VFD s dobrom koncentracijom energije oko TF, a koja istodobno sadrži visoke razine interferencije, pomoću druge VFD slabije koncentracije energije i niske razine interferencije.

4. CONCLUSION

The effects of several mathematical operations on the quadratic TFDs' representation quality have been studied in this paper on a number of nonstationary signals.

The exponentiation of the spectrogram has been shown to be a method for improving the energy concentration of LFM mono-component signals (with the bandwidth reduction of up to 2.5 times of the original bandwidth), which also ensures a significant resolution improvement in the multi-component signals case.

The exponentiation can be applied to any TFD with a low energy concentration around the IF, and lower interference amplitude values when compared to the components amplitudes. The PWVD-masked WVD has been found to have lower interference levels when compared to the spectrogram-masked WVD. This is due to the similar characteristics of the WVD and the PWVD (energy oscillations between positive and negative values) and to the fact that the PWVD can better match the characteristics of the ideal TFD, i.e. a time-frequency distribution that preserves the WVD's morphological features while suppressing the inner and outer interference terms.

A TFD characterized by a good energy concentration around the IF, and by high levels of interference, can be successfully masked by another TFD with a lower energy concentration and negligible interference levels.

5. POPIS OZNAKA

5. LIST OF SYMBOLS

vremenski signal	$s(t)$	time signal
frekvencijski signal	$S(f)$	frequency signal
vremenska varijabla	t, s	time variable
frekvencijska varijabla	f, Hz	frequency variable
vremensko-frekvencijska distribucija	$\rho(t,f)$	time-frequency distribution
energija	E, J	energy
trenutna frekvencija	$f_i(t)$	instantaneous frequency
vremenski otvor oko trenutka *t*	$w(t-\tau)$	window centered at time *t*
Fourierova transformacija na vremenskom otvoru	$F_s(t,f;w)$	Short-time Fourier transform
spektrogram	$S(t,f)$	Spectrogram
optimalni vremenski otvor	Δ	optimal time window
apsolutna pogreška TF	$\xi_{f_i(t)}$	IF bias
frekvencijski opseg	$B_i(t)$	bandwidth
vrijeme porasta	t_r	rise time
amplituda interferencije	$A_{\text{int}}(t)$	interference amplitude
analitički signal u vremenu	$z(t)$	analytic time signal
Wigner-Villova distribucija	$W_z(t,f)$	Wigner-Ville distribution
pseudo Wigner-Villova distribucija	$PW_z(t,f)$	Pseudo Wigner-Ville distribution
vremenski otvor	$h(\tau)$	time window
maksimalna amplituda bočne latice	$\|A_{\max}\|$	maximum side-lobe amplitude

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