

UDK 621.642.3:519.61:539.37

## ANALIZA GRANIČNIH STANJA POSUDE POD TLAKOM LIMIT STATE ANALYSIS OF A PRESSURE VESSEL

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**Sažetak:** Posuda pod tlakom konstruirana je za rad pri sobnim temperaturama i određenom radnom tlaku. U radu se analiziraju granična stanja takve posude po tlakom, radi potencijalne prenamjene posude. U prvome redu analizira se pojava plastičnih deformacija pri sobnoj temperaturi, a zatim i puzanje na povišenim temperaturama. Provodi se i globalna analiza čvrstoće te lokalna analiza čvrstoće.

**Ključne riječi:**

- posuda pod tlakom
- puzanje
- plastična deformacija
- metoda konačnih elemenata

**Abstract:** A pressure vessel is designed for operation at room temperature and given design pressure. The paper analyzes the limit states of such a pressure vessel for a possible change of operational conditions. In the first case, plastic deformation of the pressure vessel is analyzed, while the second case deals with creep at elevated temperatures. Both global and local stress analyses were carried out.

**Keywords:**

- pressure vessel
- creep
- plastic strain
- finite element method

### 1. UVOD

U ovome se radu analizira čvrstoća posude pod tlakom. Posuda pod tlakom projektirana je u skladu s važećim normama. To je svakodnevna situacija u industriji. Ipak, zbog različitih se razloga procesni parametri povremeno mijenjaju. U ovome slučaju analizirana su tri moguća scenarija. U prvom je smanjen radni tlak. To, naravno, ne predstavlja problem s aspekta čvrstoće posude, ali bilo bi važno doznati nova dopuštena opterećenja prirubnice u tom slučaju. Takva su opterećenja važna u projektiranju cjevovoda koji će biti priključen na posudu pod tlakom [1].

Drugi se slučaj bavi povećanim radnim tlakom u posudi. Posude pod tlakom obično se nalaze u takvim radnim uvjetima. Obično se posude pod tlakom ne koriste u tom slučaju. Ipak, budući da se u nekim ekstremnim situacijama one ipak koriste, važno je znati uvjete pri kojima će takve posude pod tlakom otkazati.

### 1. INTRODUCTION

This paper deals with the strength analysis of a pressure vessel. A pressure vessel is designed according to the current norms. This is an everyday situation in the industry. However, occasionally the process parameters change. In this particular case, three possible scenarios were analyzed. In the first, operating pressure is reduced. This, of course, does not represent a problem, but it would be interesting to know the new allowable loads on the flange in this case. Such loads are of interest in pipeline design that will be attached to the pressure vessel [1].

The second case deals with increased operational pressure. Normally, the pressure vessel would not be used in such case. Nevertheless, such situations can occur in some accidents. It is therefore of the utmost concern to know when such a pressure vessel will fail.

Treći slučaj analizira takvu posudu u novim radnim okolnostima. Radna temperatura se povećava, što dovodi do toplinskog rastezanja posude. Budući da se posuda manje-više slobodno širi, to obično ne dovodi do nekoga dodatnog napreznja. S druge strane, kod metalnih materijala javlja se puzanje prilikom dužeg izlaganja povišenim temperaturama [2]. Iako norme propisuju određene izraze za proračun čvrstoće, detaljna analiza metodom konačnih elemenata nudi mnogo bolji uvid u navedeni problem.

U skladu s tim, odabrana je karakteristična klasa posude pod tlakom, koja je projektirana i analizirana konačnim elementima pod povećanim tlakom i na povišenoj temperaturi. Opisani su dobiveni rezultati, te su izvedeni određeni zaključci.

## 2. PROJEKTIRANJE POSUDE POD TLAKOM

Odabrana posuda pod tlakom djeluje na sobnoj temperaturi, pri tlaku od 21 bar. Ispitni tlak posude iznosi 31,5 bara. Volumen posude iznosi 0,27 m<sup>3</sup>. Debljina stijenke iznosi 3,5 mm. Podnica je izvedena kao torisferična, a dodatna ojačanja podnice nisu potrebna. Materijal je nehrđajući čelik AISI 316L.

Posuda pod tlakom treba biti konstruirana u skladu s normama HRN M.E.2.251, M.E.2.252, M.E.2.253, M.E.2.254, M.E.2.256, M.E.0.021 [3].

Geometrija posude pod tlakom modelirana je u programu Catia V5R18. Dobiveni geometrijski model importiran je u FEMAP v10.01 softver i diskretiziran paraboličnim tetraedarskim elementima. Korištena su dva FEM modela: globalni i lokalni. Lokalnim modelom dobiva se bolji uvid u zonu oko prirubnice. Rubni uvjeti za lokalni model su pomaci dobiveni na odgovarajućim pozicijama globalne analize. Za proračun izrađenog modela korišten je rješavač Nastran [4].

The third case analyzes such a vessel under the new operating circumstances. Operating temperature is increased, which leads to the thermal expansion of a vessel. Since the vessel is more or less free to expand, this usually does not lead to extra stress. On the other hand, metal materials are known to creep if exposed to a prolonged load at an elevated temperature [2]. Although some closed-form expressions offer answers to the above questions, a detailed finite element analysis does offer much better insight into the problem at hand.

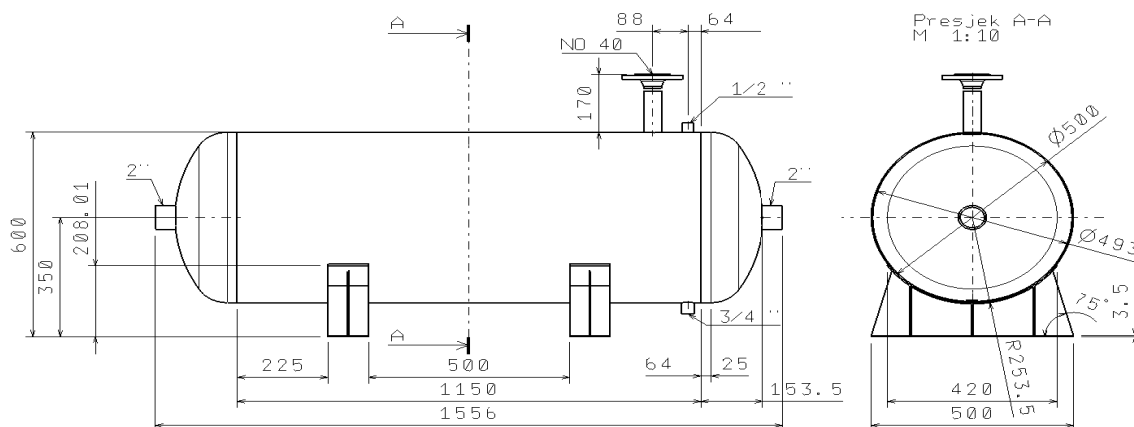
In the line with this, a typical pressure vessel is selected, designed and analyzed by the finite elements under increased pressure and increased temperature. Results are reported and some conclusions are drawn.

## 2. A DESIGN OF THE PRESSURE VESSEL

The pressure vessel at hand operates at room temperature under pressure of 21 bar. Testing pressure of the vessel is 31,5 bar. The volume is 0,27 m<sup>3</sup>. Wall thickness is 3,5 mm. The head wall has a torispherical shape and no additional reinforcements of the head wall are necessary. The material used is AISI 316L stainless steel.

The pressure vessel should be designed according to the norms HRN M.E.2.251, M.E.2.252, M.E.2.253, M.E.2.254, M.E.2.256, M.E.0.021 [3].

The geometry of the pressure vessel was modeled in Catia. The obtained solid geometry was imported into FEMAP software and discretized by the parabolic tetrahedral solid elements. Two FEM models were used: a global one and a local one. The local one was used to gain better insight into the zone around the flange. Boundary conditions for the local model were displacements obtained at relevant positions by global analysis. Upon creation of the model, the Nastran solver was used for calculations [4].



Slika 1. Dimenzije posude pod tlakom, mm  
Figure 1. Dimensions of the pressure vessel, mm

Tabela 1. Svojstva materijala posude pod tlakom pri 20 °C  
 Table 1. Pressure vessel material properties at 20 °C

Materijal / Material:	Nehrđajući čelik/Stainless steel, AISI 316L, kod/at 20 °C
Modul elastičnosti, $E$ : Young's modulus, $E$ :	193 GPa
Poissonov faktor, $\nu$ : Poisson's ratio, $\nu$ :	0,3
Naprezanje na granici tečenja, $\sigma_{0,2}$ : Yield stress, $\sigma_{0,2}$ :	468 MPa
Vlačna čvrstoća, $\sigma_M$ : Tensile strength, $\sigma_M$ :	600 MPa
Dopuštena naprezanja/ Allowable stresses	425 MPa

### 3. DOPUŠTENA OPTEREĆENJA PRIRUBNICE PRI NOVOM RADNOM TLAKU

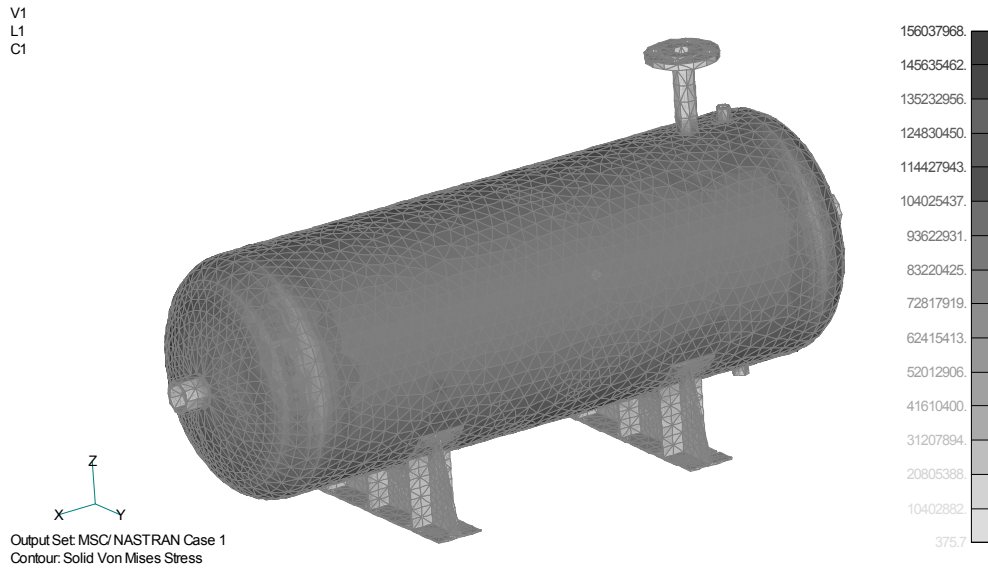
Kao što je objašnjeno u uvodu, posuda pod tlakom koristit će se pri smanjenom radnom tlaku od 11 bara. Zadatak je izračunati nova dopuštena opterećenja prirubnice. Prva analiza bila je globalna, odnosno posuda pod tlakom analizirana je kao kompletna struktura. U drugom su koraku prirubnica i dio posude izolirani te je provedena lokalna analiza naprezanja. Za rubne uvjete lokalne analize ekstrapolira se polje pomaka dobiveno iz globalne analize na rub lokalnog modela. Dobivena raspodjela ekvivalentnih naprezanja prema kriteriju von Misesa prikazana je na sl. 2 i 3. Maksimalno naprezanje dobiveno iz globalne analize iznosi 156 MPa, uz maksimalni pomak 0,3 mm. Raspodjela naprezanja toga tipa bila je očekivana. Npr. iz lokalne analize naprezanja proizlazi da se ekstremne vrijednosti naprezanja u prirubnici nalaze u zoni zavara. Raspodjela tamnih područja prema sl. 3 pokazuje da gušća mreža može dati još bolje rezultate.

Opterećenja na prirubnici unesena su kao jedinična opterećenja u svakom smjeru. Opterećenja su uvedena u dodatnom čvoru smještenom na osi prirubnice. Pomaci ostalih čvorova u toj ravnini funkcijski su povezani sa središnjim čvorom te se na taj način opterećenje prenosi u ostale čvorove. Dobivena naprezanja uspoređena su s dopuštenim naprezanjima, da bi se kasnije izračunala dopuštena opterećenja prirubnice (Table 2).

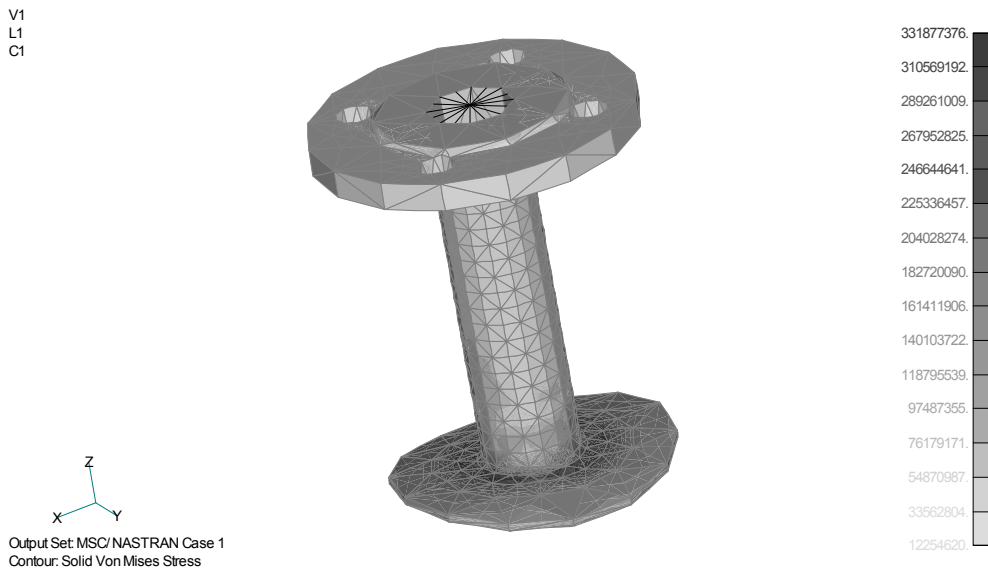
### 3. ALLOWABLE FLANGE LOADS AT NEW OPERATIONAL PRESSURE

As discussed in the introduction, the pressure vessel is to be used with the decreased operational pressure, 11 bar. The task is to calculate new allowable loads on the flange. The first analysis was global, i.e. the pressure vessel was analyzed as a complete structure. In the second step, a flange and a part of vessel were isolated and local stress analysis was performed. As boundary conditions for local analysis, the displacement field obtained from global analysis is extrapolated into the local model. Obtained von Mises stresses are presented in Figures 2 and 3. Maximal stress obtained by the global analysis was 156 MPa, while maximal displacement was 0,3 mm. Stress distribution of this type was expected. For example, local stress analysis pinpointed stress extremes in the flange welding zone. The distribution of the dark areas, Figures 3, indicates that further mesh refinement can give even better results.

The loads at the flange were taken as unit loads in each direction. The loads were introduced into the node positioned at the center of flange cross-section. Displacements of other nodes in that cross-section were constrained to that node, which lead to the distribution of loads to other nodes in the cross-section. Obtained stresses were compared to allowable stresses in order to calculate allowable loads on the flange, Table 2.



Slika 2. Von Misesova naprezanja (Pa) pri radnom tlaku od 11 bara  
Figure 2. Von Mises stress (Pa) at operating pressure 11 bar



Slika 3. Lokalna analiza naprezanja (Pa) – von Misesova naprezanja  
Figure 3. Local stress analysis (Pa) – von Mises stresses

Tabela 2. Dopuštena opterećenja prirubnice  
Table 2. Allowable loading of the flange

Dopuštena opterećenje prirubnice Allowable flange loads	Sila/Force, N			Moment/Moment, N·m		
	$F_x$	$F_y$	$F_z$	$M_x$	$M_y$	$M_z$
Maksimalna vrijednost Maximal value	1140	950	9500	170	190	1045
Minimalna vrijednost Minimal value	-950	-950	-22800	-190	-190	-950

#### 4. ANALIZA PLASTIČNOSTI

Nadalje treba analizirati određivanje tlaka koji dovodi do plastifikacije kroz čitavu debljinu stijenke. Stoga, za određivanje odgovarajućeg tlaka potrebno je povećati tlak u koracima, te tako odrediti kritičnu vrijednost [5, 6].

Za određivanje plastičnog ponašanja AISI 316L čelika izvedeno je jednoosno ispitivanje, te je uvedena u softver odgovarajuća krivulja naprezanje-deformacija, u linearnim dijelovima prema sl. 4.

Prema upravo navedenim smjernicama izvršena je analiza. Kao početni tlak plastifikacije dobiva se  $p = 45$  bar. Maksimalne vrijednosti naprezanja prema kriteriju von Misesa, raspodjela kojih je prikazana na sl. 5, pojavljuju se u podnici posude.

Od tog trenutka nadalje tlak je bio dodatno povećavan. Na kraju je stijenka posude pretrpjela plastične deformacije po cijeloj debljini stijenke (sl. 6). Takva situacija donosi i određene numeričke probleme. Problem je u tome što dio strukture ima vrlo malu krutost i stoga ne može izdržati opterećenje te se naglo širi. Granični tlak posude koji odgovara takvoj pojavi iznosi  $p = 77$  bar.

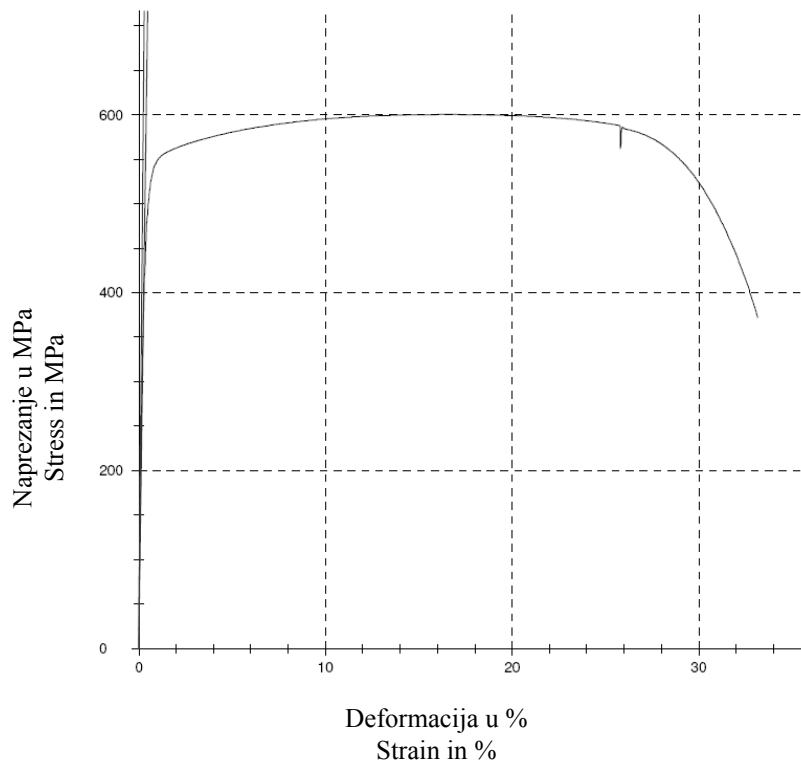
#### 4. PLASTICITY ANALYSIS

The next case to be analyzed was the determination of the pressure that will lead toward development of complete plastification throughout the wall thickness. Therefore, to obtain corresponding pressure, pressure was increased in a stepwise manner until the critical value was found [5, 6].

To describe the plastic behaviour of AISI 316L, a uniaxial test was performed and a corresponding stress-strain curve was introduced into the software as a linear piecewise curve, Figure 4.

According to the above guidelines, the analysis was performed. As an initial pressure of plastification,  $p = 45$  bar is obtained. Maximal von Mises stress values occur in the pressure vessel head and the distribution is given in Figure 5.

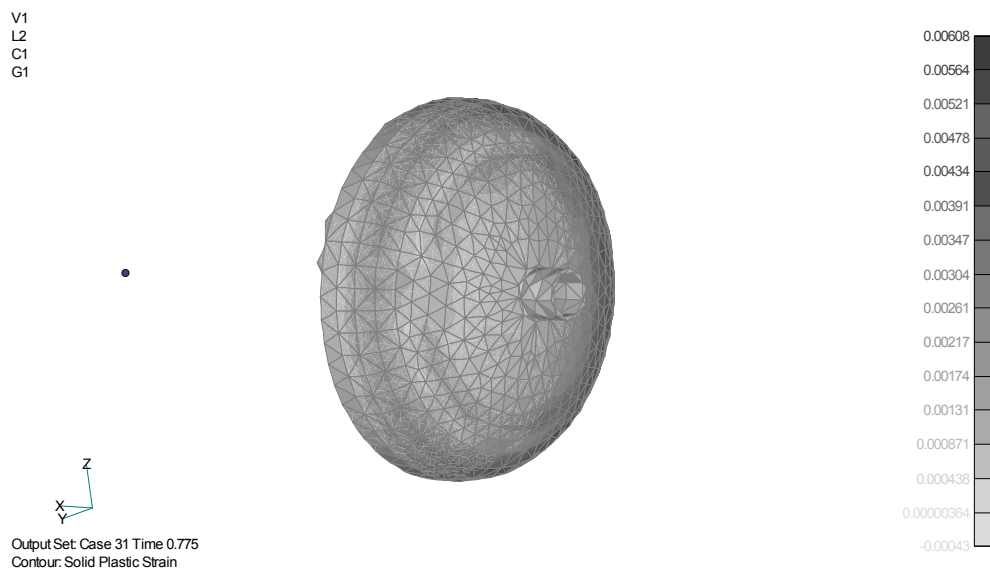
From this point onwards, pressure was further increased. At the end, the vessel wall has suffered plastic deformation throughout wall thickness (Figure 6) and calculations exhibit numerical difficulties. The problem lies in the fact that part of structure has very low stiffness and in that way cannot withstand the load and expands rapidly. The limit pressure corresponding to such behavior is  $p = 77$  bar.



Slika 4. Dijagram naprežanje-deformacija za AISI 316L čelik, pri sobnoj temperaturi  
 Figure 4. Stress-strain curve of AISI 316L steel, room temperature



Slika 5. Von Misesova naprežanja (Pa) u podnici pri 45 bara  
 Figure 5. Von Mises stress (Pa) in pressure vessel head at 45 bar



Slika 6. Ekvivalentna plastična deformacija pri 77,5 bara  
Figure 6. Plastic strain at 77,5 bar

## 5. ANALIZA PUZANJA

Svrha je ove analize odrediti koliko je vremena dopušteno posudu pod tlakom izlagati povišenoj temperaturi. Odabrana temperatura je 600 °C. Parametri materijala određeni su eksperimentalno, istim načinom kao i pri analizi plastičnosti. Primjerak je bio zagrijavan u peći, opterećen i istodobno je mjerena deformacija ekstenzometrom za visoke temperature. Izmjerena krivulja naprezanje-deformacija prikazana je na sl. 7, dok su odgovarajuća svojstva materijala navedena u tab. 3. U proračunu je korišten zakon deformacije u fazi puzanja u obliku:

$$\varepsilon_C = 765^{-8,2} \sigma^{8,2} t . \quad (1)$$

U ovoj jednadžbi treba naprezanja uvrstiti u MPa, dok je vrijeme u satima. Budući da se za radni tlak od 21 bar očekuju veće vrijednosti deformacije u fazi puzanja, analiza je izvršena pri tlaku od 11 bara.

Proračuni puzanja provedeni su za prvih 400 h. Odgovarajuća raspodjela naprezanja prema kriteriju von Misesa prikazana je na sl. 8. Raspodjela naprezanja slična je prethodnim analizama. Deformacija u fazi puzanja lokalizirana je u zonama maksimalnog naprezanja, tj. u podnici posude posebno, sl. 9. Međutim, dijagram pomaka čvora u kritičnoj zoni, sl. 10, pokazuje da su pomaci u fazi puzanja prilično mali. Također, vidljivo je da nakon 400 sati puzanja nastupa stacionarno puzanje. To znači da se deformacija puzanja u nekoj točki nakon proizvoljne količine vremena može lako aproksimirati. Npr. ako je dopušteno 1% deformacije u fazi puzanja, ta će vrijednost biti postignuta nakon:

## 5. CREEP ANALYSIS

The purpose of this analysis is to determine the allowable exposure time of the pressure vessel to an elevated temperature. The selected temperature is 600 °C. Material parameters were determined experimentally in the manner as for plastic analysis. The specimen was tested in a heat furnace while deformation was measured by the high-temperature extensometer. The measured stress-strain curve is given in Figure 7, while corresponding material properties are given in Table 3. Creep law of the form:

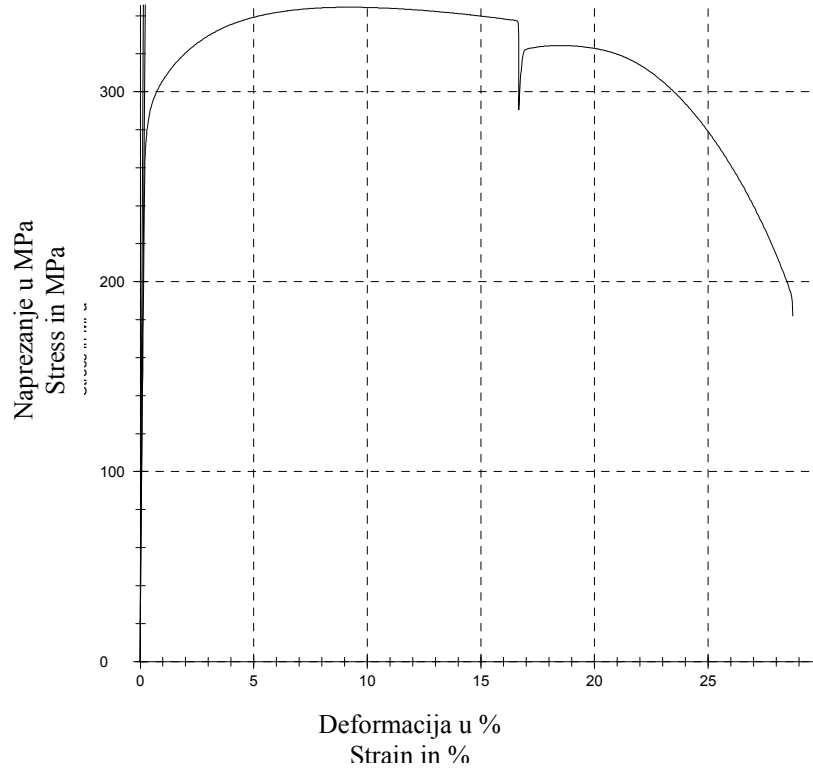
was employed. In this equation stresses should be used in MPa, while time is in hours. Since an operation pressure of 21 bar is expected to result in higher levels of creep strain, analysis was performed for the pressure of 11 bar.

Creep calculations were performed for the first 400 h. The corresponding von Mises distribution is given in Fig. 8. Stress distribution is similar to the previous analyses. Creep strain is localized to the regions of highest stresses, i.e. heads in particular, Figure 9. However, the displacement diagram of a node from the critical region, Figure 10, shows that creep strains are of a rather low level. Also, it is visible that after 400 h of creep, a state of steady creep is entered. This means that creep after an arbitrarily long time can be easily approximated. For example, if a 1% creep strain is allowed, this level will be reached after:

$$t = 765^{8,2} \sigma^{-8,2} \varepsilon_{C,dop} = 356350h \quad (2)$$

Dakle, struktura je poprilično sigurna protiv puzanja pri povišenim temperaturama.

Therefore, the structure is rather safe against exposure to elevated temperatures.



Slika 7. Dijagram naprezanje-deformacija za AISI 316L čelik, temperatura 600 °C  
Figure 7. Stress-strain curve of AISI 316L steel, temperature 600 °C



Slika 8. Von Misesova naprezanja (Pa), 400 h puzanja  
Figure 8. Von Mises stresses (Pa), 400 h of creep

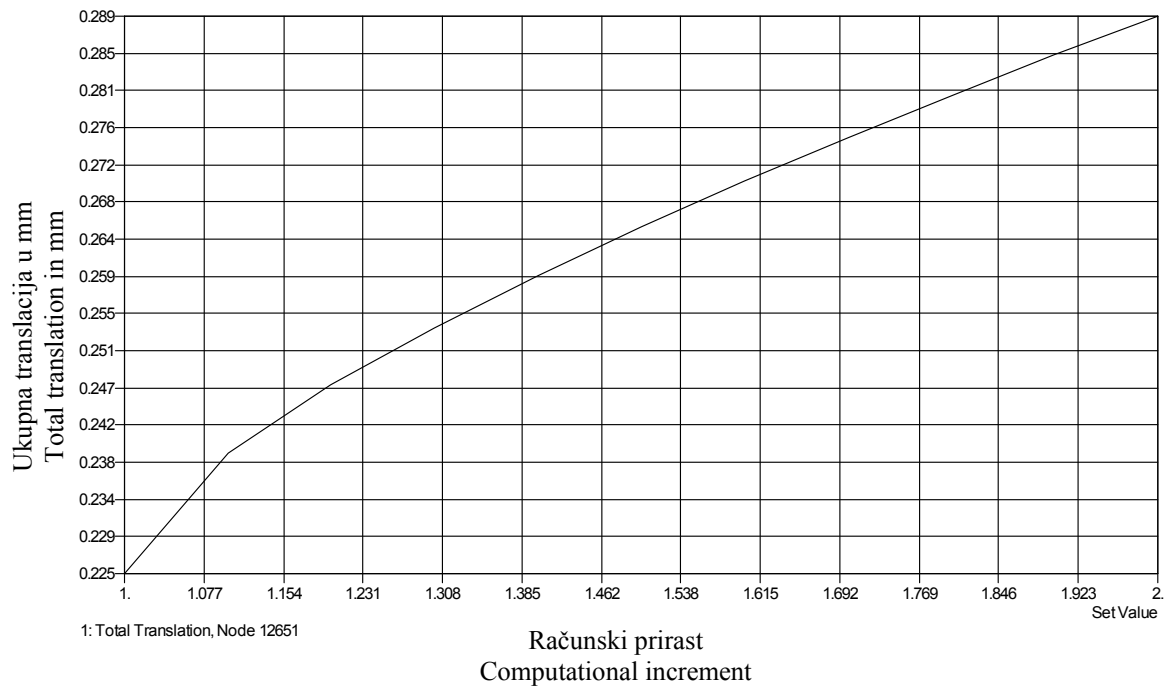




Slika 9. Von Misesova naprezanja (Pa), 400 h puzanja.  
 Figure 9. Von Mises stress (Pa), 400 h of creep

Tabela 3. Svojstva materijala posude pod tlakom pri 600 °C  
 Table 3. Pressure vessel material properties at 600 °C

Materijal / Material:	Nehrđajući čelik/Stainless steel, AISI 316L, kod/at 600 °C
Modul elastičnosti, $E$ : Young's modulus, $E$ :	130 GPa
Poissonov faktor, $\nu$ : Poisson's ratio, $\nu$ :	0,3
Naprezanje na granici tečenja, $\sigma_{0,2}$ : Yield stress, $\sigma_{0,2}$ :	288 MPa
Vlačna čvrstoća, $\sigma_M$ : Tensile strength, $\sigma_M$ :	344 MPa



Slika 10. Pomaci – puzanje u kritičnoj zoni podnice 0-400 h

Figure 10. Displacement – creep in critical zone of pressure vessel head, 0-400 h

## 5. ZAKLJUČAK

Izvršena je analiza graničnih stanja posude pod tlakom. Rad predstavlja pristup koji se može koristiti za određivanje kritične vrijednosti tlaka koji dovodi do otkazivanja posude. Uz to predstavljen je i postupak analize puzanja. Predstavljene postupci mogu se lako koristiti pri analizi raznih drugih konstrukcija posude pod tlakom, ili prilikom drugih radnih parametara.

## 6. POPIS OZNAKA

radni/ispitni tlak	$p$ , bar
modul elastičnosti	$E$ , GPa
Poissonov faktor	$\nu$ , -
naprezanje na granici tečenja	$\sigma_{0.2}$ , MPa
vlačna čvrstoća	$\sigma_m$ , MPa
dopuštena sila, po osi $x$	$F_x$ , N
dopuštena sila, po osi $y$	$F_y$ , N
dopuštena sila, po osi $z$	$F_z$ , N
dopušteni moment, oko osi $x$	$M_x$ , N·m
dopušteni moment, oko osi $y$	$M_y$ , N·m
dopušteni moment, oko osi $z$	$M_z$ , N·m
deformacija u fazi puzanja	$\varepsilon_C$ , %
naprezanje	$\sigma$ , MPa
vrijeme trajanja ispitivanja puzanja	$t$ , h
dopuštena deformacija u fazi puzanja	$\varepsilon_{C, dop}$ , %

## 5. CONCLUSIONS

A pressure vessel operating within in limit states was analyzed. The paper presents an approach that can be used to determine the critical value of pressure leading to vessel failure. Furthermore, a procedure for creep analysis was presented. Presented procedures can be easily adopted for the analysis of different pressure vessel geometries or operating parameters.

## 6. LIST OF SYMBOLS

operating/testing pressure
Young's modulus
Poisson's ratio
yield stress
tensile strength
allowable force on the flange, along $x$ axis
allowable force on the flange, along $y$ axis
allowable force on the flange, along $z$ axis
allowable moment, around $x$ axis
allowable moment, around $y$ axis
allowable moment, around $z$ axis
creep strain
stress
time duration of creep testing
allowable creep strain

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Primljeno / Received: 10.02.2009.

Prihvaćeno / Accepted: 10.07.2009.

Pregledni članak

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