

Ognjen KULJAČA<sup>1</sup>  
 Jyotirmay GADE-  
 WADIKAR<sup>1</sup>  
 Krunoslav HORVAT<sup>2</sup>

# Multilayer Neural Net Trajectory Tracking Control for Underwater Vehicle

Original scientific paper

An adaptive multilayer neural network controller for high precision maneuvering of underwater vehicles is presented. Maneuvering of underwater vehicles requires special attention to a number of factors, including thruster and vehicle's nonlinearities, couplings which exist between various degrees of freedom as well as effects of the sea currents. The neuro control system for underwater vehicle maneuvering described in this paper is based on a conventional controller supported with the so-called adaptive neural network. The adaptive neural network has two tunable layers, thus the problem of selection of proper basis is avoided.

**Keywords:** *adaptive neural network, precise maneuvering, trajectory tracking, underwater vehicle*

## Višeslojna neuronska mreža za praćenje trajektorije za ronilice

Izvorni znanstveni rad

Predstavljen je adaptivni višeslojni neuro regulator za precizno manevriranje ronilicom. Maneviranje ronilicom zahtijeva da se obrati posebna pažnja na nelinearnost procesa, međusobni utjecaj između različitih stupnjeva slobode gibanja te utjecaj morskih struja. Neuro regulacijski sustav za manevriranje ronilicom opisan u ovom radu baziran je na konvencionalnom regulatoru nadograđenim s tzv. adaptivnom neuronskom mrežom. Adaptivna mreža sastoji se od dva sloja, tako da je problem oko izbora parametara prvog sloja izbjegnuto.

**Ključne riječi:** *adaptivna neuronska mreža, održavanje trajektorije, podvodno vozilo, precizno manevriranje*

### Authors' addresses (adrese autora):

<sup>1</sup> *Alcorn State University, Systems Research Institute, School of Agriculture and Applied Sciences, Department of Advanced Technology, Simmons Technology Building, Room 113, 1000 ASU Drive 360, Alcorn State, MS 39096-7500*  
 E-mail: okuljaca@alcorn.edu; jyo@alcorn.edu

<sup>2</sup> *Brodarski Institute, Control Systems Department, Ave. V. Holjevca 20, Zagreb, Croatia*  
 E-mail: krunoslav.horvat@hrbi.hr

**Received (Primljeno):** 2009-05-11

**Accepted (Prihvaćeno):** 2009-10-05

**Open for discussion (Otvoreno za raspravu):** 2010-12-31

## 1 Introduction

The use of underwater vehicles in the inspection of the sea areas requires a high precision track-keeping along the specified route.

Consequently, the underwater vehicle is required to have good maneuvering capabilities and the control system is characterized by use of adequate position sensors and generally by the adoption of advanced control approaches.

Rigorous adaptive control results are available. The model reference approach given in [1] allows simultaneous proofs of tracking error and parameter error stability. However, all conventional adaptive control techniques assume that the unknown dynamic forces and moments are expressed as a linear function of unknown parameters and a regression matrix specific to the AUV (Autonomous Underwater Vehicle) has to be computed [1], [7]. That may not be always possible and the computation of regression matrix is usually complicated and time consuming. The possible solution might be found in using neural networks for control.

The application of neural networks to the navigation and control of underwater vehicles using backpropagation algorithm and its variants can be found in [11], [12]. However, the backpropagation algorithm is proven to have convergence and stability problems [7].

The goal of this paper is to present the main aspects of the design of a controller for precise maneuvering based on the neural network, which is embedded in a trajectory tracking control of the underwater vehicle. One of the possible approaches to the use of neural network in underwater vehicle tracking control is given in [10]. This paper demonstrates the feasibility of another approach, proposed for control of robot manipulators in [2], [7]. Control design proposed in this paper does not depend on linear dependency in parameters. No regression matrix has to be found. In contrast to [13], this paper deals with multilayer control where all the layers of neural network are tunable.

The organization of the paper is as follows. In Section 2 the basic formulation of the problem is given. Some mathematical preliminaries are given in Section 3. The neural network control algorithm is described in Section 4. The illustrative example is given in Section 5. Finally, the conclusions are given in Section 6.

## 2 Problem formulation

One of the problems in underwater vehicle's precise maneuvering is the problem of guiding the vehicle along the prescribed trajectory.

The mathematical model of the underwater vehicle in 6DOF is given by the following well known matrix expression ([1]):

$$M\dot{v} + C(v)v + D(v)v + g(\eta) + \tau_d = \tau_v \tag{1}$$

$$\dot{\eta} = J^T v$$

where:

- $M$  - system inertial matrix (incl. added mass),
- $C(v)$  - Coriolis and centripetal matrix (incl. added mass),
- $D(v)$  - matrix of hydrodynamic damping,
- $g(\eta)$  - vector of restoring forces and moments,
- $\tau_v$  - vector of control inputs,
- $\tau_d$  - vector of disturbances,
- $v = [u \ v \ w \ p \ q \ r]^T$  - vector of linear and angular velocities,
- $\eta = [x \ y \ z \ \phi \ \theta \ \psi]^T$  - position and Euler angles vector
- $J$  - velocity transformation matrix

There are several problems related to the low speed maneuvering of the underwater vehicle. We will mention the following:

- thrusters are highly nonlinear subsystems and have a significant influence on the control system dynamics,
- the hydrodynamic couplings between thrusters can be very strong,
- the effect of the sea current is very important, especially when the speed of the sea current is comparable to the vehicle speed,
- the vehicle mathematical model is complex and usually difficult to estimate (with existing nonlinearities and couplings between particular degrees of freedom).

The conventional approach adopted for the dynamic positioning controller design is the LQG control and PID control. The maneuvering performance is influenced by control of vehicle's linear and angular velocities. The problem can be resolved by use of the neural networks together with the conventional LQG or PD controller [10].

### 3 Mathematical preliminaries

Let  $R$  denote a set of real numbers,  $R^n$  a space of real n-vectors and  $R^{m \times n}$  a space of real  $m \times n$  matrices. Let  $S$  be a compact simply connected set of  $R^n$ . With map  $f : S \rightarrow R^m$ . Let us define  $C^m(S)$  the space such that  $f$  is continuous. Let  $\|\bullet\|$  be any suitable vector norm. The supremum norm of  $f(x)$  over  $S$  is defined as:

$$\sup \|f(x)\|, f : S \rightarrow R^m, \forall x \in S \tag{2}$$

Given  $A = [a_{ij}]$ ,  $B \in R^{m \times n}$  the Frobenius norm is defined by:

$$\|A\|_F^2 = \text{tr}(A^T A) = \sum_{i,j} a_{ij}^2 \tag{3}$$

#### 3.1 Neural network

Given  $x \in R^n$ , a two-layer NN (Figure 1) has a net output given by

$$y = W^T \sigma(V^T x) \tag{4}$$

where  $x = [1 \ x_1 \ \dots \ x_{N_1}]^T$ ,  $y = [y_1 \ y_2 \ \dots \ y_{N_3}]^T$  and  $\sigma(\bullet)$  the activation function. If  $z = [z_1 \ z_2 \ \dots \ z_{N_2}]^T$ , we define  $\sigma(z) = [\sigma(z_1) \ \sigma(z_2) \ \dots]^T$ . Including "1" as the first term in  $\sigma(V^T x)$  allows one to incorporate the thresholds as the first column of  $W^T$ . Then any tuning of NN weights includes tuning of thresholds as well [7].

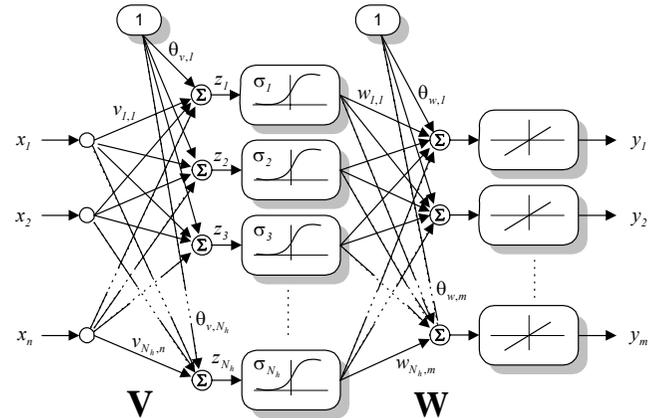


Figure 1 Neural network structure  
Slika 1 Struktura neuronske mreže

A general function  $f(x) \in C^m(S)$ ,  $x(t) \in R^n$  can be written as

$$f(x) = W^T \hat{\sigma}(V^T x) + \hat{\epsilon}(x) \tag{5}$$

with  $\epsilon(x)$  a NN functional reconstruction error.

#### 3.2 Underwater vehicle model and its properties

Underwater vehicle model in earth-fixed vector representation ([1]) is expressed as:

$$M_\eta(\eta)\dot{\eta} + C_\eta(v, \eta)\dot{\eta} + D_\eta(v, \eta)\dot{\eta} + g_\eta(\eta) + \tau_{d\eta} = \tau_\eta \tag{6}$$

Dynamics given in (6) is similar to the standard robot dynamics form and has the following properties ([1]) for any practical purposes:

Property 1: The inertia matrix  $M$  is symmetric and positive definite matrix bounded by

$$m_1 I \leq M_\eta(\eta) \leq m_2 I$$

with  $m_1, m_2$  positive known constants.

Property 2: The matrix  $\dot{M}_\eta - 2C_\eta$  is skew-symmetric

Property 3: The matrix  $D_\eta$  is real and positive definite matrix that satisfies the following inequality:

$$\sigma_{\min}(D_\eta) \|\eta\|^2 \leq \eta^T D_\eta \eta \leq \sigma_{\max}(D_\eta) \|\eta\|^2$$

with  $\sigma(D_\eta)$  as singular value of  $D_\eta$

Property 4: The matrix  $C_\eta(v, \eta)$  is bounded

Property 5: The unknown disturbance satisfies  $\|\tau_{d\eta}\| < b_d$ , with  $b_d$  known positive constant.

Given the desired trajectory  $\eta_d$  one can define the tracking error:

$$e(t) = \eta_d(t) - \eta(t) \tag{7}$$

Define filtered tracking error:

$$r = \dot{e} + Ie \tag{8}$$

where  $I = I^T > 0$  and (8) is stable system. The dynamics (6) can be now written in terms of filtered tracking error

$$M_\eta(\eta)\dot{r} = -C_\eta(v, \eta)r - D_\eta(v, \eta)r + f + \tau_{d\eta} - \tau_\eta \tag{9}$$

where nonlinear function  $f$  is:

$$f(x) = M_\eta(\eta)(\dot{\eta}_d + \Lambda e) + C_\eta(v, \eta)(\dot{\eta}_d + \Lambda e) + D_\eta(v, \eta)(\dot{\eta}_d + \Lambda e) + g_\eta(\eta) \tag{10}$$

### 4 Two-layer NN controller

The design of the controller is performed following the procedure given in [2] and [7]. A modified tuning algorithm from [2] and [7] is used to make NN robust, so no persistency of excitation is needed.

Assume that there exists constant ideal weight matrix  $W$  so that the function  $f$  in (10) can be written as:

$$f(x) = W^T \sigma(V^T x) + \varepsilon(x) \tag{11}$$

Assume that net reconstruction error satisfies  $\|\varepsilon(x)\| < \varepsilon_N(x)$  with  $\varepsilon_N(x)$  known positive constant.

#### 4.1 Controller structure

Define the NN functional estimate by

$$\hat{f}(x) = \hat{W}^T \sigma(\hat{V}^T x) \tag{12}$$

with  $\hat{W}$  and  $\hat{V}$  the current values of the NN weights. Weight estimation errors can be defined as

$$\tilde{W} = W - \hat{W}, \tilde{V} = V - \hat{V} \tag{13}$$

It is assumed that ideal weights are bounded by known values so that

$$\|Z\|_F < Z_B, Z = \begin{bmatrix} V & 0 \\ 0 & W \end{bmatrix} \tag{14}$$

Select control input as

$$\tau_\eta = \hat{W}^T \sigma(\hat{V}^T x) + K_v r - v \tag{15}$$

$$v = -K_z (\|\hat{Z}\|_F + Z_B) r \tag{16}$$

with  $K_v$  positive definite design matrix and  $K_z$  explained later in the proof. Then, the closed-loop filtered error dynamics in (6) becomes

$$M_\eta(\eta)\dot{r} = -[K_v + C_\eta(v, \eta)]r - D_\eta(v, \eta)r + W^T \sigma(V^T x) - \hat{W}^T \sigma(\hat{V}^T x) + \varepsilon + \tau_{d\eta} + v \tag{17}$$

The proposed control structure is shown in Figure 2.

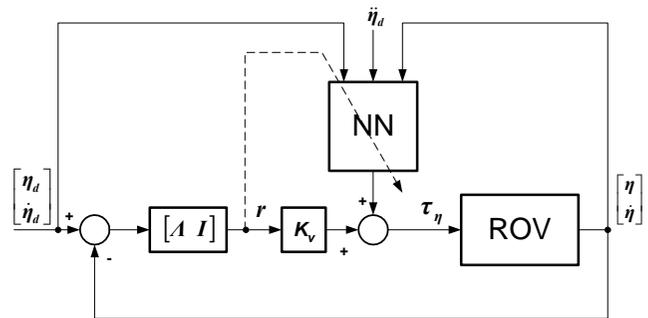


Figure 2 Tracking controller structure  
Slika 2 Struktura regulatora

Now, let us expand  $\sigma(V^T x)$  into a Taylor series around  $V^T x$  ([7]):

$$\sigma(V^T x) = \sigma(\hat{V}^T x) + \sigma'(\hat{V}^T x)\tilde{V}^T x + O(\tilde{V}^T x)^2 \tag{18}$$

where  $O(\tilde{V}^T x)^2$  are the higher order terms in Taylor series.

It is shown in [7] that the higher order Taylor series terms are bounded by:

$$\|O(\tilde{V}^T x)\| \leq c_1 + c_2 q_B \|\tilde{V}\| + c_3 \|\tilde{V}\|_F \|r\| \tag{19}$$

Let the weight tuning laws be

$$\dot{\hat{V}} = Gx(\hat{\sigma}^T \hat{W}r)^T - kG\|r\|\hat{V} \tag{20}$$

$$\dot{\hat{W}} = F\hat{\sigma}r^T W - F\hat{\sigma}'\hat{V}^T xr^T - kF\|r\|\hat{W} \tag{21}$$

with  $F$  and  $G$  any symmetric and positive definite matrices and  $k$  positive design parameter. Then the filtered tracking error and weight estimates are uniformly ultimately bounded.

*Proof:*

Define Lyapunov candidate:

$$L = \frac{1}{2} r^T M_\eta(\eta)r + \frac{1}{2} \text{tr}(\tilde{W}^T F^{-1}\tilde{W}) + \frac{1}{2} \text{tr}(\tilde{V}^T G^{-1}\tilde{V}) \tag{22}$$

After differentiating and using the dynamics (9), (10), and (17), control (15) and (16) and tuning laws (20) and (21) we obtain:

$$\dot{L} = -r^T K_v r - r^T D_\eta r + k \|r\| \text{tr}(\tilde{Z}^T (Z - \tilde{Z})) + r^T (w + v) \tag{23}$$

where:

$$w = \tilde{W}^T \hat{\sigma}^T V^T x + W^T O(\tilde{V}^T x)^2 + \varepsilon + \tau_d \tag{24}$$

and

$$\|w\| \leq C_0 + C_1 \|\tilde{Z}\|_F + C_2 \|\tilde{Z}\|_F \|r\| \tag{25}$$

where  $C_i$  are known positive constants.

With  $K_z > C_2$  we then have:

$$\dot{L} \leq -\|r\| \{ (K_{v\min} + \sigma_{\min}(D_\eta)) \|r\| - k \|\tilde{Z}\|_F (Z_B - \|\tilde{Z}\|_F) - C_0 - C_0 \|\tilde{Z}\|_F \} \tag{26}$$

where  $K_{v\min} + \sigma_{\min}(D_\eta)$  are minimum singular values of matrices  $K_v$  and  $D_\eta$ . Define  $K_{vd} = K_{v\min} + \sigma_{\min}(D_\eta)$  and  $C_3 = Z_B + C_1 k^{-1}$ . Then, after completing the squares we have:

$$\dot{L} \leq -\|r\| \left\{ K_{vd} \|r\| + k \left( \|\tilde{Z}\|_F - \frac{C_3}{2} \right)^2 - C_0 - k \left( \frac{C_3}{2} \right)^2 \right\} \tag{27}$$

Inequality (27) is always negative as long as the term in parentheses is positive, which is guaranteed as long as:

$$\|r\| > \frac{C_0 + k \left( \frac{C_3}{2} \right)^2}{K_{vd}} \tag{28}$$

$$\|\tilde{Z}\|_F > \frac{C_3}{2} + \sqrt{\frac{C_0}{k} + \left( \frac{C_3}{2} \right)^2} \tag{29}$$

Thus, Lyapunov function derivative is negative outside a compact set. According to the LaSalle extension, this demonstrates that both  $r$  and  $Z$  are ultimately unconditionally bounded.

### 5 Simulation example

The controller design procedure described above is illustrated with the mathematical model simulation example. Vehicle model used for the simulation example is the NPS Autonomous Underwater Vehicle (model controlled by forces and moments) as it is given in [9]. The desired trajectory is filtered squared signal with different frequency and amplitude for each particular DOF. The neural network input was:

$$\mathbf{x} = [\dot{\eta}_d^T \quad \ddot{\eta}_d^T \quad \eta_d^T \quad \dot{\eta}^T \quad \eta^T]^T \tag{30}$$

Simulation was performed with the following neural network parameters:  $N_w = 60$ ,  $F = \text{diag}(545)$ ,  $G = \text{diag}(73)$ ,  $k = 0.0005$ . The following gain matrix was used:

$$K_v = \text{diag}([220 \quad 220 \quad 220 \quad 220 \quad 220 \quad 220])$$

and the filtered error parameter  $\Lambda$  was  $\Lambda = \text{diag}(5)$ .

The reference trajectories and angles are given in Figures 3 to 8. Errors are shown in Figures. 9 to 14.

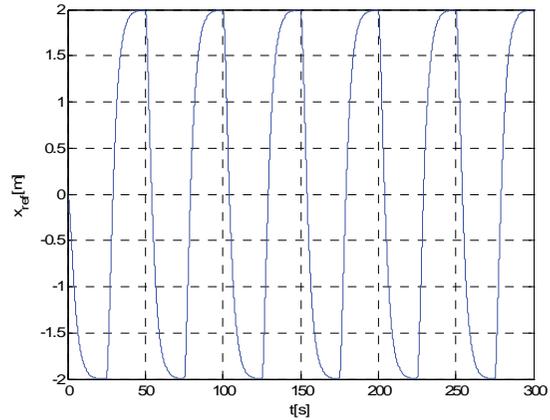


Figure 3 Reference x  
Slika 3 Referenca x

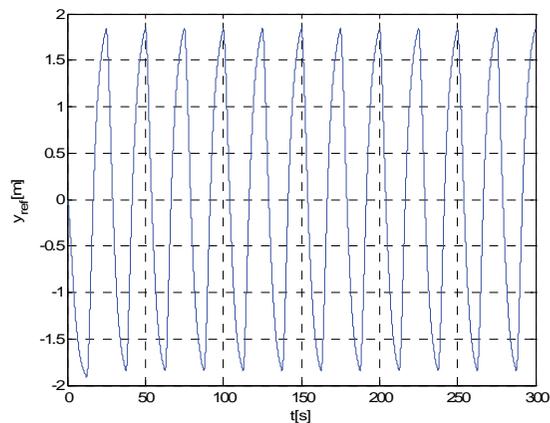
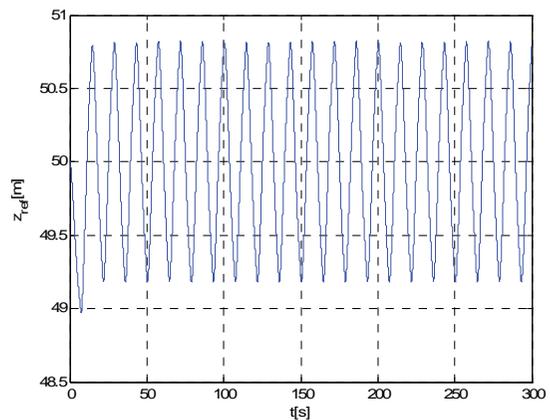


Figure 4 Reference y  
Slika 4 Referenca y

Figure 5 Reference z  
Slika 5 Referenca z



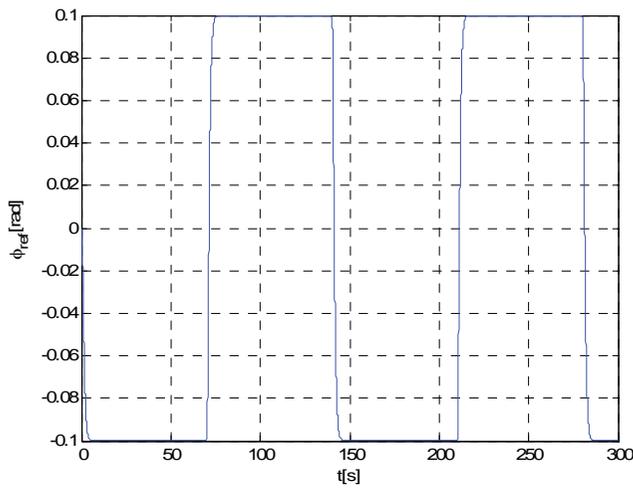


Figure 6 **Reference  $\phi$**   
Slika 6 **Referenca  $\phi$**

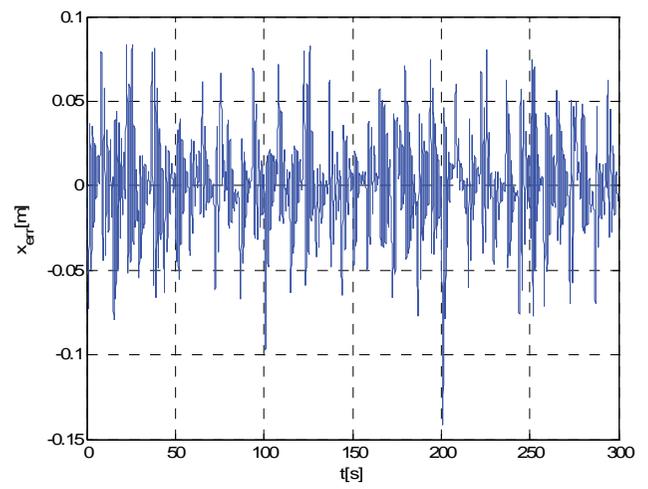


Figure 9 **Error  $x$**   
Slika 9 **Greška  $x$**

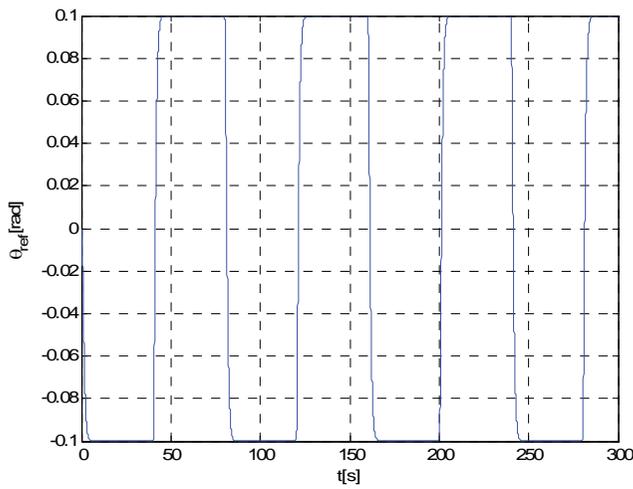


Figure 7 **Reference  $\theta$**   
Slika 7 **Referenca  $\theta$**

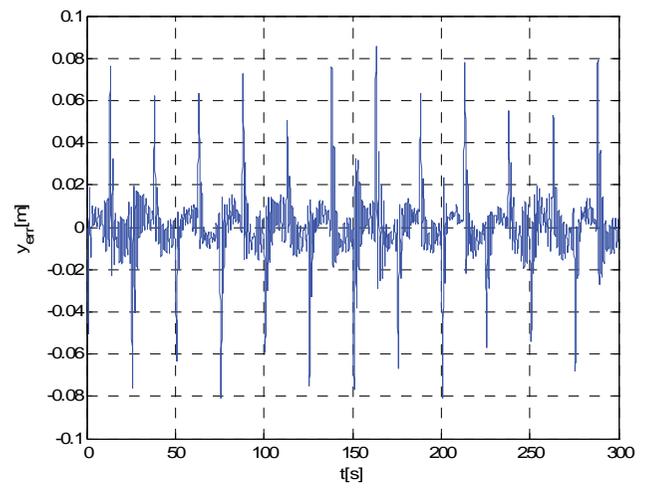
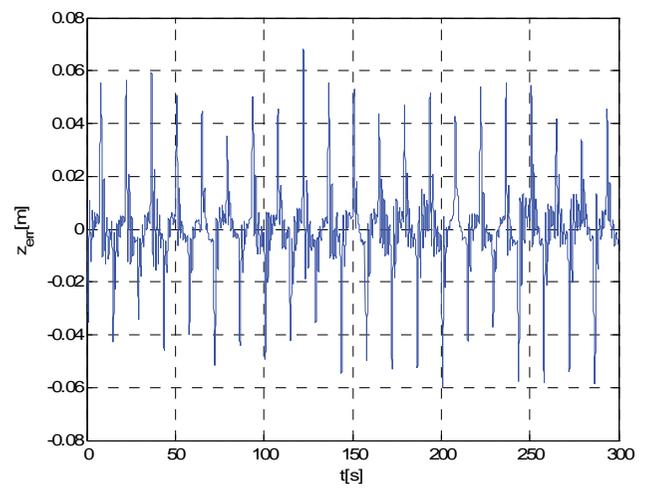
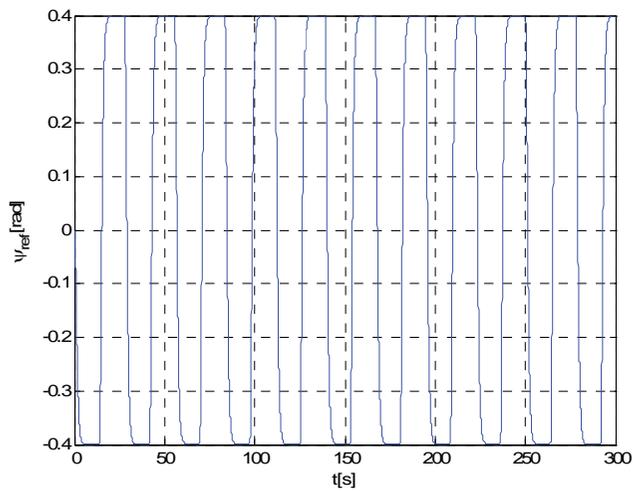


Figure 10 **Error  $y$**   
Slika 10 **Greška  $y$**

Figure 8 **Reference  $\psi$**   
Slika 8 **Referenca  $\psi$**

Figure 11 **Error  $z$**   
Slika 11 **Greška  $z$**



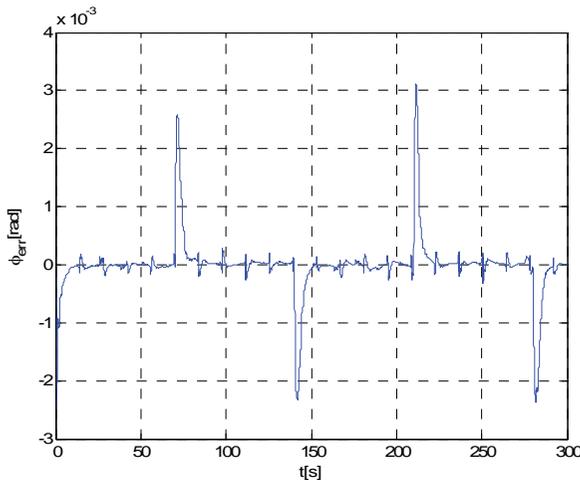


Figure 12 **Error  $\phi$**   
Slika 12 **Greška  $\phi$**

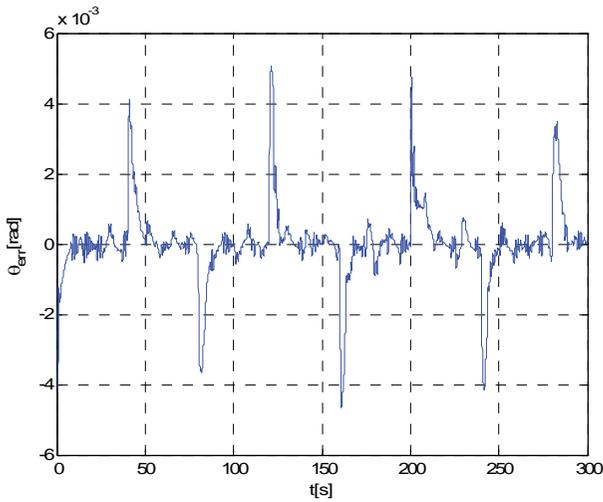


Figure 13 **Error  $\theta$**   
Slika 13 **Greška  $\theta$**

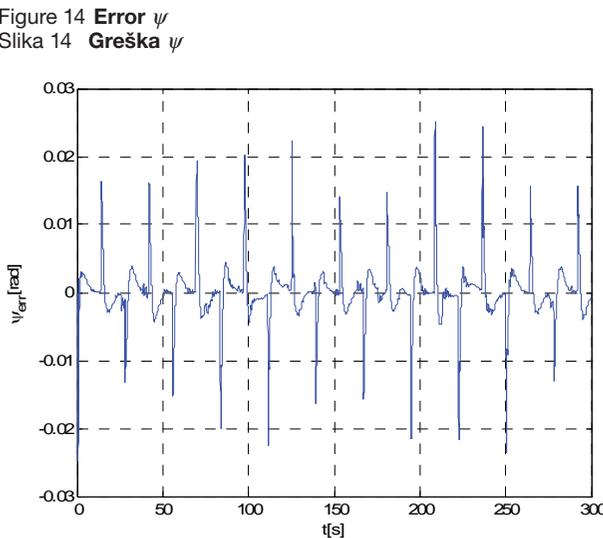


Figure 14 **Error  $\psi$**   
Slika 14 **Greška  $\psi$**

Vector of control inputs is shown in Figure 15.

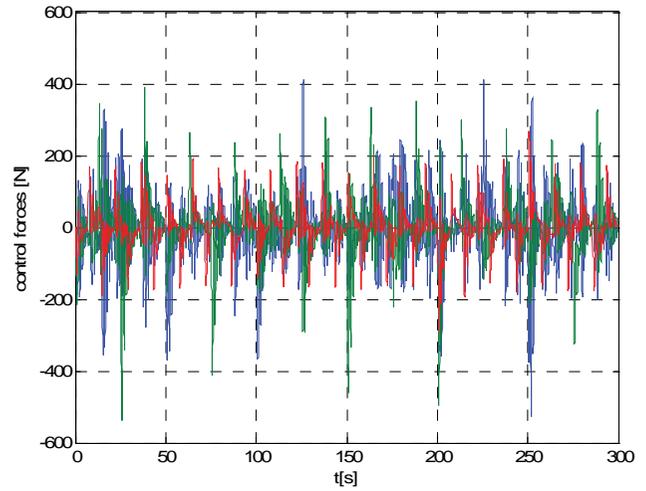


Figure 15 **Vector of control inputs - forces**  
Slika 15 **Vektor regulacijskih signala - sile**

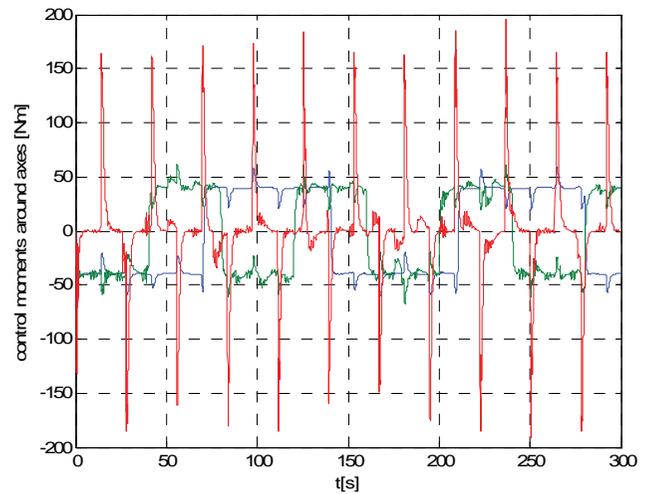


Figure 16 **Vector of control inputs - moments**  
Slika 16 **Vektor regulacijskih signala - momenti**

Simulation results show the proposed control schemes keep control errors small. Control signals are bounded, which in turns shows that neural networks weights are also bounded. This confirms the results of the proof.

### 6 Conclusion

After the presentation of neural network's adaptive mode of work some conclusions could be given.

Neural network, in combination with a conventional controller can ensure good trajectory tracking which is required during specific vehicle maneuvers.

The presented control scheme is easily added to the existing conventional vehicle dynamic positioning system.

No persistency of excitation is needed to ensure proper controller action. Stability is rigorously proven and synthesis procedure is straightforward and simple to use. No knowledge of

the details of the vehicle dynamics is needed and no regression matrices had to be found to design the controller.

Proportional gain weights can be used to drive the filtered error to small values. Neural networks do not have to be trained and weights are updated online.

### Acknowledgement

This work was supported by a grant from the Unity through Knowledge Fund, Research Cooperability Program, grant 07/07, project Applied Neuro-Fuzzy Control of Turbine Governors and Ship Rudder Systems.

### References

### References

- [1] FOSSEN, THOR I.: "Nonlinear Passive Control and Observer Design for Ships", Tutorial Workshop TT1, ECC '99, Karlsruhe, Germany, 1999.
- [2] LEWIS, F.L., LIU, K. YESILDIREK, A.: "Neural Net Robot Controller with Guaranteed Tracking Performance", IEEE Trans. on Neural Networks, Vol. 6, No. 3, May 1995.
- [3] KWAN, C.M., LEWIS, F. L., KIM, Y.H.: "Robust Neural-Network Control of Flexible-Joint Robot", Proc. of the 34<sup>th</sup> Conference on Decision & Control, New Orleans, 1995.
- [4] FOSSEN, THOR I.: "Guidance and Control of Ocean Vehicles", John Wiley and Sons, 1994.
- [5] VUKIĆ, Z., OMERDİC, E., KULJACA, LJ.: "Improved Fuzzy Autopilot for Track-Keeping", IFAC Conference on Control Applications in Marine Systems (CAMS'98), Fukuoka, Japan, 1998.
- [6] BOROVIĆ, B.: "Modern guidance and control systems of floating vehicles", M.Sc. thesis, FER, Zagreb, 1999.
- [7] LEWIS, F.L., JAGANNATHAN, S., YESILDIREK, A.: "Neural Network Control of Robot Manipulators and Non-linear Systems", Taylor & Francis 1999.
- [8] LEWIS, F.L., ABDALLAH, C. T., DAWSON, D.M.: "Control of Robot Manipulators", Macmillan 1993.
- [9] LAUVDAL, T., FOSSEN, THOR I.: "Documentation for Matlab Simulation Program for Marine and Flight Vehicles", NTH report nr. 94-43-W. 1994.
- [10] VUKIĆ, Z., BOROVIĆ, B., TOVORNIK, B.: "Adaptive Neuro Controller for a Precise Maneuvering of Underwater Vehicle", 5th IFAC Conference on Maneuvering and Control of Marine Craft, p 173-178. Aalborg, 2000.
- [11] ISHII, K., URA, T., FUJII, T.: "A feedforward neural network for identification and adaptive control of autonomous underwater vehicles", Proceedings of the IEEE Conference on Robotics and Automation, p 3216-3221, 1994.
- [12] SCHILLER, I., TENCH, K.A.: "A neural-network-based autonomous underwater vehicle guidance system", Proceedings of the IEEE Conference on AUV, p. 312-319, 1995.
- [13] BOROVIĆ, B., KULJACA, O., LEWIS, F.L.: "Neural Net Underwater Vehicle Dynamic Positioning Control", Proceedings of 9th Mediterranean Conference on Control and Automation, MED01-066, published on CD (no page numbers), Dubrovnik 2001.