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Ante ŠESTAN
Nikola VLADIMIR

Numerical Determination of Volumetric Efficiency of High-Pressure Reciprocating Pumps

Review paper

Results of experimental determination of variable-discharge high-pressure reciprocating pump performances [3, 4] show that by increasing the pressure on the pressure side of a reciprocating pump, at a constant stroke length, the discharge and volumetric efficiency of the pump decrease. Mathematical expressions defining the theoretical and the actual pump discharge have been derived on the basis of the analysis of kinematic and hydrodynamic performances of a high-pressure, single-piston single-acting pump.

The applied measuring system is described and examples of analysed measurement results are given. The Fourier approximation method is used to describe mathematically the experimental diagram of operating pressure p_1=f(t).

Assuming that physical properties of the fluid are constant, and that geometric and kinematic characteristics of a pump have not changed, the value of volumetric efficiency is defined as a function of pressure. Results of volumetric efficiency calculation are given for a series of individually performed measurements in the range from H4T25 to H4T250. Results of the numerical determination of volumetric efficiency confirm that the chosen method is acceptable since calculation results differ from experimentally determined values by less than 1%.

Key words: *Fourier series, variable-discharge reciprocating pump, volumetric efficiency*

Numeričko određivanje volumetričke korisnosti visokotlačnih klipnih pumpi

Pregledni znanstveni rad

Rezultati eksperimentalnog određivanja radnih značajki visokotlačnih klipnih pumpi promjenjive dobave [3,4] pokazuju da se s povećavanjem vrijednosti tlaka na tlačnoj strani klipne pumpe, kod konstantne duljine hoda klipa, smanjuje dobava pumpe i volumetrička korisnost. Analizom kinematičkih i hidrodinamičkih značajki rada visokotlačne, jednoklipne, jednoradne pumpe, izvedeni su matematički izrazi koji definiraju teorijsku i stvarnu dobavu. Opisan je sustav za mjerenje i prikazani su primjeri obrađenih rezultata mjerenja. Metodom aproksimacije pomoću Fourierovog reda matematički je opisan eksperimentalni dijagram radnoga tlaka p_1=f(t).

Pod pretpostavkom da su fizikalna svojstva fluida konstantna a geometrijske i kinematičke značajke pumpe nepromijenjene, vrijednost volumetričke korisnosti definirana je kao funkcija tlaka. Prikazani su rezultati proračuna volumetričke korisnosti za seriju pojedinačnih mjerenja H4T25 do H4T250. Rezultati numeričkog određivanja volumetričke korisnosti odstupaju manje od 1% od eksperimentom utvrđenih vrijednosti što potvrđuje opravdanost primijenjene metode za proračun volumetričke korisnosti na temelju eksperimentalnih podataka.

Ključne riječi: *Fourierov red, klipna pumpa promjenjive dobave, volumetrička korisnost*

Authors' address (Adresa autora):
University of Zagreb, Faculty of Mechanical Engineering and Naval Architecture,
I. Lučića 5, Zagreb, Croatia; e-mail: ante.sestan@fsb.hr; nikola.vladimir@fsb.hr

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1 Introduction

Variable-discharge reciprocating pumps found on ships are mostly used in hydraulic drive systems of ship auxiliary machinery, in boiler plants of ships as dosage pumps for the preparation of feed water, in systems for desalinization of sea water and for wastewater processing. Due to their specific application, these pumps must have a very precise and reliable control characteristic in all operating points of a hydrodynamic system. Therefore, the influence of various operating parameters on the accuracy of discharge and on the volumetric efficiency has to be determined.

The actual reciprocating pump discharge is usually defined by the following expression:

$$Q_S = Q_T - \Sigma Q_G \tag{1}$$

where: Q_S = actual pump discharge,
Q_T = theoretical pump discharge,
ΣQ_G = total leakage.

Volumetric efficiency is defined by the following equation:

$$\eta_V = \frac{Q_S}{Q_T} \tag{2}$$

The experience of using high-pressure low-discharge reciprocating pumps shows that volumetric efficiency of these pumps depends on a number of parameters, such as physical properties of the fluid, kinematic characteristics of the pump and hydrodynamic characteristics of the system.

Generally, high-pressure low-discharge reciprocating pumps have the following operating characteristics:

- range of discharge: $Q = 0.8 \div 140 \ \text{cm}^3/\text{s}$,
- range of operating pressures: $p = 0.4 - 50$ MPa,
- number of pistons: 1,
- control of discharge: by changing the stroke length,
- application: dosage of fluids of various physical properties.

Theoretical discharge of a reciprocating pump Q_T is defined by geometric and kinematic characteristics of the pump. The reciprocating pump operation implies the operation within a particular system which, in principle, consists of a suction valve and a discharge valve, a tank, piping of the suction and the discharge side, an air chamber and the associated fitting. Operating conditions are determined by hydrodynamic characteristics of the system and by physical properties of the fluid.

Results of experimental determination of variable-discharge high-pressure reciprocating pump operating characteristics [3, 4] show that by increasing the pressure value on the pressure side of a reciprocating pump, at a constant stroke length, the discharge and volumetric efficiency of the pump decrease.

On the assumption that physical properties of the fluid are constant, and geometric and kinematic characteristics of a pump have not changed, the value of volumetric efficiency is defined as a function of pressure.

The aim of this paper is to make a mathematical computer model for the numerical determination of volumetric efficiency on the basis of experimentally determined operating characteristics of a high-pressure reciprocating pump in order to prove the hypothesis about the influence of the pressure on the volumetric efficiency.

2 Definition of the theoretical and the actual discharge of a high-pressure reciprocation pump

In the analysis of the theoretical pump discharge according to [3], one starts from the assumption that the fluid flow has the property of a steady oscillatory periodic flow, which is a direct consequence of kinematic characteristics of the pump reciprocating piston mechanism. The analysis is performed on a single cylinder, single-acting pump schematically shown in Figure 1. R denotes the crank radius, S denotes the length of the connecting rod, and φ denotes the crank angle of the crankshaft. *GMT* denotes the extreme position of the piston during the discharge stroke, and *DMT* denotes the extreme position of the piston in the suction stroke. The origin of the inertial co-ordinate system is located in *GMT*, as shown in Figure 1. The pump cylinder is fitted with two non-return valves, i.e. a suction valve and a discharge valve. It is assumed that the closing and opening of the valves is instantaneous, i.e. a valve is closed or opened by the fluid flow at the moment when the piston reaches one of the extreme positions.

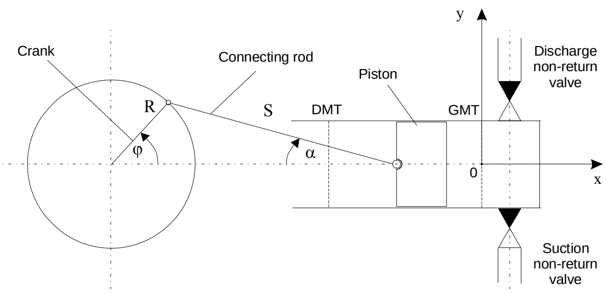

Figure 1 **Position of the piston depending on crank angle φ**
Slika 1 **Položaj klipa u funkciji kuta zakreta φ**

The position of the piston, according to Figure 1, is the function of crank angle φ and is given as follows:

$$x = R\,(1 - \cos\varphi) + S\,(1 - \cos\alpha)\,, \quad 0 \le \varphi \le 2\pi \tag{3}$$

that is:

$$x = R(1-\cos\varphi) + S\left[1-\sqrt{1-\left(\frac{R}{S}\right)^2 \sin^2\varphi}\right] \tag{4}$$

The piston velocity is given by the expression:

$$\frac{dx}{dt} = \dot{x} = -\omega \cdot R \cdot \sin\varphi \left(1 + \frac{R\cos\varphi}{\sqrt{S^2 - R^2\sin^2\varphi}}\right) \tag{5}$$

where ω represents the angular velocity defined by the expression

$$\omega = \frac{d\varphi}{dt}.$$

Further analysis assumes that ω = const.

The instantaneous value of the theoretical discharge of a single cylinder, single-acting pump is determined by the following expression:

$$Q_T(\varphi) = A_K \cdot \dot{x} = -A_K \cdot \frac{d\varphi}{dt} \cdot R \cdot \sin\varphi \left(1 + \frac{R\cos\varphi}{\sqrt{S^2 - R^2\sin^2\varphi}}\right)$$

$$\text{for} \quad \pi \le \varphi \le 2\pi \tag{6}$$

where:
$Q_T(\varphi)$ = the instantaneous value of the theoretical discharge,
A_K = the area of the piston cross-section.

Expression (6) indicates that the pump discharge has the property of a steady oscillatory periodic flow. By integrating expression (6) in the range of $\pi \le \varphi \le 2\pi$, one obtains the mean value of the theoretical pump discharge Q_T:

$$Q_T = A_K\, 2R \frac{\omega}{2\pi} \tag{7}$$

The fluid discharge is defined by the crank angle φ in the range of $\pi \le \varphi \le 2\pi$. The discharge of fluid will start after a given crank angle of $\pi \le \varphi \le \varphi_D$. The angle $(\varphi_D - \pi)$ represents the initial angle of pump discharge. The size of the initial angle of pump discharge is a function of the degree of compressibility of the fluid volume in the cylinder and in the pump chamber V_{KT} at the pressure increase from the suction pressure p_u to the discharge pressure p_i.

The instantaneous flow discharge $Q_I(\varphi)$ is expressed as:

$$Q_I(\varphi) = \begin{cases} 0 & \pi \le \varphi \le \varphi_D \\ \dot{x}A_k & \varphi_D \le \varphi \le 2\pi \end{cases} \qquad (8)$$

The initial volume V_{KT} can be expressed by the displacement coefficient C_V, which is a geometric characteristic of a pump [3]:

$$V_{KT} = (C_V + 1)2R \cdot A_K \qquad (9)$$

If x_D denotes the piston stroke length which corresponds to the size of the initial angle of pump discharge, then the volume of delivered fluid is defined by:

$$V_I = A_K \cdot (2R - x_D) \qquad (10)$$

Assuming that physical properties of the fluid are constant, and that geometric and kinematic characteristics remain unchanged, the actual discharge Q_s is a function of the discharge pressure and is expressed as:

$$Q_S = Q_T - Q_C \qquad (11)$$

where Q_C is the mean value of the flow in the pump cylinder due to the piston motion on the path x_D during the phase of compression from the suction pressure p_u to the discharge pressure p_i.

According to [1] and [2], it follows that:

$$-\frac{dp_C}{K} = \frac{dV}{V_{KT}},$$

and, using expression (9), it follows that:

$$\frac{dp_C}{dt} = \frac{K \cdot Q_C(\varphi)}{(C_V + 1) \cdot 2R \cdot A_K} \qquad (12)$$

where:
$p_c(\phi)$ = instantaneous pressure in the cylinder,
K = bulk modulus of elasticity,
$Q_C(\phi)$ = instantaneous flow in the pump cylinder.

The instantaneous flow $Q_C(\phi)$ in the compression phase, according to (12), is defined by the following expression:

$$Q_C(\varphi) = \left(\frac{dp_C}{dt}\right) \cdot \frac{(C_V + 1) \cdot 2R \cdot A_K}{K} \qquad (13)$$

According to (11), the actual flow $Q_S(\phi)$ is defined by the expression:

$$Q_S(\varphi) = -A_K \cdot \frac{d\varphi}{dt} \cdot R \cdot \sin\varphi \left(1 + \frac{R\cos\varphi}{\sqrt{S^2 - R^2\sin^2\varphi}}\right) - \left(\frac{dp_c}{dt}\right) \cdot \frac{(C_V + 1) \cdot 2R \cdot A_K}{K} \qquad (14)$$

Expressions (11) and (14) can be used for the numerical determination of actual discharge provided that the function $p_c=f(t)$ is known. Data for the numerical modelling of the function $p_c=f(t)$ have been obtained by experimental measurements carried out by a hydraulic testing device.

3 Measuring system and measurement results

Measurements were carried out by a hydraulic measuring system shown schematically in Figure 2.

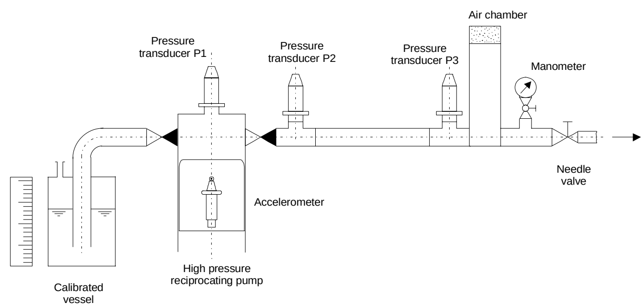

Figure 2 **Scheme of the hydraulic system with the lay-out of measuring points**
Slika 2 **Shema hidrauličkog sustava s rasporedom mjernih mjesta**

3.1 Data about the variable-discharge high-pressure reciprocating pump

A variable-discharge high-pressure reciprocating pump has been used for measurements. The pump discharge varies with a change in the stroke length at a constant number of revolutions of the pump drive shaft. Basic technical data about the pump are as follows:

Category of pump:	reciprocating-type,
Type:	single acting,
Number of pistons:	1,
Piston diameter:	22 mm,
Nominal discharge:	30 cm^3/s (at maximum stroke length)
Maximum stroke length:	40 mm,
Operating pressure:	10 MPa (at piston stroke length of $h_{max.}$),
Maximum pressure:	30 MPa (at piston stroke length of 10% $h_{max.}$),
Discharge control:	continuous, by changing the piston stroke length,
Number of operating strokes:	2.0 strokes/s,
Duration of the stroke:	0.25 s/operating stroke
Application:	dosage of liquid media.

3.2 Equipment used for measurements and for data processing

A system used for measurements and measurement data analysis, shown in Figure 3, consists of the following components:

Pressure transducers (P_1, P_2, P_3)

Manufacturer: Teledyne Taber, USA
Type: Model 2201
Measuring range: 0 - 200 bar - pressure transducer P3
0 - 300 bar – pressure transducers P1 and P2
Measurement uncertainty: < ± 0.2% of the set measuring range

Accelerometer

Manufacturer: Hottinger - Baldwin Messtechnik
Type: B 12/200
Measuring range: ± 200 m/s^2
Measurement uncertainty: < ± 0.2%

Amplifiers

Signals from transducers are conditioned by means of amplifiers of the following characteristics:
Manufacturer: Hottinger - Baldwin Messtechnik
Type: Alpha 3000
Measuring uncertainty: sensibility < 0.15%
linearity and hysteresis < 0.05%, typically 0.02%

Tape recorder

As a parallel system for recording signals from amplifiers (outputs from amplifiers are conditioned signals from transducers), a tape recorder of the following characteristics was used:

Manufacturer/type: Racal Recorders / V-Store
Linearity: < 0.3%

A PC-system used for acquisition and analysis

An Intel Pentium 133 MHz PC fitted with a system for the acquisition and analysis of signals was used. The program used for the analysis of signals was a DT VEE program, version 4.0.

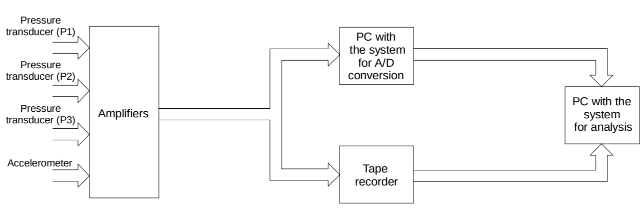

Figure 3 **Scheme of the system for measurement and analysis**
Slika 3 **Shema sustava za mjerenje i analizu**

3.3 Presentation of measurement results

Data obtained by measurements and data about the conditions in which measurements were conducted are recorded in a measuring sheet. In total, 40 measurements were carried out. Each measurement is represented by *HxTy* (e.g. *H50T100* denotes a measurement for a stroke length of 0.5 h_{max} and pressure of 100 bar). Figures 4 and 5 give examples of a measuring sheet and an experimental indicator diagram, respectively, with measurement results of the *H1T300* measurement.

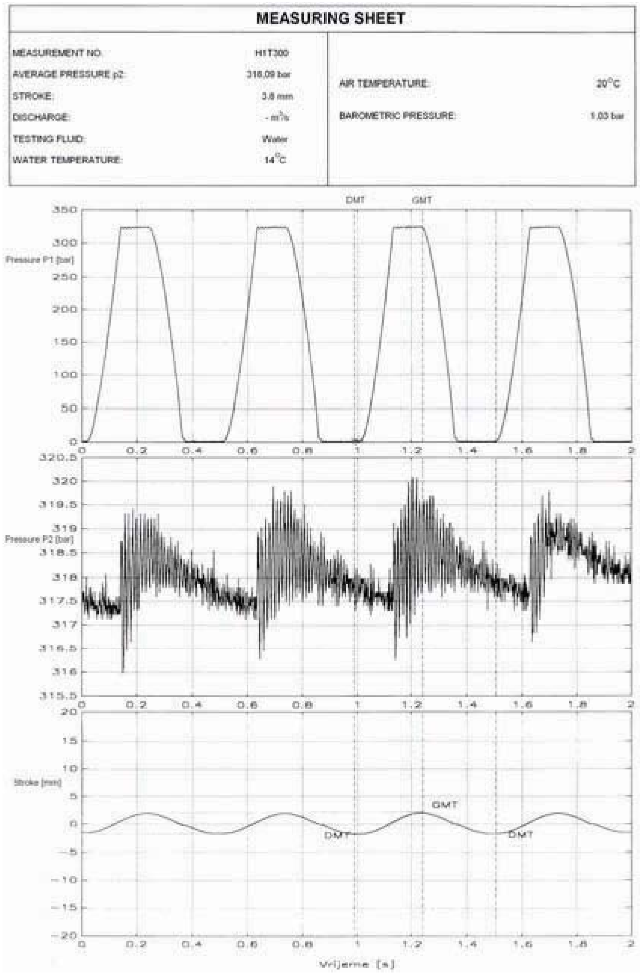

Figure 4 **Measuring sheet of the H1T300 measurement**
Slika 4 **Pojedinačni mjerni list za mjerenje H1T300**

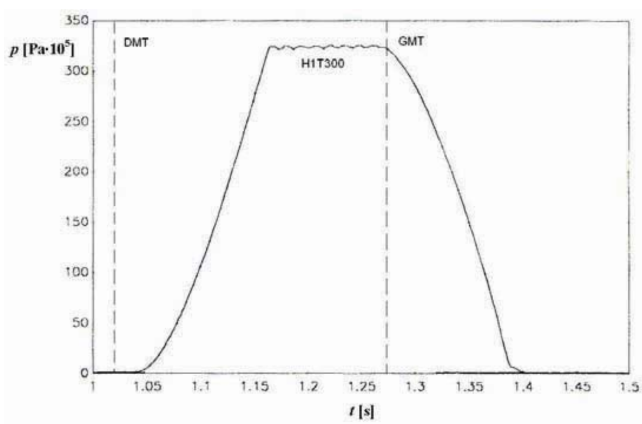

Figure 5 **Experimental p_1=*f(t)* diagram of the H1T300 measurement**
Slika 5 **Eksperimentalni dijagram p_1*(t)* za mjerenje H1T300**

4 Numerical model of pressure function

An algorithm suitable for the mathematical description and processing of experimental measurement results enables the numerical determination of actual discharge and computer analyses of the influence of various parameters on the pump operation. Results of experiments are recorded in individual measuring sheets (Figure 4) and in indicator diagrams (Figure 5), as well as in tables. Using the *AutoCAD* program, one can take from the diagram the pressure values $p_1(t)$ and the values of piston slip $h(t)$ in time intervals of $\Delta t = 1.36 \cdot 10^{-2}$ sec. The *MS Excel* program is used to sort and to process the taken data so that all output data represent instantaneous values of pressure $p_1=f(t)$ in various phases of the operating cycle. Thus, the mathematical description is simplified because instantaneous pressure variations around the mean value of the indicated pressure in transient phases of operation are removed from the diagram. A continuous $p_1=f(t)$ diagram record has the shape of a periodic function; therefore, the Fourier approximation method is used for the mathematical description of the diagram record. Using the Fourier series it is possible to approximate the $f(x)$ function as a sum of a sine and a cosine:

$$f(x) = A_0 + \sum_{n=1}^{\infty} \left[A_n \cos(nx) + B_n \sin(nx) \right] \tag{15}$$

Coefficients of the Fourier series are determined by using the algorithm generated in the Mathematica 5.2 (Wolfram Research) program. The approximation procedure for the diagram record carried out by the Mathematica 5.2 program has shown that a minimum number of the Fourier series terms required for a satisfactory description of measurement results is obtained if the Fourier series from expression (15) is developed as the sine series. In that case, only B_n coefficients of the Fourier series need to be determined. Expression (15) thus takes the following form:

$$f(x) = \sum_{n=1}^{\infty} B_n \sin(nx) \tag{16}$$

In order to enable a possible comparison between particular terms, the coefficients of sine functions of corresponding terms should be equal. Consequently, the coefficients are determined according to the time rate of change of the lowest pressure harmonic.

By replacing adequate variables, the final expression for the Fourier series terms is obtained. This expression is used to approximate the recorded diagram of the pressure function $p_1(t)$:

$$p_1(t) = \sum_{n=1}^{28} B_n \sin\left(\frac{n\pi}{\tau} t \right) \tag{17}$$

The Fourier series coefficients and terms of the function $p_1(t)$ for the *H1T300* measurement are given in Table 1. Diagram of the calculation approximation of the function $p_1(t)$ for the *H1T300* measurement is given in Figure 6.

Table 1: **The Fourier series coefficients of the function $p_1(t)$ for the H1T300 measurement**

Tablica 1: **Koeficijenti i argumenti članova Fourierovog reda funkcije $p_1(t)$ za pojedinačno mjerenje H1T300**

n	coefficients	arguments	*n*	coefficients	arguments
1	303,632	7,85398 *t*	15	-0,482535	117,81 *t*
2	-78,0277	15,708 *t*	16	2,60475	125,664 *t*
3	-35,4236	23,5619 *t*	17	-1,26895	133,518 *t*
4	-20,7396	31,4259 *t*	18	0,32515	141,372 *t*
5	-9,55087	39,2699 *t*	19	-1,465	149,226 *t*
6	14,3085	47,1239 *t*	20	1,21973	157,08 *t*
7	-0,155893	54,9779 *t*	21	0,0461599	164,934 *t*
8	-1,23169	62,8319 *t*	22	0,640203	172,788 *t*
9	-3,93797	70,6858 *t*	23	-1,33885	180,642 *t*
10	4,00239	78,5398 *t*	24	0,223542	188,496 *t*
11	0,457466	86,3938 *t*	25	-0,149621	196,35 *t*
12	3,55171	94,2478 *t*	26	0,76286	204,204 *t*
13	-2,88722	102,102 *t*	27	-0,17464	212,058 *t*
14	0,604043	109,956 *t*	28	-0,147763	219,911 *t*

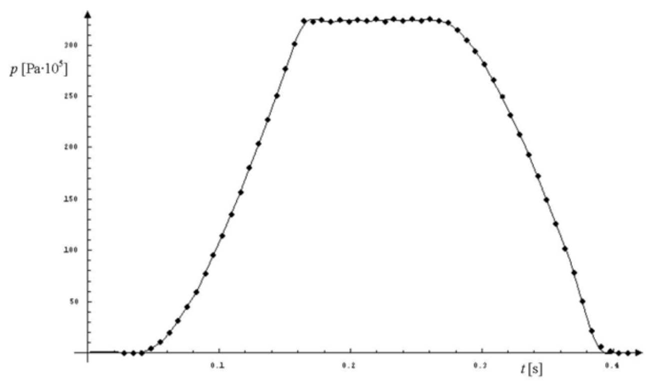

Figure 6 **Diagram of the calculation approximation of the function $p_1(t)$ for the H1T300 measurement**

Slika 6 **Dijagram proračunske aproksimacijske funkcije $p_1(t)$ za mjerenje H1T300**

5 Volumetric efficiency as a function of pressure

Assuming that physical properties of the fluid are constant, and that geometric and kinematic characteristics of a pump remain unchanged, the value of volumetric efficiency is a function of pressure. According to (2), the volumetric efficiency is defined by the following expression:

$$\eta_V = \frac{Q_S}{Q_T}$$

By integrating expression (6) in the range of $\pi \leq \varphi \leq 2\pi$ and by integrating expression (14) in the range of $\pi/\omega \leq t \leq \varphi_D/\omega$, one can calculate the values of actual discharge according to (11), and the values of volumetric efficiency according to (2). Data for a series of 40 measurements have been calculated using the MS Excel program.

Table 2 and Diagram 1 present results of individually performed measurements in the range from *H4T25* to *H4T250*. To allow comparison, data obtained by experimental measurement are also given.

Table 2 **Values of volumetric efficiency for the series of measurements from H4T25 to H4T250**
Tablica 2 **Vrijednosti volumetričke korisnosti za seriju mjerenja H4T25 do H4T250**

	H4T25	H4T50	H4T75	H4T100	H4T150	H4T200	H4T250
$\eta_{v\ calculated}$	0,941	0, 916	0,898	0,881	0,865	0,842	0,818
$\eta_{v\ experimental}$	0,934	0,915	0,896	0,878	0,860	0,840	0,815

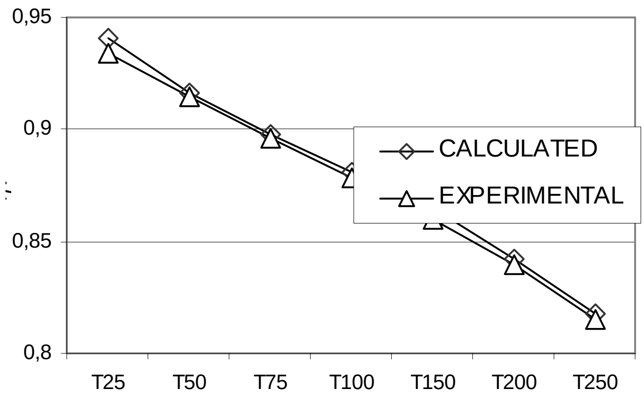

Diagram 1 **Volumetric efficiency depending on pressure for the series of measurements from H4T25 to H4T250**
Dijagram 1 **Volumetrička korisnost u ovisnosti o tlaku za seriju ispitivanja H4T25 do H4T250**

6 Conclusion

The function $p_1=f(t)$, obtained by experimental measurement, can be mathematically described by using the Fourier approximation method. Thus, numerical determination of the actual discharge and volumetric efficiency on the basis of experimental indicator diagram is made possible, together with numerical analyses of the influence of several different parameters on the pump operation. Results of the numerical determination of volumetric efficiency confirm that the chosen method is acceptable since calculation results differ from experimentally determined values by less than 1%.

Volumetric efficiency as a function of pressure is numerically determined by varying pressure values on the pressure side of the pump. Calculation results confirm that an increase in pressure on the pressure side of a reciprocating pump, at a constant stroke length, results in a decrease in the discharge and volumetric efficiency of the pump.

Index of symbols

A_K Area of the piston cross-section, m^2
C_V Coefficient of the piston discharge,
h Stroke length, m
K Bulk modulus of elasticity, N/m^2
p Gauge pressure, Pa, bar
Q Discharge, m^3/s
R Radius of the piston pump crankshaft, m
S Connecting rod length, m
t Time, s
V Volume, m^3
x Coordinate along the piston axis, m
$\dot{x}$ Pump piston velocity in the direction of x-axis, m/s
α Connecting rod angle, rad
φ Crank angle, rad
ω Angular velocity, rad/s

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