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Model-based Stochastic Inversion of Coil Impedance for Determination of Tube Inner Radius and Electromagnetic Properties

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Original scientific paper

We present a stochastic formulation of the inverse problem of determination of the inner radius and electromagnetic properties of metal tubes from the measured coil impedance. The inversion procedure is based on the Monte Carlo Markov chain method and the analytical impedance model developed and validated in our previous work. The impedance measurement uncertainty is included as the prior probability distribution. We use the Metropolis-Hasting algorithm to sample the posterior probability distributions of the tube properties. We illustrate the method using simulated measurement data sets. We corroborated from the simulation results that the requirements on electronic instrumentation and impedance measurement correspond to the targeted accuracy and precision of the tube inner radius.

Key words: Coil impedance, Inverse problem, Tube inner radius, Magnetic permeability, Electrical conductivity, Monte Carlo methods

Na modelu zasnovana stohastička inverzija impedancije zavojnice za određivanje unutarnjeg polumjera i elektromagnetskih značajki cijevi. U ovome smo radu formulirali stohastički inverzni problem određivanja unutarnjeg polumjera i elektromagnetskih značajki metalne cijevi iz izmjerene impedancije zavojnice. Inverzni je postupak zasnovan na Monte Carlo metodi Markovljevih lanaca i analitičkom modelu impedancije koji je razvijen i potvrđen u našim prethodnim radovima. Mjerna nesigurnost impedancije je uključena kao apriorna funkcija gustoće vjerojatnosti. Uzorci aposteriorne funkcije gustoće vjerojatnosti značajki cijevi su dobiveni Metropolis-Hastingsovim algoritmom. Prikazali smo metodu koristeći simulirane mjerne podatke. Zaključili smo na temelju rezultata da zahtjevi na elektroničku instrumentaciju i mjerenje impedancije zavojnice trebaju odgovarati ciljanoj točnosti i preciznosti mjerenja unutarnjeg polumjera cijevi.

Ključne riječi: impedancija zavojnice, inverzni problem, unutarnji polumjer cijevi, magnetska permeabilnost, električna specifična vodljivost, Monte Carlo metode

1 INTRODUCTION

Low-frequency electromagnetic measurement of radii and electromagnetic properties (relative magnetic permeability and electrical conductivity) of metal tubes is essential in applications such as the integrity evaluation of oil-well casings and rock resistivity measurement through casing [1–3]. The measurement methods, usually based on empirical space-frequency relations between transmitting and receiving coils, lack model-based procedures and provide only qualitative estimates of the tube properties [4, 5]

In our previous work, we presented and experimentally validated a model-based method for simultaneous measurement of inner radius and permeability-to-conductivity ratio of a tube made of conductive and ferromagnetic material [5, 6]. The tube properties are determined by fitting an impedance model, based on the Dodd-Deeds formulation, to the impedance of a coil measured at single or multiple frequencies [6–8]. The result of such a procedure is a single point in the parameter space with error analysis based on the linearized version of the impedance model. Because of the strong mutual dependence of the permeability and conductivity, the procedure is sensitive to the choice of the starting point [6].

In this paper, we present a stochastic formulation of the inverse problem of determination of the inner radius and electromagnetic properties of metal tubes from the coil impedance measured at single frequency. The theory of stochastic inversion, as laid out in Tarantola and Mosegaard [9, 10], introduces a notion of the prior information on the wanted parameters in form of probability distribution and its Bayesian combination with the probability distribution of the measurement data using a theoretical model that can have its own modeling uncertainties [11]. The solution of the inverse problem is given in form of a posterior probability distribution of the sought parameters, which is a significant improvement compared to a one-point optimization approach. The approach is quite general as it can accommodate existing knowledge (in form of prior probability distributions) obtained in previous test runs or using other measurement methods (e.g. mechanical calipers) or specifications of the tube manufacturing process. We use a Markov chain Monte Carlo (MCMC) method to obtain samples of the posterior distribution [12].

The paper is organized as follows. In Section 2, we introduce the analytical impedance model of a coil positioned inside a metal tube. The stochastic formulation of the inverse problem is given in Section 3. We present the main results of the stochastic inversion theory, implementation of the MCMC method and output analysis. In Section 4, we present the results obtained by the stochastic inversion of the simulated measurement data and discuss the requirements for the electronic instrumentation and further potential of the stochastic approach in the measurement of the tube properties. Section 5 contains the conclusions.

2 IMPEDANCE MODEL

We derived the analytical expression for impedance of an axially symmetric, one-layer coil placed coaxially inside a conductive tube in form of an integral of the modified Bessel functions. The complete derivation will not be shown here for the sake of shortness and clarity, but similar and well-explained models can be found in [5–7].

The tube is infinitely long, with inner radius r_1 and wall thickness c. The tube material is linear, isotropic and homogeneous with relative magnetic permeability μ_r and electrical conductivity σ_t. The cylindrical coil, positioned coaxially inside the tube, has length L, radius r_0, cross section area S and number of turns N. The driving current is time harmonic with frequency $\omega = 2\pi f$.

The total impedance of the coil inside the tube is sum of the impedance of the coil in air (i.e. out of tube) Z_{air} and a contribution Z_T of the tube:

$$Z = Z_{air} + Z_T. \tag{1}$$

Based on a solution of the quasistatic approximation of the magnetic vector potential equation in its axisymmetric, frequency domain form, one can calculate the voltage induced in the coil. From the induced voltage and the excitation current follows an analytical expression for Z_T:

$$Z_T = \mathrm{j}\omega \int_0^{\infty} C\left(\alpha, L, r_0, S, N\right) T\left(\alpha, \mu_r, \sigma_t, c, r_1, \omega\right) d\alpha, \tag{2}$$

where α is a separation variable for radial and axial parts of the magnetic potential equation, C depends on dimensions of the coil and T is "tube contribution" since it is the only function which depends on the tube properties (permeability μ_r, conductivity σ_t, inner radius r_1 and wall thickness c).

If the penetration depth of the eddy currents is very small comparing to the wall thickness c, then the tube contribution is no longer a function of wall thickness and a levelset of Z_T for a given r_1 can be approximated with a constant permeability-to-conductivity ratio, i.e. $PCR = \mu_r/\sigma_t = const$. [6]. Detailed derivation of (2) and the *PCR* approximation can be found in [8].

3 STOCHASTIC INVERSE PROBLEM

3.1 Theoretical background

In formulating the stochastic solution of the inverse problem, we will follow the theory as laid out by Tarantola and Mosegaard [9, 10]. We briefly introduce its basic notions.

The measurement results are given by a measurement vector $\mathbf{d}$ and an associated probability density function $\rho_{\mathrm{D}}(\mathbf{d})$ defined over the data space $\mathfrak{D}$. A probability density $\rho_{\mathrm{M}}(\mathbf{m})$ defined over the model space $\mathfrak{M}$ represents *a priori* information on the model parameters $\mathbf{m}$. Since the prior information on model parameters is independent of the measurement, we can define the joint probability density as

$$\rho_{\mathrm{M}\times\mathrm{D}}\left(\mathbf{m},\mathbf{d}\right) = \rho_{\mathrm{M}}\left(\mathbf{m}\right)\rho_{\mathrm{D}}\left(\mathbf{d}\right). \tag{3}$$

Combining (3) with a theoretical relation $\mathbf{d} = \mathbf{g}(\mathbf{m})$, and assuming that the modeling uncertainties are negligible compared to measurement uncertainties or that both uncertainties are of the Gaussian type, we can write the solution of the inverse problem in form of *a posteriori* probability density of the model parameters

$$\gamma_{\mathrm{M}}\left(\mathbf{m}\right) = \frac{1}{\nu}\rho_{\mathrm{M}}\left(\mathbf{m}\right) L\left(\mathbf{m}\right), \tag{4}$$

where $L(\mathbf{m})$ is the likelihood function $L(\mathbf{m}) = \rho_D(\mathbf{g}(\mathbf{m}))$, and ν is a normalization constant.

In order to avoid extensive exploration of the model space, one can use Markov chain Monte Carlo (MCMC) methods to obtain samples of the posterior probability density $\gamma_{\mathrm{M}}(\mathbf{m})$. More specifically, using the Metropolis-Hastings algorithm to construct a random walk whose proposals for the next update come from the prior probability density $\rho_{\mathrm{M}}(\mathbf{m})$, and if the acceptance rule is based on the ratio of the values of $L(\mathbf{m})$ in the current state and in the proposed update, one can expect that the random walk samples the *a posteriori* probability density $\gamma_{\mathrm{M}}(\mathbf{m})$ [12].

3.2 MCMC for the single-coil method

The measurement result of the single-coil method is the tube contribution Z_T at single excitation frequency. We will recast Z_T into a vector $\mathbf{d} = [\mathrm{Re}(Z_T)\ \ \mathrm{Im}(Z_T)]^{\mathrm{T}}$ and an associated probability density $\rho_{\mathrm{D}}(\mathbf{d})$ defined over the data space $\mathfrak{D}$ of all meaningful values of the resistance and reactance. We will assume that the measured values are normally distributed with a covariance matrix $\mathbf{C}_{\mathrm{D}}$ and mean $\mathbf{d}_{\mathrm{m}}$ as

$$\rho_{\mathrm{D}}\left(\mathbf{d}\right) = \frac{1}{2\pi\sqrt{\det \mathbf{C}_{\mathrm{D}}}} e^{-\frac{1}{2}(\mathbf{d}-\mathbf{d}_{\mathrm{m}})^{\mathrm{T}}\mathbf{C}_{\mathrm{D}}^{-1}(\mathbf{d}-\mathbf{d}_{\mathrm{m}})}, \qquad (5)$$

where the bivariate case is assumed for the normalization constant.

We will parameterize the model space using the properties of the tube μ_r, σ_T and r_1. However, instead of using σ_t, we will use its natural (other bases are perfectly acceptable also) logarithmic value, $\ln \sigma_t$. By doing so, the distance between two materials of different conductivities can be expressed as $|\ln(\sigma_{t1}/\sigma_{t2})|$, which is additive, invariant of scale and independent of our inclination to use conductivity or resistivity. Pairs of positive physical quantities such as conductivity-resistivity or period-frequency are called the Jeffreys quantities [9]. Since we expect to have strong dependence in form of PCR, we will also use logarithmic value of permeability. Thus, a model of tube (i.e. point in $\mathfrak{M}$) can be represented as a vector $\mathbf{m} = [\ln \mu_r\ \ \ln \sigma_t\ \ r_1]^{\mathrm{T}}$. Such parameterization does not prevent us to observe distributions of the real physical quantities in the analysis of the output of the MCMC algorithm.

We will assume that the tube properties μ_r, σ_t, r_1 and PCR must be between their respective minimal and maximal values. The region defined by these constraints forms a set

$$M_0 = \left\{ \begin{bmatrix} \ln \mu_r \\ \ln \sigma_t \\ r_1 \end{bmatrix} \middle| \begin{array}{l} (\mu_{r,min} \le \mu_r \le \mu_{r,max}) \wedge \\ (\sigma_{t,min} \le \sigma_t \le \sigma_{t,max}) \wedge \\ (r_{1,min} \le r_1 \le r_{1,max}) \wedge \\ (PCR_{min} \le PCR \le PCR_{max}) \end{array} \right\} \qquad (6)$$

The lack of any other prior information than these boundaries means that we have to choose a homogenous prior probability density of the model parameters

$$\rho_{\mathrm{M}}\left(\mathbf{m}\right) = \begin{cases} k \ \text{ if } \ \mathbf{m} \in M_0 \\ 0 \ \text{ otherwise} \end{cases} \qquad (7)$$

Distribution $\rho_{\mathrm{M}}(\mathbf{m})$ is a constant value k for $\mathbf{m} \in M_0$, because we assign probability to each region of M_0 proportional to the region's volume [9].

The proposals for the next update $\mathbf{m}^{(i+1)}$ are drawn from the transitional normal probability density $\eta\left(\mathbf{m}^{(i+1)}\right)$ with mean $\mathbf{m}^{(i)}$ and covariance matrix $\mathbf{H}\mathbf{C}_{\mathrm{T}}\mathbf{H}^{\mathrm{T}}$,

$$\mathbf{m}^{(i+1)} \sim \mathcal{N}\left(\mathbf{m}^{(i)}, \mathbf{H}\mathbf{C}_{\mathrm{T}}\mathbf{H}^{\mathrm{T}}\right), \qquad (8)$$

where the covariance matrix $\mathbf{H}\mathbf{C}_{\mathrm{T}}\mathbf{H}^{\mathrm{T}}$ is defined with

$$\mathbf{H} = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad (9)$$

$$\mathbf{C}_{\mathrm{T}} = \mathrm{diag}\left(k_{em\parallel}^2, k_{em\perp}^2, k_r^2\right). \qquad (10)$$

The form of the covariance matrix is chosen in order to obtain easier control of the random walk. The elements of $\mathbf{H}$ in (9) are such that the elements of $\mathbf{C}_{\mathrm{T}}$, $k_{em\parallel}$ and $k_{em\perp}$ correspond to a move parallel and perpendicular to the line $ln\mu_r + \ln \sigma_t = const.$, respectively. The element k_r is the standard deviation of the transition probability density for the radius component of $\mathbf{m}$.

3.3 Output analysis and numerical implementation

The result of the MCMC inversion algorithm is a sequence $\left\{\mathbf{m}^{(n)}\right\}_{n=1}^{N}$, which can be used for estimation of statistical parameters of the posterior distribution $\gamma_{\mathrm{M}}(\mathbf{m})$. Sequence $\left\{\mathbf{m}^{(n)}\right\}_{n=1}^{N}$ consists of the components $\left\{\mu_r^{(n)}\right\}_{n=1}^{N}$, $\left\{\sigma_t^{(n)}\right\}_{n=1}^{N}$ and $\left\{r_1^{(n)}\right\}_{n=1}^{N}$ which, taken individually, can be used for estimation of parameters of the marginal posterior distributions of permeability, conductivity and inner radius, respectively.

The samples $\left\{\mathbf{m}^{(n)}\right\}_{n=1}^{N}$ are mutually correlated by the definition of Markov chain. The usual strategy in obtaining the uncorrelated samples is to subsample the chain by taking every τ_f sample only [11, 12]. The subsampling value τ_f is an integrated autocorrelation time or autocovariance time and it can be calculated as

$$\tau_f = 1 + 2\sum_{s=1}^{\infty} \rho_{ff}\left(s\right), \qquad (11)$$

where $\rho_{ff}(s)$ is the normalized autocovariance function and s is the lag. In practice, the infinite sum of (11) is replaced by a sum of an appropriate length [13].

The forward model and the Monte Carlo inversion procedure were both implemented in Matlab. Number of iterations was $N = 100000$ and it took around 10 minutes to complete the task. The heaviest computational burden was N calculations of the forward impedance model.

Let Z_{Tm} be the result of the impedance measurement. This value is different from the value predicted by the

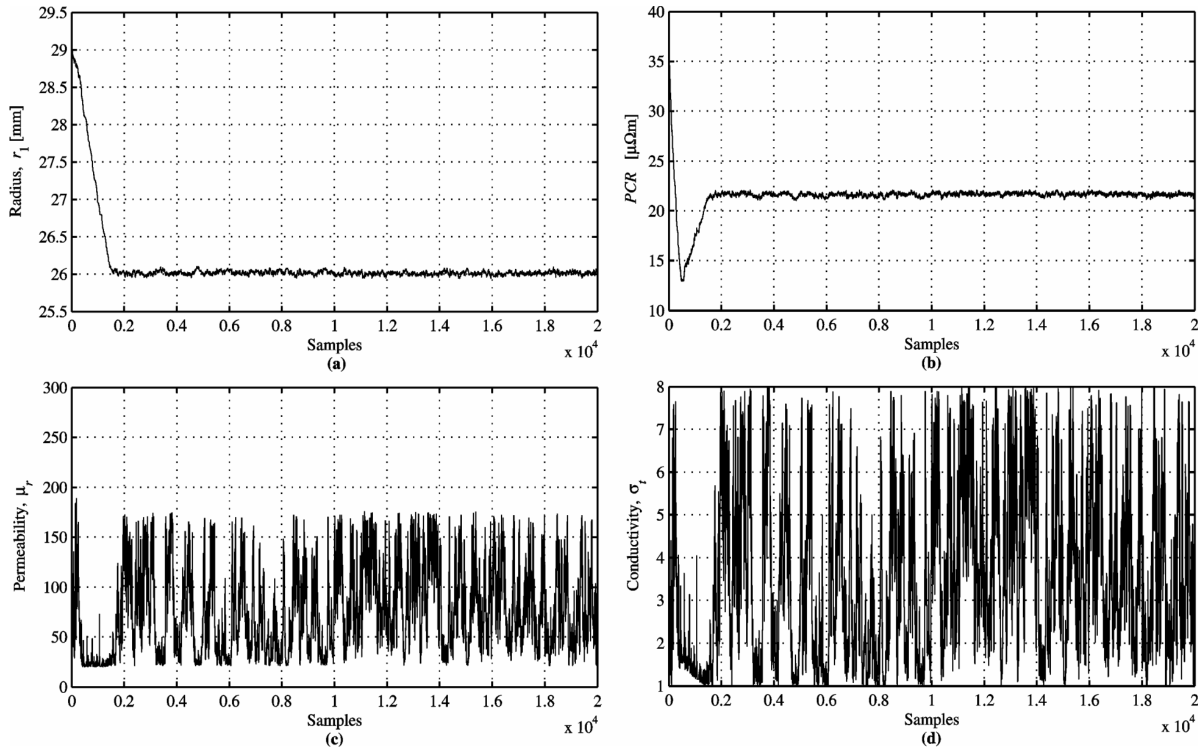

Fig. 1. Output of the MCMC algorithm as Markov chains of radius, permeability and conductivity in figures (a), (c) and (d), respectively. Markov chain of PCR in (b) is obtained from the chains in (c) and (d). Duration of the burn-in period is less than 2000 updates

model Z_{T0} due to systematic effects such as parasitic capacitances or errors in coil parameters. We analyzed several measurement situations different by their measurement uncertainties and systematic errors. The relative measurement uncertainty s_{rel} is defined as a percentage of the impedance modulus $|Z_{Tm}|$. The covariance matrix in (5) is

$$C_{\mathrm{D}} = \mathrm{diag}\left(s_{rel}^2 \left|Z_{Tm}\right|^2, s_{rel}^2 \left|Z_{Tm}\right|^2\right), \qquad (12)$$

where we assumed that the measurements of real and imaginary parts are uncorrelated and with the same uncertainty. The relative measurement error p_{rel}, which accounts for the systematic effects, is defined as a relative deviation of Z_{Tm} from the value of the tube contribution Z_{T0} as predicted by the model:

$$p_{rel} = \frac{|Z_{Tm} - Z_{T0}|}{|Z_{T0}|}. \qquad (13)$$

The simulations were made using a coil with 230 turns of copper wire, length 74 mm and radius 22 mm. The true tube parameters were $\mu_{r0} = 100$, $\sigma_{t0} = 4.6\ \mathrm{MS/m}$ and inner radius $r_{1,0} = 26\ \mathrm{mm}$. The excitation frequency was 31250 Hz.

3.4 Results and discussion

Fig. 1 depicts sequences $\left\{\mu_r^{(n)}\right\}_{n=1}^N$, $\left\{\sigma_t^{(n)}\right\}_{n=1}^N$, $\left\{PCR^{(n)}\right\}_{n=1}^N$ and $\left\{r_1^{(n)}\right\}_{n=1}^N$, where $PCR^{(n)} = \mu_r^{(n)}/\sigma_t^{(n)}$. It took about 2000 samples (burn-in period) for the chains to become stationary. The sequences for permeability, conductivity and PCR corroborate that μ_r and σ_t are mutually highly dependent. This is shown more clearly in Fig. 2, where the prior and posterior information on μ_r and σ_t are depicted (samples in the burn-in period removed). The probability distribution of PCR can be analyzed without significant loss of information in comparison to the posterior marginal joint distribution for μ_r and σ_t, as shown in Fig. 2. The random walk that samples the joint posterior distribution in the space of parameters r_1 and PCR is shown in Fig. 3.

Normalized autocovariance functions ρ_{ff} for r_1 and PCR are shown in Fig. 4. Autocovariance time τ_f is around 122 for r_1 and 133 for PCR, what indicates that taking every 150-th sample of the chain is enough in order to achieve independent samples of the posterior distribution.

From the histograms and embedded quantile-quantile

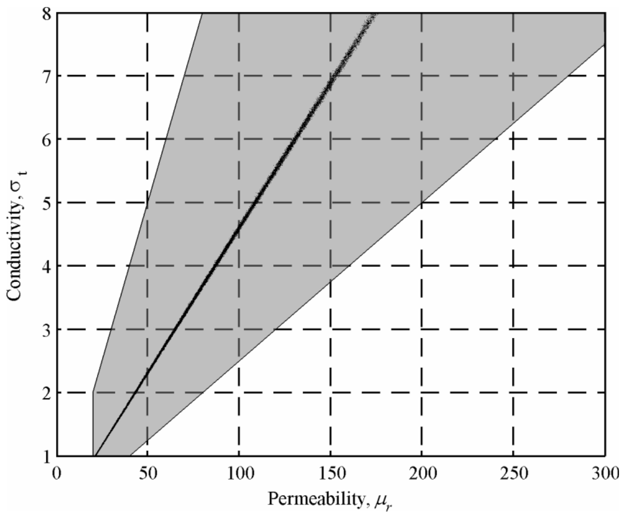

Fig. 2. Gray region represents a priori information on the electromagnetic properties. Samples drawn from the a posteriori probability distribution of the electromagnetic properties are represented with the black dots

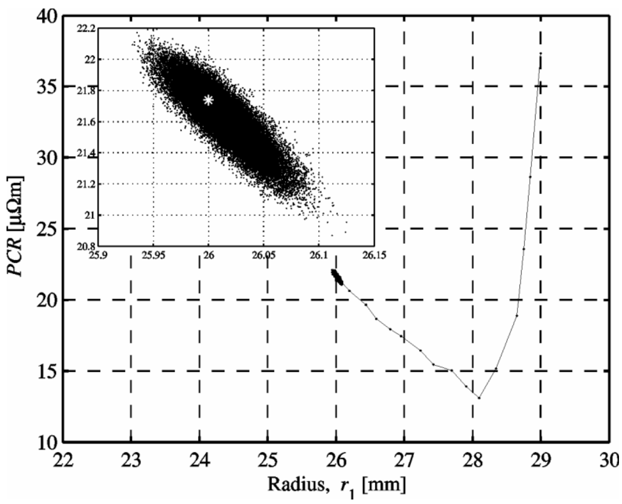

Fig. 3. Random walk in the space of parameters r_1 and PCR. The limits of the larger axes correspond to the a priori information on r_1 and PCR. The smaller axes are zoomed part of the walk after the burn-in period. The white star denotes the true properties of the tube

(Q-Q) plots in Figs. 5 and 6, one can see that the posterior distributions (after subsampling) for r_1 and PCR are similar to the normal distribution. The normality hypothesis cannot be rejected by the Kolmogorov-Smirnov test at the 5% significance level.

The main results of simulations for measurement uncertainties 0.1% and 1% and several relative errors up to 1% are given in Table 1. The acceptance ratio for all

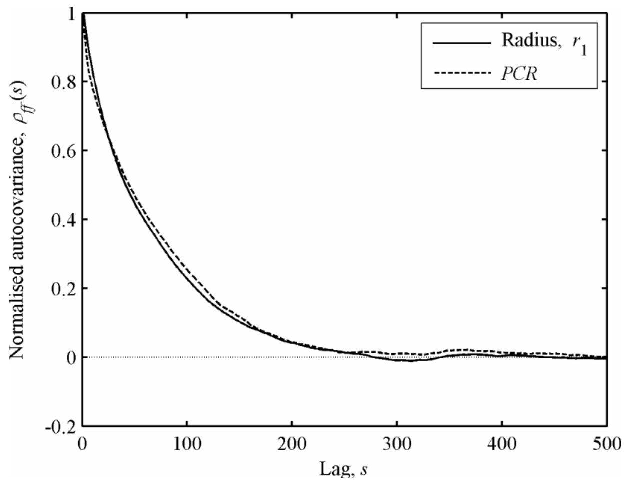

Fig. 4. Normalized autocovariance function ρ_{ff} against the lag s for r_1 and PCR

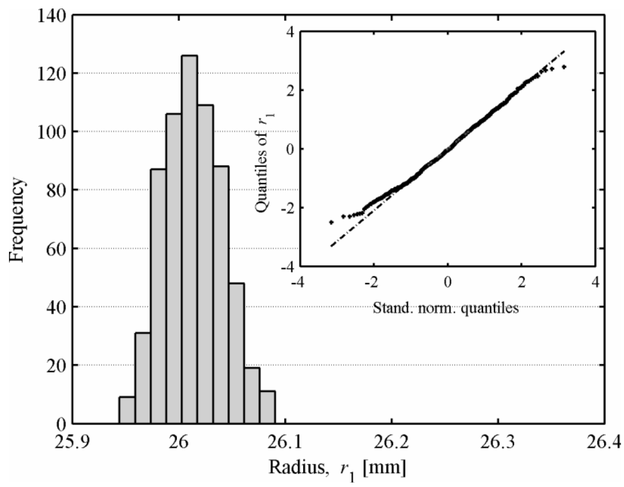

Fig. 5. Histogram of the a posteriori radius probability distribution. Smaller axes are Q-Q plot for the normalized radius probability distribution and standard normal distribution

simulations was about 39%. The parameters of the random walk in the simulations for measurement uncertainty $s_{rel} = 0.1\%$ are $k_{em\|} = 0.25$, $k_{em\perp} = 5 \cdot 10^{-3}$ and $k_r = 10^{-5}$. In case of $s_{rel} = 1\%$, the random walk parameters are $k_{em\|} = 0.25$, $k_{em\perp} = 5 \cdot 10^{-2}$ and $k_r = 10^{-4}$.

For all simulations, uncertainties $\sigma_{\bar{r}}$ and $\sigma_{\overline{PCR}}$ of the corresponding mean values of subsampled chains as estimators of the expected values are very low, what indicates that length of the chains was adequate. As expected, the measurement uncertainty has a negligible effect on the mean values of radius and PCR. The uncertainties of radius and PCR are proportional to the measurement uncertainty by a factor of about 1 for the radius and about 8 for

Table 1. The results of the MCMC simulations. The relative errors p_r and p_{PCR} are errors of the mean values with respect to the true values of radius and PCR, respectively.

s_{rel}	p_{rel}	$\bar{r}_1$ [mm]	σ_r [mm]	$\sigma_{\bar{r}}$ [mm]	$\overline{PCR}$ [$\mu\Omega$m]	σ_{PCR} [$\mu\Omega$m]	$\sigma_{\overline{PCR}}$ [$\mu\Omega$m]	τ_{ff} for r_1	τ_{ff} for PCR	p_r	p_{PCR}
0.1%	0%	26.014	0.028	0.001	21.624	0.193	0.008	123	133	0.05%	0.53%
0.1%	0.2%	25.991	0.027	0.001	21.600	0.187	0.007	139	146	0.03%	0.64%
0.1%	0.5%	25.961	0.027	0.001	21.548	0.188	0.007	128	133	0.15%	0.88%
0.1%	1%	25.910	0.027	0.001	21.477	0.187	0.009	154	167	0.35%	1.21%
1%	0%	26.023	0.204	0.005	21.593	1.173	0.031	48	38	0.09%	0.67%
1%	0.5%	25.975	0.211	0.006	21.530	1.188	0.032	50	38	0.10%	0.96%

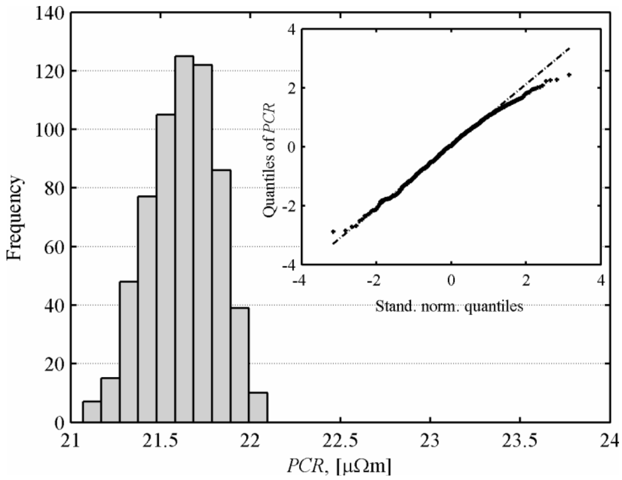

Fig. 6. Histogram of the a posteriori PCR probability distribution. Smaller axes are Q-Q plot for the normalized PCR probability distribution and standard normal distribution

PCR. Somewhat large ratio of the relative uncertainties of PCR and Z_{Tm} suggests that one can possibly find a better descriptor of the posterior distribution of μ_r and σ_t instead of the traditionally used PCR [2, 7]. The relative measurement error (due to parasitic capacitances and errors in coil parameters) has a more significant effect. The relative error in radius is for a factor 2–3 smaller than the measurement error, whereas the error in PCR is for a similar factor larger than measurement error. Therefore, any electronic instrumentation that realizes this measurement method should meet requirements on precision and accuracy of the impedance measurement at least as the targeted accuracy and precision of the tube inner radius.

4 CONCLUSIONS

We presented the stochastic formulation of the inversion of coil impedance measured at single frequency for determination of inner radius and electromagnetic properties of metal tubes. The approach is based on the Bayesian combination of the prior information on the tube properties, measured impedance and results of the impedance model. The result is obtained in form of the posterior distribution of the tube properties, which is sampled by means of Markov chain Monte Carlo method. We illustrated the method on several sets of synthetic measurement data. The simulations confirm the high correlation of the permeability and conductivity, and that the permeability-to-conductivity ratio can be used as a descriptor of their joint distribution. The requirements on electronic instrumentation and impedance measurement correspond to the targeted accuracy and precision of the tube inner radius. The stochastic formulation can accommodate different sources of the prior knowledge, e.g. specifications of the tube manufacturing process, results of the previous measurement runs, and results of other measurement methods.

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