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Particle Filters in Decision Making Problems under Uncertainty

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In problems of decision making under uncertainty, we are often faced with the problem of representing the uncertainties in a form suitable for quantitative models. Huge databases for the financial system now exist that facilitate the analysis of uncertainties representation. In portfolio management, one has to decide how much wealth to put in each asset. In this paper we present a decision making process that incorporates particle filters and a genetic algorithm into a state dependent dynamic portfolio optimization system. We propose particle filters and scenario trees as a means of capturing uncertainty in future asset returns. Genetic algorithm was used as an optimization method in scenario generation, and for determining the asset allocation. The proposed method shows better results in comparison with the standard mean variance strategy according to Sharpe ratio.

Key words: Uncertainty representation, Particle filters, Scenario trees

Čestični filtri u problemima odlučivanja s prisutnom nesigurnošću. U problemima odlučivanja u kojima je prisutna nesigurnost često se susrećemo s problemom predstavljanja nesigurnosti u obliku prikladnom za računalnu obradu. Analizu reprezentacije nesigurnosti danas nam olakšavaju velike baze podataka o financijskom sustavu. Kod upravljanja portfeljem potrebno je odlučiti koliko novca uložiti u pojedine dionice. U ovom radu predstavljen je proces odlučivanja temeljen na čestičnim filtrima i genetskom algoritmu. Pomoću razvijenog procesa odlučivanja izgrađen je sustav za dinamičko optimiranje portfelja. Čestični filtri i stabla scenarija predloženi su za predstavljanje nesigurnosti u budućim prinosima dionica. Genetski algoritam se koristi kao optimizacijska metoda u generiranju scenarija i za određivanje optimalnog portfelja. Predložena metoda uspoređena je sa standardnom mean-variance strategijom te prema Sharpeovom omjeru daje bolje rezultate.

Ključne riječi: predstavljanje nesigurnosti, čestični filtri, stabla scenarija

1 INTRODUCTION

The development and use of dynamic portfolio optimization algorithms is extremely important in financial markets. This is the result of a major growth of financial engineering, including the technological advances, globalization, increased competition, and ability to solve complex financial models [1]. The goal of portfolio optimization is to automatically determine the optimal percentage of the total investment value allocated to each asset in the portfolio [2]. Optimality is expressed in terms of return maximization or risk minimization. The core of a portfolio optimization problem is a good representation of uncertainty. Uncertainties should be represented in a form that reflects the reality and complexity of the financial system, but should also be simple enough for algorithmic implementation [3].

Uncertainty can be represented in a number of ways. One approach is to represent uncertainty by multidimensional continuous distributions or discrete distributions with large number of outcomes [4]. In both cases, the problem is how to estimate parameters of the distribution [5]. A naive method would consist of the estimation of parameters directly from the historical data. However, such an approach fails to take into account the fact that newer data has more influence on the parameters than older data. In line with that, it is important to note that the problem does not lie in modeling of historical data, but in predicting future uncertainty from the above mentioned data. The most popular approach to parameter estimation is that of Bayesian estimators, developed in [6], [7], and described in [8]. The idea of Bayesian inference is to combine prior information with sample returns. Besides parameter estimation, there is also a problem of selecting the right multivariate distribution, especially if statistical properties of uncertainty are time variant.

A different method for representing uncertainty is scenario trees. The goal of scenario trees is to represent the underlying uncertainty with a small set of discrete outcomes [1]. A scenario is a deterministic realization of all uncertain parameters. There are two approaches in generating scenario trees: simulation based and optimization based approach [9]. Simulation based approach is used in [10], [11], and [12]. Optimization based approach is introduced in [4] and used in [13]. The main idea is to generate a set of scenarios that matches some specified statistical properties of the underlying uncertainty. Those properties could be moments, co-moments, marginals, or any other relevant properties of the uncertainty. Scenario generation is done by solving an optimization problem where the goal is to minimize a measure of a distance between the statistical properties of constructed distribution and the statistical properties of the underlying uncertainty. The method can capture various kinds of uncertainties, but a realistic estimation of the statistical properties of the underlying uncertainty remains the biggest challenge in a good uncertainty representation.

Here we propose a method for uncertainty representation based on particle filters and scenario trees. Particle filter is used for estimation of the statistical properties of the underlying uncertainty in future asset returns. We have created a nonlinear model which exploits the known properties of asset returns. The parameters of the model are mean and volatility of returns, whereas with particle filter we maintain a sampled distribution of asset returns through the steps of prediction and correction. Higher moments, skewness and kurtosis, are estimated from the above mentioned distribution. Together with correlations between different assets, those properties form a set of statistical properties used for scenario generation. In scenario generation, a genetic algorithm was used as an optimization method. Based on the proposed uncertainty representation method, we have created a system for portfolio management. Generated portfolios frequently demonstrate higher returns than the ones based on a standard mean-variance strategy while maintaining the same amount of risk.

2 MODELLING APPROACH

2.1 Particle filters

Numerous problems in science require an estimation of the state of a certain system that changes over time by using a sequence of noisy measurements on the system. For example, in the financial system, it is a common task to estimate the expected value of an asset return, or the volatility of asset returns. The standard Bayesian approach to state estimation is to construct the probability density function (PDF) of the state based on all possible information, including the set of received measurements [14]. When certain constraints hold, the optimal solution is tractable. The Kalman filter and Hidden Markov model are two such solutions. When the optimal solution is intractable, there are various strategies that may help approximate the optimal solution. These approaches include extended Kalman filter, approximate grid-based filters, and particle filters.

Particle filters, introduced in [15], are a technique for implementing a recursive Bayesian filter by Monte Carlo simulations. The key idea is to represent the required density function by a set of random samples with associated weights, and to compute estimates based on these samples and weights. As the number of samples highly increases, this approximation becomes an equivalent representation to the usual functional description of the required PDF, and the particle filter approaches the optimal Bayesian estimate. For a more general description, see [16] and references therein.

To describe the algorithm, we introduce the following notation. The state vector x_k is assumed to evolve according to the following system model:

$$x_{k+1} = f_k(x_k, w_k) \tag{1}$$

where f_k is the system transition function and w_k is a zero mean, white noise sequence independent of past and current states. At discrete time steps, measurements y_k become available. These measurements are related to the state vector via the observation equation:

$$y_k = h_k(x_k, v_k) \tag{2}$$

where h_k is the measurement function and v_k is another zero mean, white noise sequence with known PDF, independent of past and present states and the system noise.

One of the particle filter algorithms proposed in the literature is sampling importance resampling (SIR) filter [15]. The assumptions required to use the SIR filter are very weak. We need to known state dynamics and measurement functions (1) and (2), and have to be able to sample realizations from the process noise distribution v_k and from the prior density $p(x_k|x_{k-1})$. Finally, the likelihood function $p(y_k|x_k)$ is necessary for pointwise evaluation (at least up to proportionality). A set of particles and weights $\{x_k^i, w_k^i\}_{i=1}^N$ is used to represent the sampled distribution $p(x_k|y_{1:k})$. The SIR filter uses resampling (elimination of particles that have small weights and concentrating on particles with large weights) at each discrete time step. An iteration of the SIR algorithm is given in Algorithm 1.

2.2 Scenario trees

The issue of modeling stochastic elements is critical to any stochastic optimization. A method to obtain the discrete outcomes for the random variables is referred to as scenario tree generation. We define a scenario as a deterministic realization of all uncertain parameters. Some scenarios may have identical history to some point. Because of that, scenarios are organized in a scenario tree (see Fig. 1). The scenario generation process should build scenarios that represent the universe of all possible outcomes – we

Algorithm 1 SIR Particle Filter: $\{x_k^i, w_k^i\}_{i=1}^N = \textbf{SIR}(\{x_{k-1}^i, w_{k-1}^i\}_{i=1}^N, y_k)$

Input: $\{x_{k-1}^i, w_{k-1}^i\}_{i=1}^N, y_k$
Output: $\{x_k^i, w_k^i\}_{i=1}^N$
for $i = 1$ **to** N **do**
  Draw $x_k^i \sim p(x_k|x_{k-1}^i)$
  Calculate $w_k^i = p(y_k|x_{k-1}^i)$
end for
Calculate total weight $t = \mathrm{SUM}(\{w_k^i\}_{i=1}^N)$
for $i = 1$ **to** N **do**
  Normalize: $w_k^i = t^{-1} w_k^i$
end for
$\{x_k^i, w_k^i\}_{i=1}^N) = \mathrm{RESAMPLE}(\{x_k^i, w_k^i\}_{i=1}^N)$
return $\{x_k^i, w_k^i\}_{i=1}^N$

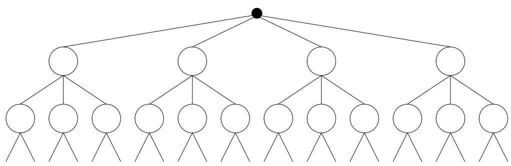

Fig. 1. An example of the scenario tree

want a representative set of scenarios. There exist different methods of scenario generation. The two most widely used ones are scenario reduction and moment matching [17].

The scenario reduction method is introduced and discussed in [18] and [19]. The goal is to eliminate scenarios that are similar or have a small probability. The method starts with a large number of scenarios, which usually result from a simulation. With the scenario reduction method, the goal is to represent the underlying distribution in an acceptable way with a reduced number of scenarios.

The second method of scenario tree generation is based on moment matching and is described in [4]. The starting point for generating the scenario tree is a description of the statistical properties of the underlying random variables. The procedure generates a scenario tree that matches those statistical properties as closely as possible. Generation of scenarios is an optimization problem where the objective function is the distance between statistical properties calculated from scenarios and specified statistical properties. If the distance is measured with a square norm, the following optimization problem needs to be solved:

$$\min \sum_i w_i(f_i(x,p) - SV_i), \ \sum_i p_i = 1, \ p_i \geq 0. \quad (3)$$

Minimization is done over vector x, which is a vector of outcomes of all underlying random variables in all scenarios, and p, which is a vector of probabilities of each scenario. $f_i(x,p)$ is a mathematical expression for calculating statistical property *i* in the scenario tree, and SV_i is the specified value of statistical property *i*. Weighting with w_i enables the emphasis of certain properties.

Since the described optimization problem is generally not convex, the solution is probably a local one. However, for most applications, it is satisfactory to have a scenario tree with properties equal to or close to the specifications. Solving of the optimization problem can be done in a number of ways, by using traditional non-convex optimization methods, or metaheuristics, like simulated annealing or genetic algorithm.

3 MODEL DESCRIPTION

The focus of this paper is the applying of a stochastic optimization method in portfolio management. Therefore, we present a model for obtaining an optimal asset allocation. To find an optimal set of weights of each asset in a portfolio, we need to represent the uncertainties from financial factors in a form suitable for algorithmic computation. We choose scenario trees as a means of capturing those uncertainties. In order to generate scenario trees, estimation of statistical properties of underlying random variables is needed. We propose particle filters for the estimation of relevant statistical properties. With this in mind, our portfolio management model consists of three independent parts:

1. Estimation of statistical properties of asset returns,
2. Generation of scenario trees,
3. Portfolio optimization.

Statistical properties of asset returns which we use are mean, standard deviation, skewness and kurtosis of the return distributions of each asset in our portfolio. The correlations between returns of different assets are also required. In order to make an estimation, the model of the financial system is developed, based on the known properties of asset returns. The estimation of the parameters of the model is done with particle filter algorithm because of the nonlinearity of the developed model.

The estimated statistical properties form the basis of the scenario generation method. We use the moment matching method described in [4]. In order to generate the scenario tree, we solve the optimization problem where the objective function is the distance between statistical properties calculated from scenarios and specified statistical properties. The solution of the resulting non-convex optimization problem is obtained from a genetic algorithm.

After generating the scenario tree, we can solve the deterministic equivalent of the stochastic asset allocation problem. The solution of the problem is a set of asset weights that maximize some utility function of wealth. Solving of the given optimization problem is done with the genetic algorithm.

The following subsections describe the parts of the model. Simultaneously, we demonstrate our approach on the example of the equity indexes of France, Germany, Japan, UK and USA in January 1975. With particle filters we estimate the distribution of returns of index values for February 1975, and then we generate scenarios that match the parameters of that distribution. It is important to note that we deal with logarithmic index returns, defined as:

$$r_k = \ln \frac{S_k}{S_{k-1}} \tag{4}$$

where S_k and S_{k-1} are current and previous index values.

3.1 Estimation of statistical properties of asset returns

In order to use particle filters for state estimation of a dynamic system, one has to build a model of the system. We use a different particle filter for different assets. Each particle filter uses mean and variance of returns of the asset as state variables. So, the state of the system at time step *k* is a vector

$$x_k = \begin{bmatrix} \mu_k \\ \sigma_k^2 \end{bmatrix} \tag{5}$$

where μ_k represents the mean of the asset returns and σ_k^2 is the variance of asset returns. The input to the system is the last known asset return r_k. The state vector x_k is assumed to evolve according to the following system model,

$$\begin{aligned} \mu_k &= \alpha\mu_{k-1} + (1-\alpha)r_k + \varepsilon_k \\ \sigma_k^2 &= \beta\sigma_{k-1}^2 + (1-\beta)r_k^2 + \eta_k \end{aligned} \tag{6}$$

where variables ε_k and η_k represent the additive Gaussian white noise. For technical reasons, samples from η_k which would result with negative σ_k^2 are ignored. Those equations follow the exponentially weighted moving average model. The output of the system is the estimated return in the $\hat{r}_{k+1}$ in the next time step. We propose the following distribution of $\hat{r}_{k+1}$,

$$f(\hat{r}_{k+1}|r_k) = \frac{C}{1 + \frac{(\hat{r}_{k+1}-\mu_k)^2}{\sigma_k^2} + \frac{((\hat{r}_{k+1}-\mu_k)^2-(r_k-\mu_k)^2)^2}{2\sigma_k^2}} \tag{7}$$

where *C* is the normalization constant.

As a result of the estimation procedure, we need estimates of the mean, standard deviation, skewness and kurtosis of the distribution $f(\hat{r}_{k+1}|r_k)$. Since particle filter maintains the distribution in a sampled form from one time step to another, estimates can be computed efficiently by using statistical estimators. There are numerous reasons for using particle filters in this particular task:

1. The system model is non-linear and we deal with a non-linear state estimation
2. The particle filter can represent the proposed distribution $f(\hat{r}_{k+1}|r_k)$ in case when the shape of the distribution is unimodal and when the shape is bimodal
3. The particle filter forms the distribution $f(\hat{r}_{k+1}|r_k)$ with importance sampling so that the estimates of its moments can be calculated efficiently by using a computer.

For the sake of the simplicity of the model, particle filters are used only in estimation of parameters of univariate distribution. Still, correlation coefficients are needed in the process of portfolio optimization. We find correlation coefficients by using statistical estimators from historical values on a time window of 60 months. The estimation problem is solved using an SIR particle filter described in Section 2. This filter uses the prior density as the importance density function. We use multinomial resampling for the resampling procedure. The quality of state estimation could be improved with other, more advanced methods, and it is a topic of an ongoing research.

The example of the distribution $f(\hat{r}_{k+1}|r_k)$ for the Japan equity index on February 1975 is shown in Figure 2. The comparison is made in Table 1. We notice that the statistical properties of this distribution differ from the properties obtained with estimation from historical data.

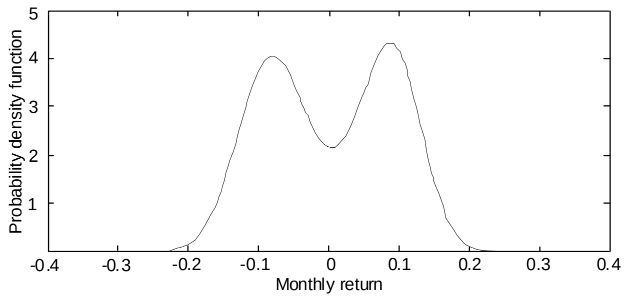

Fig. 2. Probability density function of estimated Japan equity index return for February 1975

Table 1. Comparison of statistical properties estimated with different methods

	Mean	Std. dev	Skewness	Kurtosis
Historical	0.009	0.063	-0.437	3.196
Particle F.	0.005	0.082	-0.084	1.576

3.2 Scenario tree generation

We use scenario trees for representing uncertainty in future asset returns. For the generation of the tree we use optimization method based on moment matching described in [4]. Parameters of the uncertainty distribution serve as an input to the scenario generation process. Optimization problem (3) is solved by the genetic algorithm. The output of the optimization process is the optimal set of scenarios organized in a tree, where the optimality is expressed in terms of the distance to the specified statistical properties. Naturally, the fitness function of the genetic algorithm is the distance between the properties. The genetic algorithm that solves the problem (3) uses: a rank fitness scaling, stochastic uniform selection, a modified Gaussian mutation and scattered crossover. The size of the population depends on the extent of the problem. In our example, we use 5 assets and 30 statistical properties. The usual choice for the number of scenarios, based on the discussion in [9], is 6 scenarios, which leads to 36 unknown parameters of scenarios (each scenario has a probability value and values for returns for each asset). For optimizing a function of 36 variables, we use a population of 250 candidates. To ensure that the solution found is indeed a global solution, we rerun the algorithm from different starting points.

For example, given the statistical properties in Table 2, we build a single period scenario tree that consists of six scenarios. With genetic algorithm, we obtain a perfect match. A set of six generated scenarios is given in Figure 3 where the return of each asset in every scenario is presented.

Table 2. Statistical properties of index returns for February 1975. All properties but correlations are estimated with particle filters

	Mean	Std. dev	Skewness	Kurtosis
USA	-0.022	0.035	0.153	3.227
UK	-0.048	0.104	0.064	1.355
Japan	0.005	0.082	-0.084	1.576
Germany	0.007	0.074	-0.112	1.633
France	-0.017	0.047	-0.086	2.844

Correlations of returns

	USA	UK	Japan	Germany	France
USA	1	0.508	0.320	0.337	0.398
UK		1	0.372	0.313	0.588
Japan			1	0.471	0.319
Germany				1	0.608
France					1

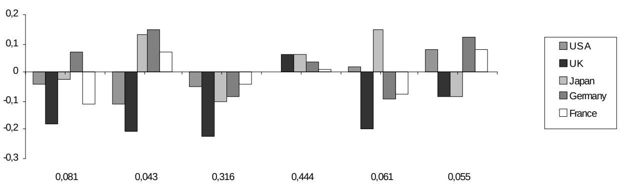

Fig. 3. Generated scenarios (with probabilities) for which the distribution properties match the properties in Table 2

3.3 Portfolio optimization

We implement the method for representing uncertainty in the example of portfolio management. In this application, the goal is to maximize the sum of the expected utility of wealth subject to budget constraints.

The optimization problem can be formulated in a following manner:

$$\begin{gathered}\max EU = \sum_s \pi_t^s f(w_t^s) \\ \text{s.t.} \quad \begin{cases} w_t^s = \sum_{i=1}^{I} r_{i,t-1}^s \delta_{i,t-1}^s \\ \sum_i \delta_{i,t}^s = 1 \\ \sum \pi_{1,t}^s = 1 \end{cases}\end{gathered} \tag{8}$$

where π_t^s is the probability that scenario s occurs at time step t; w_t^s is the wealth at time step t under scenario s; $r_{i,t}^s$ is return of asset i at time step t under scenario s; $\delta_{i,t}^s$ is the weight for asset i at time step t under scenario s. The optimization problem (8) is a deterministic equivalent of the underlying stochastic problem which we solve with genetic algorithm. The output of the optimization process is the set of weights of assets in the optimal portfolio for which the maximum of expected utility is obtained. As a fitness function we use negative utility, since the goal of genetic algorithm is function minimization. Compared to the size of the optimization problem (3), the problem (8) is simpler and easier to solve. For example, when there are 5 assets in a portfolio and 6 scenarios in a tree, problem (3) finds 36 variables, while problem (8) finds only 5 of them. For that reason, we use a population size of only 60 candidates.

Given the scenarios in Figure 3, we find the optimal weights using the logarithmic utility function. The results are reported in Table 3 and compared to the classical mean variance analysis. The difference due to different estimation methods used clearly exists.

Table 3. Weights of the optimal portfolio calculated with our model using compared to the classical mean variance portfolio

	USA	UK	Japan	Germany	France
Scenario trees	0.325	0	0.080	0.517	0.078
Classic mean-variance	0.320	0	0.462	0.218	0

4 RESULTS

The experiments are based on the data set from MSCI (Morgan Stanley Capital International). We use the total return equity indices of France, Germany, Japan, UK and the USA. Equity returns are based on the month-end

US-dollar value of the equity index for the period between January 1970 and December 2000. To verify the performance of the different portfolio models, the weights from each model are determined, and the return from holding this portfolio in the next month is calculated. In case of models that create historical estimates of parameters, those estimates are based on a window of 60 months. In each case, the out-of-sample period is from January 1975 to December 2000. Table 4 shows summary statistics for the monthly returns on the five indices and the correlations of the returns.

Table 4. Summary statistics of the data from January 1970 to January 2000

	Mean	Std. dev.
USA	0.0049	0.0446
UK	0.0060	0.0717
Japan	0.0117	0.0658
Germany	0.0065	0.0603
France	0.0060	0.0694

Correlation coefficients

	USA	UK	Japan	Germany	France
USA	1	0.5171	0.2699	0.3598	0.4405
UK		1	0.3708	0.4393	0.5440
Japan			1	0.3889	0.3922
Germany				1	0.6136
France					1

To assess the performance of the different portfolio models, we calculate the average out of sample means, volatilities and Sharpe ratios of each strategy – the mean-variance analysis, scenario trees and scenario trees with particle filters. The results are reported in Table 5. Compared with mean-variance analysis, in which the historical mean returns are taken to be the estimator of the expected returns μ, the portfolios constructed by using the model for representing uncertainty showed higher returns while maintaining the same level of volatility. In the case in which scenario trees were generated without particle filters, the key difference was the utility function used, which gave more balanced portfolios. When particle filters were used as estimators of the statistical parameters of future returns, portfolios with even higher returns were generated. Both methods clearly outperform the traditional method.

Table 5. Experiment results

	Mean	Std. dev.	Sharpe ratio
Mean-variance	0.0087	0.0465	0.1871
Scenario trees	0.0091	0.0458	0.1987
Scenario trees with PF	0.0103	0.0483	0.2133

5 CONCLUSION

In this paper a method for the uncertainty representation based on particle filters and scenario trees has been developed. Particle filters were used for estimation of statistical properties of uncertainty from historical data, and scenario trees were used as a model for uncertainty representation. The described method was included into the decision making process for dynamic portfolio optimization. Uncertainty in future asset returns, being the main problem in portfolio optimization, was captured by the proposed method. By using obtained uncertainty representation, portfolio optimization was performed by maximization of logarithmic utility function of the wealth. For the purpose of the above mentioned maximization, and for the estimation of the parameters of the scenario trees, a genetic algorithm was used.

The described method was validated by the use of the MSCI data sets. The method showed better results in comparison to the standard mean variance strategy according to Sharpe ratio. Generated portfolios frequently demonstrate higher returns than Markowitz optimal portfolios while maintaining the same amount of risk.

Future research in this area should continue along several dimensions. Firstly, in this research, a single scenario trees was used. A combination of multiple scenario trees and particle filters could result in some new enhancements. Secondly, different utility functions in portfolio optimizations could create valuable progress. Thirdly, there is no fundamental reason why 1,000 or 10,000 scenarios cannot be created by parallel and distributed computers.

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