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Conceptual Model and Evaluation of Design Characteristics in Product Development

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The paper presents an original model of evaluation in the design process, especially developed model transformation features in the conceptual design by an approach for establishing effective mapping and transformation process between product properties and domain requirements from different domains. Model transformation feature in the conceptual process of designing and the valuation model features of products is shown and described mathematically. The proposed meta-level consists of general discrete mathematical models and combinatorial representation. Special mathematical basis for the evaluation of models and combinatorial analysis for each alternative solution is derived from the theory of graphs. All variant solutions must be evaluated according to the range of the set criterions as list of requirements. Using originally developed conceptual model and evaluation model, assumptions were made for the purpose of defining the weight matrix in order to avoid errors in selecting the final solution variants. A short view of the application is given in the example.

Keywords

Conceptual design
Evaluation model
Product development
Theory of design

Model koncipiranja i vrednovanja konstrukcijskih značajki u razvoju proizvoda

Izvornoznanstveni članak

Rad prezentira originalan model vrednovanja u procesu konstruiranja, a posebno je razvijen model transformacije značajki u koncipiranju kroz uvođenje procedure preslikavanja i procesa transformacije između značajki proizvoda i domena zahtjeva iz različitih domena. Model transformacije značajki u procesu koncipiranja i model vrednovanja značajki proizvoda prikazan je i matematički opisan. Predložena meta-razina sastoji se od općih diskretnih matematičkih modela i kombinatorne reprezentacije. Posebna matematička osnova za ocjenu modela i kombinatorne analize za svaku varijantu rješenja izvedena je iz teorije grafova. Sve varijante rješenja moraju biti ocijenjene prema rasponu skupa kriterija kao liste zahtjeva. Korištenjem izvorno razvijenog modela koncipiranja i modela vrednovanja, ostvarena je pretpostavka za objektivno utvrđivanje matrice težina u cilju izbjegavanja grješke u odabiru konačne varijante rješenja. Skraćeni prikaz primjene dan je u primjeru.

Ključne riječi

Koncipiranje
Model vrednovanja
Razvoj proizvoda
Teorija konstruiranja

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1. Introduction

Product development as a necessary precondition of competitiveness in the market requires from producers a growing number of newly developed products, shorter development period, quality and price that is demanded by an ever-gowing fastidious market. Generating large numbers of principle solution variants in the conceptual phase of a product as a technical system, it is necessary to evaluate every single solution in respect to criteria defined by design task. In the conceptual phase, the designer has to evaluate acceptability of a selected solution with valid arguments, because the result of evaluation directly influences further product development. Models of conceptual solutions in papers have been given by more authors with relatively poor informatics content and without showing mathematically formalized procedures of design characteristics transformation in the conceptual phase (*KF*). This is the reason why it is needful to set a mathematically formalized model of conceptual design and model of design characteristics evaluation. This is necessary to obtain diversity in application, objectivity and comparability of results. The work [1] gives recommendations for goodness evaluation of design solutions; and papers [2-3] are serious contribution in this area. In the popers of [4−7] a display of result of research in the area of design characteristics evaluation are given with emphasized stage of formalism. Late in 1996 in the USA the first workshops in the area of DBD (Decision Based Design) took place. Development of product and

Symbols/Oznake

U	- universal set–as axiomatic approach designing - univerzalni skup - kao aksiomatski pristup konstruiranju
P	- predicate of the element of set U - predikat elemenata skupa U
u	- working steps of design process - radni koraci procesa konstruiranja
U'	- subset of U - podskup skupa U
KF	- conceptual phase in design proces - faza koncipiranja u procesu konstruiranja
ΔR	- domain of residum - ostatak domene
KE	- set of design elements - skup konstrukcijskih elemenata
ΔP_{KB}	- domain of unused properties of design elements - domena neiskorištenih svojstava konstrukcijskih elemenata
KZ	- requirements of customer - zahtjevi kupca (korisnika)
BD_{n}	- requirements of biological domain - zahtjevi biološke domene
FZ	- functional requirements - funkcionalni zahtjevi
ZF_{z}	- requirements of physical domain - zahtjevi fizikalne domene
ZK_{z}	- requirements of design domain - zahtjevi konstrukcijske domene
V_{r}	- variants solutions - varijantna rješenja
$\chi_{\mathrm{ZK}\rightarrow\mathrm{KE}}$	- characteristic functions in discreet matrix - karakteristična funkcija u diskretnim matricama
v_{ij}	- properties of variants solutions - svojstva varijanti rješenja
$\mathbf{M}$	- matrix principle variants solutions - matrica principijelnih varijanti rješenja
$e_{\mathrm{KE}j,\mathrm{p}}$	- properties of available j-th design elements - svojstva varijanti rješenja
$\wedge$	- the conuction matrix - matrica konjukcije
φ	- function of transformation requirements in model - funkcija transformacije zahtjeva u modelu
$[\mathbf{B}]_{J\times K}$	- operator matrix of linear transformation of functional requests - matrica operator linearne transformacije funkcionalnih zahtjeva
δ_{jk}	- Kronecker symbol - Kronecker-ov simbol
Δ	- deviation of mapping in transformation process - odstupanje u preslikavanju kroz proces transformacije
$inv.inc\ (F)$	- invariantainconsistency of variant solutions - invarijanta inkonzistentnosti varijanti rješenja
$[\mathbf{C}]_{\mathrm{k}} = \tilde{\mathbf{C}}$	- matrix of criterions - matrica kriterija
$[\mathbf{R}]$	- matrix product variants solutions and criterions - matrica relacije uređaja varijanti i kriterija
$\mathbf{W}$	- weight matrix - matrica težina
$Q^{w}_{rk}\left(o^{w}_{nrk}\right)$	- matrix of appreciation weights - matrica ocijenjenih težina
G_{i}	- symbol of graph - oznaka za graf
$[\mathbf{B}]$	- incidence matrix - matrica incidencije
$\mathbf{X}$	- vector of the potential - vektor potencijala
$\mathbf{F}$	- function of flow - funkcija toka

process makes up one such area of human activities with constant important decision-making. Therefore, research in this area and the system of approach to evaluation in the conceptual phase of technical product, makes possible the growth of objectivity level in the argumentative selection of variant conceptual solution as an acceptable solution. Definition and implementation of evidence for a formalized model of conceptual design and for a system of criterions are the principle for description of developed evaluation model based on the mathematical model application of theory of graphs. Axiomatic design theory [7], gives a rational base for model definition of conceptual product design as a technical system. On this basis, with simultaneously use of elements of propositional and predicative logic, algebra judgment, vector algebra, matrix calculation and theory of graphs, in this work the synergy of conceptual model, decision about weight criterion values and evaluation model has been realized. Use of theory of analogy between generated different concepts, which are the group connectional solutions, provides a defining of systems of criterions and use of evaluation model. The represented algorithm of evaluation procedure completely adds on to the model of conceptual design. A model of decision about values for every criterion as the level of design characteristics fulfilment for a group of generalized variant solutions.

2. Model of transformation design parameters in Conceptual Design

The aim of contractual research is a method of actions and instructions which allows us to mathematically describe the activity of design. The intention of this paper was to show, in the larger or lesser level, formalized algorithms of evaluation with the goal of finding appropriate regularity for generating of principle solution variants in some design task. Process planning in the solution of constructional task performs analyses and syntheses procedures of characteristics of products as the technical system from the list of requirements to generated conceptual solutions. A conceptual solution represents generated answers from conceptual process on contractual tasks in surroundings of technical system. In that process of transformation, from an idea to a principal solution within the technical system, there is of interaction between the significance of a technical system and the process of a concept which generates principal solution variants. In this way, we achieve that with an approach of concept that a product is a technical system with different constructional tasks can be solved inside the defined structure of technical system and modelled conceptual algorithm. Such interaction develops by analysis and synthesis in relation to the constructional characteristics of the defined constructional task and characteristics of technical system. The technical system with its own structure has a crucial influence on the constructional solution because it is necessary to achieve the solution coordinated with surroundings as a system. Use of axiomatic design in product development according to [7] is the assumption achieved for mathematical formalization of a design process. With the model of transformation requirements in conceptual design and model of evaluation we wish to set, prove and show a formalized concept mathematical model. Since, in a conceptual phase, we generate the functional structure of principal solution variants of specified contractual task, analysis of copying is necessary and demands transformation in conceptual model and defining relations within conceptual model with the aim of achieving the most successive, complete and with all the technical system integrated concepts of a principal solution variant. Applying a set theory, mathematical logic and statement algebra the conceptual model in this work shows interactions between relations in a conceptual system for generating principal variants of solution defined by the contractual task. The mathematical notion of setting is suitable for describing the relations between variables within a conceptual model. Let's define design process as a set U (universal set), and its elements as sub sets, and as such let them represent working steps of design process as axiomatic approach. Let us assume that every sub-set of design set is part of the design process in solving a design task; this represents the working unit and allows separate analyses. The universal set must satisfy two requirements:

- It must have working steps of design process in solving the defined design task.
- It must not have unnecessary elements irrelevant for solving the design task.

Then design process can be shown as:

$$U = \left\{ u_1, \ldots, u_n \right\}. \tag{1}$$

With predicate P of element set U (sub set of set U). We'll mark the property of elements of U. Then with the predicate P elements of set U are marked as working steps of design, unambiguously defining the elements of U, and set U becomes universal set of design process as a system. By defining the predicate P with elements from U universally conceder all elements that have property P then we can write:

$$U = \left\{ u \middle| P(u) \right\}. \tag{2}$$

According to that, predicate P is axiomatically defined as:

$$\left\langle u \mapsto \textit{working steps of design process} \right\rangle. \tag{3}$$

Then the set U as a design process is mathematically formalized and completely defined. Conception is element of set U; the concept is an integral part of design process in solving design task. With this approach concept as element of set U we mark it as U' like as conceptual phase (*KF*) in the design process. Then it is U' real sub set of set U and then goes:

- Ire flexibility $\rightarrow U' \not\subset U'$ (4)
- Asymmetric $\rightarrow U' \subset U \wedge U \not\subset U'$ (5)

Structure and elements of a technical system have a direct influence on the design process, and with that in bound and on concept as its integral part, which is design solution a resulting product. A conceptual model as a part of design process in action of copying and transformation demands an ordering party through domains of technical system in set of design significances demands represents transformation system. Elements of technical system are defined as domains which are the starting point for solving the design task. Depending on different design task and difference of functional demands, different domains of technical system are turned on during conducting of conceptual model. For further analysis, we presume that a technical system consists of a domain that we can lead as physical domain, biological domain, chemical

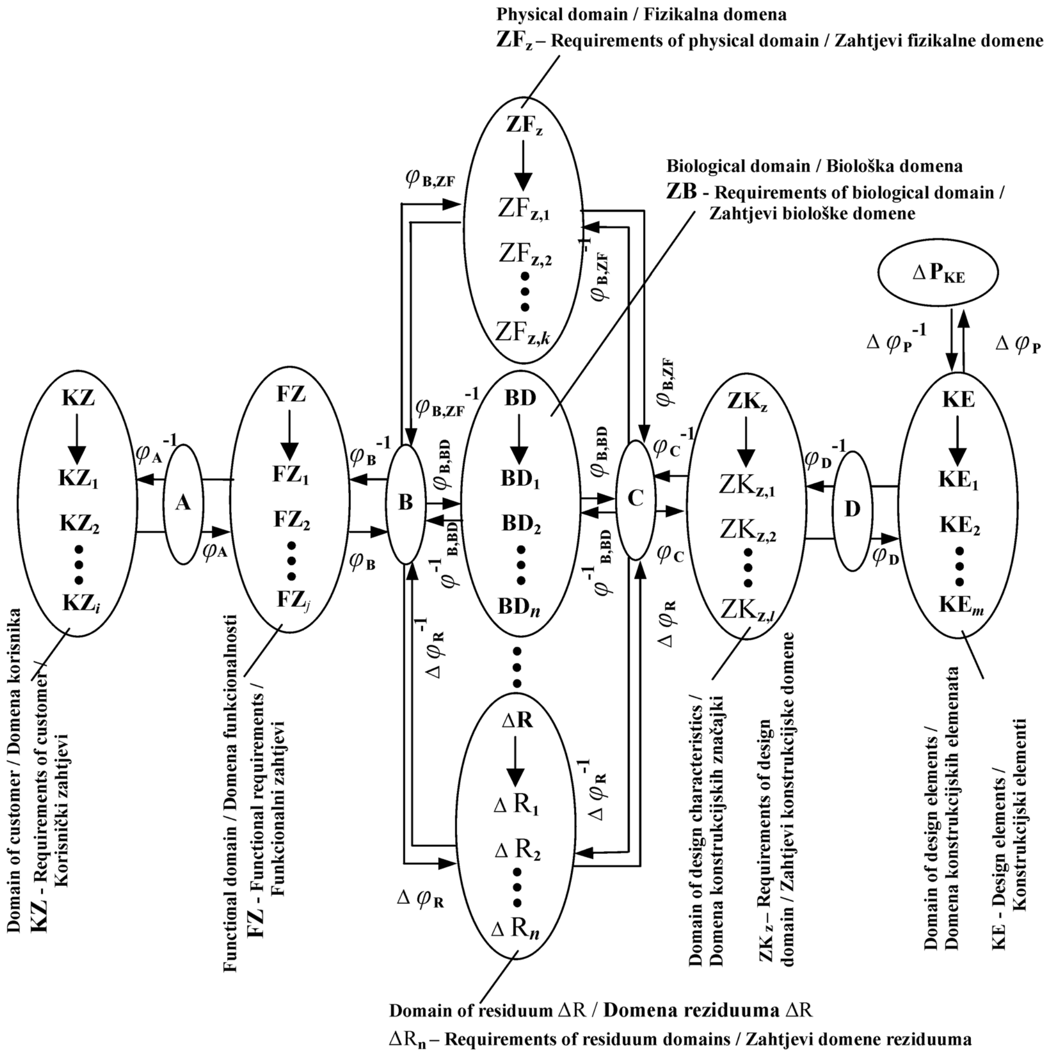

Figure 1. Review transformation of requirements in the conceptual model of product development as technical system

Slika 1. Prikaz transformacije zahtjeva u modelu koncipiranja razvoja proizvoda kao tehničkog sustava

domain, sociological domain and so on … ΔR domain (Figure 1, different products require different domains, which is not the limit, but the advantage was presented by the model) [8]. In this kind of domain introductions of technical system, the intention is not to determine all domains. It is important to point to their existence and dynamical change on structure and content. In that way, different design tasks can be determined and can present the answer in described need. In that sense ΔR presents residuum domain and all contents of technical system that are not like the domains explicitly provided, but due to content of design task are activated on the aim of its solution. This content results in a new product need. This kind of model can be seen in Figure 1. It is important to emphasize that the functional requests are not only just physical, but in their difference they belong to other domains. Introducing ΔR domain it is possible to identify different functional requests by their content and scientific-specialized association in function of defined design task. The content of ΔR domain is defined by the difference of design tasks and derived functional requests. In the following statement, on an example of physical domain, the model of requests identification from the functional domain is described. Analogous, it also applies to other elements from residuum domain. In the process of design characteristics identification, using the domain of design elements, a domain of *KE* design elements and a domain of unused (properties) predicates (ΔP_{KE}) of partial design elements from a domain of design elements are defined. A domain of design elements is a separate system, defined by its own structure and by the elements as a subsystem. Every design element is defined by its own functional structure, predicate, shape etc. and as such, it presents its own technical system. It is a sub set of an entire technical system from all design elements. In the domain generation process of principal solution variants (ΔP_{KE}), tracking of partial design elements and usability of their characteristics in the realization of requests of design characteristics is allowed. With the reversible method it is possible to realize expected or necessary improvement of principal solution variants toward the grade of predicate utilization of design elements. These elements make a functional structure of generated concepts. The conceptual model is defined by this expression:

$$U' = KF = \{KZ, FZ, ZF_z, \Delta R, ZK_z, KE, \Delta P_{KE}\} \quad (6)$$

$$\varphi_A, \varphi_A^{-1}, \varphi_B, \varphi_B^{-1}, ..., \varphi_D, \varphi_D^{-1} . \quad (7)$$

These functions are unambiguous and inverse. They present the analysis of transformation process of identification requirements for product, as the technical system in a conceptual model. A function $(\varphi_D, \varphi_D^{-1})$ is the iterative functions of model realization of principal solution variants as the system $V = \{V_1,..., V_r\}$ and presents a synthesis of a conceptual model. Applying the axioms of union for the conceptual set of any random production elements *x, y, z, t* sub sets *KZ, FZ, ZF$_z$, ZK$_z$* and *KE*, we can write:

$$\forall x \exists y \forall z \left[z \in y \Leftrightarrow \exists t (t \in x \wedge z \in t) \right] . \quad (8)$$

Then *KF* is an undivided set and becomes the union:

$$KZ \cup FZ \cup ZF_z \cup ZK_z \cup KE \Rightarrow$$

$$\{KZ, FZ, ZF_z, ZK_z, KE\} = \bigcup_{i=1}^{5} = KF . \quad (9)$$

Which means that for any set *KZ, FZ, ZF$_z$, ZK$_z$* and if KE exists, a set of elements that are the sub set of elements *KZ, FZ, ZF$_z$, ZK$_z$* and *KE* also exists, then their union is the *KF* set.

For sets *KZ, FZ, ZF$_z$, ZK$_z$* and *KE* we say that they are disjointed sets and then we have:

$$KZ \cap FZ \cap ZF_z \cap ZK_z \cap KE = \varphi , \quad (10)$$

$$\left| \bigcup_{i=1}^{5} S_i \right| = \sum_{i=1}^{5} |S_i| \in KF \subseteq U$$
$$\forall S \to KZ, FZ, ZF_z, ZK_z, KE . \quad (11)$$

Therefore, by concept *KF* a set of principal solution variants *V* is defined:

$$V = \{V_1,..., V_r\}. \quad (12)$$

The domain of design *KE* elements, with the iterative copying function $(\varphi_D, \varphi_D^{-1})$, forms a set of solution variants:

$$ZK_z xKE = \left\{ (ZK_{z,i}, KE_j) \middle| \begin{matrix} ZK_{z,i} \in ZK_z \wedge \\ KE_j \in KE; \\ i = 1,.l, j = 1,.m \end{matrix} \right\} \Rightarrow V_r . \quad (13)$$

In that way we make a copy of ZK_z set on the KE set. ZK_z set is a domain, and *KE* set is a co-domain. Characteristic functions (known in discreet matrix) $\chi_{ZK_z \mapsto KE}\left(P\left(D\left(ZK_z\right)\right)\right)$ enables testing if some element of set satisfies *KE* predicates (characteristics) of ZK_z set for *r*-th solution variant of *KF* subset from a *U* set:

$$\chi_{ZK_z \mapsto KE} : U \to \{0,1\} \Rightarrow$$

$$\chi_{ZK_z \mapsto KE} \begin{pmatrix} (ZK_{z,1},...,ZK_{z,l}) \\ \mapsto (KE_1,...,KE_m) \end{pmatrix} == \begin{cases} 1 \to V_r \subseteq KF \\ 0 \to V_r \not\subset KF \end{cases} . \quad (14)$$

Applying algebra judgment as the intuitive statements for characteristic control in a concept of a set of solution variants, it is possible to examine a transformation of a ZK_z set, using *KE* set in a set of variant solutions $V_r \subseteq KF \subseteq U$. Every variant solution must be consistent with all the elements of ZK_z set, modelled by the *KE* set elements. Then, a set of variant solution predicates V_r defined on the ZK_z set, in relation to the *KE* set is defined by this expression:

$$P(ZK_z \to KE) = \{P_i(ZK_{z,i} \to KE)\}; i=1,...,l . \quad (15)$$

Every *r*-th solution variant of the set of solution variants V_r is defined by a predicate set. Consequently, it is possible to write a set of acceptable solution variants V_r as the equation (16):

$$V_r\left(P\left(ZK_{z,l}\right)\right) = \left[V_r\left(P_i\left(ZK_{z,i} \mapsto KE\right)\right)\right]_{1\times r}; i = 1,...,l =$$
$$= \begin{Bmatrix} V_1\left(P_1\left(ZK_{z,1} \mapsto KE\right),...,P_l\left(ZK_{z,l} \mapsto KE\right)\right) \\ \cdot \\ V_r\left(P_1\left(ZK_{z,1} \mapsto KE\right),...,P_l\left(ZK_{z,l} \mapsto KE\right)\right) \end{Bmatrix} \quad (16)$$

For variant solution V_1 sing applied bisection, we define:

$$V_1\left(P_1\left(ZK_{z,1}\right),..,P_l\left(ZK_{z,l}\right)\right) = \begin{bmatrix} P_1\left(ZK_{z,1}\right) \\ \cdot \\ P_1\left(ZK_{z,1}\right) \end{bmatrix} \left[KE_1 \cdot KE_m\right], \quad (17)$$

$$V_1\left(P_1\left(ZK_{z,1}\right),...,P_l\left(ZK_{z,l}\right)\right) =$$
$$= \begin{bmatrix} P_1\left(ZK_{z,1}, KE_1\right) & \cdot & P_1\left(ZK_{z,1}, KE_m\right) \\ \cdot & \cdot & \cdot \\ P_l\left(ZK_{z,l}, KE_1\right) & \cdot & P_l\left(ZK_{z,l}, KE_m\right) \end{bmatrix} , \quad (18)$$

$$V_1\left(P_1\left(ZK_{z,1}\right),..,P_l\left(ZK_{z,l}\right)\right) = \begin{bmatrix} v_{11} & \cdot & v_{1m} \\ \cdot & \cdot & \cdot \\ v_{l1} & \cdot & v_{lm} \end{bmatrix} . \quad (19)$$

The conceptual structure of solution variant V_1 in expression v_{ij}, $i=1,...,l$; $j=1,...,m$ created with modeling of $ZK_{z,i}$ characterised by KE_j element. Every KE_j element from the KE set has n elements with its predicates; therefore it is a subset in a KE set. This way the possibility of combinatorial counting in the modeling of principal solution variants is insured. From the disjointed characteristic of ZK_z and KE sets, and independent of KE_j; $j=1,...,m$ elements, we use restrictive criterion. Therefore, different requests of design characteristics could not be modeled by placing the same m-th design elements, instead of different design elements in the aim of making solution variants. Restrictiveness defines only diagonal components of conceptual matrix of solution variants V_1 , and others are null. Then the copying iterative function is defined by modeling of principled solution variants with the following expression:

$$\varphi_D : P_i\left(ZK_{z,i}\right) \mapsto KE_j\left(e_{KEj,p}\right); \quad i=1,...,l;\ j=1,...,m;\ p=1,...,n. \tag{20}$$

Predicate P_1 of design characteristic $ZK_{z,i}$ can be copied at any p-th element ($e_{KEj,\,p}$) from the j-th KE_j set. Every predicate has at its disposal $m\times n$ iterations for a verification from which ($e_{KEj,\,p}$) design element of j-th KE_j set is fitted predicate accomplished. Regardless of where “to send” $ZK_{z,1} \equiv P_1$, we can send the element $ZK_{z,2} \equiv P_2$ to $m\times n$ methods, etc. up to $ZK_{z,l} \equiv P_l$. In equation (16), every matrix component becomes sub matrix with n components for l-th predicate. To every matrix component in equations (17, 18, 19) a characteristic function is assigned for a function $\varphi_D\left(P_i \mapsto KE_j\left(e_{KEj,p}\right)\right)$:

$$_D\chi_{\varphi\left(P_i \mapsto KE_j\right)} : U \to \left\{ \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} true = i \\ false = l \end{Bmatrix} \right\} \Rightarrow \begin{cases} 1 \to \varphi \subset V_r \\ 0 \to \varphi \not\subset V_r \end{cases} \tag{21}$$

All unused predicates of particular design elements make a predicate domain $\Delta P_{KE} = \sum_{j=1}^{m} P\left[KE_j \sum_{p=1}^{n} P\left(e_{KEj,p}\right)\right]$.

Modeling matrix of principle solution variants V_1,..., V_r determined by a set of predicates P_1,..., P_l by copying iterative function (φ_D) in a set of design elements $KE_j(e_{KEJ,\,p})$, $j=1,...,m$; $p=1,...,n$ becomes:

$$\mathbf{M} = \left[m\right]_{r\times l} = \begin{bmatrix} V_1\left(m_{11},...,m_{1l}\right) \\ \cdot \\ V_r\left(m_{r1},...,m_{rl}\right) \end{bmatrix}. \tag{22}$$

The conceptual matrix becomes a matrix of l-th predicate incidental for every r-th principled solution variant of the m-th KE set elements. Every matrix row component for individual solution variant presents combinations of nth set elements for a set of predicates P_1,..., P_l: $KE_j(e_{KEJ,\,p})$, $j=1,...,m$; $p=1,...,n$. Used intuitive logical attributes are term and assessment. With the logical conjunction operation over all FZ set elements in relation to ZF_z set elements, we perform identification and implement copying function with the aim of prove copying. It is possible to define conjunction operation like this: “conjunction value is true if and only if all conjunction elements are true”. Therefore:

$$\chi_{FZ \mapsto ZFz} : U \to \left\{ \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \Leftrightarrow \begin{Bmatrix} true = i \\ false = l \end{Bmatrix} \right\} \Rightarrow \begin{cases} 1 \to \quad FZ \mapsto ZF_z \\ 0 \to FZ \mapsto ZF_z = \varphi \end{cases} \tag{23}$$

Then the conjunction matrix of copying characteristics is determined by:

$$\left[\wedge\left(FZ \mapsto ZF_z\right)\right]_{j\times k} = \begin{bmatrix} FZ_1kZF_{z,1} & \cdot & FZ_1kZF_{z,k} \\ \cdot & \cdot & \cdot \\ FZ_jkZF_{z,1} & \cdot & FZ_jkZF_{z,k} \end{bmatrix} = \begin{bmatrix} i_{11} & l_{12} & \cdot & l_{1k} \\ l_{21} & i_{22} & \cdot & l_{2k} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ l_{j1} & l_{j2} & \cdot & i_{jk} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \cdot & 0 \\ 0 & 1 & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdot & 1 \end{bmatrix} \tag{24}$$

Every second value of any proposition $\wedge_{j\times k}$ that differs from $i \vee 1$, discards identification FZ with ZF_z. In that way, the copying process is unambiguously defined, for the purpose of unambiguous and independent identification of all elements from FZ set with the elements of ZF_z set. Simultaneously, we realize a procedure of independent joining. A conjunction esures that one element of ZF_z set, using the copying function, identifies with only one element of ZF_z set. In this way, we avoid characteristics opposing. Let us observe two judgments, the functional requests of FZ set and requests of physical characteristics ZF_z set:

$$\left.\begin{matrix} E\left(FZ\right) \to "FZ_i; i=1,...,j" \\ H\left(ZF_z\right) \to "ZF_{z,i'}; i'=1,...,m" \end{matrix}\right\} \to \to E\left(FZ_i\right) \Rightarrow H\left(ZF_{z,i'}\right). \tag{25}$$

The predicate from FZ set is a functional request for all FZ_i from that set. Predicate from ZF_z set is physical characteristic request for all $ZF_{z,i'}$ from that set. Accordingly, for every $FZ_i \in FZ$ predicate of functional request applies, and for every $ZF_{z,i'} \in ZF_z$ predicate of physical characteristic. Then implication judgment (25) is true only if:

$$E(FZ_i) \Rightarrow H(ZF_{z,i'}) \Rightarrow \forall i = i'; (\forall FZ_i \in FZ \mapsto \forall ZF_{z,i'} \in ZF_z). \quad (26)$$

From condition $E(FZ_i) \Rightarrow H(ZF_{z,i'})$ follows:

$$\forall i = i'; (\forall FZ_i \in FZ \mapsto \forall ZF_{z,i'} \in ZF_z). \quad (27)$$

In that case (26) is a true judgment. If we introduce a second quantification and then write:

$$\exists i \neq i'; (\exists FZ_i \in FZ \mapsto \exists ZF_{z,i'} \in ZF_z) \quad (28)$$

$$E(FZ_i) \Rightarrow H(ZF_{z,i'}) \Rightarrow \exists i \neq i'; (\exists FZ_i \in FZ \mapsto \exists ZF_{z,i'} \in ZF_z). \quad (29)$$

Using condition $E(FZ_i) \Rightarrow H(ZF_{z,i'})$ is possible to write:

$$\exists i \neq i'; (\exists FZ_i \in FZ \mapsto \exists ZF_{z,i'} \in ZF_z). \quad (30)$$

Then (29) is untrue judgment. In this case placed, conducted and proved transformations that are not necessary (but can be dependent on design task), all functional requests of physical characteristic. Residuum domain provides the possibility for other testing by forming of new equations (23, 30), applying analogy like one for physical domain, as long as all-functional requirements have not obtained their own belonging. We have proved interacting independence and unambiguousness of copying function φ_B of conjunction operation:

$$\varphi_B : [FZ]_j \xrightarrow{f} [ZF_z]_k \Rightarrow [FZ]_j \xrightarrow{\wedge} [ZF_z]_k. \quad (31)$$

Analogously, for other copying functions of elements of conceptual process (Figure 1), using analogy from equations (22) and (31), and other equations in displayed evidence, to determine conjunctions matrixes. It is possible to write transformation using vector's algebra:

$$\varphi_{FZ \mapsto ZF_z} : FZ(FZ_1, ..., FZ_j) \mapsto ZF_z(ZF_{z,1}, ..., ZF_{z,k}). \quad (32)$$

Expression from above written in an equation form:

$$\varphi_{FZ \mapsto ZF_z} = \varphi_B : [\mathbf{FZ}] = [\mathbf{B}][\mathbf{ZF_z}]. \quad (33)$$

Matrix $[\mathbf{B}]_{j\times k}$ is operator matrix of linear transformation of functional requests of *FZ* set in requests of physical characteristics ZF_z. Analogy applies to other transformations inside the conceptual model. Then the process of transformation inside the conceptual model is presented by integral equation:

$$[\mathbf{KF}] = [\mathbf{D}][\mathbf{C}][\mathbf{B}][\mathbf{A}][\mathbf{KZ}] = [\mathbf{OK}][\mathbf{KZ}]. \quad (34)$$

The conceptual matrix operator [**OK**] connects all transformations created in *KF* set. Then we can define a conception as the finally dimensioning vector space *KF*, where every subset *KF*, *FZ*, ZF_z, ZK_z and of conceptual set *KF* consists of final number of elements. Transformation matrixes are linear operators over the vector conceptual space. Operator matrix of conjunction in equation (22) is a unit matrix [**I**]. Then conjunction matrixes (22), using Kronecker symbol **δ** can be written as:

$$[\wedge(FZ \mapsto ZF_z)]_{j\times k} = [\mathbf{I}] = [\delta_{jk}(FZ \mapsto ZF_z)]. \quad (35)$$

Analogy applies to other transformations in model, which ensures reversibility and model interactivity:

$$\varphi^{-1}_{KZ \mapsto FZ} : [\mathbf{FZ}] = [\mathbf{A}]^{-1}[\mathbf{KZ}]. \quad (36)$$

Starting requirements determined by a design task in *r*-th solution variant, are not representative at all. Principal solution variant does not reflect customer demands. In a general case the following expression is valid:

$$|\mathbf{KZ}| \neq |\mathbf{FZ}| \wedge / \vee |\mathbf{FZ}| \neq |\mathbf{ZF_z}| \wedge / \vee |\mathbf{ZF_z}| \neq \neq |\mathbf{ZK_z}| \wedge / \vee |\mathbf{ZK_z}| \neq |\mathbf{KE_z}| \quad (37)$$

Then model equations with the appreciation of equation (36) give the possibility of system reconfiguration and better solution achievement. Using this formalism, it is possible to observe where a deviation from set design task appears in the process of characteristic transformations of principal solution. In that way we verify, in every phase of conception, an entire system of information as elements of sets *KF* with a goal of achieving variants of conception solutions that contain all characteristics from desired solution. That kind of difference (36) influences the efficacy of design solution, as the answer on the identify requests through the transformation process in *KF* set. The degree of difference in a positive and negative sense can be determined by the introduction of deviation Δ between individual transformation vectors as the operator that talks about the inconsistency of components of conceptual process. Deviation Δ, as the entity characteristic for this difference according to the equation (36), is defined as:

$$\Delta(\mathbf{KZ},\mathbf{FZ}) = \frac{|\mathbf{KZ}| - |\mathbf{FZ}|}{|\mathbf{FZ}|}. \quad (38)$$

Graphical representation of the former equation (analogous for other domains) is on the angle between two vectors of conceptual process. Conceptual space is bounded vector space, apropos real Euclidean space $\Re^2$. Higher value of angle, agree of higher inconsistency and conversely, in that way in its entire conceptual inconsistency treats evaluation inconsistency of variant solutions in accordance with expression [9]:

$$inv.inc(F)=\frac{\|F-B\cdot X(F)\|_2}{\|B\cdot X(F)\|_2}. \quad (39)$$

Therefore, the conceptual model is described and the assumption for the application of formalized method of assessment and evaluation is created.

3. The procedure to define weight in evaluation conceptual design

In conceptual phase of design processes, usually a large number of variant solutions exist. Variant solutions are shown as the final set of objects in matrix form $[\mathbf{V}]_r=\mathbf{V}$. All variant solutions cannot be included in design elaboration but must be evaluated according to the range of meeting the set criterions. The set of criterions can be shown in matrix vector $[\mathbf{C}]_k=\tilde{\mathbf{C}}$. The level of fulfilling the set criterion verifies every variant solution, and this solution gets the numerical number or weight criterion for the given variant solution. The experts evaluate the weight factors, and these experts are the evaluators who are relevant for the design problem. In this article, the procedure of evaluating weight factors of some criterions for variant solutions is mathematically defined as a matrix equation. Using the unification and formalism, we create an assumption for objective defining of weight matrix [**W**] in order to avoid arbitrariness and subjectivity, and to affect the selection of final variant solution.

3.1. Decision making model based on grading approach

The designer must make decisions or draw conclusions during the conceptual phase, and this is not always easy because of the lack of necessary information and time limitations (restrictions). The importances of quick evaluation of notional solutions, which are generated during the design process, are known from earlier. However, the problem appears during the clearly defined decision making procedure, and during the defining procedure. The designer must be objective while considering some of the alternative design solutions. The design process defined mapping process in space domain process as variables of design process to object domain with design elements DE_s (Figure 1). The model of identification variant solutions with all criterions is shown in Figure 2. The matrix equation of identification has the following form in equation (40):

$$inv.inc(F)=\frac{\|F-B\cdot X(F)\|_2}{\|B\cdot X(F)\|_2}. \quad (40)$$

The matrix **R** represents matrix of weight **W** in a real process. The matrix weight **W** is shown as:

$$\mathbf{W}=\begin{bmatrix} w_{11} & \cdot & w_{1k} \\ \cdot & \cdot & \cdot \\ w_{r1} & \cdot & w_{rk} \end{bmatrix}. \quad (41)$$

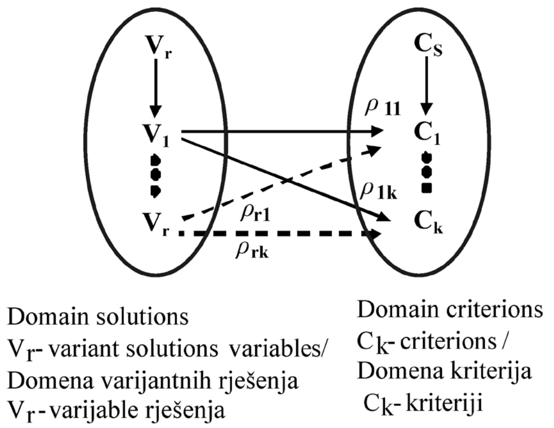

Figure 2. The model of identification variant solutions with all criterions of evaluation

Slika 2. Model identifikacije varijantnih rješenja s kriterijima vrednovanja

In accordance with those equations matrix of appreciation weight can be shown:

$$Q^w_{rk}\left(o^w_{nrk}\right)=\begin{bmatrix} Q^w_{11} & \cdot & Q^w_{1k} \\ \cdot & \cdot & \cdot \\ Q^w_{r1} & \cdot & Q^w_{rk} \end{bmatrix}. \quad (42)$$

The global form of the appreciation of *n* evaluators by decision making of weight factors for all criterions *k* and any *r* variants are given by:

$$\mathbf{Q^N_{rk}}\left[o_n(f_i)\right]^k_r=\mathbf{W}[w]^k_r. \quad (43)$$

The model with estimated weight factors for *r* variants of conceptual design as well as *k* criterions leads toward mathematically formalization presented by equations (43).

3.2. Evaluation model with Potential Method (PM)

All parameters, which have influence on the variant solutions in the conceptual design (Figure 1), are seated on the different levels of hierarchic structure. Hierarchic structure of an evaluation model is given with limited number of levels:

$$H_i,\ \forall i=1,\dots,n; \quad (44)$$

In the evaluation model, every hierarchic level of analyzed parameters of the conceptual model presented by one graph, uses the potential method - PM [8-9]:

$$G_i = (V, R), \forall i = 1,...,n \wedge H_n \mapsto n. \quad (45)$$

The presented evaluation model is defined by comparison of an ordered pair of elements of hierarchic structure and presented by graph G_i, which is determined by the relation:

$$(v_i, u_j) \in \rho_{ij} \subset R \Rightarrow (u_j, v_i) \in \rho_{ji} \not\subset R,$$
$$\rho_{ji} \neq \rho_{ij}, i = 1,.,n; j = 1,.,m. \quad (46)$$

Graph G_i of each level is uniquely determined by incidence matrix **B**:

$$[B] = \alpha \rightarrow [\cdots \ \overset{v}{-1} \ \overset{u}{1} \ \cdots]. \quad (47)$$

The numerical value of graph element is the potential of the element. The element of graph is the node, to which the evaluation parameter is assigned. The potential of any node **X** presents the numerical value, whose analyzed element has its own hierarchic level related to other elements on the same level. Real function $\mathbf{X} : V \rightarrow \Re$ in the set of nodes as a set of design parameters are presents the potential of each element which is included in the design process. Real function $\mathbf{F} : R \rightarrow \Re$ in the set of arcs is the flow of preferences transformation. Potential of the elements **X** and the flow **F** are determined as the vector of n elements:

$$\begin{pmatrix} n := cardV \rightarrow [\mathbf{X}] = \begin{bmatrix} X_1 \\ \cdot \\ \cdot \\ X_n \end{bmatrix}, m := cardR \rightarrow \\ \rightarrow [\mathbf{F}] = \begin{bmatrix} F_1 \\ \cdot \\ \cdot \\ F_m \end{bmatrix} \end{pmatrix} \rightarrow \quad (48)$$
$$\rightarrow \mathbf{F} = \mathbf{B} \cdot \mathbf{X}$$

If the direction of the preference is defined as $F_\alpha = F_{ji} := log_a\, a_{ij}, a \succ 1$ and vector of the potentials as **X**, then the vector of priorities w and its components w_i are determined by:

$$w = a^X, a \succ 0 \rightarrow w_i := a^{X_i}, i = 1,...,n \rightarrow$$
$$w_i = a^{X_i} = a^{\frac{1}{n}\sum_{j=1}^{n} F_{ji}} = a^{\frac{1}{n}\sum_{j=1}^{n} log_a a_{ji}} = \left(\prod_1^n a_{ij}\right)^{\frac{1}{n}}, i = 1,..n. \quad (49)$$

The evaluation model presented in this work includes mathematical formalisms of theory of graphs [8]. How to use the theory of graphs to assess the acceptability level of one shipbuilding component with butt-welded cruciform joints [10] is presented as an example in this paper (Figure 3). The radius of roundness at the place of weld toe was the only design parameter that was changed. It could be expected that under nominal loading, stress concentration will appear in the vicinity of toes. Therefore, an optimal shape of toe line due to stress distribution by finite elements analysis was analyzed. Actually, stress distribution with its peak value can be used as a parameter of goodness of component shaping. In this paper, an optimal design of one characteristic shipbuilding structure with butt weld cruciform has been investigated.

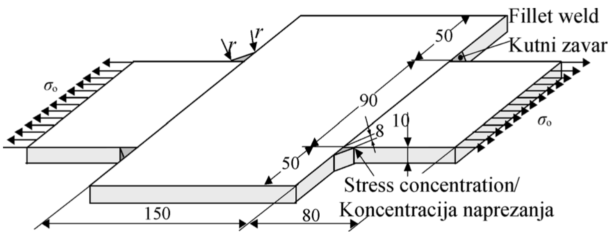

Figure 3. Thin-walled welded shipbuilding component

Slika 3. Zavarena komponenta tankostjene brodske konstrukcije

Thin-walled characteristic shipbuilding component with butt welded cruciform joints has been chosen by [11] to investigate the influence of size of roundness radius, which is measured at the weld face between the filler metal and the parent metal on the stress distribution. The geometry of the component and fillet welds with the throat thickness of 8 mm. Mechanical properties of material and its chemical composition are given in the ref. [11]. It is clear that stress rises at the place where the fillet weld is attached to the component body. All details of finite elements modelling of 1/4 of the component are given in the [10]. Stress distribution was calculated by finite elements analysis for five assumed radiuses of roundness (R = 0, 1, 2, 4 and 8 mm). It was shown that stress concentration factor (SCF) value becomes lower with radius of roundness increasing. More rounded fillet weld at the place of touching with the parent metal causes not only SCF decreasing, but also removing the stress peak from the weld toe.

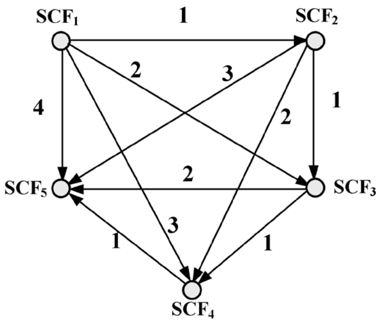

Figure 4. The graph $G_r(r_1,...,r_5)$ of preferences between SCFs

Slika 4. Graf $G_r(r_1,...,r_5)$ preferencija između faktora koncentracije naprezanja

The model of identification variant solutions of five size of roundness radius with all criterions and decision making of weight factors for all criterions k and *r* variants are given in PhD thesis [8]. The first step in assessing any particular variant solution is to create a hierarchical structure with all criteria arranged by levels. Here, all performed component designs are loaded with the same nominal stress and the only criterion was the radius of roundness. Of course, the greater value of R needs untoward additional mechanical work and more deposit, but such detail performance increases carrying capacity of the whole component. In this paper, well-known theory of graphs is applied. Generally, a directed and oriented graph *G* is determined toward (45), where V is the set of nodes, which are here *SCF*s in the function of the radiuses roundness ($r_1 \ldots r_5$) and R is the set of relations between elements of the set V. The graph *G* is determined by matrix of incidence **B**. Let us define the graph $G_\alpha(SCF_1,..., SCF_5)$ where the relation between particular *SCF*s (r_1) is quantitatively given by preferences α (Figure 4). The components of potential vector were determined by difference of potentials of the set of nodes V [6]:

$$y_\alpha = \frac{X_u - X_v}{\alpha(u,v)}, \tag{50}$$

where *u* and *v* are the components of the set of nodes V.

The set of potentials for all *SCF*s in the function of *r* can be shown as matrix:

$$\left[X_{SCF}(r_1,..,r_5)\right] = \begin{bmatrix} -1,90 \\ -1,00 \\ -0,10 \\ 1,00 \\ 2,00 \end{bmatrix}. \tag{51}$$

The flow *F* is the result of the potentials difference of criteria, given by $F: R \rightarrow \Re$. Thus, the flow can be written as F = B·X. The normal integral F is every solution of equation $\mathbf{B}^T \cdot \mathbf{B} \cdot \mathbf{X} = \mathbf{B}^T \cdot \mathbf{F}$, $\Sigma X_i = 0$. Let us define the matrix **A** as $A_\alpha = B_\alpha^{\ T} \cdot B_\alpha$. Abovementioned equation $B_\alpha^T \cdot B_\alpha \cdot X_\alpha = \Delta F_r$ can be written as $A_\alpha \cdot X_\alpha = \Delta F_\alpha$. Hence, the solution of the flow difference of the normal integral *F* is given by:

$$A_\alpha \cdot X_\alpha = \Delta F_\alpha \rightarrow \begin{bmatrix} -9,48 \\ -5,00 \\ -0,52 \\ 5,00 \\ 10,00 \end{bmatrix} = \Delta F_\alpha;$$

$$\left[W_\alpha\right] = \begin{bmatrix} w_{\alpha_1} \\ w_{\alpha_2} \\ w_{\alpha_3} \\ w_{\alpha_4} \\ w_{\alpha_5} \end{bmatrix} = \begin{bmatrix} 0,035 \\ 0,065 \\ 0,121 \\ 0,260 \\ 0,520 \end{bmatrix}. \tag{52}$$

Weight components vector influenced the priorities between analyzed *SCF*s depending on different radiuses of roundness. Norm vector of priorities must satisfy the condition $\Sigma\ w_i = 1$. In our example, the weight matrix of the set of *SCF*s is calculated by the equation (52). According to this equation it can be concluded that w_{SCF5}=0,52 presents the greatest value of the component of norm vector of weights. This means that performed variant solution with the greatest radius of roundness are most acceptable solution. Of course, this conclusion should be extended with variation of other important design parameters to be able to make decision with highest accuracy.

4. Conclusion

A new approach to conceptual design and decision-making model presented here includes all complexity of product development as the technical system. It is based on mathematical formalisms of the goodness of some variant solution toward given criteria. The proposed model assures clearly defined procedure for grades assignment, with the aim to rank all variant solutions. Highest value of the grade obtained by evaluation procedure means the best solution. Such an approach in original form unites conceptual phases of design and evaluation of alternative solutions, which makes the time of product development shorter. Working examples of applying these grading models may be found in the reference [8]. The theory of graphs was successfully applied to assess the goodness of proposed variant solutions in the conceptual phase of design. In this way, the final decision is quantitatively argued and the procedure of choice of most acceptable solution is accelerated. However, the procedure given in this work should be understood just as the base for further complex investigation of all influenced design parameters in the product development.

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