Multivariable Controller Tuning for Non-square Systems with RHP Zeros by Genetic Algorithm

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A simple genetic algorithm is applied by simulation for the tuning of centralized and decentralized controllers for linear non-square multivariable systems with RHP zeros. All the loops are first tuned independently using Ziegler-Nichols method based on closed loop oscillation. A detuning factor is used to obtain the range of controller parameters. This approximate range of the controller parameters is used to improve the search of genetic algorithm. This method is applied to a MIMO non-square coupled distillation column (3 inputs and 2 outputs) system.[1] Both centralized and decentralized controllers are designed by using the genetic algorithm optimization method. The proposed centralized controller tuned by the genetic algorithm gives the best performance when compared to the decentralized controller and other analytical methods. These proposed ranges of controller parameters for genetic algorithm can make the convergence faster than the method proposed for multivariable square systems by Vlachos *et al.*[2,3]

Key words:
Non-square systems with RHP zero, centralized controller and decentralized controller, genetic algorithm

Introduction

Processes with an unequal number of inputs and outputs often arise in the chemical industry. Such non-square systems usually have more inputs than outputs. Examples[1] for non-square systems are (3 input, 2 output) coupled distillation column, Shell standard control problem, (7 input, 5 output) system, crude distillation unit (5 input, 4 output) system, etc. A common approach towards the control of non-square processes is to first square up or square down the system through the addition or removal of appropriate inputs (manipulated variables) or outputs (controlled variables) in order to obtain a square system matrix. Then the usual control design methods can be employed to achieve the design specifications. But none of the alternatives are desirable. Adding unnecessary outputs to be measured can be costly, while deleting some of the inputs may require a larger variation in the inputs in order to achieve the desired control. There are two design structures for the non-square system. One of them is the centralized controller and the other is the decentralized controller. The centralized controller would use feedback from all the measured outputs to manipulate each input. Whereas the decentralized controller would pair one output with one (or more) inputs and implement feedback based control. Even though the decentralized controller can lead to high degree of interactions, it is popular in industries due to the following reasons. The decentralized controllers are easy to implement, they are easy for the operator to understand and the operator can easily retune the controllers to take into account the variations in the process conditions.

Sarma and Chidambaram[4] have extended the two simple design methods, namely Davison's method[5] and Tanttu & Lieslehto method[6] for designing centralized controller meant for square systems to non-square systems and have also compared them with the independent design procedure for robust decentralized controllers for non-square systems reported by Loh and Chiu.[7]

Vlachos *et al.*[2] have proposed a simple genetic algorithm for designing decentralized PI controllers for the square systems. Genetic algorithms are search and optimization procedures that are motivated by the principles of natural genetics and natural selection. The powerful capacity of genetic algorithm[8] in locating the global optimal solution is used in the design of PI controllers. In this method, the different performance criteria are combined into a single objective function using the penalty function method. The range of values for the proportional and integral term of controller is given randomly. The disadvantage of this method is that, a large number of computations are involved and time consuming. Sadasivarao and Chidambara[9,10] have shown that the approximate initial guess for controller settings used

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in the genetic algorithm is able to converge faster than the method proposed by Vlachos *et al.*[5] In the present study, these approximate ranges are used to design centralized and decentralized PI controllers for non-square systems. The present study is applied by simulation to the coupled distillation column[1] (3 input, 2 output) system.

Genetic algorithm

A simple genetic algorithm[11] is used in this work. The binary alphabet and gray coding were used for encoding the controller parameters. A generation gap of 0.9, linear rank-based fitness assignment with a selective pressure of two, proportionate selection with stochastic universal sampling, single-point crossover and fitness based reinsertion is used as chosen by Vlachos *et al.*[2] The size of the population chosen to be 80, so that the search space is attacked at many points simultaneously, thus resulting a faster convergence to the global optimum. The initially selected population is left to evolve for 50 generations. 20 bit string element is used for the encoding of each of the controller parameters. The crossover and mutation probabilities are chosen to be 0.45 and 0.01, respectively.

In the present study, Ziegler-Nichols tuning method[12] is used for the systems having stability limit and the direct synthesis tuning method is used for the systems having no stability limit. For the open loop transfer function models (first order and second order), which have no stability limit on K_c, the ranges of K_c and T_I are selected by extending $K_{c,des}$, 5 times and $T_{I,des}$, 10 times in both directions (i.e lower limit of proportional gain is taken as $K_{c,des}/5$ and upper limit as $5 \cdot K_{c,des}$). If the individual transfer function models have a stability limit, then the lower limit of proportional gain is taken as 0 and the upper limit as $K_{c,des}/1.5$, the lower limit of T_I is taken as $T_{I,des}/2$ and no upper limit is placed on T_I, so that the system will not go to an unstable region. The parameters are encoded as K and K/T in the algorithm. Given the range of the controller parameters, the genetic algorithm will generate initial guesses randomly in that range and evaluate the objective function. This way, the unstable region is eliminated from the region of search. This approximate range for the genetic algorithm is able to reduce the number of computations and the algorithm converges faster.

Objective function

Objective function (sum of deviation from performance criteria) is the mathematical way of representing the desired response. The method of Vlachos *et al.*[2] is followed to represent the objective function. The objective function[2] for the ith output and the jth set point pattern can be expressed as

$$J_{ij}(K_c, T_I) = \int_0^{t_{max}} [\max\{f_{ij}^{(l)}(t) - y_i(t), 0\} + \max\{y_i(t) - f_{ij}^{(u)}(t), 0\}]\, dt \qquad (1)$$

where K_c and T_I are parameters (proportional gain and integral time) of the n number of PI controllers associated with the n loops of the multivariable system. $f_{ij}^{(l)}$ and $f_{ij}^{(u)}$ are the user defined lower and upper boundaries of the region representing the performance objectives for the ith output and the jth set point pattern respectively. A unit step change is given to one of the set points (jth set point) by keeping the remaining set points constant and the performances of all outputs are evaluated using the above equation. The results are weighted and added together to form a single performance. The procedure is repeated for the other set points (j varies from 1 to n) and the final objective function is represented as

$$J_0(K_c, T_I) = \max_{1 \le j \le n} \left\{ \sum_{i=1}^{n} w_{ij} J_{ij}(K_c T_I) \right\} \qquad (2)$$

J_{ij} denotes the objective function for the ith output and the jth set point. w_{ij} denotes the weighing factor of the objective function for the ith output and the jth set point. The higher the weighing factor, the more important the corresponding component of the objective function becomes. In case of the centralized control system, K_c and T_I are the parameters of the n x n size of PI controllers matrix. In the present case, w_{ij} is taken as 1 when $i = j$ and is taken as 0.25 when $i \neq j$, to give more importance to set point tracking objectives.

Controllers for non-square systems with RHP zeros

In the multi-input and multi-output system, interaction and location of transmission zero are important.[12,13] These RHP zeros impose limitations on stability and controllability of the system. These affect both the amplitude and phase angle. The extra phase lag that is added by the RHP zero contributes to the instability and makes control difficult. So the controller design for the non-square system which is having positive zeros in the individual scalar systems is more concern in the present example. Few reported methods are available for non-minimum phase systems to design multivariable square systems, which involves complicated control strategies and lengthy calculations. The optimization method (genetic algorithm) is applied to design the centralized and decentralized controllers for non-square systems with RHP zeros.

Design example

The proposed ranges for the genetic algorithm is applied to a non-square coupled distillation column to design both the centralized and decentralized PI controllers. These two design settings are compared with the method suggested by Sarma and Chidambaram[4] to design the PI controller settings for non-square system. A coupled distillation column (3 input, 2 output) studied by Levein and Morari[1] is described by,

$$\mathbf{G}(s)=\begin{bmatrix} \dfrac{0.052\,e^{-8s}}{19.8s+1} & \dfrac{-0.03(1-15.8s)}{108s^2+63s+1} & \dfrac{0.012(1-47s)}{181s^2+29s+1} \\ \dfrac{0.0725}{890s^2+64s+1} & \dfrac{-0.0029(1-560s)}{293s^2+51s+1} & \dfrac{0.0078}{42.3s+1} \end{bmatrix} \quad (3)$$

Here the outputs are mole fraction of ethanol in distillate (y_1) and mole fraction of water in bottoms (y_2), and manipulated variables are distillate flow rate (u_1), steam flow rate (u_2), product fraction from the side column (u_3).

The performance objectives are chosen as follows:

Peak overshoot: $\leq$ 20 %

Settling time: $\leq$ 50 min

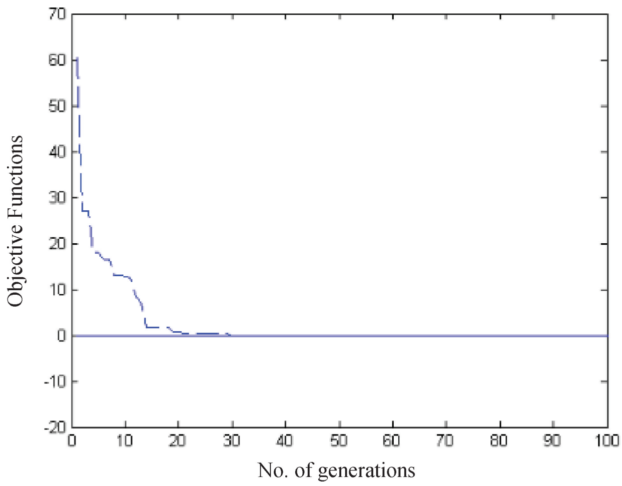
Loop coupling: $\leq$ 50 %, $0 \leq t \leq 75$ min

$\leq$ 5 %, $25 \leq t \leq 125$ min.

In the Vlachos *et al.*[2] method, the range of controller parameters is chosen randomly (proportional term −50 to 50 and integral term 0.1 to 100), the algorithm takes nearly 40 generations to achieve the desired objective function, whereas the proposed genetic algorithm is able to converge faster and even first generation itself the desired objective function is reached (refer to Fig. 1) and the optimal controller settings are achieved.

Centralized controllers

The centralized controller is designed for the coupled distillation column system by using the simple genetic algorithm.[11] For convenience in representing the ranges, the parameters are encoded as $K_{c,ij}$ and $K_{c,ij}/T_{I,ii}$ in the algorithm. Since six PI controllers are used for the centralized control system for the above example, 12 parameters are to be estimated. The approximate ranges for the controller parameters are calculated and used in the genetic algorithm to get the optimum controller parameters.



Legend: solid line − proposed genetic algorithm
dashed − Vlachos method

Fig. 1 – *Convergence of genetic algorithm. Objective function versus no. of generations.*

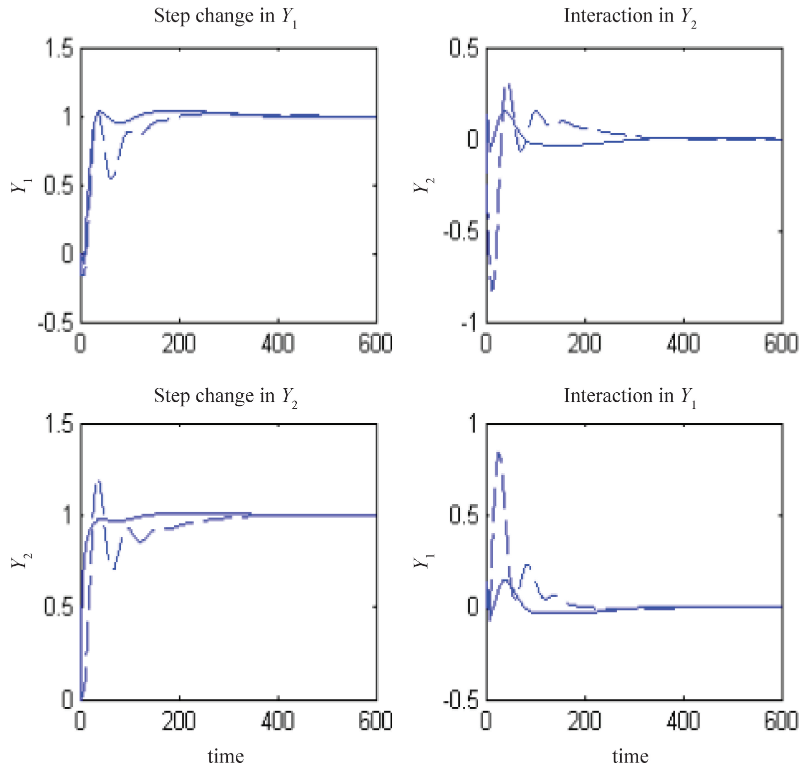
The system response is shown in Fig. 2 for unit step change in set point. The optimum controller parameters are given below as proportional and integral gain form

$$\begin{bmatrix} 7.5748+\dfrac{4.1371}{s} & 10.8965+\dfrac{9.9103}{s} \\ 4.4523+\dfrac{1.2653}{s} & 33.5059+\dfrac{21.8233}{s} \\ -34.505-\dfrac{2.5336}{s} & 36.725+\dfrac{1.5431}{s} \end{bmatrix}$$

Decentralized controller

In this decentralized controller, first the pairing has to be done using the Block relative gain (BRG). Block relative gain[14] has to be calculated for all possible pairings. BRG is equal to unity for the best pairing and for lesser interaction. The first output variable y_1 is paired with u_2 and y_2 is paired with u_1 and u_3. This pairing leads to a less interaction and a unity BRG. The optimal closed loop response for the decentralized controller is shown in Fig. 2. The resulting decentralized controller parameters designed by the simple genetic algorithm is shown below:

$$\begin{bmatrix} -23.8095-\dfrac{0.8275}{s} & 0 \\ 0 & 26.88+\dfrac{0.3798}{s} \\ 0 & 0.1+\dfrac{0.1787}{s} \end{bmatrix}$$



Legend: solid – – – centralized controller, dashed – decentralized controller

Fig. 2 – *Performance comparisons of centralized and decentralized controllers. For unit step change in y_1 and y_2 (servo problem).*

Comparison criterion of controller performance

There are three ways the performance of the controller system can be measured: integral of absolute error (IAE), integral square error (ISE), and integral of time weighted absolute error (ITAE). In multivariable systems, different criterions have been considered to compare the performance of the controllers. Stephanopoulos[15] has discussed that ISE should be used to suppress large errors, while ISE should be used to suppress small errors and errors that persist for long times, ITAE criterion should be used,

$$\mathrm{IAE} = \int_0^\infty |\, y_s(t) - y(t) |\, \mathrm{d}t \qquad (4)$$

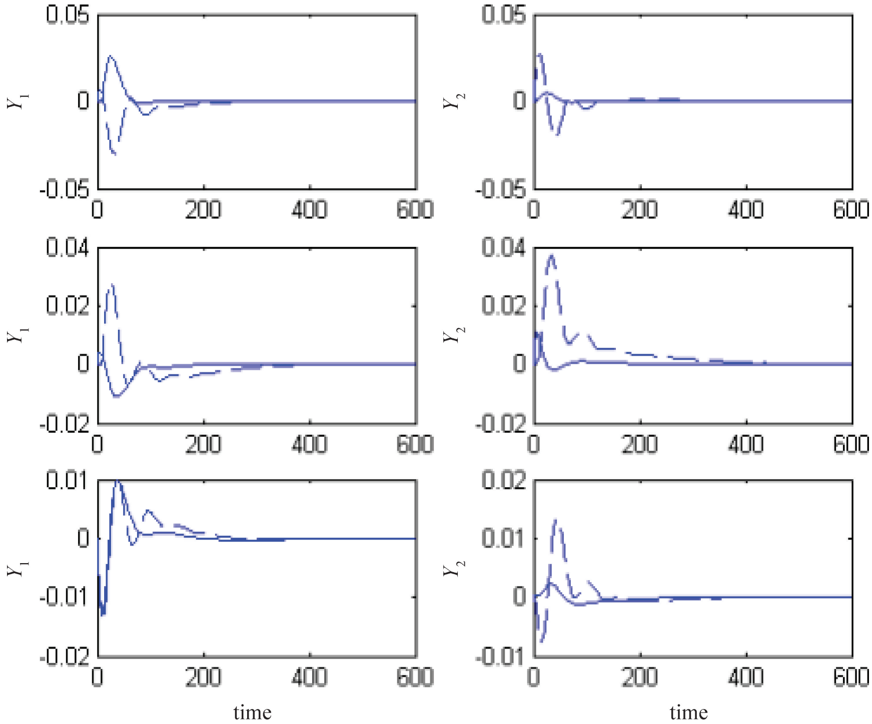
$$\mathrm{ISE} = \int_0^\infty [y_s(t) - y(t)]^2\, \mathrm{d}t \qquad (5)$$

In the multivariable system, most of the controller settings give comparable responses. To distinguish the responses, IAE or ISE values should be compared. It is to be noted that while comparing two controller settings, the ISE values in response may be lower, but at the same time, it may be higher in interaction. In this case, the criterion used is better which considers the minimization of sum of the IAE or ISE values in both the response and the interaction.

Comparison of controller performance for coupled distillation column

The performances of centralized and decentralized controllers designed by optimization method (Genetic algorithm) were compared along with the analytical controllers designed by Sarma and Chidambaram.[4] The results have shown that the centralized controllers designed by the optimization technique performed better than the decentralized controllers designed by the optimization technique and previously reported analytical methods. Fig. 2 compares the response of y_1 and y_2 for unit step change in set point y_1 and y_2 respectively. It also compares the interaction in y_2 and y_1 for unit step change in set point y_1 and y_2 respectively. It is clear that the interaction is lower in the centralized controller performance compared to that of the decentralized controller. And also the settling time is less for the centralized controller performance than that of the decentralized controller. It is also shown that decentralized controllers give a larger overshoot than do centralized controllers.

The performances of the designed controllers were also compared for change in load variables. It



Legend: solid – centralized controller, dashed – decentralized controller

Fig. 3 – *Performance comparisons of centralized and decentralized controllers for unit step change in load variables v_1, v_2 and v_3. The ith row figures are for change in load variables V_i.*

is assumed that the load transfer function matrix is same as that of the process transfer function matrix. Fig. 3 compares the output variable y_1 and y_2 for unit step change in load variable v_1, v_2 and v_3 respectively. In the regulatory problem also, the centralized controllers gave better response i.e., less overshoot than did the decentralized controllers.

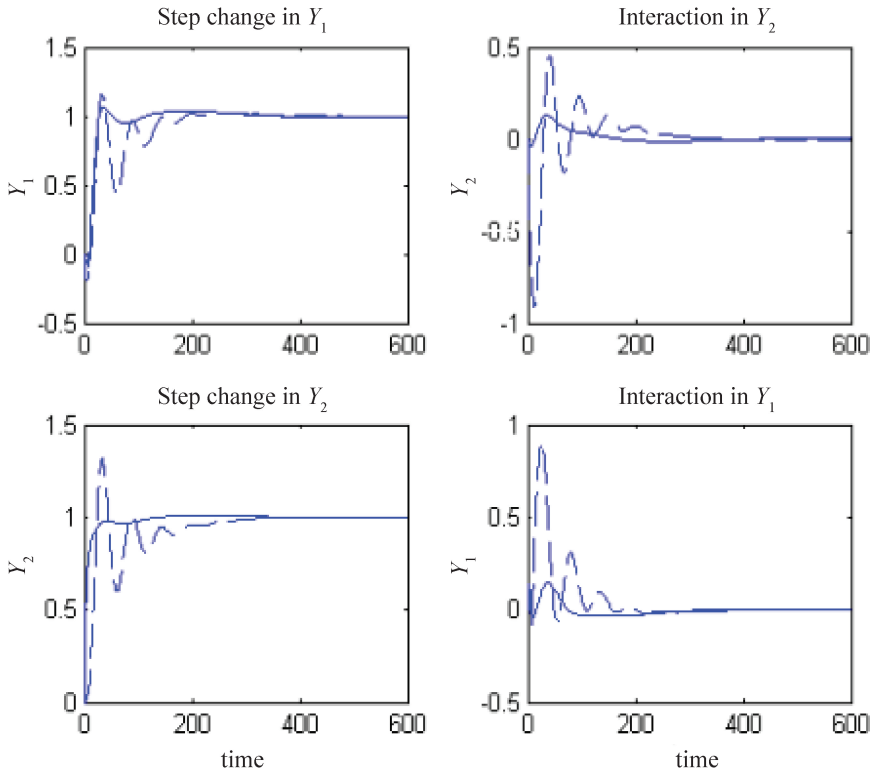
ISE values for the centralized controller and the decentralized controller are given in Table 1a along with those values from Sarma and Chidambaram[4] for the servo problem, and in Table 1b for the regulatory problem. For the servo problem, the sum of ISE values are lower for the centralized controller when compared to the decentralized controller and the other proposed method. It is noted that for the regulatory problem also, centralized controller gives lower ISE values when compared to the others.

Table 1a – *ISE values for servo problem GA – genetic algorithm optimization technique. D-Davison, LC-Loh and Chiu (Reference for SI no. 3 and 4 – Sarma and Chidambaram).*

Method	Step in	ISE values		Sum of ISE
		Y_1	Y_2	
1. centralized controller (GA)	Y_1	7.5381	0.18	7.72
	Y_2	0.8468	2.25	3.09
2. decentralized (GA)	Y_1	14.225	10.68	24.91
	Y_2	6.3149	9.42	15.74
3. D method centralized	Y_1	33.04	12.41	45.45
	Y_2	3.11	11.46	14.57
4. LC method decentralized	Y_1	44.99	9.91	54.90
	Y_2	19.64	28.34	47.98

Table 1b – *ISE values for regulatory problem GA – genetic algorithm optimization technique. D-Davison, LC-Loh and Chiu.*

Method	Step in	ISE values		Sum of ISE
		Y_1	Y_2	
1. decentralized controller (GA)	V_1	0.0202	0.0168	0.037
	V_2	0.0172	0.0404	0.057
	V_3	0.0048	0.0045	0.009
2. centralized controller (GA)	V_1	0.0147	0.0005	0.015
	V_2	0.0040	0.0012	0.005
	V_3	0.0041	0.0002	0.004
3. D method centralized	V_1	0.0319	0.0465	0.078
	V_2	0.01	0.0056	0.156
	V_3	0.0075	0.0046	0.013
4. LC method decentralized	V_1	0.041	0.107	0.148
	V_2	0.036	0.021	0.057
	V_3	0.0065	0.0045	0.011



Legend: solid – centralized controller, dashed – decentralized controller

Fig. 4 – *Controller performance of perturbed system for unit step change y_1 and y_2*

Robustness studies

Robustness studies were carried out for this system by increasing the individual element gain of the transfer function model by 10 %. The same controller settings as previously obtained were used. Performance of the centralized controller and the decentralized controller for the perturbed system is shown in Fig. 4. From the figure, it is clear that the performance of the centralized controllers and the decentralized controllers is similar to the corresponding performance of the perfect parameter system.

Conclusion

A genetic algorithm optimization technique was used to design centralized and decentralized controller for linear non-square MIMO systems with RHP zero. A simulation study is given for a (3 input, 2 output) non-square coupled distillation column. The simulation results indicate that in the presence of uncertainty, the two types of controller structure achieve a robust performance. By comparing the ISE values, centralized controller designed by the optimization method gave the lowest ISE values for both servo and regulatory problems when compared to the decentralized controller and other analytical methods. In addition, the inclusion of approximate ranges for the PI control parameters into the genetic algorithm improved the convergence of the algorithm significantly.

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