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Application of Joint Time Frequency Analysis for Early Crack Detection on One-stage Gear Drive

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Original scientific paper

The relationships between the existence of gear teeth cracks as a cause and symptoms emerging as housing vibrations that can be detected with signals measured was investigated numerically and experimentally. Experimental measurements on one-stage gear drives have been carried out involving gears built in with different notch length in tooth root. Signals measured as accelerations on gear box house have been analysed with STFT. A new computer programme DSP_Studio has been developed for detecting and measuring the diagnostic signals, and simultaneously for analysing the effect of crack in tooth root. Numerical simulation model of simple gear pair, which incorporates simulation of crack presence in altered tooth stiffness, has been conducted to confirm experimentally obtained results. The experimental results corroborate with the numerical results. The results derived represent an important contribution to early crack detection on one-stage gear drives.

Keywords
Diagnostics
Gear wheel
Frequency analysis
Injuries
Time signal
Vibrations

Ključne riječi
Dijagnostika
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Upotreba vremensko frekvencijske analize za rano otkrivanje pukotina na jednostupanjskim zupčastim pogonima

Izvornoznanstveni članak

Sljedeći rad raspravlja o istraživanju povezanosti pukotine u korijenu zuba, kao uzrok, i simptomima koji se pojavljuju u obliku vibracija na kućištu te ih je moguće očitavati u izmjerenim signalima. U tu svrhu su bila obavljena eksperimentalna mjerenja na izrađenom jednostupanjskom zupčastom pogonu u kom su bili ugrađeni zupčanici sa različito dugim pukotinama u korijenu zuba. Signali, izmjereni u obliku ubrzavanja na kućištu pogona, bili su analizirani u vremenskom, frekvencijskom, kepstralnom te vremensko-frekvencijskom prostoru. Da bi obuhvatili dijagnostičke signale i analizu utjecaja pukotine zuba izradili smo vlastiti kompleksan računalni program DSP_studio, koji predstavlja novost i važan doprinos na području tehničke dijagnostike. Da bi potvrdili rezultate dobivene eksperimentima, izveli smo numeričku simulaciju modela jednostavnog zupčanog para u kojem je bila simulirana prisutnost pukotine u promijenjenom slijedu krutosti zuba. Ustanovili smo dobro slaganje eksperimentalno i numeričko dobivenih rezultata. Dobiveni rezultati predstavljaju doprinos k ranom otkrivanju pukotina na zupčastim pogonima.

1. Introduction

Gears and gear assemblies are well known mechanical elements, with established quality manufacturing procedures and analytical methods for their service-life predictions. However, there is an ever present need for monitoring of gear status during its operation, recording of relative changes and critical conditions, in-time discovery of the developing damage and damage tracing due to material fatigue. The most common method used today is the measurement of accelerations on the gear housing. The main advantage of this method lies in the fact that one can reliably determine the eigen frequencies in the frequency spectrum of the gear assembly from the number of gear revolutions, number of gear teeth, the number of roller bearings and their geometry. This provides for easy determination of any gear damage by observing the changes in the frequency spectrum as the damage develops. With modern measuring equipment it is even possible to exactly determine the type, the position and the extent of the damage. No major changes to the gear assembly are required [5] and [6]. The measured signals are often so complex that it is impossible to determine directly the relationships between the recorded symptoms and the causes (damage) [3].

First publications from the field of early crack detection date back to the second half of the eighties [10], in which the application of an existing digital signal

Symbols/Oznake

D_z	- engagement damping, Ns/m - prigušenje
F	- force, N - sila
J	- moment of inertia, kg·m^2 - moment inercije
k_z	- tooth stiffness, N/(mm·μm) - krutost zuba
M	- torque moment, N·m - moment
r	- radius, m - radius
t	- time, s - vrijeme
T	- kinetic energy, kg·m^2·s^{-2} - kinetička energija
V	- potential energy, kg·m^2·s^{-2} - potencijalna energija
P	- Rayleight disipations function - Rayleightjeva disipacijska funkcija
x	- displacement, m - pomik
μ	- dynamic viscosity, Pa·s - dinamička viskoznost
ρ	- density, kg·m^{-3} - gustoća
φ	- general characteristic - općenita značajka
ω	- angular velocity, rad·s^{-1} - kutna brzina

Indices/Indeksi

1	- gear 1 - zubčanik 1
2	- gear 2 - zubčanik 2

processing technique for calculating the amplitude and phase modulation of the tooth meshing vibration of a gear from the time domain average of the vibration was extended so that the amplitude and phase of the changes in the vibration due to a fatigue crack in a tooth could be determined [11]. In the nineties many new algorithms and technologies for signal processing especially in time-frequency domain were developed for data compression, e.g. digital images, which started to be applied also in the field of early damage detection. Wang and McFadden [12] were the first who established the importance of time-frequency distribution and algorithms for digital image processing for monitoring the presence of local damages on gear teeth. The development of Daubechies's systematic technique for generating time-finite orthonormal wavelet functions by means FIR (Finite Impulse Response) filter banks [13] brought about many possibilities for the use of wavelet transformation also in the filed of early damage detection [14] and [15]. Most of expensive investigations were carried out for military industry and deal with the applications of newly developed algorithms of digital signal processing for early damage detection. For efficient crack detection it is necessary to detect those changes in diagnostic signals which do not overlap with changes due to other damages present. During the mesh of two gears the number of teeth in mesh changes periodically, which results in the changed teeth stiffness. This presents one of permanently present vibration sources in the gear pair, even if the gear pair is manufactured faultlessly. Early damage to a gear, such as local spalling or fatigue cracking, causes a change in the associated vibration signal only when the affected teeth are in mesh [10]. At the beginning and/or the end of the mesh, due to the altered tooth stiffness, the tooth vibrates for a very short time, which can be seen in frequency domain as the increase in harmonics amplitudes in high frequency area. On the basis of the analysis of numerically generated and experimentally measured diagnostic signals on gears with notches, characteristic changes occurring due to the altered stiffness of teeth in mesh are presented in this paper/as follows.

2. Numerical generation of diagnostic signal

The procedure of the numerical generation of diagnostic signal includes the building of mechanical replacement model, derivation of motion equations (mathematical model) and simulation, i.e. solving the system of motion equations.

2.1. Mechanical replacement model

The selection of replacement model poses a general problem in almost all fields of engineering. In selecting the replacement model, particularly the aim of researches is important. On that basis we can decide on the type and number of freedom steps and on which sources of excitation are worth being taken into account. With the increase of the number of freedom steps the replacement models become more complex and more complicated. As a result, computer processing of the motion equations becomes problematic if a suitable computer is not available.

For our case of modelling, we selected a replacement model of the gear pair shown in Figure 1.

The model in Figure 1a has two freedom steps φ_1 and φ_2. In modelling, the external moment M_1, the variable tooth stiffness k_z and tooth engagement damping d_z can be taken into account for excitation sources. Clearance between teeth was not taken into account in the model. With reduction to the engagement line (Figure 1b) the model becomes only one step of freedom [5].

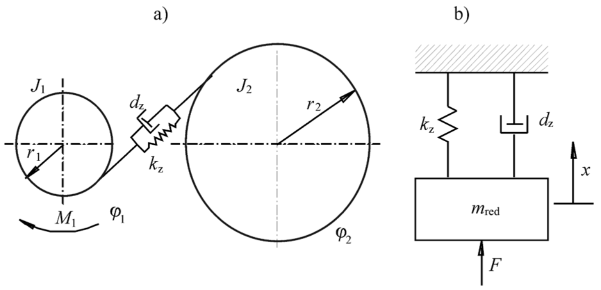

Figure 1. Mechanical replacement model of gear pair with a) two masses and b) reduced to the engagement line

Slika 1. Mehanički nadomjesni model jednostavnog zupčanog para (a) dvomasni model, (b) jednomasni model

2.1.1. Modelling of crack in tooth root

The sole excitation source considered was altering tooth stiffness which also facilitated the modelling of crack presence. Tooth stiffness inside the system presents periodical source of excitation (Figure 2a) regardless the fact that system is burdened with constant torque moment.

The form of tooth stiffness altered due to the presence of simulated crack must correspond to the greatest extent to real conditions during meshing. For this purpose, different forms of the altered tooth stiffness were investigated. The results show that there were no significant differences among the generated signal, as well as that the reduced tooth stiffness resulting from the presence of the simulated crack can be modelled in the form of one or two triangular or rectangular impulses [6]. In the present paper the double rectangular form is applied (Figure 2a).

Figure 2b shows the whole process of tooth stiffness. It is obvious that by each tooth rotation (every 19 teeth) the repetition of series of impulses takes place.

2.2. Mathematical model

For the numerical generation of diagnostic signals the 2nd order Lagrange's equation are used, which can be written in the main form as:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} + \frac{\partial V}{\partial q} + \frac{\partial P}{\partial \dot{q}} = F, \tag{1}$$

in which q is generalised coordinate

From the above equation, two motion equations were derived [16].

$$J_1\ddot{\phi}_1 + \left(\dot{\phi}_1 r_1 + \dot{\phi}_2 r_2\right) r_1 d_z + \left(\phi_1 r_1 + \phi_2 r_2\right) r_1 k_z(t) = M_1, \tag{2}$$

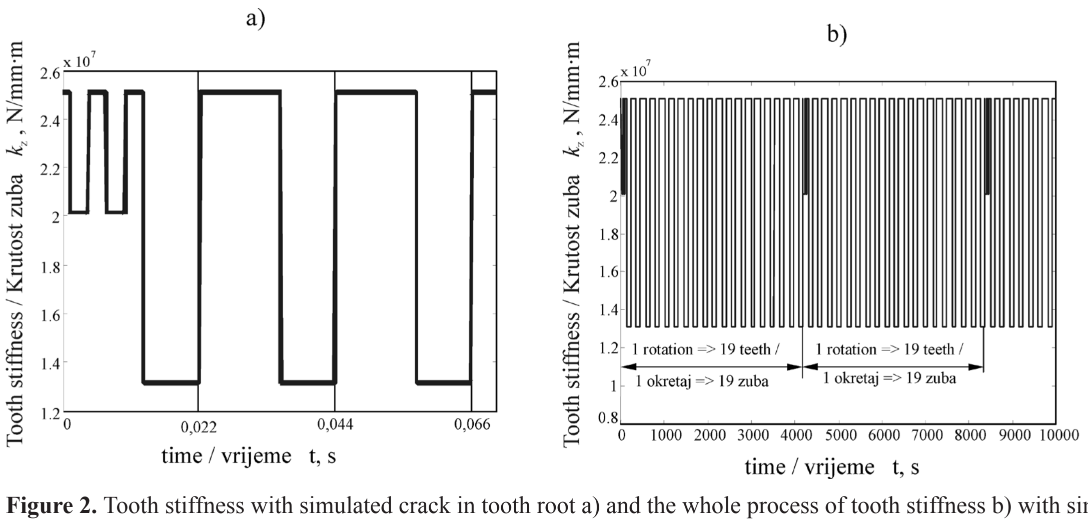

Figure 2. Tooth stiffness with simulated crack in tooth root a) and the whole process of tooth stiffness b) with simulated crack in tooth root (m=4 mm, z_1=19, z_2=34, α=20°, β=0°, n_1=1430 rpm)

Slika 2. Krutost zuba bez pukotine u korijenu zuba (a) i cjelokupan tijek krutosti zuba b) sa pukotinom u korijenu zuba (m=4 mm, z_1=19, z_2=34, α=20°, β=0°, n_1=1430 rpm)

$$J_2\ddot{\phi}_2 + \left(\dot{\phi}_1 r_1 + \dot{\phi}_2 r_2\right) r_2 d_z + \left(\phi_1 r_1 + \phi_2 r_2\right) r_2 k_z\left(t\right) = M_2. \quad (3)$$

Due to the periodically altered tooth stiffness, this is a linear system of equations with periodic coefficients. The periodical functions can be presented as the coordinates of linear oscillating system. Thus the linear periodic system is transferred into the non-linear system, connected to linear system of equations. Linear systems with periodic coefficients are always a special case of non-linear systems [17]. By system reduction to the meshing line, we get the equation of an oscillating one-mass pendulum. (Figure 1b).

$$m_{\text{red}} \cdot \ddot{x} + d_z \cdot \dot{x} + k_z\left(t\right) \cdot x = F. \quad (4)$$

The programme package *Matlab*© [7] was used to solve the system of motion equations. Figure 3 shows the graphical described mathematical model. The whole excitation was modelled as a sum of tooth stiffness without crack and influence of simulated crack in tooth root.

Damping value was during all simulations carried out set to be 1, which precludes its influence on the values of amplitudes gained. Simulation time was 0.1 seconds; therefore more than two turnings of gear took place.

2.3. Signals obtained by numerical simulations

Figure 4 shows the calculated signal from simulation without simulated crack (Figure 4a) and with simulated crack in toot root (Figure 4b).

Grid lines on x-axis are drawn on the distance of 453 Hz, which is equal the engagement frequency fz. This means that grid lines appear on the harmonic of engagement frequency. A comparison of signals shows visual changes in time and frequency domain. We can observe that 8th and 9th harmonics on Figure 5a) are significantly higher by the 3624 Hz and 4077 Hz. The major change in frequency domain is traced at the amplitude of 8 and 9th harmonics (frequency 3624 Hz) and also at several higher harmonics. The analysis of signals measured was carried out up to 4096 Hz only so that the higher values changes cannot be mutually compared.

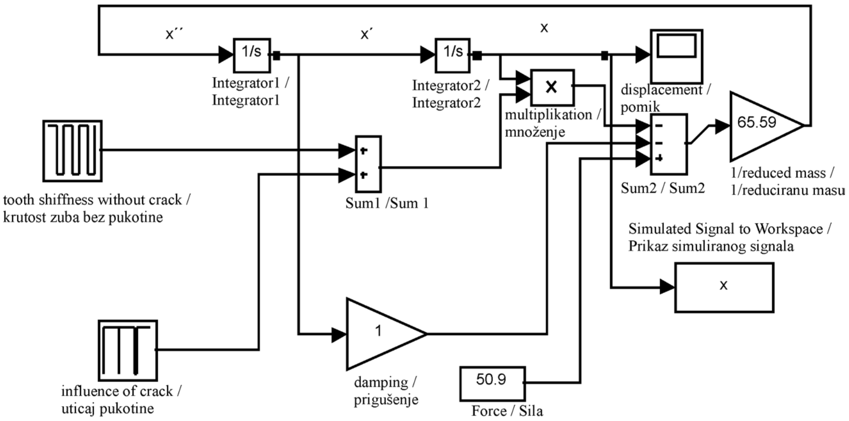

Figure 3. Mathematical model

Slika 3. Matematički model

3. Signal measurement

The testing site can be seen in the Figure 5. One step gear drive is placed in the middle of the measurement line. On the both sides are the motors. On the input side of the gear drive is the normal motor and on the output side of the gear drive is the servomotor, which is used as a break. The torque is measured on the output side with the special indicator. For shaft axes misalignment correction a clutch

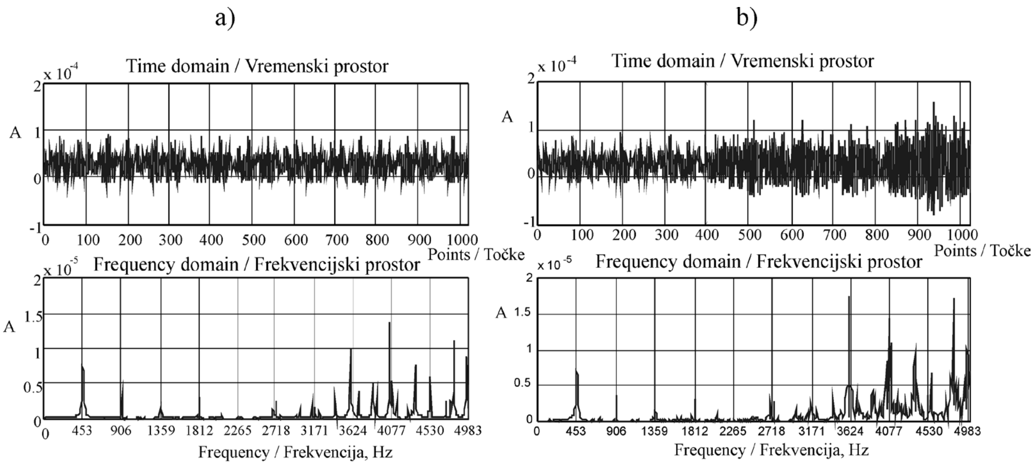

Figure 4. Calculated signal in time and frequency domain - with (a) and without (b) simulated crack in tooth root

Slika 4. Vremenski tijek signala i njegov frekvencijski spektar - sa (a) i bez (b) prisutnosti pukotine with rubber plugs was used and was installed between the torque indicator and the output side of a gear drive. To achieve different conditions a frequency regulator of the driving motor was used.

The measuring system is based on the LABVIEW measuring site. The site comprises a personal computer DEWEPORT 2000 (processor Pentium 166 MHz, 32 Mb RAM), measuring internal bus computer card NI AT 2150c specially adapted for this kind of measurements, two accelerometers Hewlett Packard (PCB Piezotronics) PCBI-353A16 and LABVIEW 4.0 software, which runs under Windows'98 and has been used for data analysis. The frequency range of accelerometers is from 1 Hz to 20 kHz with +/- 5 % error margin.

The Hewlett Packard (PCB Piezotronics) calibrator model 394B06 has been used to calibrate the accelerometers and the complete measuring system.

The subject of experimental measurements was a one-stage gear drive type EZ5.B3.132 C250.

From the viewpoint of fracture mechanics the fatigue crack cannot possibly be replaced with a notch (due to different path of crack extension, different stress field (Figure 6)…). But the crack and notch have common properties in worsening the root of tooth and hence diminishing its stiffness during pressure, which plays an important role in vibroacoustics.

From the viewpoint of vibroacoustics analysis, fatigue crack can be replaced by the notch, since both cause equal effect of decreasing in tooth stiffness during engagement. This can be measured with sensors on gear housing. Figure 7 illustrated pinion with the notch of 2 mm.

The sampling frequency was equal to 10 kS/sec. Apart from the notch lengths the following operating parameters, i.e. constant output torque Mt_2=90 Nm, constant output number of gear revolutions n_2=800 min^{-1}, have not been varied during the signal measurements.

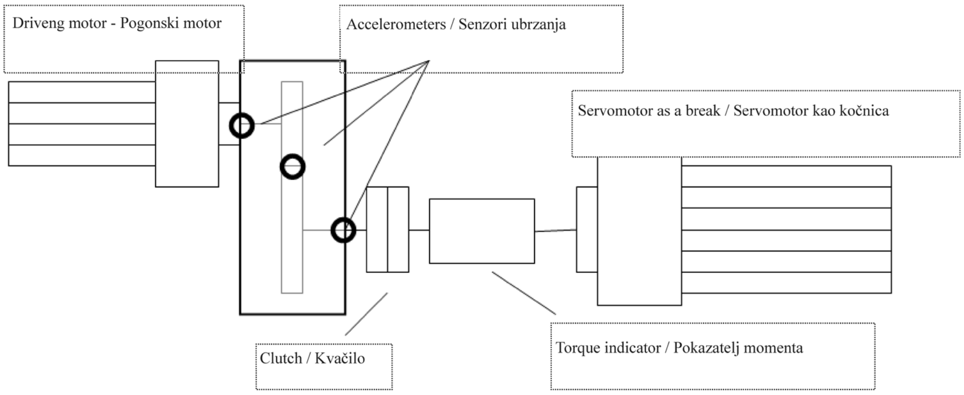

Figure 5. Testing setup

Slika 5. Sustav za testiranje

To determine the characteristic changes in the signal due to fatigue cracks in a tooth root three identical pinions (m=4, z=19) have been manufactured with notches of lengths a = 1, 2 and 3 mm. Now the outstanding question is whether the fatigue crack can be replaced with a notch?

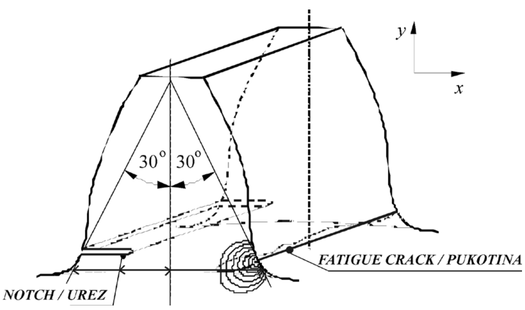

Figure 6. Difference between Fatigue Crack and Notch

Slika 6. Usporedba pukotine i ureza

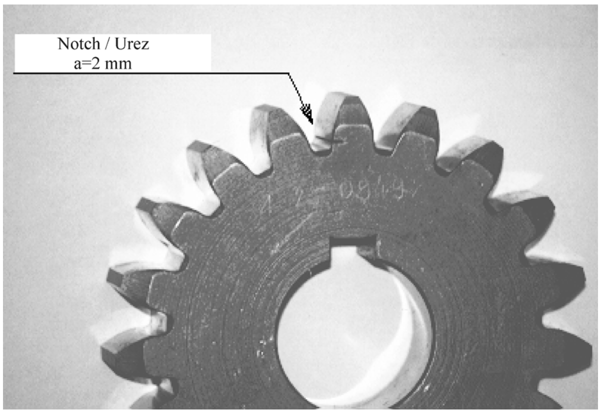

Figure 7. Tooth with notch length of 2 mm

Slika 7. Zupčanik sa urezom dubine 2 mm

2.3. Signals obtained by experimental measurements

Figure 8 shows the measured signals from the experiment without notch (Figure 8a) and with notch in toot root (Figure 8b).

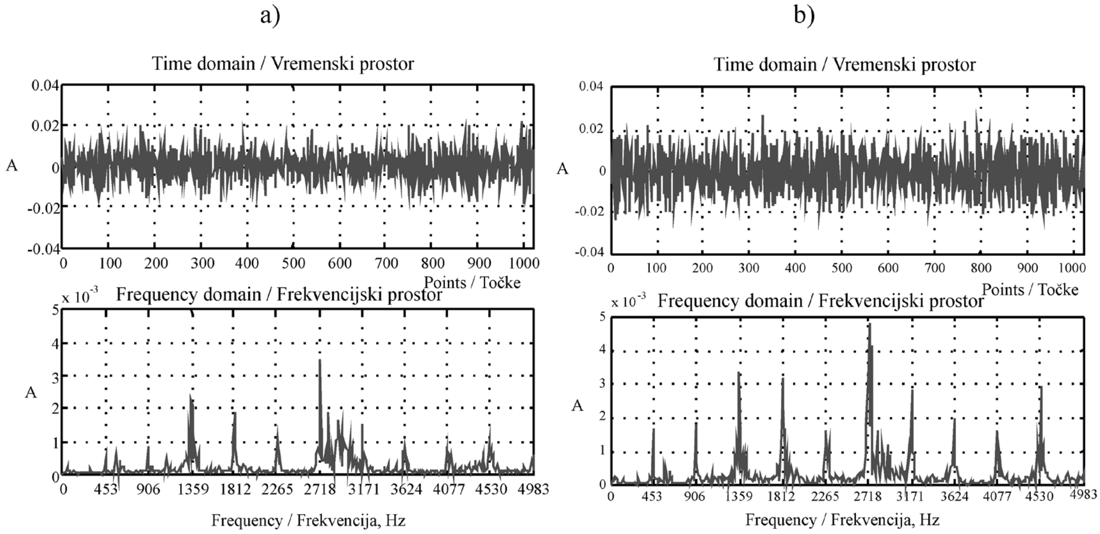

Figure 8. Measured signals in time and frequency domain – with (a) and without (b) notch in tooth root

Slika 8. Izmjeren vremenski tijek signala i njegov frekvencijski spektar - sa (a) i bez (b) prisutnosti ureza

4. Results of JTFA Analysis of numerical and experimentally obtained signals

4.1. Program DSP_studio

A new computer program DSP_studio has been developed with aid of LabView computer system and programming language G [8] for the purpose of signal recording, representation and determination of the signal characteristics in the time domain, frequency domain and Joint Time Frequency analysis (JTFA) analysis.

$$STFT(t,\omega) = \int_{-\infty}^{+\infty} x(\tau)\gamma^{*}_{t,\omega}(\tau)\mathrm{d}\tau = \int_{-\infty}^{+\infty} x(\tau)\gamma^{*}(\tau - t)\mathrm{e}^{-j\omega t}\mathrm{d}\tau. \tag{5}$$

The STFT (Short Time Frequency) analysis of signals measured was performed by JTFA Module within the Programme DSP_ Studio. The bases of the Module are built-in parts of the Joint Time-Frequency Analyser 3.1a from the National Instruments Programme. This programme facilitates the analysis of acquired data through acquisition cards or data stored on disks. Certain parts of stored signals can be analysed regarding the time or number of acquired points. Program draws a two-dimensional spectrogram by using the results of analysis. Spectrogram is in fact a triple-dimensional picture, where the third dimension is presented in different colours or grey tones. The first dimension is time measured in seconds. Its length is conditioned with the duration of signal being traced. The second dimension (y-axis) is represented by the frequencies measured in Hertz. Its length depends on pass-band which belongs to Nyquist frequency, which is equal to half of sample frequency. The third dimension (different colours) stands for the energy density present on the small rectangle area, which is defined by the time interval Δt in the direction of x-axis and frequency interval Δf in the direction of y-axis. Higher the energy density at given frequency and time is, the darker is the colour of the point in spectrogram [2] and [9].

Due to presence of crack in tooth root, its stiffness decreases which gives rise to higher amplitudes of tooth vibration. This can be detected in higher energy released which is manifested in spectrogram through darker tones.

4.2. STFT of measured signals

Figure 9 shows the spectrogram of signal detected on gear without crack in tooth root. Figure 10 shows the spectrogram of signal detected on gear with 2-mm crack length in tooth root. Both analyses were made with Hanning window, which proved to be the most suitable tool along the whole band pass. The window length was 64 points.

A comparison of both pictures shows the differences by higher harmonics of engagement frequency. The increase in amplitudes is apparent mostly up to 9th harmonics of basic frequency. In the interval between 7 - 9th harmonics it is possible to observe periodical repetition of engagement frequency of tooth with crack.

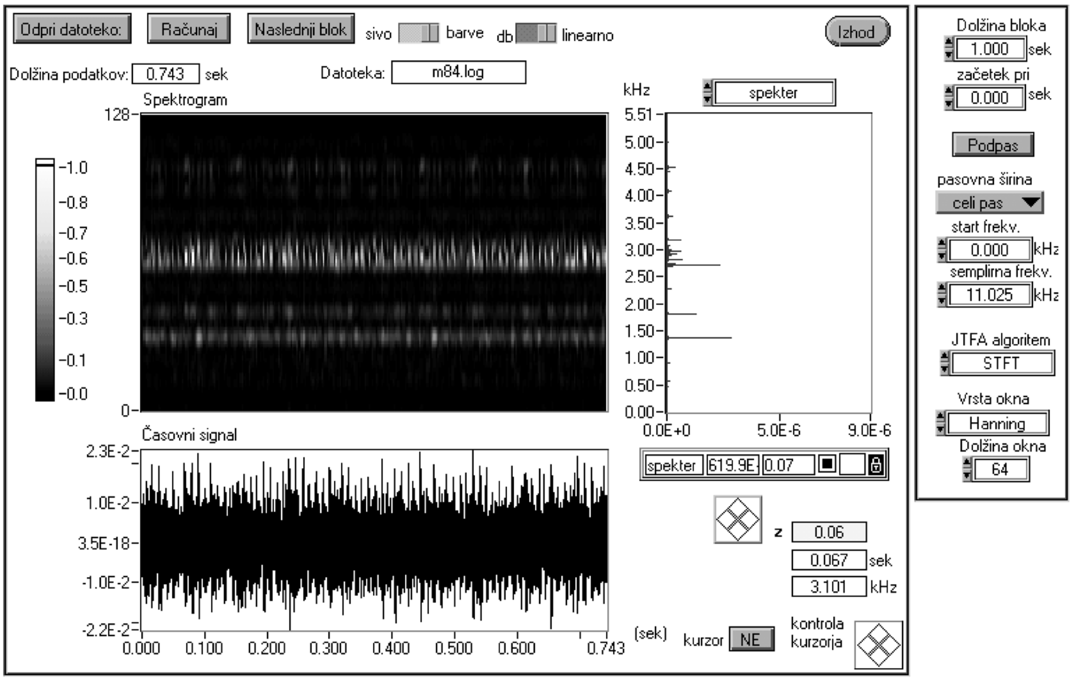

Figure 9. Spectrogram of signal detected on gear without notch in tooth root

Slika 9. Spektrogram signala snimanog na zupčaniku bez ureza

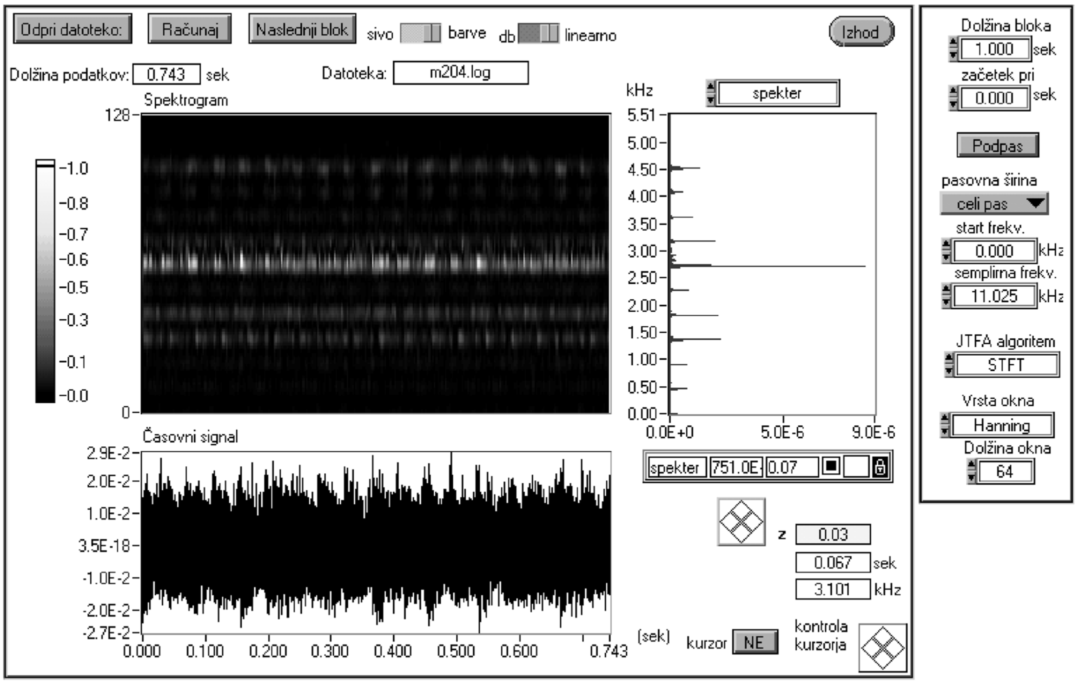

Figure 10. Spectrogram of signal detected on gear with notch a = 2 mm in tooth root

Slika 10. Spektrogram signala snimanog na zupčaniku sa urezom a=2 mm

Figure 11 is a detail from previous picture which demonstrates the presence of crack in tooth root in spectrogram.

Theoretical values of marked times are:

Engagement frequency f_z of gear with 19 tooth by 1430 rpm amounts to 452,8 Hz. Engagement time can be calculated following the equation:

$$t_{\text{tooth}} = \frac{1}{f_z} = \frac{1}{452{,}8} = 0{,}0022 \text{ sec.} \tag{6}$$

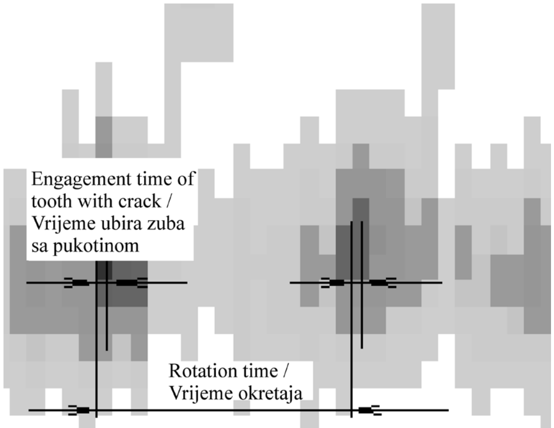

Figure 11. The presence of crack in tooth root in spectrogram (a detail from previous picture)

Slika 11. Prikaz posljedica pukotine u korijenu zuba na slici spektograma (detalj iz prethodne slike)

Rotation time can be obtained from frequency of shaft rotation f_g, which amounts to 23,83 Hz by 1430 rpm.

$$t_{rot} = \frac{1}{f_g} = \frac{1}{23.83} = 0.042 \text{ sec.} \tag{7}$$

The time values of tooth rotation obtained with measurements corroborate with the calculated values.

4.3. STFT of calculated signals

STFT was performed using also the numerically-obtained signals. Figure 12 shows the spectrogram of signal obtained by simulation without tooth crack.

Figure 13 demonstrates the spectrogram of signal obtained by simulation with tooth crack. A comparison of those two spectrogrames with the spectrogrames obtained on measured signals suggests the similarity around the area of the 8th harmonics (the same periodical colour tones).

The results obtained from measurements and the results calculated from numerical simulation corroborate very well.

The results derived suggest that the application of STFT makes possible detecting the repetition of local damages in spectrogram, which is not otherwise feasible when using classical Fourier transformation.

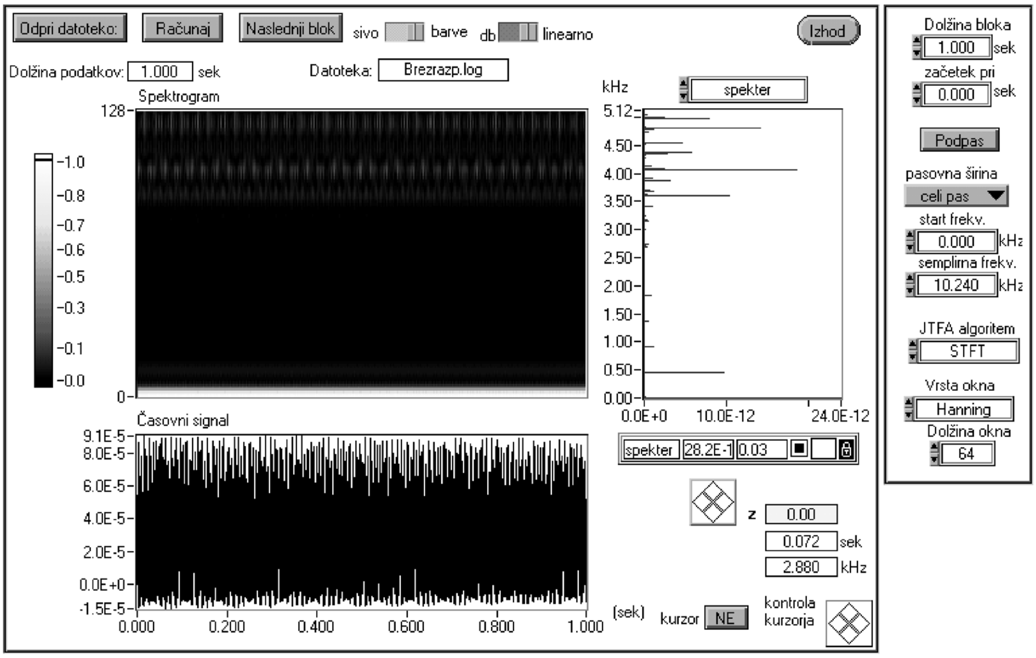

Figure 12. Spectrogram of signal obtained by simulation without tooth crack

Slika 12. Spektogram odaziva simulacije bez pukotine na zubu

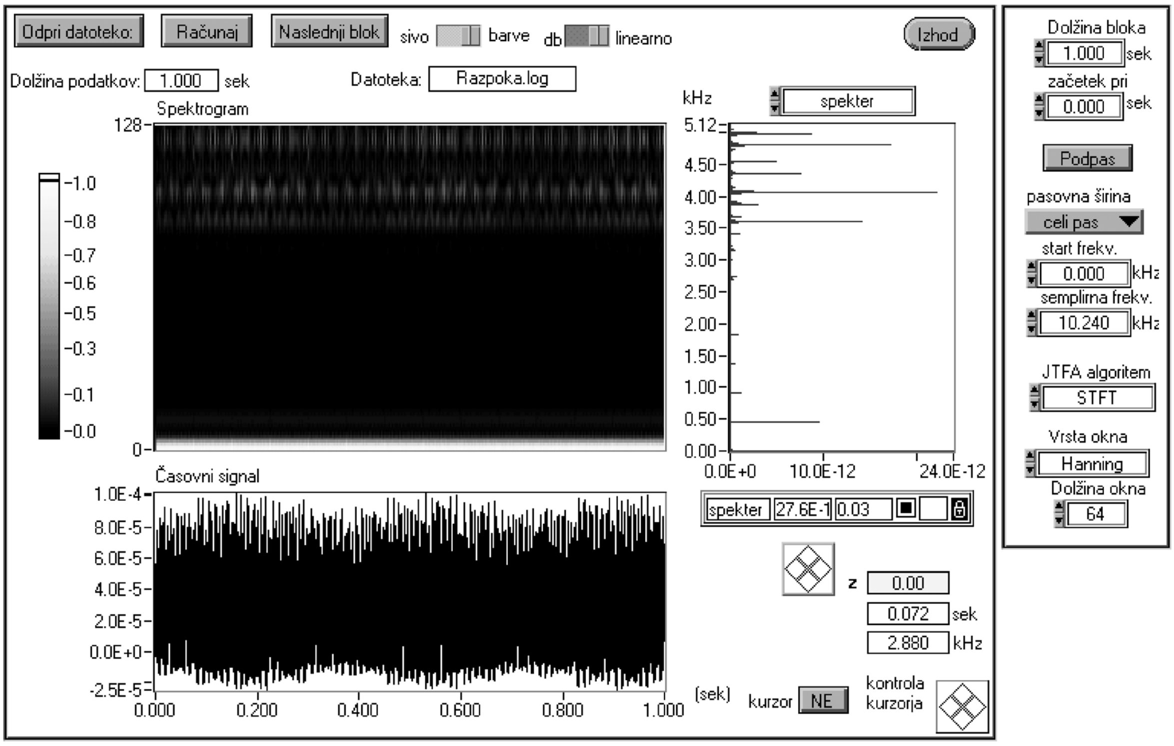

Figure 13. Spectrogram of signal obtained by simulation with tooth crack

Slika 13. Spektogram odaziva simulacije sa pukotinom na zubu

5. Conclusion

Each damage (also fatigue crack), which occurs in gear drive leads to certain changes in diagnostic signals that can be measured on the gear housing. The goal of vibroacoustics is to detect those changes, in our case occurring due to the presence of crack. With the STFT analysis we established that the changes in the spectrograms of numerically generated signals and those in the spectrogram of experimentally measured signals on the gears with the notch were similar. This shows that the altered tooth stiffness is the common characteristic value which plays an important role in detecting changes occurring due to notches and, consequently, cracks.

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