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THE ITERATIVE MULTIOBJECTIVE METHOD IN OPTIMIZATION PROCESS PLANNING

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Preliminary notes

Estimation of production time, delivery term, production costs etc., are some of the key problems of unit production. In the previous research strong correlation was discovered between the features of the product drawing and production time, which has resulted in 8 regression equations. They were realized using stepwise multiple linear regression. Since the optimization of these regression equations did not fully define the most frequent requirements, multiobjective optimization was applied. The applied criteria included: minimum production time, maximum work costs/total costs ratio for a group of workpieces. The group was created using specific classifiers that defined similar workpieces. An iterative STEP method with seven decision variables within a group was applied, and the groups with a high index of determination were selected. Independent values that maximize the work costs/total costs ratio and minimize production times were determined. The obtained regression equations of time production parts and work costs/total costs ratio are included in the objective functions to reduce production time and increase work costs/total costs at the same time. The values of decision variables that minimize production time and maximize work costs/total costs ratio were determined. As the solution of the described problem, multicriteria iterative STEP method was applied.

Key words: *multiobjective method for optimization, production time, regression analysis*

Iterativna višekriterijalna metoda u optimiranju tehnološkog procesa

Prethodno priopćenje

Procjena vremena izrade, roka isporuke, troškova izrade, itd. neki su od ključnih problema komadne proizvodnje. U prethodnom istraživanju uočena je jaka korelacijska veza između značajki nacrta proizvoda i vremena izrade koja je rezultirala s 8 regresijskih jednadžbi. One su realizirane primjenom postupne višestruke linearne regresije. Kako optimiranje tih regresijskih jednadžbi nije u potpunosti definiralo najčešće zahtjeve, primijenjena je višekriterijalna optimizacija. Kriteriji su bili: minimalno vrijeme izrade, maksimalan omjer troškova rada prema sveukupnim troškovima za grupu izradaka. Grupa je kreirana posebnim klasifikatorima koji su odredili slične izratke. Primijenjen je iterativni STEP model od sedam varijabli odluka unutar grupe, a odabrane su grupe s visokim indeksom determinacije. Određene su vrijednosti neovisnih varijabli maksimizirajući omjer troškova rada i ukupnih troškova te minimiziranjem komadnog vremena. Dobivene regresijske jednadžbe komadnog vremena izrade pozicija i omjer troškova rada prema ukupnim troškovima uključeni su u objektne funkcije kako bi se reduciralo komadno vrijeme izrade te istovremeno povećao omjer troškova rada prema ukupnim troškovima. To je odredilo vrijednosti varijabli odlučivanja koje minimiziraju komadno vrijeme i maksimiziraju omjer troškova rada prema ukupnim troškovima. Kao rješenje opisanog problema primijenjena je višekriterijalna interaktivna STEP metoda.

Ključne riječi: *komadno vrijeme, regresijska analiza, višekriterijalna metoda optimizacije*

1 Introduction
Uvod

Predicting events, fate of individuals, nations, rulers, health, success in warfare – has always been the focus of interest of all cultures and civilizations. If something could not be reached by ratio (reason), attempts were made to reach it in the sphere of irrational. Mystics, religious prophets, charismatic people with exceptional powers or qualities, people who were able to predict the future, either as sorcerers, astrologers, astronomers, palmists or as economic, stock-exchange, political and geo-strategic analysts, futurists were and still are appreciated in society. This is either due to curiosity, the need for decision-making, the desire for economic stability, good health, or due to fear of the future.

In the turbulent, global and neo-liberal market there is a pronounced need for predicting economic trends either in the microsphere or at the macroeconomic level. Defining comparative criteria for performance evaluation of companies in production strategies is an essential element of strategic considerations of the management of individual companies. Defining of long-term business objectives includes also defining of the range of products that have or will have a place in the market. Optimization of technological parameters in production for the purpose of cost reduction or production time shortening is often the subject of interest of numerous researchers and articles. The use of numerous methods of operational research and artificial intelligence are some of the approaches to the given problem. Of course, these are almost always partial approaches because of the complexity of the problem. The managements of companies on the other hand insist on as exact (comprehensive) as possible assistance in decision making, directing researchers to the area of business intelligence by defining broader areas of interest. In times of crisis, recession, and in the 'normal' business conditions as well, managements are constantly confronted with the same questions: how to reduce production times, delivery, production cycle; how to 'cut' all expenses including the costs of product manufacturing, and how to increase own share of the market pie; how to increase productivity; how to balance the productivity of all jobs during the process, especially when cycle production is concerned; how to increase the ratio of productive/unproductive time or cost; how to increase utilization of capacities, how to increase company profits… Such questions are a constant nightmare of all managements of manufacturing companies. Our experiences and numerous experiences of others as well, and following of economic trends in Croatia and wider have motivated us to start research in this area. Since a considerable number of research works and papers are dealing with optimization of technological parameters, we have decided to focus our attention on the relationship between product features (geometry, complexity, quantity,...) and production times and costs [1, 2, 3, 4, 5]. It has been proved that it is possible to make estimation of production time applying classification, group technology, stepwise multiple linear regression as the basis for accepting or rejecting of orders, based on 2D [2, 3] drawings, and the set basis for automatic retrieval of features from the background of 3D objects (CAD: Pro/E, CATIA) and their transfer to regression models [6, 7]. Of course, certain constraints have been set: application of standardized production times from technical documentation or estimations made using CAM software (CATIA, PRO/E, CamWorks), type of production equipment/technological documentation determines whether it will be single- or low-batch production. Initial steps have been taken regarding medium-batch, large-batch or mass production.

It has been assumed (relying on experience) that small companies (SMEs) in Croatia make decision about acceptance of production (based on customer's design solution of the product, delivery deadlines and manufacturing costs imposed by the customer - PICOS concept: automotive industry VW, GM) on the basis of free intuitive assessment due to the lack of time and experts. This often results in wrong estimates.

Since during the process of privatization in Croatia numerous large companies in the field of mechanical engineering disappeared, the newly created companies are "doomed" to work mainly for large international companies, providing only their work, without own share in innovativeness, without brand or patents and without transfer of new technologies. If the optimization of regression curves is to be applied (independent variables - product features, dependent variable – production time), it is hard to explain what it would mean for the minimum or maximum production time for a given group of products. The minimum production time could mean a higher productivity, but we do not know about the profit. The maximum production time could suggest that a higher occupancy of capacities may mean higher earnings, although it may not be so. This dual meaning has led us to introduce multiple objective optimization for a new class of variables that differently classify our products. A response variable (dependent variable) can assume several meanings: maximum profit per product, minimum delivery time (related to production time, and also to organizational waste of time, production balancing...), ratio of the production cost and the costs of product materials, ratio of the production cost and the ultimate production cost. Thus, the problem-solving approach has become more complex, and is no longer a mere result of intuition and heuristics, but more exact assessment of 'common' optimum for more set criteria.

2 Previous research

Prethodno istraživanje

In the first part of the research of possible relationship between 2D product features and production time, regression equations were obtained for the considered groups of geometrically and technologically similar products. The research was limited to the following: workpiece initial shape – round bar, classical machine tools, small batch production (based on original technological documentation of the former largest machine tools manufacturer 'Prvomajska' in Croatia and in ex-Yugoslavia until the year 1990), and customary sequence of operations. The values of independent variables (50!) were taken from "classical" paper drawings and technological standards. Of course, a certain degree of subjectivity is present in defining work norms and setting of norms for machining of some parts. Some subjectivity of the people working in the Department of Time and Work Study in "Prvomajska" Machine Tool Factory (until 1990) could be assumed, because several employees were dealing with time assessment issues. At the same time, work norms for workers performing certain operations were often very low in order to provide overreaching of the work norms and higher wages for direct workers, proving thus the much proclaimed loyalty to the "working class" and success of the established system of "self-management" in the Yugoslav type of socialism with a "human face". Therefore, having all this in mind, a systematic error was taken into consideration in the estimation of time standards. One of the co-authors of this paper (Antolić) was for some time the technical director of INAS company, a small successor of "Prvomajska"

Table 1 *Presentation of created regression equations 2D*
Tablica 1. *Prikaz sačinjenih regresijskih jednadžbi 2D*

No	Shape of product - representative of product group	Regression equations	Index of determination, r^2	Relative error, %	Comment on regression equation
1	Whole sample	$t = -11.69 + 16.95x_{45} + 1.22\,x_{40} + 0.54\,x_{47} + 127.47x_{22} - 3.24x_{18} + 0.15x_{32} + 0.03x_6$	0.736552	30.74	Model is developed with procedure in advance. Three independent variables are omitted x_8, x_{19} and x_{33}.
2	Round bars	$t = 55.47 + 22.43x_{45} + 1.162\,x_{40} + 0.43x_{11} + 1.61x_{50} - 5.41x_8 - 3.26x_{18} + 1.78x_{42}$	0.74285	30.95	Model is developed with procedure in advance.Two independent variables are omitted x_1 and x_{26}.
3	Shafts	$t = 6.13 + 0.83x_2 + 1.27x_{39} - 3.30x_8 + 5.51x_{46} - 6.86x_{18} + 0.09x_6 + 124.33x_{22}$	0.807626	25.90	Model covers more narrower field of rotational parts. It gives better results than No. 2.
4	Discs	$t = -5.17 + 0.73x_{47} + 0.93x_{40} + 5.25x_{20} + 0.52x_{24} + 139.11x_{30} + 0.23x_{32} - 0.51x_{33}$	0.809405	24.24	Simillar results as in No. 3.
5	Discs-with fine machining	$t = -60.78 + 0.59x_{47} + 047x_9 + 0.74x_1 + 0.25x_{10} + 0.84x_{39} + 291.07x_{25} + 5.9x_{15}$	0.985057	8.01	Model covers more narrower field of rotational parts. It gives better results than all the previous models.
6	Rotational parts	$t = -37.11 + 0.94x_{40} + 0.03x_{29} + 319.22x_{26} + 0.13x_{23} + 114.67x_{43} - 80.98x_{45} - 0.46x_6$	0.893321	27.06	Model is better than No. 2 as a result of higher degree of homogenization of data. Solution is better with omitted variable,x_2 and included variables x_6, x_{23}, x_{43} and x_{45}.
7	Flat bars	$t = -10.96 + 0.58x_{40} + 34.50x_{45} + 218.42x_{22} - 5.48x_{50} + 185.03x_{26} + 0.39x_9 - 0.50x_{49}$	0.900332	15.92	Constraints are greater for all variables so results are better. Narrow field of homogenization.
8	Sheet metals	$t = 0.47 + 1.27x_{40} + 137.45x_{45} - 13.23x_{43} - 0.70x_{43} + 0.28x_4 + 0.05x_6 + 3.91x_{16}$	0.900823	24.04	Model is characterized by the presence of complex variables x_{40}, x_{43}, x_{45}

Machine Tool Factory, which finally ceased to exist in 2009. Thus, the used technological documentation for classical milling machines (420 positions) is from that source. By classification of products, according to the BTP, 8 regression equations for 8 groups of products were obtained. The main grouping criteria were the features (geometrical, tolerance, hardness) from the technical drawings and for each product the production time was used (technological and auxiliary time). However, since today is the time of 3D modeling, CNC, and machining centers, the initial research for the development of automatic retrieval of product features from 3D models was conducted. Using CAM software, for these 3D models technological time was calculated in order to obtain regression equations for the estimation of production time. Thus, the following was obtained in Table 1.

$$Y = 28{,}77308 + 8{,}277896x_{19} - 0.16359K_s - 1.46341f_{ea} - 50{,}8704x_{45} + 0{,}000324x_{44} + 0.002462x_{43} \quad (1)$$

$2{,}00 < x_{19} < 8{,}00$ – tolerance of dimension line of the part (2)

$13{,}00 < K_s < 46{,}00$ – all dimension lines (3)

$9{,}00 < f_{ea} < 25{,}00$ – features of 3D model (4)

$0{,}174 < x_{45} < 0{,}584$– mass of the part (5)

$4\,063{,}80 < x_{44} < 74\,724{,}50$ – volume of the part (6)

$6\,660{,}70 < x_{43} < 28\,131{,}30$ – superficial area (7)

$45{,}00 < Y < 111{,}00$ – production time. (8)

Error between estimation by regression and calculated production time for each part (-5,64 %; + 4,32 %).

Table 2 *Overview of new classifiers of products*
Tablica 2. *Pregled novih klasifikatora proizvoda*

CLASSIFIERS W1 – W5				
W1 (material)	W2 (shape)	W3 (according to max. product dimension)	W4 (complexity) BA – number of dimension lines	W5 (treatment complexity)
1 - Polymers 2 - Aluminium and aluminium alloys 3 - Copper and copper alloys 4 - Non-alloy steel 5 - Alloy steel	1 - Rotational (round bars, round tubes, hexagons, plates) 2 - Prismatic (plates, flat, rectangular tubes) 3 - Profile (L, U, I, Z, C) 4 - Sheet-metal (foils, strips, sheets) 5 - Complex	1 - mini (V<120) 2 - midi (120<V<400) 3 - standard (400<V1<1 000) 4 - kilo (1 000 <V<2 000) 5 - mega (V>2 000 mm)	1 - very simple BA≤5 2 - simple 5>BA≤10 3 - average 11>BA≤25 4 - complex 25>BA≤75 5 - very complex BA>75	1 - very rough 2 - rough 3 - medium 4 - fine 5 - very fine treatment

- Conditions were determined on the basis of the data range on the number of dimension lines of the considered sample of 415 elements. A classifier which is being developed is based on 5 basic product features:
- W1- MATERIAL (quality of material)
- W2 - SHAPE (prevailing shape of product)
- W3 - SIZE (according to the product maximum dimension)
- W4 - COMPLEXITY (with respect to the number of tips, edges, surfaces; number of dimension lines in 2D model …)
- W5 - TREATMENT COMPLEXITY (requirements regarding surface, roughness, measurement tolerances, shape tolerances and position tolerances)

It was found that the optimization of regression equations, in order to obtain minimum or maximum production times was insufficient with respect to the needs in real production. Thus, the aim was to obtain, by considering a series of regression equations, the optimum for multiobjective optimization (minimal production time, labor cost/material cost ratio or labor cost/total cost ratio for the selected group of products. As multiobjective optimization requires the same variables (x_1,...x_7), it was necessary to make new grouping of the basic set (302 workpieces) using new classifiers.

The conditions were defined based on the range of data about the number of dimension lines on the considered sample of 415 elements. A classifier that is being developed is based on 5 basic workpiece features. For the purpose of the research, a group of workpieces (W1-W5) 41113 was selected for further analysis. The code 41113 means: steel – rotational – small – very simple – commonly complex - workpieces. From the available database, the minimum and maximum values for independent variables, and dependent variable (Z_1-production time), and derived variable Z_2 were taken.

Two regression equations, Z_1 (production time) and Z_2 (labor cost/total cost ratio), were selected. For them multiobjective optimization was also performed. In order to use the same types of variables, new grouping was made using specifically adjusted classifiers. *Workpiece classification according to the criterion of complexity* was done semi-automatically by setting conditions on certain features of drawings (basic roughness, the finest roughness requirement, the narrowest tolerance of measures, the narrowest tolerance of shape or position (geometry), number of all roughness and geometry requirements in the drawing. Each of these 6 criteria based on its specific conditions is assigned a value ranging from 1 to 5. The obtained result is rounded to integer (e.g. 3,49 is W=3, and 3,51 is W=4), and this integer (in the range from 1 to 5) becomes complexity criterion coefficient (the fifth digit in the code).

Conditions were set regarding:

1. basic roughness (common for all surfaces that are not separately specified) – unit of measure is *R*a (surface roughness)
2. finest roughness requirement (specified in the drawing) - unit of measure is *R*a, it was so indicated in 2D drawings (roughness requirement)
3. narrowest tolerance of measures (mm) (measurements requirement)
4. narrowest tolerance of diameter (unit of measure is IT – diameter requirement)
5. narrowest tolerance of shape or position (geometry requirement)
6. number of all requirements on roughness and geometry

Table 3 *Minimum and maximum values of selected variables*
Tablica 3. *Minimalne i maksimalne vrijednosti odabranih varijabli*

PRODUCT TYPE 41113									
min	**2.90**	**0.100**	**1.00**	**11.21**	**0.22**	**0.0132**	**0.001**	**6**	**11.09**
max	**100.00**	**0.400**	**5.00**	**19.63**	**12.50**	**0.3972**	**0.820**	**33**	**2 524.33**
arithmetic mean	28.75	0.388	1.63	11.63	3.47	0.1177	0.105	17.75	406.88
standard deviation	21.87	0.061	1.1 3	1.71	3.02	0.1151	0.189	8.83	641.74
mode	36.00	0.400	1.00	11.21		0.0735	0.048	12	
range	97.10	0.300	4.00	8.41	12.28	0.3840	0.819	27	2 513.24
sum	689.90	9.300	39.00	279.06	83.18	2.8249	2.518	426	9 765.21
variable	***X*1**	***X*2**	***X*3**	***X*4**	***X*5**	***X*6**	***X*7**	***Y*1**	***Y*2**
Variable description	Product outer diameter	Narrowest tolerance of measures	Scale of the drawing	Material mass/strength ratio	Wall thickness/length ratio	Material surface area	Material mass	Production time	Work cost/material cost ratio
unit of measure jedinica mjere	mm	mm	number broj	*	number broj	dm^2	kg	h 10^{-2}	number broj

Table 4 *Results of stepwise multiple linear regression*
Tablica 4. *Rezultati postupne višestruke linerarne regresije*

Regression Statistics	Dependent variable - production time Z_1	Regression Statistics	Dependent variable- work costs/ultimate costs ratio Z_2
Multiple *R*	0.92212166	Multiple *R*	0.99207
R Square	0.85030835	*R* Square	0.984202
Adjusted *R* Square	0.78481826	Adjusted *R* Square	0.977291
Standard Error	4.09742037	Standard Error	0.002725
Observations	24.0	Observations	24.0
Z_1	Coefficients	Z_2	Coefficients
Intercept	-13.490042	Intercept	0.990439
X Variable 1	0.86652065	X Variable 1	0.000238
X Variable 2	-0.1993556	X Variable 2	-0.0039
X Variable 3	0.75343156	X Variable 3	0.00046
X Variable 4	1.41593567	X Variable 4	0.000794
X Variable 5	-1.8669075	X Variable 5	-0.00107
X Variable 6	4.83640676	X Variable 6	-0.04466
X Variable 7	51.274031	X Variable 7	-0.08551

specified in the drawing (i.e. how many surfaces are to be particularly finely treated and how many surfaces have special tolerances concerning the shape or position (in relation to another surface; roughness and geometry requirement.

3 Description of the Objective Model
Opis objektnog modela

The general multiobjective optimization problem with n decision variables, m constraints and p objectives is [8]:

$$\text{maximize } Z(x_1,x_2,...,x_n) = \\ = \left[Z_1(x_1,x_2,...,x_n), Z_2(x_1,x_2,...,x_n),...Z_p(x_1,x_2,...,x_n)\right] \quad (9)$$

$$\text{s.t. } g_i(x_1,x_2,...,x_n) \le 0, \; i=1,2,...,m \\ x_j \ge 0, \; j=1,2,...,n \quad (10)$$

where $Z(x_1, x_2, \ldots, x_n)$ is the multiobjective objective function and $Z_1(\;)$, $Z_2(\;)$, $Z_p(\;)$ are the p individual objective functions. Benayoun [9] (1971) developed the step method as an iterative technique that should converge to the best-compromise solution in no more than p iterations, where p is the number of objectives. The method is based on a geometric notion of best, i.e., the minimum distance from an ideal solution, with modifications of this criterion derived from a decision maker's (DM) reactions to a generated solution. The method begins with the construction of a payoff table. The table is found by optimizing each of the p objectives individually, where the solution to the k^{th} such individual optimization, called x^k, gives by definition the maximum value for the k^{th} objective, which is called M_k (i.e., $Z_k(x^k) = M_k$). The values of the other p - 1 objectives implied by x^k are shown in the k^{th} row of the payoff table. The payoff table is used to develop weights on the distance of a solution from the ideal solution. The step method employs the ideal solution, which has components M_k for k = 1, 2, ... , p. The ideal solution is generally infeasible. The λ, metric is used to measure distance from the ideal solution. The distance is scaled by a weight based on the range of objective Z_k and the feasible region is allowed to change at each iteration of the algorithm. The basic problem in the step method is:

$$\text{Min}\ \lambda \qquad (11)$$

$$\Pi_k \left[M_k - Z_k(x)\right] - \lambda \leq 0, \quad k = 1,2..., p \qquad (12)$$

$$x \in F_d^i \quad \lambda \geq 0 \qquad (13)$$

where F_d^i is the feasible region at the i^{th} iteration and λ is used to indicate that the original metric has been modified. Initially, $F_d^0 = F_d$; i.e., at the start of the algorithm the original feasible region is used in (13). The weights π_k in (12) are defined as:

$$\Pi_k = \frac{\alpha_k}{\sum_1^k \alpha_k} \qquad (14)$$

where

$$\alpha_k = \frac{M_k - n_k}{M_k} \left[\sum_{j=1}^{n} (c_j^k)^2\right]^{-\frac{1}{2}} \qquad (15)$$

where n_k is the minimum value for the k^{th} objective; i.e. it is the smallest number in the k^{th} column of the payoff table. The c_j^k are objective function coefficients, where it is assumed that each objective is linear.

$$Z_k(x) = c_1^k x_1 + c_2^k x_2 + ... + c_n^k x_n,\ k = 1,2,..., p. \qquad (16)$$

The solution of (11) to (13) with F_d in (13) yields a non-inferior solution x(0), which is closest, given the modified metric in (14), to the ideal solution. The decision maker (DM) is asked to evaluate this solution. If it is satisfactory, the method terminates; if it is unsatisfactory, then the decision maker specifies an amount ΔZ^*_k by which objective k^* may be decreased in order to improve the level of unsatisfactory objectives, where objective k^* is at a more than satisfactory level. A problem with a new feasible region in decision space is then solved. A solution is feasible to the new problem, $x \in F_d^{i+1}$, if and only if the following three conditions are satisfied:

$$x \in F_d^i \qquad (17)$$

$$Z_k(x) \geq Z_k(x^i) \quad \forall k \neq k^* \qquad (18)$$

$$Z_{k^*}(x) \geq Z_{k^*}(x^i) - \Delta Z_{k^*}. \qquad (19)$$

For the new problem $\alpha_{k^*} = 0, \pi_{k^*} = 0$, and the other π_k are recomputed from (14) for $k \neq k^*$. The problem in (11) to (13) is then resolved with $i = i + 1$, and size $\pi_{k^*} = 0$, (12) includes constraints for $k \neq k^*$ only. The solution to the new problem yields a new non-inferior solution, which the decision maker evaluates. The method continues until the decision maker is satisfied, which the authors claim occurs in fewer than p iterations.

4 Results of the Multiobjective Analysis
Rezultati višekriterijalne analize

On the basis of considerations of regression functions in previous sections, the problem of multiobjective optimization with minimization of the objective functions Z_1 and Z_2 with related constraints (equations (20) to (22)) is defined.

$$\text{Min } Z_1 = -13{,}49004192 + 0{,}866520652 \cdot \boldsymbol{x_1} - 0{,}199355601 \cdot \boldsymbol{x_2} + 0{,}753431562 \cdot \boldsymbol{x_3} + 1{,}415935668 \cdot \boldsymbol{x_4} - 1{,}866907529 \cdot \boldsymbol{x_5} + 4{,}836406757 \cdot \boldsymbol{x_6} - 51{,}27403107 \cdot \boldsymbol{x_7} \qquad (20)$$

$$\text{Min } Z_2 = -0{,}990438731 - 0{,}000238475 \cdot \boldsymbol{x_1} + 0{,}003897645 \cdot \boldsymbol{x_2} - 0{,}00045981 \cdot \boldsymbol{x_3} - 0{,}000794225 \cdot \boldsymbol{x_4} + 0{,}0010738 \cdot \boldsymbol{x_5} + 0{,}044664232 \cdot \boldsymbol{x_6} + 0{,}085514412 \cdot \boldsymbol{x_7} \qquad (21)$$

$$\boldsymbol{x_1} \leq 100;\ \boldsymbol{x_2} \leq 0{,}4;\ \boldsymbol{x_3} \leq 5{,}0;\ \boldsymbol{x_4} \leq 19{,}63;\ \boldsymbol{x_5} \leq 12{,}50;\ \boldsymbol{x_6} \leq 0{,}3972;\ \boldsymbol{x_7} \leq 0{,}820 \qquad (22)$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$$

In equations (20) and (21) Z_1 represents variable *T*, and Z_2 variable *TU/TR*. It should be mentioned that for the needs of consistency of the objective functions Z_1 and Z_2, for the objective function Z_2 (equation (21)) the signs of the coefficients of variables and of the free member have been changed. The values of objective functions Z_1 and Z_2 in the extreme points of the set of possible solutions (feasible region) are given in Table 3. It is visible from the table that that there is no common set of points (x_1,... x_7) where both functions Z_1 and Z_2 have extreme (maximum) values, and thus the need for optimization of the given problem is justified.

On the basis of the data given in Table 5 the data for the first payoff table (Table 6) have been selected, which is necessary for the calculation of the first compromise solution.

***Table 5** Values of the decision variables and the objective functions*
***Tablica 5.** Vrijednosti varijabli odlučivanja i objektnih funkcija*

Extreme point	Decision variables							Objective functions	
	x_1	x_2	x_3	x_4	x_5	x_6	x_7	$Z_1(x_1...x_7)$	$Z_2(x_1...x_7)$
A	100	0	0	0	0	0	0	**73,1620**	-1,0143
B	0	0,4	0	0	0	0	0	-13,5698	-0,9889
C	0	0	5	0	0	0	0	-9,7229	-0,9927
D	0	0	0	19,63	0	0	0	14,3048	-1,0060
E	0	0	0	0	12,50	0	0	-36,8264	-0,9770
F	0	0	0	0	0	0,3972	0	-11,5690	-0,9727
G	0	0	0	0	0	0	0,820	-55,5347	**-0,9203**

Table 6 *First payoff table*
Tablica 6. *Prva payoff tablica*

Point of optimal solution X^k	Ideal values (M_k) of objective functions (Z_k) for X^k	
	$M_1=Z_1(X^k)$	$M_2=Z_2(X^k)$
X^1=(100,0,0,0,0,0,0)	**73,1620**	−1,0143
X^2=(0,0,0,0,0,0,0.820)	−55,5347	− **0,9203**

where k=1...2. In accordance with equations (14) and (15) coefficients of equation (12) are calculated, which is shown by the expressions (23) through (26).

$$\alpha_1 = \frac{73.1620-(-1.0143)}{73.1620}\cdot\frac{1}{\sqrt{2659.3}} = 1.0139\cdot 0.0194 = 0.0197 \tag{23}$$

$$\alpha_2 = \frac{-55.5347-(-0.9203)}{-55.5347}\cdot\frac{1}{\sqrt{0.0093}} = 0.9834\cdot 10.3695 = 10.1974 \tag{24}$$

$$\Pi_1 = \frac{\alpha_1}{\alpha_1+\alpha_2} = \frac{0.0197}{0.0197+10.1974} = 0.0019 \tag{25}$$

$$\Pi_2 = \frac{\alpha_2}{\alpha_1+\alpha_2} = \frac{10.1974}{0.0197+10.1974} = 0.9981 \tag{26}$$

Arranging the obtained equations, the problem of multiobjective optimization has been practically reduced to the problem of single-objective optimization where the variable λ is minimized according to equation (11). The set of equations for the calculation of the first compromise solution of the given problem is shown in Table 6, and the results of decision variables (x_1,...x_7) and objective functions Z_1 and Z_2 are given in Table 8.

Table 7 *Set of equations of the first compromise solution*
Tablica 7. *Set jednadžbi prvog kompromisnog rješenja*

Min λ
$-\lambda - 0.016463892\cdot\mathbf{x_1} + 0.003787756\cdot\mathbf{x_2} - 0.014315200\cdot\mathbf{x_3} - 0.026902778\cdot\mathbf{x_4} + 0.035471243\cdot\mathbf{x_5} - 0.091891728\cdot\mathbf{x_6} + 0.974206590\cdot\mathbf{x_7} \le -1.6465$
$-\lambda + 0.000238022\cdot\mathbf{x_1} - 0.003890239\cdot\mathbf{x_2} + 0.000458936\cdot\mathbf{x_3} + 0.000792716\cdot\mathbf{x_4} - 0.001071760\cdot\mathbf{x_5} - 0.044579370\cdot\mathbf{x_6} - 0.085351935\cdot\mathbf{x_7} \le -0.070005466$
$\mathbf{x_1} \le 100; \mathbf{x_2} \le 0.4; \mathbf{x_3} \le 5.0; \mathbf{x_4} \le 19.63; \mathbf{x_5} \le 12.50; \mathbf{x_6} \le 0.3972; \mathbf{x_7} \le 0.820;$

Since in the given problem there are two objective functions, it is necessary to make calculation of the second compromise solution, and thus the previous equations for Z_1 and Z_2 become new constraints shown in equations (27) and (28)

Table 8 *Results of the first compromise solution*
Tablica 8. *Rezultati prvog kompromisnog rješenja*

$\mathbf{x_1}$=100; $\mathbf{x_2}$=0.4; $\mathbf{x_3}$=1.0; $\mathbf{x_4}$=12.0428; $\mathbf{x_5}$=12.5; $\mathbf{x_6}$=0.3962;
$\mathbf{x_7}$=9999998E-4; λ=7.128304E−2;
Min Z

$$0.866520652\cdot\mathbf{x_1} - 0.199355601\cdot\mathbf{x_2} + 0.753431562\cdot\mathbf{x_3} + 1.415935668\cdot\mathbf{x_4} - 1.866907529\cdot\mathbf{x_5} + 4.836406757\cdot\mathbf{x_6} - 51.27403107\cdot\mathbf{x_7} \le 82.90614192 \tag{27}$$

$$-0.000238475\cdot\mathbf{x_1} + 0.003897645\cdot\mathbf{x_2} - 0.00045981\cdot\mathbf{x_3} - 0.000794225\cdot\mathbf{x_4} + 0.0010738\cdot\mathbf{x_5} + 0.044664232\cdot\mathbf{x_6} + 0.085514412\cdot\mathbf{x_7} \le -0.001061269 \tag{28}$$

Since the value $Min\ Z_1(x_1,...x_7)$= 69.4161, it has been decided that the previous value for M_1 =73.1620 is to be reduced for the value of 33.1620, and thus the new value for M_1=40. The second payoff table is given below.

Table 9 *Second payoff table*
Tablica 9. *Druga payoff tablica*

Point of optimal solution X^k	Ideal values (M_k) of objective functions (Z_k) for X^k	
	$M_1=Z_1(X^k)$	$M_2=Z_2(X^k)$
X^1=(100,0,0,0,0,0,0)	73,1620 − 33,1620=**40**	−1,0143
X^2=(0,0,0,0,0,0,0.820)	−55,5347	−**0,9203**

where k=1...2. In accordance with equations (14) and (15), coefficients of equation (12) are calculated, which is shown by the expressions (29) through (32). Since only the value of variable M_1 has been changed, the values of equations (30) and (32) remain the same as in the case of calculation of the first compromise solution.

$$\alpha_1 = \frac{40-(-1.0143)}{40}\cdot\frac{1}{\sqrt{2659.3}} = 1.0254\cdot 0.0194 = 0.0199 \tag{29}$$

$$\alpha_2 = \frac{-55.5347-(-0.9203)}{-55.5347}\cdot\frac{1}{\sqrt{0.0093}} = 0.9834\cdot 10.3695 = 10.1974 \tag{30}$$

$$\Pi_1 = \frac{\alpha_1}{\alpha_1+\alpha_2} = \frac{0.0199}{0.0199+10.1974} = 0.0019 \tag{31}$$

$$\Pi_2 = \frac{\alpha_2}{\alpha_1+\alpha_2} = \frac{10.1974}{0.0197+10.1974} = 0.9981 \tag{32}$$

As in the case of the first compromise solution, by arranging the obtained equations, the problem of multiobjective optimization has been reduced to the problem of single-objective optimization where the variable λ is minimized according to equation (11). The set of equations for the calculation of the second compromise solution of the given problem is shown in Table 10, and the results of decision variables (x_1,...x_7) and objective functions Z_1 and Z_2 are given in Table 11.

Table 10 *Set of equations of the second compromise solution*
Tablica 10. *Set jednadžbi drugog kompromisnog rješenja*

Min λ
$-\lambda - 0.001646389\cdot\mathbf{x_1} + 0.000378776\cdot\mathbf{x_2} - 0.001431520\cdot\mathbf{x_3} - 0.002690278\cdot\mathbf{x_4} + 0.003547124\cdot\mathbf{x_5} - 0.009189173\cdot\mathbf{x_6} + 0.097420659\cdot\mathbf{x_7} \le -0.101631080$
$-\lambda + 0.000238022\cdot\mathbf{x_1} - 0.003890239\cdot\mathbf{x_2} + 0.000458936\cdot\mathbf{x_3} + 0.000792716\cdot\mathbf{x_4} - 0.001071760\cdot\mathbf{x_5} - 0.044579370\cdot\mathbf{x_6} - 0.085351935\cdot\mathbf{x_7} \le -0.070005466$
$\mathbf{x_1} \le 100; \mathbf{x_2} \le 0.4; \mathbf{x_3} \le 5.0; \mathbf{x_4} \le 19.63; \mathbf{x_5} \le 12.50; \mathbf{x_6} \le 0.3972; \mathbf{x_7} \le 0.820;$
$0.866520652\cdot\mathbf{x_1} - 0.199355601\cdot\mathbf{x_2} + 0.753431562\cdot\mathbf{x_3} + 1.415935668\cdot\mathbf{x_4} - 1.866907529\cdot\mathbf{x_5} + 4.836406757\cdot\mathbf{x_6} - 51.27403107\cdot\mathbf{x_7} \le 82.90614192$
$-0.000238475\cdot\mathbf{x_1} + 0.003897645\cdot\mathbf{x_2} - 0.00045981\cdot\mathbf{x_3} - 0.000794225\cdot\mathbf{x_4} + 0.0010738\cdot\mathbf{x_5} + 0.044664232\cdot\mathbf{x_6} + 0.085514412\cdot\mathbf{x_7} \le -0.001061269$

***Table 11** Results of the second compromise solution*
***Tablica 11.** Rezultati drugog kompomisnog rješenja*

$\boldsymbol{x_1}$= 3.37147; $\boldsymbol{x_2}$= 0.3711865; $\boldsymbol{x_3}$= 4.553035;
$\boldsymbol{x_4}$= 18.92068; $\boldsymbol{x_5}$= 0.2269908; $\boldsymbol{x_6}$= 0.2826709;
$\boldsymbol{x_7}$= 2.965111E–2; λ = 7.682257E–2;
Min $\boldsymbol{Z_1}(x_1,...x_7)$= 19.0013
Min $Z_2(x_1,...x_7)$= –0.9915
Max $\boldsymbol{Z_2}(x_1,...x_7)$= 0.9915

5 Conclusion
Zaključak

The paper presents research on the development of a model for the estimation of production time for unit production or medium size batch production. As a result, eight regression equations were obtained. They show estimation of the production time as a function of geometrical and technological characteristics of a homogeneous group of products that were grouped using logical operators. Using specifically developed 5 classifiers at 5 levels, on the sample taken from the real production a homogenous group was formed which resulted in a regression equation showing dependence between production time (Z_1) and 7 independent variables (x_1,...x_7). After that, the dependence between the work costs/total costs ratio (Z_2) and independent variables (x_1,...x_7) is shown in another regression equation. The optimization part of the work considers the possibility of application of standard STEP method as multiobjective optimization approach in optimization of production problems, where the objective functions are obtained by regression model. The results obtained by application of STEP method indicate that its application is possible in the optimization of decision variables of the given objective functions. It is evident that the results of both objective functions are within the statistical range, i.e. Min $Z_1(x_1,...x_7)$ = 19.0013 and Max $Z_2(x_1,...x_7)$ = 0.9915, and thus it is not necessary to introduce a new payoff table to find a new compromise (feasible) solution. The following can be concluded: it is cost-effective to manufacture products with minimum outside diameter (x_1), maximum (wider range) tolerance (x_2), maximum scale (x_3), maximum strength/mass ratio (x_4), minimum of wall thickness/length ratio (x_5), maximum product surface area (x_6) and minimum mass of material (x_7).

6 References
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