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HYERS-ULAM STABILITY OF A GENERALIZED HOSSZÚ FUNCTIONAL EQUATION

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Abstract. In this paper, we investigate the Hyers-Ulam stability of a generalized Hosszú functional equation, namely $f(x+y-\alpha xy)+g(xy) = h(x)+k(y)$, where f, g, h, k are functions of a real variable with values in a Banach space.

1. Introduction

Given an operator T and a solution class $\{u\}$ with the property that $T(u) = 0$, when does $\|T(v)\| \leq \varepsilon$ for an $\varepsilon > 0$ imply that $\|u - v\| \leq \delta(\varepsilon)$ for some u and for some $\delta > 0$? This problem is called the stability of the functional transformation (ref. [12]). A great deal of work has been done in connection with the ordinary and partial differential equations. If f is a function from a normed vector space into a Banach space, and $\|f(x + y) - f(x) - f(y)\| \leq \varepsilon$, Hyers [3] proved that there exists an additive function A such that $\|f(x)-A(x)\| \leq \varepsilon$ (cf. [11]). If $f(x)$ is a real continuous function of x over $\mathbf{R}$, and $|f(x+y)-f(x)-f(y)| \leq \varepsilon$, it was shown by Hyers and Ulam [5] that there exists a constant k such that $|f(x)-kx| \leq 2\varepsilon$. Taking these results into account, we say that the additive Cauchy equation $f(x+y) = f(x)+f(y)$ is stable in the sense of Hyers and Ulam. The interested reader should refer to the books by Hyers, Isac and Rassias [4] and by Jung [6] for an indepth account on the subject of stability of functional equations.

Let Y be a Banach space and $\mathbf{R}$ be the set of real numbers. A function $f : \mathbf{R} \to Y$ is said to be additive if it satisfies

$$f(x + y) = f(x) + f(y)$$

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for all $x, y \in \mathbf{R}$.

In [10], Kannappan and Sahoo determined the general solution $f, g, h, k : \mathbf{R} \to \mathbf{R}$ of the functional equation

$$f(x + y - \alpha xy) + g(xy) = h(x) + k(y) \tag{1.1}$$

for all $x, y \in \mathbf{R}$ (see also [1]). Here α is a priori chosen parameter. If $\alpha = 1$, then (1.1) is a pexiderized version of Hosszú functional equation, namely

$$f(x + y - xy) + f(xy) = f(x) + f(y).$$

If $\alpha = 0$, then (1.1) reduces to

$$f(x + y) + g(xy) = h(x) + k(y).$$

This functional equation was studied in [10] to characterize Cauchy differences that depend on the product of arguments.

The following three results are needed to establish the main results of this paper. The first result is due to Hyers [3].

Theorem 1.1. *Let $f : E_1 \to E_2$ be a function between Banach spaces such that*

$$\|f(x + y) - f(x) - f(y)\| \le \varepsilon$$

for some $\varepsilon \ge 0$ and for all $x, y \in E_1$. Then there exists a unique additive function $A : E_1 \to E_2$ satisfying

$$\|f(x) - A(x)\| \le \varepsilon$$

for any $x \in E_1$. Moreover, if $f(tx)$ is continuous in t for each fixed $x \in E_1$, then the function A is linear.

The next result was established by Găvruta [2] concerning the Hyers-Ulam stability of Hosszú's functional equation (cf. [9]).

Theorem 1.2. *Let Y be a Banach space and suppose that $f : \mathbf{R} \to Y$ satisfies the functional inequality*

$$\|f(x + y - xy) + f(xy) - f(x) - f(y)\| \le \varepsilon$$

for all $x, y \in \mathbf{R}$ with $\varepsilon \ge 0$. Then there exist a unique additive function $A : \mathbf{R} \to Y$ and a constant $b = f(1) - A(1) \in Y$ such that

$$\|f(x) - A(x) - b\| \le 9\,\varepsilon$$

for all $x \in \mathbf{R}$.

The following result was established by Jung and Sahoo [7] (see also [8]).

Theorem 1.3. *Let Y be a Banach space. If a function $f : \mathbf{R} \to Y$ satisfies the inequality*

$$\|f(xy) + f(x + y) - f(xy + x) - f(y)\| \le \varepsilon$$

for some $\varepsilon \geq 0$ *and for all* $x, y \in \mathbf{R}$*, then there exists a unique additive function* $A : \mathbf{R} \to Y$ *such that*

$$\|f(x) - A(x) - f(0)\| \leq 12\varepsilon$$

for all $x \in \mathbf{R}$.

In this paper, we prove the Hyers-Ulam stability of the equation (1.1) by using Theorem 1.1 due to Hyers [3], Theorem 1.2 due to Găvruta [2], and Theorem 1.3 due to Jung and Sahoo [7].

2. Main result

In the following theorem we show the Hyers-Ulam stability of the functional equation (1.1) for the case $\alpha = 0$. For the sake of convenience, we shall write the functional equation (1.1) as $f(x+y) - g(xy) = h(x) + k(y)$.

Theorem 2.1. *Let* Y *be a Banach space. If functions* $f, g, h, k : \mathbf{R} \to Y$ *satisfy the functional inequality*

$$\|f(x+y) - g(xy) - h(x) - k(y)\| \leq \varepsilon \tag{2.1}$$

for some $\varepsilon \geq 0$ *and for all* $x, y \in \mathbf{R}$*, then there exist unique additive functions* $A_1, A_2 : \mathbf{R} \to Y$ *such that for all* $x \in \mathbf{R}$

$$\|g(x) - 2A_1(x) - \delta_2\| \leq 144\,\varepsilon,$$

$$\|f(x) - A_1(x^2) - A_2(x) - \delta_1\| \leq \frac{153}{2}\,\varepsilon,$$

$$\|h(x) - A_1(x^2) - A_2(x) - \delta_3\| \leq \frac{155}{2}\,\varepsilon,$$

$$\|k(x) - A_1(x^2) - A_2(x) - \delta_4\| \leq \frac{155}{2}\,\varepsilon,$$

where $\delta_1, \delta_2, \delta_3, \delta_4$ *are constants in* Y *satisfying* $\|\delta_1 - \delta_2 - \delta_3 - \delta_4\| \leq \frac{\varepsilon}{2}$.

Proof. Letting $x = 0$ in (2.1), we get

$$\|f(y) - k(y) - b_1\| \leq \varepsilon \tag{2.2}$$

where $b_1 = g(0) + h(0)$. Putting $y = 0$ in (2.1), we have

$$\|f(x) - h(x) - b_2\| \leq \varepsilon \tag{2.3}$$

where $b_2 = g(0) + k(0)$. Finally, letting $x = 0$ and $y = 0$ in (2.1), we obtain

$$\|f(0) - g(0) - h(0) - k(0)\| \leq \varepsilon. \tag{2.4}$$

Using (2.1), (2.2) and (2.3), we see that

$$\begin{aligned}
&\|f(x+y) - g(xy) - f(x) - f(y) + b_1 + b_2\| \\
&\quad = \quad \| f(x+y) - g(xy) - h(x) - k(y) \\
&\qquad\qquad + \ h(x) - f(x) + b_2 + k(y) - f(y) + b_1 \| \\
&\quad \leq \quad \| f(x+y) - g(xy) - h(x) - k(y) \| \\
&\qquad\qquad + \| h(x) - f(x) + b_2 \| + \| k(y) - f(y) + b_1 \| \\
&\quad \leq \quad 3\,\varepsilon.
\end{aligned}$$

Hence we have

$$\|f(x+y) - g(xy) - f(x) - f(y) + b_1 + b_2\| \leq 3\,\varepsilon \tag{2.5}$$

for all $x, y \in \mathbf{R}$. Defining

$$\phi(x) = f(x) - b_1 - b_2 \tag{2.6}$$

and using (2.6) in inequality (2.5), we have

$$\|\phi(x+y) - g(xy) - \phi(x) - \phi(y)\| \leq 3\,\varepsilon \tag{2.7}$$

for all $x, y \in \mathbf{R}$. From (2.7), we see that

$$\|\phi(x+y+z) - g(xz+yz) - \phi(x+y) - \phi(z)\| \leq 3\,\varepsilon, \tag{2.8}$$

$$\|\phi(x+y+z) - g(xy+xz) - \phi(x) - \phi(y+z)\| \leq 3\,\varepsilon, \tag{2.9}$$

$$\|\phi(y+z) - g(yz) - \phi(y) - \phi(z)\| \leq 3\,\varepsilon. \tag{2.10}$$

Now using (2.7), (2.8), (2.9) and (2.10), we obtain

$$\begin{aligned}
&\|g(xy+xz) + g(yz) - g(xy) - g(xz+yz)\| \\
&\quad \leq \quad \| \phi(x+y) - g(xy) - \phi(x) - \phi(y)\| \\
&\qquad\qquad + \|\phi(x+y+z) - g(xz+yz) - \phi(x+y) - \phi(z) \| \\
&\qquad\qquad + \|\phi(x) + \phi(y+z) + g(xy+xz) - \phi(x+y+z) \| \\
&\qquad\qquad + \|\phi(y) + \phi(z) + g(yz) - \phi(y+z) \| \\
&\quad \leq \quad 12\,\varepsilon
\end{aligned}$$

which is

$$\|g(xy+xz) + g(yz) - g(xy) - g(xz+yz)\| \leq 12\,\varepsilon \tag{2.11}$$

for all $x, y, z \in \mathbf{R}$. Letting $z = 1$ in (2.11), we have

$$\|g(xy+x) + g(y) - g(xy) - g(x+y)\| \leq 12\,\varepsilon$$

for all $x, y \in \mathbf{R}$. From Theorem 1.3, we see that

$$\|g(x) - A(x) - \delta_2\| \leq 144\,\varepsilon \tag{2.12}$$

where $A:\mathbf{R}\to Y$ is a unique additive map and $\delta_2 = g(0)$. Now writing $A = 2A_1$ in (2.12), where A_1 is an additive map and uniquely determined by A, we have

$$\|g(x) - 2A_1(x) - \delta_2\| \le 144\,\varepsilon \tag{2.13}$$

for all $x \in \mathbf{R}$.

Letting $y = -x$ in (2.7), we have

$$\|\phi(0) - g(-x^2) - \phi(x) - \phi(-x)\| \le 3\,\varepsilon \tag{2.14}$$

for all $x \in \mathbf{R}$. Now using (2.13) and (2.14), we see that

$$\begin{aligned} &\big\|\phi(x) + \phi(-x) - 2A_1(x^2) + g(0) - \phi(0)\big\| \\ &\quad\le\ \big\|\,\phi(x) + \phi(-x) + g(-x^2) - \phi(0)\,\big\| + \big\|\,g(-x^2) - 2A_1(-x^2) - g(0)\,\big\| \\ &\quad\le\ 147\,\varepsilon. \end{aligned}$$

Thus we have

$$\big\|\phi(x) + \phi(-x) - 2A_1(x^2) + g(0) - \phi(0)\big\| \le 147\,\varepsilon \tag{2.15}$$

for all $x \in \mathbf{R}$.

Replacing x by $-x$ and y by $-y$ in (2.7), we obtain

$$\|\phi(-(x+y)) - g(xy) - \phi(-x) - \phi(-y)\| \le 3\,\varepsilon \tag{2.16}$$

for all $x, y \in \mathbf{R}$. From (2.7) and (2.16), we observe that

$$\begin{aligned} &\|\phi(x+y) - \phi(-(x+y)) - \phi(x) + \phi(-x) - \phi(y) + \phi(-y)\| \\ &\quad\le\ \|\,\phi(x+y) - g(xy) - \phi(x) - \phi(y)\,\| \\ &\qquad + \|\,\phi(-x) + \phi(-y) + g(xy) - \phi(-(x+y))\,\| \\ &\quad\le\ 6\,\varepsilon \end{aligned}$$

for all $x, y \in \mathbf{R}$. Defining $F:\mathbf{R}\to Y$ by

$$F(x) = \phi(x) - \phi(-x) \qquad \forall x \in \mathbf{R} \tag{2.17}$$

and using this F in the last inequality we have the functional inequality

$$\|F(x+y) - F(x) - F(y)\| \le 6\,\varepsilon$$

for all $x, y \in \mathbf{R}$. By Theorem 1.1, there is a unique additive function $A_0 : \mathbf{R}\to Y$ such that

$$\|F(x) - A_0(x)\| \le 6\,\varepsilon \tag{2.18}$$

for all $x \in \mathbf{R}$. Writing $A_0 = 2A_2$ in (2.18), where $A_2:\mathbf{R}\to Y$ is an additive map and then using (2.17), we have

$$\|\phi(x) - \phi(-x) - 2A_2(x)\| \le 6\,\varepsilon \tag{2.19}$$

for all $x \in \mathbf{R}$.

Using (2.15) and (2.19), we see that

$$\begin{aligned}
&\big\|2\phi(x) - 2A_1(x^2) - 2A_2(x) + g(0) - \phi(0)\big\| \\
&\quad \le \big\|\,\phi(x) + \phi(-x) - 2A_1(x^2) + g(0) - \phi(0)\big\| + \|\phi(x) - \phi(-x) - 2A_2(x)\,\| \\
&\quad \le 153\,\varepsilon
\end{aligned}$$

for all $x \in \mathbf{R}$. Hence

$$\left\|\phi(x) - A_1(x^2) - A_2(x) + \frac{1}{2}[g(0) - \phi(0)]\right\| \le \frac{153}{2}\,\varepsilon. \tag{2.20}$$

Since $\phi(x) = f(x) - b_1 - b_2 = f(x) - 2g(0) - h(0) - k(0)$, from (2.20) we have

$$\big\|f(x) - A_1(x^2) - A_2(x) - \delta_1\big\| \le \frac{153}{2}\,\varepsilon \tag{2.21}$$

where $\delta_1 = \frac{1}{2}\,[f(0) + g(0) + h(0) + k(0)]$. We can easily prove the uniqueness of A_2 satisfying the inequality (2.21).

Next, using (2.2) and (2.21), we see that

$$\begin{aligned}
&\big\|k(x) - A_1(x^2) - A_2(x) + b_1 - \delta_1\big\| \\
&\quad \le \|\,k(x) - f(x) + b_1\,\| + \big\|f(x) - A_1(x^2) - A_2(x) - \delta_1\,\big\| \\
&\quad \le \frac{155}{2}\,\varepsilon
\end{aligned}$$

for all $x \in \mathbf{R}$. Hence

$$\big\|k(x) - A_1(x^2) - A_2(x) - \delta_4\big\| \le \frac{155}{2}\,\varepsilon$$

where $\delta_4 = \frac{1}{2}\,[f(0) - g(0) - h(0) + k(0)]$.

Finally, using (2.3) and (2.21), we see that

$$\begin{aligned}
&\big\|h(x) - A_1(x^2) - A_2(x) + b_2 - \delta_1\big\| \\
&\quad \le \|\,h(x) - f(x) + b_2\,\| + \big\|f(x) - A_1(x^2) - A_2(x) - \delta_1\,\big\| \\
&\quad \le \frac{155}{2}\,\varepsilon
\end{aligned}$$

for all $x \in \mathbf{R}$. Hence

$$\big\|h(x) - A_1(x^2) - A_2(x) - \delta_3\big\| \le \frac{155}{2}\,\varepsilon$$

where $\delta_3 = \frac{1}{2}\,[f(0) - g(0) + h(0) - k(0)]$.

In view of the inequality (2.4), it is easy to check that the constants $\delta_1, \delta_2, \delta_3, \delta_4$ satisfy $\|\delta_1 - \delta_2 - \delta_3 - \delta_4\| = \frac{1}{2}\,\|f(0) - g(0) - h(0) - k(0)\| \le \frac{\varepsilon}{2}$. Now the proof of the theorem is complete. ∎

Remark 2.2. The above theorem gives a Hyers-Ulam stability as well as the general solution of the original equation. We may put $\varepsilon = 0$ in Theorem 2.1 to get the general solution of the functional equation (1.1) with $\alpha = 0$: The functions $f, g, h, k : \mathbf{R} \to Y$ satisfy the functional equation $f(x+y) - g(xy) =$

$h(x)+k(y)$ if and only if there exist additive functions $A_1, A_2 : \mathbf{R} \to Y$ and constants $\delta_1, \delta_2, \delta_3 \in Y$ such that

$$\begin{aligned} f(x) &= A_1(x^2) + A_2(x) + \delta_1 + \delta_2 + \delta_3, \\ g(x) &= 2A_1(x) + \delta_1, \\ h(x) &= A_1(x^2) + A_2(x) + \delta_2, \\ k(x) &= A_1(x^2) + A_2(x) + \delta_3. \end{aligned}$$

In the following theorem, we treat the stability of the functional equation (1.1) when the parameter $\alpha \neq 0$.

Theorem 2.3. *Let Y be a Banach space. If functions $f, g, h, k : \mathbf{R} \to Y$ satisfy the functional inequality*

$$\|f(x+y-\alpha xy) + g(xy) - h(x) - k(y)\| \leq \varepsilon \tag{2.22}$$

for some $\varepsilon \geq 0$ and for all $x, y \in \mathbf{R}$, then there exists unique additive function $A : \mathbf{R} \to Y$ such that for all $x \in \mathbf{R}$

$$\begin{gathered} \|f(x) - A(\alpha x) - a\| \leq 54\,\varepsilon, \\ \|h(x) - A(\alpha x) - a - b_1\| \leq 55\,\varepsilon, \\ \|k(x) - A(\alpha x) - a - b_2\| \leq 55\,\varepsilon, \\ \|g(x) - A(\alpha^2 x) - a - b_1 - b_2\| \leq 57\,\varepsilon, \end{gathered}$$

where $a = f\left(\frac{1}{\alpha}\right) - A(1)$, $b_1 = g(0) - k(0)$, and $b_2 = g(0) - h(0)$.

Proof. Letting $y = 0$ in (2.22), we get

$$\|f(x) - h(x) + b_1\| \leq \varepsilon \tag{2.23}$$

where $b_1 = g(0) - k(0)$. Putting $x = 0$ in (2.22), we have

$$\|f(y) - k(y) + b_2\| \leq \varepsilon \tag{2.24}$$

where $b_2 = g(0) - h(0)$. Using (2.22), (2.23) and (2.24), we see that

$$\begin{aligned} &\|f(x+y-\alpha xy) + g(xy) - f(x) - f(y) - b_1 - b_2\| \\ &\quad = \|\, f(x+y-\alpha xy) + g(xy) - h(x) - k(y) \\ &\qquad + \ h(x) - f(x) - b_1 + k(y) - f(y) - b_2 \,\| \\ &\quad \leq \|\, f(x+y-\alpha xy) + g(xy) - h(x) - k(y) \,\| \\ &\qquad + \|\, h(x) - f(x) - b_1 \,\| + \|\, k(y) - f(y) - b_2 \,\| \\ &\quad \leq 3\,\varepsilon. \end{aligned}$$

Hence

$$\|f(x+y-\alpha xy) + g(xy) - f(x) - f(y) - b_1 - b_2\| \leq 3\,\varepsilon \tag{2.25}$$

for all $x, y \in \mathbf{R}$. Since $\alpha \neq 0$, substituting $y = \frac{1}{\alpha}$ in (2.25), we obtain

$$\left\| g\left(\frac{x}{\alpha}\right) - f(x) - b_1 - b_2 \right\| \leq 3\,\varepsilon \tag{2.26}$$

for all $x \in \mathbf{R}$. Now replacing x by αx in (2.26), we have

$$\|g(x) - f(\alpha x) - b_1 - b_2\| \le 3\,\varepsilon \tag{2.27}$$

for all $x \in \mathbf{R}$. From (2.25) and (2.27), we have

$$\begin{aligned}
&\|f(x+y-\alpha xy) + f(\alpha xy) - f(x) - f(y)\| \\
&\quad = \ \| f(x+y-\alpha xy) + g(xy) - f(x) - f(y) - b_1 - b_2 \\
&\qquad\quad + \ f(\alpha xy) - g(xy) + b_1 + b_2 \,\| \\
&\quad \le \ \| f(x+y-\alpha xy) + g(xy) - f(x) - f(y) - b_1 - b_2 \,\| \\
&\qquad\quad + \| f(\alpha xy) - g(xy) + b_1 + b_2 \,\| \\
&\quad \le \ 6\,\varepsilon.
\end{aligned}$$

Thus we have

$$\|f(x+y-\alpha xy) + f(\alpha xy) - f(x) - f(y)\| \le 6\,\varepsilon \tag{2.28}$$

for all $x, y \in \mathbf{R}$. Replacing x by $\frac{x}{\alpha}$ and y by $\frac{y}{\alpha}$ in (2.28), we obtain

$$\left\| f\left(\frac{x+y-xy}{\alpha}\right) + f\left(\frac{xy}{\alpha}\right) - f\left(\frac{x}{\alpha}\right) - f\left(\frac{y}{\alpha}\right) \right\| \le 6\,\varepsilon. \tag{2.29}$$

Defining $\psi : \mathbf{R} \to Y$ by

$$\psi(x) = f\left(\frac{x}{\alpha}\right) \qquad \forall x \in \mathbf{R} \tag{2.30}$$

and using it in (2.29), we see that

$$\|\psi(x+y-xy) + \psi(xy) - \psi(x) - \psi(y)\| \le 6\,\varepsilon$$

for all $x, y \in \mathbf{R}$. Hence by Theorem 1.2, there exists a unique additive map $A : \mathbf{R} \to Y$ such that for all $x \in \mathbf{R}$

$$\|\psi(x) - A(x) - a\| \le 54\,\varepsilon, \tag{2.31}$$

where $a = \psi(1) - A(1)$. Thus from (2.30) and (2.31), we obtain

$$\|f(x) - A(\alpha x) - a\| \le 54\,\varepsilon, \tag{2.32}$$

where $a = f\left(\frac{1}{\alpha}\right) - A(1)$.

From (2.23) and (2.32), we get

$$\begin{aligned}
&\|h(x) - A(\alpha x) - a - b_1\| \\
&\quad \le \ \| h(x) - f(x) - b_1 \,\| + \|f(x) - A(\alpha x) - a \,\| \\
&\quad \le \ 55\,\varepsilon
\end{aligned}$$

for all $x \in \mathbf{R}$. Similarly, from (2.24) and (2.32), we have

$$\begin{aligned}
&\|k(x) - A(\alpha x) - a - b_2\| \\
&\quad \le \ \| k(x) - f(x) - b_2 \,\| + \|f(x) - A(\alpha x) - a \,\| \\
&\quad \le \ 55\,\varepsilon.
\end{aligned}$$

Finally, from (2.27) and (2.32), we get

$$\begin{aligned}
&\left\|g(x) - A(\alpha^2 x) - a - b_1 - b_2\right\| \\
\leq\ & \| g(x) - f(\alpha x) - b_1 - b_2 \| + \left\|f(\alpha x) - A(\alpha^2 x) - a\right\| \\
\leq\ & 57\,\varepsilon.
\end{aligned}$$

The proof of the theorem is now complete. □

Remark 2.4. If we put $\varepsilon = 0$ in Theorem 2.3, we can obtain the general solution of the original functional equation of (2.22): The functions $f, g, h, k : \mathbf{R} \to Y$ satisfy the functional equation (1.1) with $\alpha \neq 0$ if and only if there exists an additive function $A : \mathbf{R} \to Y$ and constants $\delta_1, \delta_2, \delta_3 \in Y$ such that

$$\begin{aligned}
f(x) &= A(\alpha x) - \delta_1 + \delta_2 + \delta_3, \\
g(x) &= A(\alpha^2 x) + \delta_1, \\
h(x) &= A(\alpha x) + \delta_2, \\
k(x) &= A(\alpha x) + \delta_3.
\end{aligned}$$

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