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ON SYMMETRIC BLOCK DESIGNS (45,12,3) WITH INVOLUTORY AUTOMORPHISM FIXING 15 POINTS

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Abstract. All symmetric block designs (45,12,3) admitting an involutory automorphism fixing 15 points are classified. The orders of their groups are also determined.

1. Introduction and preliminaries

An involutory automorphism of a symmetric block designs (45,12,3) can fix at most 15 points. Here we are dealing with this case. The aim of this article is to prove the following

Theorem 1.1. *Let D be a symmetric block design (45,12,3) admitting an involutory automorphism fixing 15 points. Then D is isomorphic to one of the 28 designs listed below in the Table 2. All of the 28 designs from the Table 2 are mutualy non-isomorphic. However, there are two among them which are dual of each other, and 26 are self-dual. These 28 symmetric block designs have groups of orders* 2^1, 2^2, 2^3, 2^4, $2^2 \cdot 3$, $2^3 \cdot 3$, $2^3 \cdot 3^3$, $2^4 \cdot 3^3$, $2^4 \cdot 3^4$ *and* $2^7 \cdot 3^4 \cdot 5$.

This result was obtained by means of combinatorial and group theoretical methods and with the help of a computer.

In the following we use the term design for symmetric block designs. We begin by recalling some basic facts related to designs (see for example [1], [2], [4]).

Definition 1.2. *Let $D = (P, B, I)$ be an incidental structure with point set P, line set B and incidence relation $I \subseteq P \times B$. For $x \in B$,* $\mathrm{P} \in P$ *denote*

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$< x >= \{Q \in P \mid (Q,x) \in I\}$,
$< \mathrm{P} >= \{y \in B \mid (\mathrm{P},y) \in I\}$,
$|x| = | < x > |$, $|\mathrm{P}| = | < \mathrm{P} > |$.

A (v,k,λ)*-design,* $v,k,\lambda \in \mathbf{N}$*,* $k > \lambda$ *is an incidence structure* $D = (P,B,I)$ *such that:*

(i) $|P| = |B| = v = k(k-1)/\lambda + 1$,
(ii) $|x| = |\mathrm{P}| = k$,
(iii) $| < x > \cap < y > | = | < \mathrm{P} > \cap < Q > | = \lambda$, *for all* $x, y \in B$, $\mathrm{P}, Q \in P$, *with* $x \neq y$, $\mathrm{P} \neq Q$.

For two designs D_1 and D_2 an isomorphism of D_1 onto D_2 is a bijection which maps points onto points and lines onto lines preserving the incidence. Similarly, dual isomorphisms are bijections which map points onto lines and lines onto points and preserve the incidences. The full automorphism group of D is denoted by $\mathrm{Aut}\, D$.

Definition 1.3. *Let* D *be a* (v,k,λ)*-design and* G *an automorphism group of* D*. For* $x \in B$, $\mathrm{P} \in P$, $g \in G$*, we denote by* xg, $\mathrm{P}g$ *the* g*-images of* x *and* P*, and by* $xG = \{xg \mid g \in G\}$, $\mathrm{P}G = \{\mathrm{P}g \mid g \in G\}$*, the* G*-orbits of* x *and* P*, respectively. By a known result the number* t *of point orbits equals the number of line orbits. Denote corresponding orbits by* B_i, P_r, $0 \leq i, r \leq t-1$ *and* $|B_i| = \Omega_i$, $|P_r| = \omega_r$*.*
From the above definitions, it follows immediately that

(i) $B = \bigcup_{i=0}^{t-1} B_i$, $P = \bigcup_{r=0}^{t-1} P_r$,

(ii) $\sum_i \Omega_i = \sum_r \omega_r = v$.

Let $x \in B_i$, $\mathrm{P} \in P_r$, $\gamma_{ir} = | < x > \cap P_r|$, $\Gamma_{ir} = | < \mathrm{P} > \cap B_i|$*. Then, obviously one can easily see, that* γ_{ir} *and* Γ_{ir} *do not depend on the choice of* x *and* P*, respectively.*

The introduced cardinalities satisfy some important relations:

Lemma 1.4. *It is:*

$$\Omega_i \gamma_{ir} = \omega_r \Gamma_{ir}, \tag{1}$$

$$\sum_{r=0}^{t-1} \gamma_{ir} \Gamma_{jr} = \lambda \Omega_j + \delta_{ij} n; \quad \sum_{i=0}^{t-1} \Gamma_{ir} \gamma_{is} = \lambda \omega_s + \delta_{rs} n, \tag{2}$$

δ_{ij}, δ_{rs} *being the correspondent Kronecker symbols, and* $n = k - \lambda$ *the order of* D*. Because of* (1), (2) *can be rewritten as:*

$$(3)\qquad \sum_{r=0}^{t-1} \frac{\Omega_j}{\omega_r}\gamma_{ir}\gamma_{jr} = \lambda\Omega_j + \delta_{ij}n; \ \sum_{i=0}^{t-1} \frac{\Omega_i}{\omega_r}\gamma_{ir}\gamma_{is} = \lambda\omega_s + \delta_{rs}n.$$

Definition 1.5. *We mark*

$$[B_i, B_j] = \sum_{r=0}^{t-1} \gamma_{ir}\Gamma_{jr} \quad \textit{and} \quad [P_r, P_s] = \sum_{i=0}^{t-1} \Gamma_{ir}\gamma_{is}$$

and call these expressions the orbit products.

Definition 1.6. *Given a* (v, k, λ)*-design and* $G \leq Aut D$ *with corresponding* G*-orbits distributions, the matrices* (γ_{ir}), (Γ_{ir}) *are called the orbital structures of* D *with respect to* G*, for lines and points, respectively. Obviously, the orbital structures of* D *are uniquely determined up to the order of rows and columns. We call* $\gamma_i \equiv (\gamma_{i0}, \ldots, \gamma_{i\,t-1})$ *and* $\Gamma_r \equiv (\Gamma_{0r}, \ldots, \Gamma_{t-1\,r})$ *the orbital structures for lines in* B_i *and points in* P_r*, respectively.*

Let us mark the points of P_r by $r_0, r_1, \ldots, r_{\omega-1}$ and the lines of B_i by $\widehat{1}_0, \widehat{1}_1, \ldots, \widehat{1}_{\Omega-1}$. Now, for each orbit P_r the automorphism group G is represented as a permutation group on the indicies $0, \ldots, \omega_{r_i} - 1$. The analogous holds for the line orbits.

In the matrix (γ_{ir}), let us mark their parts as follows: fixed part of the set of lines fixed in D by $< \rho >$, unfixed part of the set of lines fixed in D by $< \rho >$, fixed part of the set of lines unfixed in D by $< \rho >$ and unfixed part of the set of lines unfixed in D by $< \rho >$, by $F(\rho)$, $G(\rho)$, $CF(\rho)$ and $CG(\rho)$ respectively.

Definition 1.7. *For* $\rho \in Aut D$ *denotes the sets of points and lines in* D *fixed by* $< \rho >$*, by* $F_\rho(P)$ *and* $F_\rho(B)$*.*

Remark 1.8. By a known result $|F_\rho(P)| = |F_\rho(B)| \equiv F_\rho$, and by [6] $F_\rho \leq k + \sqrt{k - \lambda}$.

Lemma 1.9. *Let* $\rho \in Aut D$, $|\rho| = p$, p *a prime. Denote* $F_\rho(P) = F(P)$, $F_\rho(B) = F(B)$, $F_\rho = F$*. Let* N_i, M_j *be the sets of orbital stucture lines belonging to the* ρ*-fixed lines with* i *fixed points and* ρ*-nonfixed lines with* j *fixed points, respectively. Their cardinalities are marked with* N_i *and* M_j*. Counting the appearences of fixed points and their mutual connections, we* *have (s. also* [2], [3]*):*

$$\sum_i N_i = F,\ \ p\sum_i M_j = v - F \tag{4}$$

$$\sum_i iN_i + p\sum_j jM_j = F \cdot k \tag{5}$$

$$\sum_i \binom{i}{2} N_i + p\sum_j \binom{j}{2} M_j = \binom{F}{2}\lambda. \tag{6}$$

Hereby, $i \equiv k \pmod p$*,* $i \le F, j \le \lambda$*.*

The analogous equalities are valid for dual sets N'_i, M'_j and their cardinalities N'_i, M'_j.

LEMMA 1.10. *Let* P_r *be a* ρ*-fixed point of* D*, and* $N_i^{(r)}$*,* $M_j^{(r)}$ *the subsets of* N_i*,* M_j *containing* P_r*. Mark* $|N_i^{(r)}| = n_i^{(r)}$*,* $|M_j^{(r)}| = m_j^{(r)}$*. Counting the appearences of* P_r *and its connections with other fixed points we get:*

$$\sum_i n_i^{(r)} + p\sum_j m_j^{(r)} = k \tag{7}$$

$$\sum_i (i-1)n_i^{(r)} + p\sum_j (j-1)m_j^{(r)} = (F-1)\lambda. \tag{8}$$

Similarly:

$$\sum_{P_r \in F(P)} n_i^{(r)} = iN_i,\ \ \sum_{P_r \in F(P)} m_j^{(r)} = jM_j. \tag{9}$$

Again, the analogous is valid for dual objects.

LEMMA 1.11. *Let* D *be a nontrivial* (v, k, λ)*-design, in which the involution* ρ *fixes* F *points.*

Let $f_\infty(i)$ *be the number of fixed points of the orbital line* B_i*,* $f_\infty(i) = |< x > \cap F(P)| = |i^F|$*,* $x \in B_i$*, and* $f_\infty(i,j)$ *the number of common fixed points of the orbital lines* B_i *and* B_j*,* $f_\infty(i,j) = |< x > \cap < y > \cap F(P)| = |(i,j)^F|$*,* $x \in B_i$*,* $y \in B_j$*. The following is valid:*

$$\sum_{r=F}^{G-1} \gamma_{ir} = k - f_\infty(i),\ \ \textit{for}\ \ i \in \{0, 1, \ldots, G-1\},\ \ G = F + \frac{v-F}{p} \tag{10}$$

$$\sum_{r=F}^{G-1} \gamma_{ir}(\gamma_{ir} - 1) = \lambda - f_\infty(i),\ \ \textit{for}\ \ i \in \{0, 1, \ldots, G-1\} \tag{11}$$

$$\sum_{r=F}^{G-1} \gamma_{ir}\gamma_{jr} = 2(\lambda - f_\infty(i,j)),\ \ \textit{for}\ \ i,j \in \{0, 1, \ldots, G-1\},\quad i \neq j. \tag{12}$$

PROOF. It follows from Definitions 1.2 and 1.7, and Lemma 1.4 (see also Beutelspacher ([1]), page.211. □

DEFINITION 1.12. *Let for the construction of the symmetric (v,k,λ)-design, admitting an automorphism ρ which has the prime order p, the substructures $F(\rho)$ and $G(\rho)$ of the orbit structure (γ_{ir}) be already constructed.*

Let us consider the substructure R_{r+u} of the orbital structure (γ_{ir}), which consists of the set P_r already constructed r fixed lines and the set Q_u already constructed u unfixed lines, $R_{r+u} = \mathrm{P}_r \cup Q_u$.
For the construction of an unfixed part of a new layer x (fixed or unfixed line), we choose the subset P_s of P_r, $s \le r$, and the subset Q_t of Q_u, $t \le u$, $P_s = \{l_1,\dots,l_s\}$, $Q_t = \{z_1,\dots,z_t\}$, $s+t = j-1$, i.e. the subset R_{j-1} of R_{r+u}, $R_{j-1} = \mathrm{P}_s \cup Q_t = \{l_1,\dots,l_s,z_1,\dots,z_t\} = \{r_1,\dots,r_s,r_{s+1},\dots,r_{j-1}\}$,

$$r_m = \begin{cases} l_m, & \text{for} \quad 1 \le m \le s, \\ z_{m-s}, & \text{for} \quad s+1 \le m \le j-1. \end{cases}$$

The sets are defined as follows:

$$V = \{F, F+1, \dots, G-1\}, \tag{13$_1$}$$

$$\mathbf{B}_i^{(j-1)} = \left\{ q \in V \mid \sum_{m=1}^{j-1} \gamma_{r_m q} = i \right\}, \tag{13$_2$}$$

and the cardinalities:

$$B_i^{(j-1)} = |\mathbf{B}_i^{(j-1)}|, \tag{13$_3$}$$

$$B_i^j = \sum_{q \in \mathbf{B}_i^{(j-1)}} \gamma_{x,q}. \tag{13$_4$}$$

Obviously:

$$V = \bigcup_i \mathbf{B}_i^{(j-1)}, \tag{14$_1$}$$

$$\sum_i B_i^{(j-1)} = G - F = \frac{v-F}{p}. \tag{14$_2$}$$

REMARK 1.13. $B_i^{(j-1)}$ is the number of unfixed points, which in the choosed $j-1$ layers appear i times.

B_j^i is the number of unfixed points of the searched layer x, which in the choosed $j-1$ layers appear i times.

LEMMA 1.14. *Let D be a (v,k,λ)-design admitting an automorphism ρ which has prime order p fixing F points.*

For searched the layer x, which has $f_\infty(x)$ fixed points, from Definition 1.12 it holds that:

$$0 \leq B_i^j \leq p \cdot B_i^{(j-1)}, \tag{15}$$

$$\sum_i B_i^j = k - f_\infty(x), \tag{16}$$

$$\sum_i i \cdot B_i^j = p\left[(j-1)\lambda - \sum_{m=1}^{j-1} f_\infty(r_m, x)\right]. \tag{17}$$

PROOF. Following the equality $(13)_3$ and $(13)_4$ we get:

$$B_i^j = \sum_{q \in B_i^{(j-1)}} \gamma_{x,q} \leq \sum_{q \in B_i^{(j-1)}} p = p \cdot B_i^{(j-1)},$$

i.e. the inequalities (15) hold.

According to (10), we have:

$$\sum_{q=F}^{G-1} \gamma_{xq} = k - f_\infty(x), \quad \text{for} \quad x \in \{0, 1, \ldots, G-1\}. \tag{18}$$

Separating and grouping the members in the expression (18), according to $(13)_1$–$(13)_4$ and $(14)_1$ we get:

$$\sum_{q \in V} \gamma_{xq} = \sum_i \sum_{q \in B_i^{(j-1)}} \gamma_{xq} = \sum_i B_i^j = k - f_\infty(x),$$

i.e. the equation (16) holds.

According to (12), we have:

$$\sum_{q \in V} \gamma_{xq}\gamma_{r_m q} = p[\lambda - f_\infty(r_m, x)], \quad \text{for} \quad r_m \in R_{j-1}. \tag{19}$$

Adding $j-1$ equalities from the system (19), for the searched layer x and for $j-1$ choosen layers, the following is obtained employing $(13)_1$–$(13)_4$ and $(14)_1$:

$$\begin{aligned}
\sum_{m=1}^{j-1}\sum_{q\in V}\gamma_{xq}\gamma_{r_m q} &= \sum_{q\in V}\gamma_{xq}\sum_{m=1}^{j-1}\gamma_{r_m q} = \\
&= \sum_{i}\sum_{q\in \mathbf{B}_i^{(j-1)}}\gamma_{xq}\sum_{m=1}^{j-1}\gamma_{r_m q} = \\
&= \sum_{i}\sum_{q\in \mathbf{B}_i^{(j-1)}}\gamma_{xq}\cdot i = \\
&= \sum_{i} i \sum_{q\in \mathbf{B}_i^{(j-1)}}\gamma_{xq} = \sum_i i\cdot B_i^j = \\
&= \sum_{m=1}^{j-1} p[\lambda - f_\infty(r_m,x)] = \\
&= p\left[\lambda(j-1) - \sum_{m=1}^{j-1} f_\infty(r_m,x)\right],
\end{aligned}$$

i.e. (17) holds. □

Remark 1.15.

1. Lemma 1.14 is most frequently used when the subset R_{j-1} has some special property (meaning maximality or minimality) in relation to the searched layer x or the subset R_{j-1} has some such special property, for example, all the layers from R_{j-1} have maximum intersections with the choosed layer x in their fixed parts.
2. Lemma 1.14 gives necessary conditions for determining cardinalities γ_{ij}, using the obtained B_i^j.

The algorithm we use for constructing all designs with parameters (v,k,λ) admitting the given automorphism of the group G is described in more details in [3]. Essentially, it consists of the following:

ALGORITHM (V.Ćepulić) Let $D=(P,B,I)$ be a (v,k,λ)-design and $G\le \mathrm{Aut}\,D$. Let $\mathrm{P}_1,\ldots,\mathrm{P}_t$ and $\mathrm{B}_1,\ldots,\mathrm{B}_t$ be the G-orbits of points and lines in the given order. At first we build, layer by layer, possible orbital structures $\Gamma=(\gamma_{ir})$ and, after that, the designs themselves by "indexing" the obtained structures, that is by finding the subsets P_{ir} of P_r which satisfy all the conditions for D in constituting line orbit representatives. The identification and elimination of isomorphic orbital structures, as well as of isomorphic and dually isomorphic designs, based on the introduction of some lexicographical ordering among them, is an important final step in both stages of construction.

2. Proof of the Theorem

2.1. *Construction of the orbital structure.* Now we assume that D is a design with the parameters (45,12,3) admitting an involutory automorphism which fixes 15 points.

The construction of the orbital structure will be done without using a computer.

Using the properties of the fixed part of unfixed lines, this part is constructed first and its duality, unfixed part of the fixed lines, and only then the fixed part of the fixed lines. Finally, are used in the construction of the unfixed part of the unfixed lines modified equalities (16) and (17) which offer more concise results.

We shall begin by determinating the parameters N_i, M_j, $n_i^{(r)}$ and $m_j^{(r)}$.

2.1.1. *Determination of the parameters of the orbital structure.*

Lemma 2.1. *Let D be a (45,12,3)-design admitting an involutory automorphism fixing 15 points. Then it holds:*

$$N_6 = 15,\ N_i = 0 \quad else \quad M_3 = 15,\ M_1 = 0. \tag{20}$$

Proof. By Lemma 1.9 we have $i \equiv 12 \pmod 2$. For $y \in M_j$, it is valid that $|y \cap y\rho| = 3$ and therefore $j \in \{1, 3\}$. Both fixed lines must have three common fixed points, which means that the fixed lines must have at least one common fixed point. Hence, $N_0 = 0$.
Lemma 1.9 offers the following equations for the parameters N_i, M_j:

$$N_{12} + N_{10} + N_8 + N_6 + N_4 + N_2 = 15 \tag{21}$$

$$M_3 + M_1 = 15 \tag{22}$$

$$12N_{12} + 10N_{10} + 8N_8 + 6N_6 + 4N_4 + 2N_2 + 6M_3 + 2M_1 = 180 \tag{23}$$

$$66N_{12} + 45N_{10} + 28N_8 + 15N_6 + 6N_4 + N_2 + 6M_3 = 315. \tag{24}$$

Building the expression $\frac{1}{8}[2(24) + 24(21) + 42(22) - 9(23)] = 0$, we get:

$$6N_{12} + 3N_{10} + 2N_8 + 2N_2 + 3M_1 = 0,$$

which yields $N_{12} = N_{10} = N_8 = N_2 = M_1 = 0$. Now, from the above equations it follows immediately that $N_6 = M_3 = 15$, $N_4 = 0$. □

Lemma 2.2. *It is true that $n_6^{(r)} = 6$, $m_3^{(r)} = 3$ for all $\mathrm{P}_r \in F(P)$.*

Proof. In view of Lemma 1.11, the equations of Lemma 1.10 are reduced to:

$$n_6^{(r)} + 2m_3^{(r)} = 12, \quad n_6^{(r)} + 4m_3^{(r)} = 42.$$

Hence $n_6^{(r)} = 6$, $m_3^{(r)} = 3$, as the only possible solution. □

2.1.2. *Construction of the orbital structures $G(\rho)$ and $CF(\rho)$.*

LEMMA 2.3. *The substructure $G(\rho)$ as a part of D, is a semiprojective plane with the parameters (15,3,[1]).*

PROOF. For $x \in B$, denoted $x^F =< x > \cap F(P)$. Let $Q = B \setminus F(B)$, and $q_i, q_j \in Q$, $q_i \neq q_j$.
It is valid that $|q_i^F \cap q_j^F| \leq 1$, for $i \neq j$, as otherwise it would be that $q_i^F \cap q_j^F \subseteq \{a, b\}$, where a, b are fixed points. It is not valid, because in this case points a, b would lie on four common lines contradicting $\lambda = 3$.

Having in mind that $M_3 = 15$, $m_3^{(r)} = 3$, for $\forall\, r \in \{0, 1, \ldots, 14\}$, we can see that the structure $G(\rho)$ is a semiprojective plane with the parameters (15,3,[1]). □

LEMMA 2.4. *The substructure $CF(\rho)$ is a semiprojective plane with the parameters (15,3,[1]) using full orbits of the unfixed points as the points of its structure.*

PROOF. All unfixed lines have three fixed points, thus all of the full orbits of unfixed points appear only in $CF(\rho)$, each by $\lambda = 3$ times.

All of the 15 fixed lines has three full orbits of the unfixed points, due to $N_6 = 15$.

The intersection of two fixed lines can contain at the most one full orbit. Hence, the structure $CF(\rho)$ is a semiprojective plane with the parameters (15,3,[1]), using full orbits of unfixed points as the points of its structure. □

LEMMA 2.5. *Up to isomorphism there is only one part of the design D, corresponding the substructure $G(\rho)$. Marking the fixed points in the orbital structures of lines as $0, 1, 2, \ldots, 14$, lexicographically first substructure is:*

$0'$	0	1	2
$1'$	0	3	4
$2'$	0	5	6
$3'$	1	7	8
$4'$	1	9	10
$5'$	2	11	12
$6'$	2	13	14
$7'$	3	7	11
$8'$	3	9	13
$9'$	4	8	12
$10'$	4	10	14
$11'$	5	7	14
$12'$	5	9	12
$13'$	6	8	13
$14'$	6	10	11

Proof. Let us consider three fixed points P_1, P_2, P_3, positioned on the same line in $G(\rho)$. As $\lambda = 3$ and $n_6^{(r)} = 6$, each two of them are positioned on one additional line from $F(\rho)$, and together they cover at least $3 \cdot 6 - 3 \cdot 1 = 15$ fixed lines, i.e. each of the unfixed lines contains at least one of them. Maximally one of the points P_1, P_2, P_3 can be situated on the set of lines in $G(\rho)$ through any of other fixed points.

If two of them shoud appear there, then because of $\lambda = 3$, this point would appear together with the points P_1, P_2, P_3 on maximum $1 + 1 + 3 = 5$ lines in $F(\rho)$.

As pass these three points all the fixed lines, it is in a contradiction to $n_6^{(r)} = 6$.

Because of $m_3^{(r)} = 3$, each of the points P_1, P_2, P_3, is also situated on two lines in $G(\rho)$. Each of the 6 lines contains two of the rest of 12 fixed points. As each of them can appear only once at most, each must appear once exactly.

It has thus been proved, that for any line in $G(\rho)$, the rest of the 6 lines containing some of its fixed points, contain also all the other fixed points, each of them once exactly. On the basis of this, it is easy to show that, up to isomorphism, $G(\rho)$ is uniquely determined, as formulated in the sayings of the Lemma. □

Lemma 2.6. *Using full orbits of the unfixed points as the points of the structure $CF(\rho)$, the structure is uniquely determined up to isomorphism, and is dually isomorphic to the structure for $G(\rho)$.*

Proof. Because $M_1 = 0$, full orbits of unfixed points appear only on fixed lines and, as $\lambda = 3$, each of them on three lines.

For three fixed lines which contain the same unfixed orbit of the points, it is easy to prove that each of the other fixed lines contains exactly one of the 6 remaining unfixed orbits laying on these lines.

This property is analogue to that from Lemma 2.5, and similar deduction leads to the claim of Lemma 2.6. □

2.1.3. *Construction of the structure $F(\rho)$.*

Lemma 2.7. *With the substructures $G(\rho)$ and $CF(\rho)$, already constructed up to isomorphism there are two structures $F(\rho)$, both of them with the parameters (15,6,[1,3]) i.e. with 15 points and lines, such that 6 points lay on each of the lines, and that 6 lines pass through each of the points, and with the lines intersecting at one or three points.*

Proof. **(I)** First we prove that, with the substructures $G(\rho)$ and $CF(\rho)$ already constructed, there is at least one substructure of three fixed lines, passing through the same three fixed points, referred to as "searched substructure" in the rest of the text.

It is proved (I) by the contradiction, i.e. by the impossible of construction, with substructures $G(\rho)$ and $CF(\rho)$, already constructed of such a structure $F(\rho)$ in which the "searched substructure" would be omitted. For that purpose the substructures of $F(\rho)$ and $G(\rho)$ containing fixed point i, $i \in \{0, 1, \ldots, 14\}$ will be observed. They will be denoted with $F(\rho)_i$ and $G(\rho)_i$, respectively. Let $i = 0$.

(a) Distribution of fixed points in the substructure $F(\rho)_0$

As is proved by Lemma 2.5, for $G(\rho)_0$ we have:

$$G(\rho)_0 = \begin{array}{lccccccccc} \text{fixed points:} & & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ q_1^F & \ldots & 1 & 1 & 1 & & & & \\ q_2^F & \ldots & 1 & & & 1 & 1 & & \\ q_3^F & \ldots & 1 & & & & & 1 & 1 \end{array}$$

where q_1, q_2 and q_3 are unfixed lines passing through the point 0. Let us denote:

$$F(\rho)_0 = \{p_i^F \mid 0 \le i \le 5\}, \quad \text{where } p_i,\ i = 0, 1, 2, 3, 4, 5,$$

are fixed lines passing through the point 0.

Therefore, the point 0 appears 6 times, the points from the set $C = \{1, 2, 3, 4, 5, 6\}$ ones, and the points from the set $D = \{7, 8, 9, 10, 11, 12, 13, 14\}$ three times, at 36 locations in $F(\rho)_0$.

(b) The properties of fixed points of the set c in the substructure $F(\rho)_0$

FIRST PROPERTY: None of following pairs of fixed points 1 2, 3 4 and 5 6 can participate in any of the fixed lines p_i, $0 \le i \le 5$, because on the remaining 9 fixed lines every point from a pair should appear five times more, and on one of them both should appear, and in the fourth time already, which is contradicts $\lambda = 3$.

SECOND PROPERTY: At least four fixed lines from $F(\rho)_0$ have nonempty intersection with the set C. Else, at least three lines have empty inersection with C, and in this way fixed points from D would participate in least three fixed lines from $F(\rho)_0$, which is impossible, as eight points from D can't be arranged in three blocks of the lenght 5. In this was the intersections of the blocks contain two points maximum.

Thus, for the distribution of fixed points from the set C we have the following five cases:

FIRST CASE: 3+1+1+1+0+0

$C:$ 1	2	3	4	5	6	
1		1		1		$(p_0)_c$
	1					$(p_1)_c$
			1	1		$(p_2)_c$
					1	$(p_3)_c$

SECOND CASE: 2+2+1+1+0+0

$C:$ 1	2	3	4	5	6
1			1		
	1		1		
				1	
					1

THIRD CASE: 2+2+1+1+0+0

$C:$ 1	2	3	4	5	6	
1		1				$(p_0)_c$
	1			1		$(p_1)_c$
			1			$(p_2)_c$
					1	$(p_3)_c$
						$(p_4)_c$

FOURTH CASE: 2+1+1+1+1+0

$C:$ 1	2	3	4	5	6
1		1			
	1				
			1		
				1	
					1

FIFTH CASE: 1+1+1+1+1+1

$C:$ 1	2	3	4	5	6	
1						$(p_0)_c$
	1					$(p_1)_c$
		1				$(p_2)_c$
			1			$(p_3)_c$
				1		$(p_4)_c$
					1	$(p_5)_c$

where the symbol "$a+b+c+d+e+f$" denotes that the line p_0 contains a points from C, ..., and that the line p_5 contains f points from C, and $(p_i)_C = p_i \cap C$.

(c) Non-existence of the structure $F(\rho)$ without the shearched substructure.

FIRST CASE:

FIXED POINTS

p_i^F	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14
p_0^F	1	1		1		1									
p_1^F	1		1												
p_2^F	1			1											
p_3^F	1							1							
p_4^F	1							1	1	1	1	1			
p_5^F	1							1	1				1	1	1

The lines p_1, p_2 and p_3 have each 4 more fixed points from D, and they can't avoid additional intersection with the lines p_4 and p_5, which in D have 5 fixed points each. It means that the lines p_1, p_2 and p_3 have each two fixed points from the set $\{7, 8, 9, 10, 11\}$ and two from the set $\{7, 8, 12, 13, 14\}$. The other two of the mentioned four are situated in the supplement-$\{12, 13, 14\}$, relevantly $\{9, 10, 11\}$. Consequently, the lines p_1, p_2 and p_3 do not contain any of the points 7 and 8. Therefore, the points 7 and 8, should both be situated on the line p_0, which then with the lines p_4 and p_5 gives the "SEARCHED SUBSTRUCTURE".

The proof is developed similarly in the other cases.

(**II**) Construction of substructures $F(\rho)$.

For first three of fixed lines p_0, p_1 and p_2 can be supposed

$$p_0 \cap p_1 \cap p_2 = \{0, 1, 2\},$$

and on the basis of this their fixed parts are constructed i.e. the part of $F(\rho)_0$. (see first three of the fixed lines in Table 1).

Branching starts at the construction of the fixed lines p_3, p_4 and p_5 from $F(\rho)_0$. In the part constructed of $F(\rho)_0$, the fixed points of the set $E = \{3, 4, 5, 6, 7, 8, 9, 10, 11\}$ are divided in three equally valid groups:

$$3\,4\,5 \quad 6\,7\,8 \quad 9\,10\,11.$$

Furthermore, because of the structure $G(\rho)_0$, the fixed point 0 in $F(\rho)_0$ comes six times, eight fixed points from D come three times each, and six fixed points from C one time each.

Because of the assumption $p_0 \cap p_1 \cap p_2 = \{0, 1, 2\}$, additional six fixed points should be found, which would appear three times each. The points from the set $H = \{12, 13, 14\}$ cannot be taken in to consideration. Namely, if, for example, the point 12 should participate three times (meaning on the lines p_3, p_4 and p_5), then additional five points from E could not be realised two times each. Obviously two points from the set H cannot take part in the situation.

The participation of all of the three points from H leads to the contradiction $p_3 \cap p_4 \cap p_5 = \{0, 12, 13, 14\}$.

Hence, it remains that only the points from the mentioned three equally valid groups can appear three times each in $F(\rho)_0$.

There are only two alternatives: $2+2+2$ or $3+3+0$ where the symbol "$a+b+c$" means that from the first group comes a, from the second group b, and from the third group c points three times each in $F(\rho)_0$ ($3+2+1$ can not be considerated as otherwise two of the fixed lines would have exactly two common fixed points).

(Hence, either all of these three groups participate in $F(\rho)_0$ with two points three times each, or two points from the two groups participate three times each, and from the third group none participates three times in $F(\rho)_0$.) It is easy to prove that, taking in to consideration the necessary incidences i.e. that in that part of the lines, they intersect in one or three points, the first choice $2+2+2$ gives the first SUBSTRUCTURE $F(\rho)$ (see for example the substructure $F(\rho)$ on the orbital structure $S_{1,1}$, Table 1), and the second choise $3+3+0$ gives the second SUBSTRUCTURE $F(\rho)$ (see the part $F(\rho)$ of $S_{2,1}$, Table 1), having in mind that the fixed point 1, i.e. that with $F(\rho)_1$ is treated in the same way as $F(\rho)_0$, and in the same way as $F(\rho)_2$.

We have thus proved that under given assumptions there are two substructures for $F(\rho)$, both obviously having the parameters (15,6,[1,3]). ∎

2.1.4. *Constructions of the substructures $F(\rho) \cup G(\rho) \cup CF(\rho)$.*

LEMMA 2.8. *Up to isomorphism there are two substructures $F(\rho) \cup G(\rho)$.*

PROOF. Follows immediately from Lemmas 2.5 and 2.7 ∎

LEMMA 2.9. *Up to isomorphism there are two substructures $F(\rho) \cup CF(\rho) \cup G(\rho)$.*

PROOF. Follows immediately from the symmetry of the schemes $F(\rho)$, uniqueness of $G(\rho)$ and duality of the schemes $G(\rho)$ and $CF(\rho)$, where in $CF(\rho)$, as usually, full 2-orbits are used as the points of the structure. ∎

REMARK 2.10. Let us mark lexicographically the first substructure with S_1 (see the substructure $F(\rho) \cup CF(\rho) \cup G(\rho)$ on orbit structures $S_{1,1}$ and $S_{1,3}$, Table 1) and the second with S_2 (see the substructure $F(\rho) \cup CF(\rho) \cup G(\rho)$ on orbit structures $S_{2,1}$, $S_{2,2}$ and $S_{2,5}$, Table 1).

2.1.5. *Construction of substructures $CG(\rho)$ for S_1 and S_2.*

LEMMA 2.11. *Eleven* ORBIT STRUCTURES $S_{1,1}$, $S_{1,2}$, $S_{1,3}$, $S_{1,4}$, $S^*_{1,4}$, $S_{1,5}$, $S^*_{1,5}$, $S_{1,6}$, $S_{1,7}$, $S^*_{1,7}$, $S_{1,8}$ *are obtained for the first* SUBSTRUCTURE S_1 *corresponding to the schemes* $(A/A)_1$, $(A/A)_2$, $(A/A)_3$, $(A/B)_1$, $(B/A)_1$, $(A/C)_2$, $(C/A)_2$, D/D, E/F, F/E *and* G/G *for* $CG(\rho)$, *where* $S_{i,j}$ *and* $S^*_{i,j}$ *are mutually dual substructures.*

PROOF. For each unfixed line x of the orbit substructure S_1 there is a unique set $P_3 = \{p_1, p_2, p_3\}$ of three fixed lines p_1, p_2 and p_3, which in pairs have a maximum number (2) of common fixed points from the set $\{\mathrm{P}_1, \mathrm{P}_2, \mathrm{P}_3\}$, where P_1, P_2 and P_3 are fixed points of the line x.

From the orbit substructure S_1 (see Table 1) we can see that the lines from the set P_3 have one common unfixed point. Thus according to Definition 1.12, for the searched line x, there is:

$$j = 4,\ P_3 = \{p_1, p_2, p_3\},\ B_0^{(3)} = 8,\ B_2^{(3)} = 6,\ B_4^{(3)} = 0,\ B_6^{(3)} = 1.$$

According to Lemma 1.14, and taking into consideration that in $CG(\rho)$ there are no full point orbits, the following is valid:

$$0 \le B_i^j \le 1 \cdot B_i^{(j-1)},\ \text{hence}$$
$$0 \le B_0^4 \le 8,\ 0 \le B_2^4 \le 6,\ B_4^4 = 0,\ 0 \le B_6^4 \le 1.$$

and:

$$B_0^4 + B_2^4 + B_6^4 = 9,$$
$$B_2^4 + 3B_6^4 = 3,$$

from which two options are possible for each unfixed line x:

1. possibility: $B_0^4 = 8,\ B_2^4 = 0,\ B_6^4 = 1;$
2. possibility: $B_0^4 = 6,\ B_2^4 = 3,\ B_6^4 = 0.$

These solutions will be used for the construction of unfixed parts of all 15 unfixed lines, for both matrices S_1 and S_2. The first possibility gives an unique solution for the unfixed line, and the second gives four solutons.

Thus for the first unfixed line there are 5 options, and for the first and the second unfixed line together only 7 options.

For each of these 7 options we can obtain a single solutions for the first, the second and the third unfixed line, and these 7 options for three unfixed lines will be marked with A, B, C, D, E, F and G.

On the basis of Lemma 1.14, unfixed parts of unfixed lines are obtained, and using the symmetry of the whole scheme already obtained, parallel and unfixed parts of the unfixed points are constructed. Some of these schemes branch out (resulting in the indecies), and some disappear.
We must emphasize that for the first three of the unfixed lines, as for the first three of the unfixed points, there were the same 7 options A, B, C, D, E, F and G, and only some of them could be arranged. (for example, the option A for the lines with the options A, B and C for the points, in symbols A/A, A/B i A/C, etc.)

It can be the illustrated with the schemes $(A/A)_1$, $(A/B)_1$ which show the sequence of their construction. Let us observe the first three of the unfixed lines and the first three of the unfixed points on these schemes. The symbols "_ _" designate that then, according to Lemma 1.14, the fourth line is constructed, and the symbols ":" that the fourth point is constructed, etc. (for more details see [5] in which, according to Definition 1.12 and Lemma 1.14, the orbit structures for the designes (15,7,3), (25,9,3) and (31,10,3) are constructed admitting an involutory automorphism). ∎

A/A=(A/A)1

	0	1	2	3	4	5	6	7	8	9	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$
0	1	0	0	0	1	1	0	1	1	1	1	1	1	0	0
1	0	1	1	1	0	0	0	1	1	1	1	0	0	1	1
2	0	1	1	0	1	1	1	0	0	0	0	1	1	1	1
3	0	1	0	1_	0_	1_	1_	0_	1_	1_	0_	1_	0_	1_	1
4	1	0	1	0:	1_	0_	1_	0_	1_	0_	1_	1_	1_	1_	0
5	1	0	1	1:	0:	1_	0_	1_	0_	1_	1_	0_	1_	0_	1
6	0	0	1	1:	1:	0:	1_	1_	0_	0_	1_	0_	1_	1_	1
7	1	1	0	0:	0:	1:	1:	1_	0_	1_	0_	1_	1_	0_	1
8	1	1	0	1:	1:	0:	0:	0:	1_	1_	1_	1_	0_	1_	0
9	1	1	0	1:	0:	1:	1:	1:	1:	1_	0_	0_	1_	1_	0
$\bar{0}$	1	1	0	0:	1:	1:	1:	0:	1:	0:	1_	1_	0_	0_	1
$\bar{1}$	1	0	1	1:	1:	0:	0:	1:	1:	0:	1:	1_	0_	0_	1
$\bar{2}$	1	0	1	0:	1:	1:	1:	1:	0:	1:	0:	0:	1_	1_	0
$\bar{3}$	0	1	1	1:	1:	0:	1:	0:	1:	1:	0:	0:	1:	1	0
$\bar{4}$	0	1	1	1:	0:	1:	1:	1:	0:	0:	1:	1:	0:	0	1

A/A=(A/A)1

0	1	2	3	4	5	6	7	8	9	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$
1	0	0	0	1	1	0	1	1	1	1	1	1	0	0
0	1	1	1	0	0	0	1	1	1	1	0	0	1	1
0	1	1	0	1	1	1	0	0	0	0	1	1	1	1
0	0	1	1_	1_	0_	1_	0_	1_	0_	1_	1_	0_	1_	1
1	1	0	0:	0_	1_	1_	0_	1_	1_	0_	1_	1_	1_	0
1	0	1	1:	0:	1_	0_	1_	0_	1_	1_	0_	1_	0_	1
0	1	1	1:	0:	1:	1_	1_	0_	1_	0_	0_	1_	1_	1
1	0	1	0:	1:	0:	1:	1_	0_	0_	1_	1_	1_	0_	1
1	1	0	1:	1:	0:	0:	0:	1_	1_	1_	1_	0_	1_	0
1	0	1	1:	1:	0:	0:	1:	1:	0_	1_	0_	1_	1_	0
1	1	0	0:	1:	1:	1:	0:	1:	0:	1_	1_	0_	0_	1
1	1	0	1:	0:	1:	0:	1:	1:	1:	0:	1_	0_	0_	1
1	0	1	0:	1:	1:	1:	1:	0:	1:	0:	0:	1_	1_	0
0	1	1	1:	1:	0:	1:	0:	1:	1:	0:	0:	1:	1	0
0	1	1	1:	0:	1:	1:	1:	0:	0:	1:	1:	0:	0	1

Similar is valid for S_2:

LEMMA 2.12. *For the second* SUBTRUCTURE S_2 *eleven possible* ORBIT STRUCTURES *are obtained* $S_{2,1}$, $S_{2,2}$, $S_{2,3}$, $S^*_{2,3}$, $S_{2,4}$, $S^*_{2,4}$, $S_{2,5}$, $S^*_{2,5}$, $S_{2,6}$, $S^*_{2,6}$ *and* $S_{2,7}$.

PROOF. Similar to the proof of Lemma 2.11. ∎

REMARK 2.13. Without the use of a computer, it was proved that all of the 22 orbit structures are isomorhic with 5 orbit structures (γ_{ir}): $S_{1,1}$, $S_{1,3}$, $S_{2,1}$, $S_{2,2}$ and $S_{2,5}$, for which can easily be shown to be self-dual. However, nonisomorhic character of the remaining 5 orbit structures was not proved. They are listed layer by layer in the Table 1. All the data submitted in the paper, were obtained without the use of a computer!

2.2. *Construction of block designes.* The indexing of the orbit structures obtained was done according to ALGORITHM by V. Ćepulić. There are exactly 28 designes to isomorhism (see Table 2). All of the 28 designs from the Table 2 are mutualy non-isomorphic. However, there are two among them which are dual of each other, and 26 are self-dual. These 28 symmetric block designs have groups of orders 2^1, 2^2, 2^3, 2^4, $2^2 \cdot 3$, $2^3 \cdot 3$, $2^3 \cdot 3^3$, $2^4 \cdot 3^3$, $2^4 \cdot 3^4$ and $2^7 \cdot 3^4 \cdot 5$ (see Table 2).

Furthermore, indexing shows that the 5 orbit structures mentioned are mutually non-isomorphic.

The most important conclusion is that in this case all of the designes are classified. The proof of the Theorem is thus conclusive.

TABLE 1
THE ORBIT STRUCTURES

S1,1

	fixed points															unfixed points														
	0	1	2	3	4	5	6	7	8	9	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	0	1	2	3	4	5	6	7	8	9	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$
0	1	1	1	1	1	1	.	.	.	.	.	.	.	.	.	2	2	2	.	.	.	.	.	.	.	.	.	.	.	.
1	1	1	1	.	.	.	1	1	1	.	.	.	.	.	.	.	.	.	2	2	2	.	.	.	.	.	.	.	.	.
2	1	1	1	.	.	.	.	.	.	1	1	1	.	.	.	.	.	.	.	.	.	2	2	2	.	.	.	.	.	.
3	1	.	.	1	1	.	1	1	.	.	.	.	1	.	.	.	.	.	.	.	.	2	.	.	2	2	.	.	.	.
4	1	.	.	1	1	.	.	.	.	1	1	.	.	1	.	.	.	.	2	.	.	.	.	.	.	.	2	2	.	.
5	1	.	.	.	.	.	1	1	.	1	1	.	.	.	1	2	.	.	.	.	.	.	.	.	.	.	.	.	2	2
6	.	1	.	1	.	1	1	.	1	.	.	.	1	.	.	.	.	.	.	.	.	.	2	.	.	.	2	.	2	.
7	.	1	.	1	.	1	.	.	.	1	.	1	.	1	.	.	.	.	.	2	.	.	.	.	2	.	.	.	.	2
8	.	1	.	.	.	.	1	.	1	1	.	1	.	.	1	.	2	.	.	.	.	.	.	.	.	2	.	2	.	.
9	.	.	1	.	1	1	.	1	1	.	.	.	1	.	.	.	.	.	.	.	.	.	.	2	.	.	.	2	.	2
$\bar{0}$	.	.	1	.	1	1	.	.	.	.	1	1	.	1	.	.	.	.	.	.	2	.	.	.	.	2	.	.	2	.
$\bar{1}$	.	.	1	.	.	.	.	1	1	.	1	1	.	.	1	.	.	2	.	.	.	.	.	.	2	.	2	.	.	.
$\bar{2}$	.	.	.	1	.	.	1	.	.	1	.	.	1	1	1	.	.	2	.	.	2	.	.	2	.	.	.	.	.	.
$\bar{3}$	.	.	.	.	1	.	.	1	.	.	1	.	1	1	1	.	2	.	.	2	.	.	2	.	.	.	.	.	.	.
$\bar{4}$	.	.	.	.	.	1	.	.	1	.	.	1	1	1	1	2	.	.	2	.	.	2	.	.	.	.	.	.	.	.
0	1	.	.	.	.	1	.	.	.	.	.	.	.	.	1	1	.	.	.	1	1	.	1	1	1	1	1	1	.	.
1	1	.	.	.	.	.	.	.	1	.	.	.	.	1	.	.	1	1	1	.	.	.	1	1	1	1	.	.	1	1
2	1	.	.	.	.	.	.	.	.	.	.	1	1	.	.	.	1	1	.	1	1	1	.	.	.	.	1	1	1	1
3	.	1	.	.	1	.	.	.	.	.	.	.	.	.	1	.	1	.	1	.	1	1	.	1	1	.	1	.	1	1
4	.	1	.	.	.	.	.	1	.	.	.	.	.	1	.	1	.	1	.	1	.	1	.	1	.	1	1	1	1	.
5	.	1	.	.	.	.	.	.	.	.	1	.	1	.	.	1	.	1	1	.	1	.	1	.	1	1	.	1	.	1
6	.	.	1	1	.	.	.	.	.	.	.	.	.	.	1	.	.	1	1	1	.	1	1	.	.	1	.	1	1	1
7	.	.	1	.	.	.	1	.	.	.	.	.	.	1	.	1	1	.	.	.	1	1	1	.	1	.	1	1	.	1
8	.	.	1	.	.	.	.	.	.	1	.	.	1	.	.	1	1	.	1	1	.	.	.	1	1	1	1	.	1	.
9	.	.	.	1	.	.	.	1	.	.	.	1	.	.	.	1	1	.	1	.	1	.	1	1	1	.	.	1	1	.
$\bar{0}$	.	.	.	1	.	.	.	.	1	.	1	.	.	.	.	1	1	.	.	1	1	1	.	1	.	1	1	.	.	1
$\bar{1}$	.	.	.	.	1	.	1	.	.	.	.	1	.	.	.	1	.	1	1	1	.	.	1	1	.	1	1	.	.	1
$\bar{2}$	.	.	.	.	1	.	.	.	1	1	.	.	.	.	.	1	.	1	.	1	1	1	1	.	1	.	.	1	1	.
$\bar{3}$	.	.	.	.	.	1	1	.	.	.	1	.	.	.	.	.	1	1	1	1	.	1	.	1	1	.	.	1	1	.
$\bar{4}$	.	.	.	.	.	1	.	1	.	1	.	.	.	.	.	.	1	1	1	.	1	1	1	.	.	1	1	.	.	1

S1,3

fixed points															unfixed points														
0	1	2	3	4	5	6	7	8	9	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	0	1	2	3	4	5	6	7	8	9	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$
1	1	1	1	1	1	.	.	.	.	.	.	.	.	.	2	2	2	.	.	.	.	.	.	.	.	.	.	.	.
1	1	1	.	.	.	1	1	1	.	.	.	.	.	.	.	.	.	2	2	2	.	.	.	.	.	.	.	.	.
1	1	1	.	.	.	.	.	.	1	1	1	.	.	.	.	.	.	.	.	.	2	2	2	.	.	.	.	.	.
1	.	.	1	1	.	1	1	.	.	.	.	1	.	.	.	.	.	.	.	.	2	.	.	2	2	.	.	.	.
1	.	.	1	1	.	.	.	.	1	1	.	.	1	.	.	.	.	2	.	.	.	.	.	.	.	2	2	.	.
1	.	.	.	.	.	1	1	.	1	1	.	.	.	1	2	.	.	.	.	.	.	.	.	.	.	.	.	2	2
.	1	.	1	.	1	1	.	1	.	.	.	1	.	.	.	.	.	.	.	.	.	2	.	.	.	2	.	2	.
.	1	.	1	.	1	.	.	.	1	.	1	.	1	.	.	.	.	.	2	.	.	.	.	2	.	.	.	.	2
.	1	.	.	.	.	1	.	1	1	.	1	.	.	1	.	2	.	.	.	.	.	.	.	.	2	.	2	.	.
.	.	1	.	1	1	.	1	1	.	.	.	1	.	.	.	.	.	.	.	.	.	.	2	.	.	.	2	.	2
.	.	1	.	1	1	.	.	.	.	1	1	.	1	.	.	.	.	.	.	2	.	.	.	.	2	.	.	2	.
.	.	1	.	.	.	.	1	1	.	1	1	.	.	1	.	.	2	.	.	.	.	.	.	2	.	2	.	.	.
.	.	.	1	.	.	1	.	.	1	.	.	1	1	1	.	.	2	.	.	2	.	.	2	.	.	.	.	.	.
.	.	.	.	1	.	.	1	.	.	1	.	1	1	1	.	2	.	.	2	.	.	2	.	.	.	.	.	.	.
.	.	.	.	.	1	.	.	1	.	.	1	1	1	1	2	.	.	2	.	.	2	.	.	.	.	.	.	.	.
1	.	.	.	.	1	.	.	.	.	.	.	.	.	1	1	.	.	.	1	1	.	1	1	1	1	1	1	.	.
1	.	.	.	.	.	.	.	1	.	.	.	.	1	.	.	1	1	1	.	.	.	1	1	1	1	.	.	1	1
1	.	.	.	.	.	.	.	.	.	.	1	1	.	.	.	1	1	.	1	1	1	.	.	.	.	1	1	1	1
.	1	.	.	1	.	.	.	.	.	.	.	.	.	1	.	1	.	1	.	1	1	.	1	1	.	1	.	1	1
.	1	.	.	.	.	.	1	.	.	.	.	.	1	.	1	.	1	.	1	.	1	.	1	.	1	1	1	1	.
.	1	.	.	.	.	.	.	.	.	1	.	1	.	.	1	.	1	1	.	1	.	1	.	1	1	.	1	.	1
.	.	1	1	.	.	.	.	.	.	.	.	.	.	1	.	.	1	1	1	.	1	1	.	.	1	.	1	1	1
.	.	1	.	.	.	1	.	.	.	.	.	.	1	.	1	1	.	.	.	1	1	1	.	1	.	1	1	.	1
.	.	1	.	.	.	.	.	.	1	.	.	1	.	.	1	1	.	1	1	.	.	.	1	1	1	1	.	1	.
.	.	.	1	.	.	.	1	.	.	.	1	.	.	.	1	1	.	1	.	1	.	1	1	.	1	1	.	.	1
.	.	.	1	.	.	.	.	1	.	1	.	.	.	.	1	1	.	.	1	1	1	.	1	1	.	.	1	1	.
.	.	.	.	1	.	1	.	.	.	.	1	.	.	.	1	.	1	1	1	.	.	1	1	1	.	.	1	1	.
.	.	.	.	1	.	.	.	1	1	.	.	.	.	.	1	.	1	.	1	1	1	1	.	.	1	1	.	.	1
.	.	.	.	.	1	1	.	.	.	1	.	.	.	.	.	1	1	1	1	.	1	.	1	.	1	1	.	.	1
.	.	.	.	.	1	.	1	.	1	.	.	.	.	.	.	1	1	1	.	1	1	1	.	1	.	.	1	1	.

S2,1

	fixed points															unfixed points														
	0	1	2	3	4	5	6	7	8	9	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	0	1	2	3	4	5	6	7	8	9	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$
0	1	1	1	1	1	1	.	.	.	.	.	.	.	.	.	2	2	2	.	.	.	.	.	.	.	.	.	.	.	.
1	1	1	1	.	.	.	1	1	1	.	.	.	.	.	.	.	.	.	2	2	2	.	.	.	.	.	.	.	.	.
2	1	1	1	.	.	.	.	.	.	1	1	1	.	.	.	.	.	.	.	.	.	2	2	2	.	.	.	.	.	.
3	1	.	.	1	1	.	1	1	.	.	.	.	1	.	.	.	.	.	.	.	.	2	.	.	2	2	.	.	.	.
4	1	.	.	1	.	1	1	.	1	.	.	.	.	1	.	.	.	.	.	.	.	.	2	.	.	.	2	2	.	.
5	1	.	.	.	1	1	.	1	1	.	.	.	.	.	1	.	.	.	.	.	.	.	.	2	.	.	.	.	2	2
6	.	1	.	1	1	.	.	.	.	1	1	.	1	.	.	.	.	.	2	.	.	.	.	.	.	.	2	.	2	.
7	.	1	.	1	.	1	.	.	.	1	.	1	.	1	.	.	.	.	.	2	.	.	.	.	2	.	.	.	.	2
8	.	1	.	.	1	1	.	.	.	.	1	1	.	.	1	.	.	.	.	.	2	.	.	.	.	2	.	2	.	.
9	.	.	1	.	.	.	1	1	.	1	1	.	1	.	.	2	.	.	.	.	.	.	.	.	.	.	.	2	.	2
$\bar{0}$	.	.	1	.	.	.	1	.	1	1	.	1	.	1	.	.	2	.	.	.	.	.	.	.	.	2	.	.	2	.
$\bar{1}$	.	.	1	.	.	.	.	1	1	.	1	1	.	.	1	.	.	2	.	.	.	.	.	.	2	.	2	.	.	.
$\bar{2}$	.	.	.	1	.	.	1	.	.	1	.	.	1	1	1	.	.	2	.	.	2	.	.	2	.	.	.	.	.	.
$\bar{3}$	.	.	.	.	1	.	.	1	.	.	1	.	1	1	1	.	2	.	.	2	.	.	2	.	.	.	.	.	.	.
$\bar{4}$	.	.	.	.	.	1	.	.	1	.	.	1	1	1	1	2	.	.	2	.	.	2	.	.	.	.	.	.	.	.
0	1	.	.	.	.	.	.	.	.	1	.	.	.	.	1	1	1	.	1	1	.	.	.	1	1	1	1	1	.	.
1	1	.	.	.	.	.	.	.	.	.	1	.	.	1	.	1	.	1	1	.	1	.	1	.	1	1	.	.	1	1
2	1	.	.	.	.	.	.	.	.	.	.	1	1	.	.	.	1	1	.	1	1	1	.	.	.	.	1	1	1	1
3	.	1	.	.	.	.	1	.	.	.	.	.	.	.	1	1	1	.	.	.	1	1	1	.	1	.	1	.	1	1
4	.	1	.	.	.	.	.	1	.	.	.	.	.	1	.	1	.	1	.	1	.	1	.	1	.	1	1	1	1	.
5	.	1	.	.	.	.	.	.	1	.	.	.	1	.	.	.	1	1	1	.	.	.	1	1	1	1	.	1	.	1
6	.	.	1	1	.	.	.	.	.	.	.	.	.	.	1	.	.	1	1	1	.	1	1	.	.	1	.	1	1	1
7	.	.	1	.	1	.	.	.	.	.	.	.	.	1	.	.	1	.	1	.	1	1	.	1	1	.	1	1	.	1
8	.	.	1	.	.	1	.	.	.	.	.	.	1	.	.	1	.	.	.	1	1	.	1	1	1	1	1	.	1	.
9	.	.	.	1	.	.	.	1	.	.	.	1	.	.	.	1	1	.	1	.	1	.	1	1	1	.	.	1	1	.
$\bar{0}$	.	.	.	1	.	.	.	.	1	.	1	.	.	.	.	1	1	.	.	1	1	1	.	1	.	1	1	.	.	1
$\bar{1}$	.	.	.	.	1	.	1	.	.	.	.	1	.	.	.	1	.	1	1	1	.	.	1	1	.	1	1	.	.	1
$\bar{2}$	.	.	.	.	1	.	.	.	1	1	.	.	.	.	.	1	.	1	.	1	1	1	1	.	1	.	.	1	1	.
$\bar{3}$	.	.	.	.	.	1	1	.	.	.	1	.	.	.	.	.	1	1	1	1	.	1	.	1	1	.	.	1	1	.
$\bar{4}$	.	.	.	.	.	1	.	1	.	1	.	.	.	.	.	.	1	1	1	.	1	1	1	.	.	1	1	.	.	1

S2,2

fixed points															unfixed points														
0	1	2	3	4	5	6	7	8	9	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	0	1	2	3	4	5	6	7	8	9	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$
1	1	1	1	1	1	.	.	.	.	.	.	.	.	.	2	2	2	.	.	.	.	.	.	.	.	.	.	.	.
1	1	1	.	.	.	1	1	1	.	.	.	.	.	.	.	.	.	2	2	2	.	.	.	.	.	.	.	.	.
1	1	1	.	.	.	.	.	.	1	1	1	.	.	.	.	.	.	.	.	.	2	2	2	.	.	.	.	.	.
1	.	.	1	1	.	1	1	.	.	.	.	1	.	.	.	.	.	.	.	.	2	.	.	2	2	.	.	.	.
1	.	.	1	.	1	1	.	1	.	.	.	.	1	.	.	.	.	.	.	.	.	2	.	.	.	2	2	.	.
1	.	.	.	1	1	.	1	1	.	.	.	.	.	1	.	.	.	.	.	.	.	.	2	.	.	.	.	2	2
.	1	.	1	1	.	.	.	.	1	1	.	1	.	.	.	.	.	2	.	.	.	.	.	.	.	2	.	2	.
.	1	.	1	.	1	.	.	.	1	.	1	.	1	.	.	.	.	.	2	.	.	.	.	2	.	.	.	.	2
.	1	.	.	1	1	.	.	.	.	1	1	.	.	1	.	.	.	.	.	2	.	.	.	.	2	.	2	.	.
.	.	1	.	.	.	1	1	.	1	1	.	1	.	.	2	.	.	.	.	.	.	.	.	.	.	.	2	.	2
.	.	1	.	.	.	1	.	1	1	.	1	.	1	.	.	2	.	.	.	.	.	.	.	.	2	.	.	2	.
.	.	1	.	.	.	.	1	1	.	1	1	.	.	1	.	.	2	.	.	.	.	.	.	2	.	2	.	.	.
.	.	.	1	.	.	1	.	.	1	.	.	1	1	1	.	.	2	.	.	2	.	.	2	.	.	.	.	.	.
.	.	.	.	1	.	.	1	.	.	1	.	1	1	1	.	2	.	.	2	.	.	2	.	.	.	.	.	.	.
.	.	.	.	.	1	.	.	1	.	.	1	1	1	1	2	.	.	2	.	.	2	.	.	.	.	.	.	.	.
1	.	.	.	.	.	.	.	.	1	.	.	.	.	1	1	1	.	1	1	.	.	.	1	1	1	1	1	.	.
1	.	.	.	.	.	.	.	.	.	1	.	.	1	.	1	.	1	1	.	1	.	1	.	1	1	.	.	1	1
1	.	.	.	.	.	.	.	.	.	.	1	1	.	.	.	1	1	.	1	1	1	.	.	.	.	1	1	1	1
.	1	.	.	.	.	1	.	.	.	.	.	.	.	1	1	1	.	.	.	1	1	1	.	1	.	1	.	1	1
.	1	.	.	.	.	.	1	.	.	.	.	.	1	.	1	.	1	.	1	.	1	.	1	.	1	1	1	1	.
.	1	.	.	.	.	.	.	1	.	.	.	1	.	.	.	1	1	1	.	.	.	1	1	1	1	.	1	.	1
.	.	1	1	.	.	.	.	.	.	.	.	.	.	1	.	.	1	1	1	.	1	1	.	.	1	.	1	1	1
.	.	1	.	1	.	.	.	.	.	.	.	.	1	.	.	1	.	1	.	1	1	.	1	1	.	1	1	.	1
.	.	1	.	.	1	.	.	.	.	.	.	1	.	.	1	.	.	.	1	1	.	1	1	1	1	1	.	1	.
.	.	.	1	.	.	.	1	.	.	.	1	.	.	.	1	1	.	1	.	1	.	1	1	.	1	1	.	.	1
.	.	.	1	.	.	.	.	1	.	1	.	.	.	.	1	1	.	.	1	1	1	.	1	1	.	.	1	1	.
.	.	.	.	1	.	1	.	.	.	.	1	.	.	.	1	.	1	1	1	.	.	1	1	1	.	.	1	1	.
.	.	.	.	1	.	.	.	1	1	.	.	.	.	.	1	.	1	.	1	1	1	1	.	.	1	1	.	.	1
.	.	.	.	.	1	1	.	.	.	1	.	.	.	.	.	1	1	1	1	.	1	.	1	.	1	1	.	.	1
.	.	.	.	.	1	.	1	.	1	.	.	.	.	.	.	1	1	1	.	1	1	1	.	1	.	.	1	1	.

S2,5

	fixed points															unfixed points														
	0	1	2	3	4	5	6	7	8	9	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$	0	1	2	3	4	5	6	7	8	9	$\bar{0}$	$\bar{1}$	$\bar{2}$	$\bar{3}$	$\bar{4}$
0	1	1	1	1	1	1	.	.	.	.	.	.	.	.	.	2	2	2	.	.	.	.	.	.	.	.	.	.	.	.
1	1	1	1	.	.	.	1	1	1	.	.	.	.	.	.	.	.	.	2	2	2	.	.	.	.	.	.	.	.	.
2	1	1	1	.	.	.	.	.	.	1	1	1	.	.	.	.	.	.	.	.	.	2	2	2	.	.	.	.	.	.
3	1	.	.	1	1	.	1	1	.	.	.	.	1	.	.	.	.	.	.	.	.	2	.	.	2	2	.	.	.	.
4	1	.	.	1	.	1	1	.	1	.	.	.	.	1	.	.	.	.	.	.	.	.	2	.	.	.	2	2	.	.
5	1	.	.	.	1	1	.	1	1	.	.	.	.	.	1	.	.	.	.	.	.	.	.	2	.	.	.	.	2	2
6	.	1	.	1	1	.	.	.	.	1	1	.	1	.	.	.	.	.	2	.	.	.	.	.	.	.	2	.	2	.
7	.	1	.	1	.	1	.	.	.	1	.	1	.	1	.	.	.	.	.	2	.	.	.	.	2	.	.	.	.	2
8	.	1	.	.	1	1	.	.	.	.	1	1	.	.	1	.	.	.	.	.	2	.	.	.	.	2	.	2	.	.
9	.	.	1	.	.	.	1	1	.	1	1	.	1	.	.	2	.	.	.	.	.	.	.	.	.	.	.	2	.	2
$\bar{0}$	.	.	1	.	.	.	1	.	1	1	.	1	.	1	.	.	2	.	.	.	.	.	.	.	.	2	.	.	2	.
$\bar{1}$	.	.	1	.	.	.	.	1	1	.	1	1	.	.	1	.	.	2	.	.	.	.	.	.	2	.	2	.	.	.
$\bar{2}$	.	.	.	1	.	.	1	.	.	1	.	.	1	1	1	.	.	2	.	.	2	.	.	2	.	.	.	.	.	.
$\bar{3}$	.	.	.	.	1	.	.	1	.	.	1	.	1	1	1	.	2	.	.	2	.	.	2	.	.	.	.	.	.	.
$\bar{4}$	.	.	.	.	.	1	.	.	1	.	.	1	1	1	1	2	.	.	2	.	.	2	.	.	.	.	.	.	.	.
0	1	.	.	.	.	.	.	.	.	1	.	.	.	.	1	1	1	.	1	1	.	.	.	1	1	1	1	1	.	.
1	1	.	.	.	.	.	.	.	.	.	1	.	.	1	.	1	.	1	1	.	1	.	1	.	1	1	.	.	1	1
2	1	.	.	.	.	.	.	.	.	.	.	1	1	.	.	.	1	1	.	1	1	1	.	.	.	.	1	1	1	1
3	.	1	.	.	.	.	1	.	.	.	.	.	.	.	1	1	1	.	.	.	1	1	1	.	1	.	1	.	1	1
4	.	1	.	.	.	.	.	1	.	.	.	.	.	1	.	.	1	1	1	.	.	1	.	1	.	1	1	1	.	1
5	.	1	.	.	.	.	.	.	1	.	.	.	1	.	.	1	.	1	.	1	.	.	1	1	1	1	.	1	1	.
6	.	.	1	1	.	.	.	.	.	.	.	.	.	.	1	.	.	1	1	1	.	1	1	.	.	1	.	1	1	1
7	.	.	1	.	1	.	.	.	.	.	.	.	.	1	.	1	.	.	.	1	1	1	.	1	1	.	1	1	1	.
8	.	.	1	.	.	1	.	.	.	.	.	.	1	.	.	.	1	.	1	.	1	.	1	1	1	1	1	.	.	1
9	.	.	.	1	.	.	.	1	.	.	.	1	.	.	.	1	1	.	1	.	1	.	1	1	1	.	.	1	1	.
$\bar{0}$	.	.	.	1	.	.	.	.	1	.	1	.	.	.	.	1	1	.	.	1	1	1	.	1	.	1	1	.	.	1
$\bar{1}$	.	.	.	.	1	.	1	.	.	.	.	1	.	.	.	1	.	1	1	1	.	.	1	1	.	1	1	.	.	1
$\bar{2}$	.	.	.	.	1	.	.	.	1	1	.	.	.	.	.	.	1	1	1	.	1	1	1	.	1	.	.	1	.	1
$\bar{3}$	.	.	.	.	.	1	1	.	.	.	1	.	.	.	.	.	1	1	1	1	.	1	.	1	1	.	.	1	1	.
$\bar{4}$	.	.	.	.	.	1	.	1	.	1	.	.	.	.	.	1	.	1	.	1	1	1	1	.	.	1	1	.	1	.

Table 2

THE LIST OF THE DESIGNS (45,12,3), admitting an involutory automorphism: (15 16)(17 18)(19 20)(21 22)(23 24)(25 26)(27 28)(29 30) (31 32)(33 34)(35 36) (37 38)(39 40)(41 42)(43 44)

1. design (I. orbit structure)

0	1	2	3	4	5	15	16	17	18	19	20
0	1	2	6	7	8	21	22	23	24	25	26
0	1	2	9	10	11	27	28	29	30	31	32
0	3	4	6	7	12	27	28	33	34	35	36
0	3	4	9	10	13	21	22	37	38	39	40
0	6	7	9	10	14	15	16	41	42	43	44
1	3	5	6	8	12	29	30	37	38	41	42
1	3	5	9	11	13	23	24	33	34	43	44
1	6	8	9	11	14	17	18	35	36	39	40
2	4	5	7	8	12	31	32	39	40	43	44
2	4	5	10	11	13	25	26	35	36	41	42
2	7	8	10	11	14	19	20	33	34	37	38
3	6	9	12	13	14	19	20	25	26	31	32
4	7	10	12	13	14	17	18	23	24	29	30
5	8	11	12	13	14	15	16	21	22	27	28

0	5	14	15	23	25	29	31	33	35	37	39
0	8	13	17	19	21	29	31	34	36	41	43
0	11	12	17	19	23	25	27	38	40	42	44
1	4	14	17	21	26	27	32	33	37	42	43
1	7	13	15	19	24	27	32	35	38	39	41
1	10	12	15	20	21	25	30	34	35	40	43
2	3	14	19	22	24	28	29	35	40	42	43
2	6	13	15	18	26	27	29	34	37	40	44
2	9	12	15	18	21	24	31	33	36	38	42
3	7	11	15	17	22	26	30	31	33	40	41
3	8	10	15	17	24	25	28	32	36	37	44
4	6	11	15	20	22	23	29	32	36	38	43
4	8	9	15	19	23	26	28	30	34	39	42
5	6	10	17	20	22	24	27	31	34	39	42
5	7	9	17	20	21	26	28	29	35	38	44

The order of the automorphism group is 16

2. design

0	5	14	15	23	25	29	31	33	35	37	39
0	8	13	17	19	21	29	31	34	36	41	43
0	11	12	17	19	23	25	27	38	40	42	44
1	4	14	17	21	26	27	32	33	37	42	43
1	7	13	15	19	24	27	32	35	38	39	41
1	10	12	15	20	21	25	30	34	35	40	43
2	3	14	19	22	24	28	29	35	40	42	43
2	6	13	15	18	26	27	29	34	37	40	44
2	9	12	15	18	21	24	31	33	36	38	42
3	7	11	15	17	22	26	30	31	34	39	42
3	8	10	15	17	24	25	28	32	36	37	44
4	6	11	15	20	22	23	29	32	36	38	43
4	8	9	15	19	23	26	28	30	33	40	41
5	6	10	17	20	22	24	27	31	33	40	41
5	7	9	17	20	21	26	28	29	35	38	44

The order of the automorphism group is 12

3. design

0	5	14	15	23	25	29	31	33	35	37	39
0	8	13	17	19	21	29	31	34	36	41	43
0	11	12	17	19	23	25	27	38	40	42	44
1	4	14	17	21	26	27	32	33	37	42	43
1	7	13	15	19	24	27	32	35	38	39	41
1	10	12	15	20	21	26	29	34	35	40	44
2	3	14	19	22	24	28	29	35	40	42	43
2	6	13	15	18	25	27	30	34	37	40	43
2	9	12	15	18	21	24	31	33	36	38	42
3	7	11	15	17	22	26	30	31	33	40	41
3	8	10	15	17	24	25	28	32	36	37	44
4	6	11	15	20	22	23	29	32	36	38	43
4	8	9	15	19	23	26	28	30	34	39	42
5	6	10	17	20	22	24	27	31	34	39	42
5	7	9	17	20	21	25	28	30	35	38	43

The order of the automorphism group is 432

5. design

0	5	14	15	23	25	29	31	33	35	37	39
0	8	13	17	19	21	29	31	34	36	41	43
0	11	12	17	19	23	25	27	38	40	42	44
1	4	14	17	21	26	27	32	33	37	42	43
1	7	13	15	20	23	27	32	36	38	39	41
1	10	12	15	20	21	26	29	34	35	40	44
2	3	14	19	21	24	27	30	35	39	41	44
2	6	13	15	18	25	27	30	34	37	40	43
2	9	12	15	18	21	24	31	33	36	38	42
3	7	11	15	17	22	26	30	31	34	39	42
3	8	10	15	17	24	25	28	32	35	38	43
4	6	11	15	19	22	24	29	32	36	37	44
4	8	9	15	19	23	26	28	30	33	40	41
5	6	10	17	20	22	24	27	31	33	40	41
5	7	9	17	20	21	25	28	30	36	37	44

The order of the automorphism group is 432

6. design

0	5	14	15	23	25	29	31	33	35	37	39
0	8	13	17	19	21	29	31	34	36	41	43
0	11	12	17	19	23	25	27	38	40	42	44
1	4	14	17	22	25	28	32	34	37	41	44
1	7	13	15	19	24	27	32	35	38	39	41
1	10	12	15	20	21	25	30	34	35	40	43
2	3	14	19	22	24	28	29	35	40	42	43
2	6	13	15	18	26	27	29	34	37	40	44
2	9	12	15	17	21	24	32	33	36	37	42
3	7	11	15	17	22	26	30	31	34	39	42
3	8	10	15	18	24	25	28	31	36	38	44
4	6	11	15	20	22	23	29	32	36	38	43
4	8	9	15	19	23	26	28	30	33	40	41
5	6	10	17	20	22	24	27	31	33	40	41
5	7	9	17	20	21	26	28	29	35	38	44

The order of the automorphism group is 48

7. design (II. orbit structure)

0	5	14	15	23	25	29	31	33	35	37	39
0	8	13	17	19	21	29	31	34	36	41	43
0	11	12	17	19	23	25	27	38	40	42	44
1	4	14	17	21	26	27	32	33	37	42	43
1	7	13	15	20	23	27	32	36	38	39	41
1	10	12	15	20	21	26	29	34	35	40	44
2	3	14	19	21	24	27	30	35	39	41	44
2	6	13	15	18	25	27	30	34	37	40	43
2	9	12	15	18	21	24	31	33	36	38	42
3	7	11	15	17	22	26	30	31	35	38	43
3	8	10	15	17	24	25	28	32	33	40	41
4	6	11	15	19	22	24	29	32	34	39	42
4	8	9	15	19	23	26	28	30	36	37	44
5	6	10	17	20	22	24	27	31	36	37	44
5	7	9	17	20	21	25	28	30	34	39	42

The order of the automorphism group is 216

8. design

0	5	14	15	23	25	29	31	33	35	37	39
0	8	13	17	19	21	29	31	34	36	41	43
0	11	12	17	19	23	25	27	38	40	42	44
1	4	14	17	21	26	27	32	33	37	42	43
1	7	13	15	20	23	27	32	36	38	39	41
1	10	12	15	20	21	26	29	34	35	40	44
2	3	14	19	21	24	27	30	35	39	41	44
2	6	13	15	18	25	27	30	34	37	40	43
2	9	12	15	18	21	24	31	33	36	38	42
3	7	11	15	17	22	26	30	31	35	38	43
3	8	10	15	17	24	25	28	32	34	39	42
4	6	11	15	19	22	24	29	32	33	40	41
4	8	9	15	19	23	26	28	30	36	37	44
5	6	10	17	20	22	24	27	31	36	37	44
5	7	9	17	20	21	25	28	30	33	40	41

The order of the automorphism group is 216

9. design

0	5	14	15	23	25	29	31	33	35	37	39
0	8	13	17	19	21	29	31	34	36	41	43
0	11	12	17	20	24	25	27	37	40	42	43
1	4	14	17	22	25	28	32	33	38	41	43
1	7	13	15	20	24	28	31	36	38	39	42
1	10	12	15	19	21	25	30	33	36	40	44
2	3	14	19	22	23	27	30	36	39	42	43
2	6	13	15	18	26	27	29	33	38	40	43
2	9	12	15	17	21	23	32	34	35	38	42
3	7	11	15	17	22	26	29	32	36	37	44
3	8	10	15	18	24	25	27	32	34	39	41
4	6	11	15	20	22	23	30	31	34	40	41
4	8	9	15	19	24	26	28	30	35	37	43
5	6	10	17	19	22	24	27	31	35	38	44
5	7	9	17	20	21	26	27	30	33	39	41

The order of the automorphism group is 4

x1. (III. orbit structure)

0	3	5	6	8	13	29	30	37	38	39	40
0	4	5	7	8	14	31	32	41	42	43	44
1	3	4	9	10	12	21	22	37	38	41	42
1	3	5	9	11	13	23	24	33	34	43	44
1	4	5	10	11	14	25	26	35	36	39	40
2	6	7	9	10	12	15	16	39	40	43	44
2	6	8	9	11	13	17	18	35	36	41	42
2	7	8	10	11	14	19	20	33	34	37	38
3	6	9	12	13	14	19	20	25	26	31	32
4	7	10	12	13	14	17	18	23	24	29	30
5	8	11	12	13	14	15	16	21	22	27	28

0	9	14	15	17	21	23	31	33	35	37	39
0	10	13	15	19	21	25	29	34	36	41	43
0	11	12	17	19	23	25	27	38	40	42	44
1	6	14	15	17	26	28	30	34	38	41	44
1	7	13	15	19	24	28	32	35	37	40	42
1	8	12	17	19	22	30	32	33	36	39	43
2	3	14	19	22	24	27	29	35	39	41	44
2	4	13	17	22	26	27	31	34	37	40	43
2	5	12	15	24	26	29	31	33	36	38	42
3	7	11	15	18	22	25	30	31	33	40	41
3	8	10	15	18	23	26	27	32	35	38	43
4	6	11	15	20	22	23	29	32	36	37	44
4	8	9	15	20	24	25	27	30	34	39	42
5	6	10	17	20	21	24	27	32	33	40	41
5	7	9	17	20	22	25	28	29	35	38	43

The order of the automorphism group is 432. This design is isomorphic with 3. design x2.

0	9	14	15	17	21	23	31	33	35	37	39
0	10	13	15	19	21	25	29	34	36	41	43
0	11	12	17	19	23	25	27	38	40	42	44
1	6	14	15	17	26	28	30	34	38	41	44
1	7	13	15	19	24	28	32	35	37	40	42
1	8	12	17	19	22	30	32	33	36	39	43
2	3	14	19	22	24	27	29	35	39	41	44
2	4	13	17	22	26	27	31	34	37	40	43
2	5	12	15	24	26	29	31	33	36	38	42
3	7	11	15	18	22	25	30	31	33	40	41
3	8	10	15	18	23	26	27	32	36	37	44
4	6	11	15	20	22	23	29	32	35	38	43
4	8	9	15	20	24	25	27	30	34	39	42
5	6	10	17	20	21	24	27	32	33	40	41
5	7	9	17	20	22	25	28	29	36	37	44

The order of the automorphism group is 432. This design is isomorphic with 5. design

x3.

0	9	14	15	17	21	23	31	33	35	37	39
0	10	13	15	19	21	25	29	34	36	41	43
0	11	12	17	19	23	25	27	38	40	42	44
1	6	14	15	17	26	28	30	34	38	41	44
1	7	13	15	19	24	28	32	35	37	40	42
1	8	12	17	20	21	30	32	33	36	40	43
2	3	14	19	22	23	28	30	36	39	42	43
2	4	13	17	22	26	27	31	34	37	40	43
2	5	12	15	24	26	29	31	33	36	38	42
3	7	11	15	18	22	25	30	31	33	40	41
3	8	10	15	18	23	26	27	32	35	38	43
4	6	11	15	20	22	23	29	32	36	37	44
4	8	9	15	20	24	25	27	30	34	39	42
5	6	10	17	19	22	24	27	32	33	39	41
5	7	9	17	20	22	25	28	29	35	38	43

The order of the automorphism group is 16. This design is isdomorphic with 1. design

10. design

0	9	14	15	17	21	23	31	33	35	37	39
0	10	13	15	19	21	25	29	34	36	41	43
0	11	12	17	19	23	25	27	38	40	42	44
1	6	14	15	17	26	28	30	34	38	41	44
1	7	13	15	19	24	28	32	35	37	40	42
1	8	12	17	19	22	30	32	33	36	39	43
2	3	14	19	22	24	27	29	35	39	41	44
2	4	13	17	22	26	27	31	34	37	40	43
2	5	12	15	24	26	29	31	33	36	38	42
3	7	11	15	18	22	25	30	31	34	39	42
3	8	10	15	18	23	26	27	32	36	37	44
4	6	11	15	20	22	23	29	32	35	38	43
4	8	9	15	20	24	25	27	30	33	40	41
5	6	10	17	20	21	24	27	32	34	39	42
5	7	9	17	20	22	25	28	29	36	37	44

The order of the automorphism group is 1296

11. design

0	9	14	15	17	21	23	31	33	35	37	39
0	10	13	15	19	21	25	29	34	36	41	43
0	11	12	17	19	23	25	27	38	40	42	44
1	6	14	15	17	26	28	30	34	38	41	44
1	7	13	15	19	24	28	32	35	37	40	42
1	8	12	17	20	21	30	32	33	36	40	43
2	3	14	19	22	23	28	30	36	39	42	43
2	4	13	17	22	26	27	31	34	37	40	43
2	5	12	15	24	26	29	31	33	36	38	42
3	7	11	15	18	22	25	30	31	33	40	41
3	8	10	15	18	23	26	27	32	36	37	44
4	6	11	15	20	22	23	29	32	35	38	43
4	8	9	15	20	24	25	27	30	34	39	42
5	6	10	17	19	22	24	27	32	33	39	41
5	7	9	17	20	22	25	28	29	36	37	44

The order of the automorphism group is 16

12. design

0 9 14 15 17 21 23 31 33 35 37 39
0 10 13 15 19 21 25 29 34 36 41 43
0 11 12 17 19 23 25 27 38 40 42 44
1 6 14 15 17 26 28 30 34 38 41 44
1 7 13 15 20 23 28 32 36 37 40 42
1 8 12 17 20 22 29 31 34 35 40 43
2 3 14 19 21 24 28 30 35 40 42 43
2 4 13 17 22 25 28 32 33 38 39 43
2 5 12 15 24 26 29 31 33 36 38 42
3 7 11 15 18 22 25 30 31 33 40 41
3 8 10 15 18 23 26 27 32 35 38 43
4 6 11 15 19 22 24 29 32 35 37 44
4 8 9 15 20 24 25 27 30 34 39 42
5 6 10 17 20 21 24 27 32 33 40 41
5 7 9 17 19 22 26 27 30 36 37 43

The order of the automorphism group is 4

13. design

0 9 14 15 17 21 23 31 33 35 37 39
0 10 13 15 19 21 25 29 34 36 41 43
0 11 12 17 19 23 25 27 38 40 42 44
1 6 14 15 17 26 28 30 34 38 41 44
1 7 13 15 20 23 28 32 36 37 40 42
1 8 12 17 20 22 29 31 34 35 40 43
2 3 14 19 21 24 28 30 35 40 42 43
2 4 13 17 22 25 28 32 33 38 39 43
2 5 12 15 24 26 29 31 33 36 38 42
3 7 11 15 18 22 25 30 31 34 39 42
3 8 10 15 18 23 26 27 32 35 38 43
4 6 11 15 19 22 24 29 32 35 37 44
4 8 9 15 20 24 25 27 30 33 40 41
5 6 10 17 20 21 24 27 32 34 39 42
5 7 9 17 19 22 26 27 30 36 37 43

The order of the automorphism group is 4

x4.

0 9 14 15 17 21 23 31 33 35 37 39
0 10 13 15 19 21 25 29 34 36 41 43
0 11 12 17 19 23 25 27 38 40 42 44
1 6 14 15 17 26 28 30 34 38 41 44
1 7 13 15 20 24 27 31 36 38 39 42
1 8 12 17 20 22 29 31 34 35 40 43
2 3 14 19 22 24 27 29 35 39 41 44
2 4 13 17 22 25 28 32 33 38 39 43
2 5 12 15 24 25 30 32 34 35 37 42
3 7 11 15 18 22 25 30 31 33 40 41
3 8 10 15 18 23 26 27 32 35 38 43
4 6 11 15 20 22 23 29 32 36 37 44
4 8 9 15 19 24 26 28 29 33 40 42
5 6 10 17 20 21 24 27 32 33 40 41
5 7 9 17 19 22 26 27 30 36 37 43

The order of the automorphism group is 16. This design is isomorphic with 11. design

14. design

0 9 14 15 17 21 23 31 33 35 37 39
0 10 13 15 19 21 25 29 34 36 41 43
0 11 12 17 19 23 25 27 38 40 42 44
1 6 14 15 18 25 28 30 33 38 41 44
1 7 13 15 20 24 27 31 36 38 39 42
1 8 12 17 20 21 30 32 33 36 40 43
2 3 14 19 22 23 28 30 36 39 42 43
2 4 13 17 21 26 28 32 34 38 39 44
2 5 12 15 24 25 30 32 34 35 37 42
3 7 11 15 17 22 26 30 31 34 40 41
3 8 10 15 18 23 26 27 32 35 38 43
4 6 11 15 20 22 23 29 32 36 37 44
4 8 9 15 19 24 26 28 29 33 40 42
5 6 10 17 19 22 24 27 32 33 39 41
5 7 9 17 20 22 25 28 29 35 38 43

The order of the automorphism group is 12

15. design

0	9	14	15	17	21	23	31	33	35	37	39
0	10	13	15	19	21	25	29	34	36	41	43
0	11	12	17	19	23	25	27	38	40	42	44
1	6	14	15	18	25	28	30	33	38	41	44
1	7	13	15	20	24	27	31	36	38	39	42
1	8	12	17	20	21	30	32	33	36	40	43
2	3	14	19	22	23	28	30	36	39	42	43
2	4	13	17	21	26	28	32	34	38	39	44
2	5	12	15	24	25	30	32	34	35	37	42
3	7	11	15	17	22	26	30	31	34	40	41
3	8	10	15	18	23	26	27	32	36	37	44
4	6	11	15	20	22	23	29	32	35	38	43
4	8	9	15	19	24	26	28	29	33	40	42
5	6	10	17	19	22	24	27	32	33	39	41
5	7	9	17	20	22	25	28	29	36	37	44

The order of the automorphism group is 12

16. design

0	9	14	15	17	21	23	31	33	35	37	39
0	10	13	15	19	21	25	29	34	36	41	43
0	11	12	17	19	23	26	27	38	40	41	44
1	6	14	15	17	25	28	30	34	38	42	44
1	7	13	15	19	24	27	32	35	37	40	42
1	8	12	17	20	22	29	31	34	35	40	43
2	3	14	19	22	24	28	29	35	39	41	44
2	4	13	17	22	25	27	32	33	38	39	43
2	5	12	15	24	26	29	31	33	36	38	42
3	7	11	15	18	22	25	30	31	33	40	41
3	8	10	15	18	23	26	28	32	35	38	43
4	6	11	15	20	22	23	29	32	36	37	44
4	8	9	15	20	24	26	27	30	34	39	41
5	6	10	17	20	21	24	28	32	33	40	41
5	7	9	17	19	22	26	28	30	36	37	43

The order of the automorphism group is 8

17. design

0	9	14	15	17	21	23	31	33	35	37	39
0	10	13	15	19	21	25	29	34	36	41	43
0	11	12	17	19	23	26	27	38	40	41	44
1	6	14	15	17	25	28	30	34	38	42	44
1	7	13	15	20	23	27	32	36	37	40	42
1	8	12	17	20	22	29	31	34	35	40	43
2	3	14	19	21	24	27	30	35	40	42	43
2	4	13	17	22	25	27	32	33	38	39	43
2	5	12	15	24	26	29	31	33	36	38	42
3	7	11	15	18	22	25	30	31	33	40	41
3	8	10	15	18	23	26	28	32	35	38	43
4	6	11	15	19	22	24	29	32	35	37	44
4	8	9	15	20	24	26	27	30	34	39	41
5	6	10	17	20	21	24	28	32	33	40	41
5	7	9	17	19	22	26	28	30	36	37	43

The order of the automorphism group is 2

18. design

0	9	14	15	17	21	23	31	33	35	37	39
0	10	13	15	19	21	25	29	34	36	41	43
0	11	12	17	20	24	25	27	38	39	41	44
1	6	14	15	18	26	27	30	33	38	41	43
1	7	13	15	20	24	28	32	35	37	40	41
1	8	12	17	19	22	30	32	34	35	39	43
2	3	14	19	22	23	27	29	35	40	41	44
2	4	13	17	22	25	28	31	33	38	40	43
2	5	12	15	24	26	29	31	34	35	38	42
3	7	11	15	18	22	25	29	32	33	39	42
3	8	10	15	17	23	26	28	32	36	38	44
4	6	11	15	19	22	24	30	31	36	37	44
4	8	9	15	20	23	25	27	30	34	40	42
5	6	10	17	19	21	24	27	32	33	40	42
5	7	9	17	20	22	26	27	29	36	37	43

The order of the automorphism group is 8

19. design (IV. orbit structure)

0	9	14	15	17	21	23	31	33	35	37	39
0	10	13	15	19	21	25	29	34	36	41	43
0	11	12	17	19	23	25	27	38	40	42	44
1	6	14	15	17	26	28	30	34	38	41	44
1	7	13	15	19	24	28	32	35	37	40	42
1	8	12	17	19	22	30	32	33	36	39	43
2	3	14	19	22	24	27	29	35	39	41	44
2	4	13	17	22	26	27	31	34	37	40	43
2	5	12	15	24	26	29	31	33	36	38	42
3	7	11	15	18	22	25	30	31	35	38	43
3	8	10	15	18	23	26	27	32	33	40	41
4	6	11	15	20	22	23	29	32	34	39	42
4	8	9	15	20	24	25	27	30	36	37	44
5	6	10	17	20	21	24	27	32	35	38	43
5	7	9	17	20	22	25	28	29	33	40	41

The order of the automorphism group is 216

20. design

0	9	14	15	17	21	23	31	33	35	37	39
0	10	13	15	19	21	25	29	34	36	41	43
0	11	12	17	19	23	25	27	38	40	42	44
1	6	14	15	17	26	28	30	34	38	41	44
1	7	13	15	19	24	28	32	35	37	40	42
1	8	12	17	19	22	30	32	33	36	39	43
2	3	14	19	22	24	27	29	35	39	41	44
2	4	13	17	22	26	27	31	34	37	40	43
2	5	12	15	24	26	29	31	33	36	38	42
3	7	11	15	18	22	25	30	31	35	38	43
3	8	10	15	18	23	26	27	32	34	39	42
4	6	11	15	20	22	23	29	32	33	40	41
4	8	9	15	20	24	25	27	30	36	37	44
5	6	10	17	20	21	24	27	32	35	38	43
5	7	9	17	20	22	25	28	29	34	39	42

The order of the automorphism group is 216

21. design

0	9	14	15	17	21	23	31	33	35	37	39
0	10	13	15	19	21	25	29	34	36	41	43
0	11	12	17	20	24	25	27	37	40	42	43
1	6	14	15	18	26	27	29	33	38	42	43
1	7	13	15	20	24	28	31	35	38	40	41
1	8	12	17	19	22	29	31	33	36	40	44
2	3	14	19	22	23	27	30	35	40	41	43
2	4	13	17	22	25	28	32	33	38	39	43
2	5	12	15	24	26	30	32	33	36	37	41
3	7	11	15	18	22	25	29	32	35	37	44
3	8	10	15	17	23	26	28	32	34	40	42
4	6	11	15	19	22	24	30	31	34	39	42
4	8	9	15	20	23	25	27	30	36	38	44
5	6	10	17	19	21	24	27	32	35	38	44
5	7	9	17	20	22	26	27	29	34	39	41

The order of the automorphism group is 4

22. design (V. orbit structure)

0	9	14	15	17	21	23	31	33	35	37	39
0	10	13	15	19	21	25	29	34	36	41	43
0	11	12	17	19	23	25	27	38	40	42	44
1	6	14	15	17	26	28	30	34	38	41	44
1	7	13	17	19	22	28	32	35	37	40	43
1	8	12	15	19	24	30	32	33	36	39	42
2	3	14	19	22	24	27	29	35	39	41	44
2	4	13	15	24	26	27	31	34	37	40	42
2	5	12	17	22	26	29	31	33	36	38	43
3	7	11	15	18	22	25	30	31	33	40	41
3	8	10	15	18	23	26	27	32	35	38	43
4	6	11	15	20	22	23	29	32	36	37	44
4	8	9	17	20	22	25	27	30	34	39	43
5	6	10	17	20	21	24	27	32	33	40	41
5	7	9	15	20	24	25	28	29	35	38	42

The order of the automorphism group is 8

23. design

0	9	14	15	17	21	23	31	33	35	37	39
0	10	13	15	19	21	25	29	34	36	41	43
0	11	12	17	19	23	25	27	38	40	42	44
1	6	14	15	17	26	28	30	34	38	41	44
1	7	13	17	19	22	28	32	35	37	40	43
1	8	12	15	20	24	29	31	34	35	40	42
2	3	14	19	22	24	27	29	35	39	41	44
2	4	13	15	24	25	28	32	33	38	39	42
2	5	12	17	22	26	29	31	33	36	38	43
3	7	11	15	18	22	25	30	31	33	40	41
3	8	10	15	18	23	26	27	32	35	38	43
4	6	11	15	20	22	23	29	32	36	37	44
4	8	9	17	20	22	25	27	30	34	39	43
5	6	10	17	20	21	24	27	32	33	40	41
5	7	9	15	19	24	26	27	30	36	37	42

The order of the automorphism group is 8

24. design

0	9	14	15	17	21	23	31	33	35	37	39
0	10	13	15	19	21	25	29	34	36	41	43
0	11	12	17	19	24	26	27	37	40	41	44
1	6	14	15	18	25	27	30	33	38	41	44
1	7	13	17	19	22	28	31	36	38	39	44
1	8	12	15	19	24	30	32	34	35	39	42
2	3	14	19	22	23	28	30	35	40	41	43
2	4	13	15	23	26	27	31	34	38	40	42
2	5	12	17	21	26	30	32	33	36	38	43
3	7	11	15	17	22	25	29	32	33	40	42
3	8	10	15	18	23	26	28	32	36	37	44
4	6	11	15	20	22	24	30	31	36	37	43
4	8	9	17	20	21	25	28	30	34	40	44
5	6	10	17	20	22	23	27	32	34	39	41
5	7	9	15	20	24	26	28	29	35	38	41

The order of the automorphism group is 2

25. design

0	9	14	15	17	21	23	31	33	35	37	39
0	10	13	15	19	21	25	29	34	36	41	43
0	11	12	17	20	24	25	27	38	39	41	44
1	6	14	15	18	25	28	29	33	38	42	44
1	7	13	17	19	22	28	32	36	37	39	44
1	8	12	15	20	23	30	32	33	36	40	41
2	3	14	19	22	23	27	29	35	40	41	44
2	4	13	15	23	26	27	32	34	38	39	42
2	5	12	17	22	26	29	31	33	36	38	43
3	7	11	15	17	22	25	30	31	34	40	42
3	8	10	15	18	24	26	27	31	36	37	44
4	6	11	15	20	22	24	29	32	35	37	43
4	8	9	17	20	21	26	28	29	34	40	44
5	6	10	17	19	21	24	27	32	33	40	42
5	7	9	15	19	24	26	28	30	35	38	41

The order of the automorphism group is 2

26. design

0	9	14	15	17	21	23	31	33	35	37	39
0	10	13	15	19	22	25	29	33	36	41	43
0	11	12	17	19	24	26	27	37	40	41	44
1	6	14	15	18	26	27	29	33	38	42	44
1	7	13	17	19	22	28	32	35	38	39	44
1	8	12	15	19	24	30	31	34	36	39	42
2	3	14	19	22	23	27	30	35	40	42	43
2	4	13	15	23	26	28	31	34	38	40	41
2	5	12	17	21	26	30	32	33	36	38	43
3	7	11	15	17	21	25	29	32	34	40	42
3	8	10	15	18	24	26	28	32	35	37	43
4	6	11	15	20	22	23	30	32	36	37	44
4	8	9	17	20	22	26	27	29	34	39	43
5	6	10	17	20	22	24	28	31	33	40	42
5	7	9	15	20	24	25	27	30	35	38	41

The order of the automorphism group is 2

27. design

0	9	14	15	17	21	23	31	33	35	37	39
0	10	13	15	19	22	25	29	33	36	41	43
0	11	12	17	19	24	26	27	38	39	41	44
1	6	14	15	18	26	27	30	34	37	41	43
1	7	13	17	20	21	27	31	36	38	40	43
1	8	12	15	19	24	29	31	34	35	40	42
2	3	14	19	21	24	28	30	36	39	42	43
2	4	13	15	23	25	27	32	34	38	39	42
2	5	12	17	22	25	30	31	34	36	37	44
3	7	11	15	17	22	26	30	32	33	40	42
3	8	10	15	18	23	26	28	31	36	38	44
4	6	11	15	20	21	24	29	32	36	37	44
4	8	9	17	20	22	26	28	29	34	39	43
5	6	10	17	19	21	23	28	32	34	40	41
5	7	9	15	20	24	25	28	30	35	38	41

The order of the automorphism group is 2

28. design

0	9	14	15	17	21	23	31	33	35	37	39
0	10	13	15	19	22	25	29	33	36	41	43
0	11	12	17	20	24	26	27	37	40	41	43
1	6	14	15	18	26	28	29	34	37	42	43
1	7	13	17	19	21	28	32	35	38	40	43
1	8	12	15	20	23	30	32	33	36	40	42
2	3	14	19	21	24	27	30	36	39	42	43
2	4	13	15	23	26	27	32	34	38	39	41
2	5	12	17	21	25	29	32	34	36	37	44
3	7	11	15	18	21	25	30	31	34	40	41
3	8	10	15	17	24	26	28	31	36	38	44
4	6	11	15	19	22	24	30	32	35	37	44
4	8	9	17	20	22	25	28	30	34	39	43
5	6	10	17	19	22	23	27	31	34	40	42
5	7	9	15	20	24	25	27	29	35	38	42

The order of the automorphism group is 4

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