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A NOTE ON BLECHER'S CHARACTERIZATION OF HILBERT C*-MODULES

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Abstract. By a theorem of D. P. Blecher, a Hilbert C*-module V over a C*-algebra $\mathcal{A}$ (faithfully and nondegenerately represented on a Hilbert space $\mathcal{H}$) is characterized by a certain Hilbert space $\mathcal{H}_V$, such that V can be embedded in the algebra $B(\mathcal{H}, \mathcal{H}_V)$ of bounded operators between $\mathcal{H}$ and $\mathcal{H}_V$. In this paper it is shown: 1. For a Hilbert C*-module over a C*-algebra of compact operators the Hilbert space $\mathcal{H}_V$ coincides with a Hilbert subspace of the module, which characterizes all adjointable operators on the module. 2. For any Hilbert C*-module V, its strict completion can be realized in the algebra $B(\mathcal{H}, \mathcal{H}_V)$.

1. Introduction

The aim of this paper is to show two uses of the Hilbert space $\mathcal{H}_V$ from the following characterization of Hilbert C*-modules:

Theorem 1.1 (D. P. Blecher, [3]). *Let $\mathcal{A} \subseteq \mathbf{B}(\mathcal{H})$ be a nondegenerate C*-algebra and let V be a right Banach $\mathcal{A}$-module (and an operator space). V is a Hilbert $\mathcal{A}$-module (with the same norm-, resp. operator space structure) if and only if the following three conditions are satisfied:*
(i) $\mathcal{H}_V = V \otimes_{h\mathcal{A}} \mathcal{H}^c$ is a (column) Hilbert space;
(ii) $\phi : V \to \mathbf{B}(\mathcal{H}, \mathcal{H}_V)$, defined by $\phi(x)(\xi) = x \otimes_{\mathcal{A}} \xi$, is a (complete) isometry;
(iii) $\phi(x)^* \phi(x) \in \mathcal{A}$ for all $x \in V$.

If this is the case, the (unique[1]) inner product on V turning it into a Hilbert $\mathcal{A}$-module is given by

$$< x \mid y >= \phi(x)^* \phi(y).$$

[1] E.C. Lance has shown in [7] that there is a 1-1 correspondence between norm and inner product of Hilbert $\mathcal{A}$-modules.

In this paper it is shown that Blecher's Hilbert space $\mathcal{H}_V$ for Hilbert C*-modules over C*-algebras of compact operators coincides with a Hilbert subspace V_e of such a module V, known for several good properties. Further, it is shown that the V-strict completion $\mathcal{M}(V)$ of V, which is a generalization of the notion of a multiplier algebra from C*-algebra theory to Hilbert C*-modules, can be realized in $\mathbf{B}(\mathcal{H}, \mathcal{H}_V)$ for any Hilbert C*-module V.

Let us first shortly explain the objects in Blecher's theorem (for more details, see [3]). The Hilbert space $V \otimes_{h\mathcal{A}} \mathcal{H}^c$ is the module tensor product $V \otimes_{\mathcal{A}} \mathcal{H}^c$ of V and the Hilbert column space $\mathcal{H}^c$, treated as a left $\mathcal{A}$-module, completed with respect to the Haagerup norm. Elementary tensors in $V \otimes_{\mathcal{A}} \mathcal{H}^c$ are denoted by $x \otimes_{\mathcal{A}} \xi$ $(x \in V, \xi \in \mathcal{H})$. $\mathcal{H}^c$ is isometric to the Hilbert space $\mathcal{H}$ (that's why their elements are identified) equipped with an additional operator space[2] structure, setting $\mathcal{H}^c = B(\mathbf{C}, \mathcal{H})$.

The inner product of the Hilbert space $\mathcal{H}_V$ from Blecher's theorem is given on elementary tensors by

$$(x \otimes_{\mathcal{A}} \xi \mid y \otimes_{\mathcal{A}} \eta)_{\mathcal{H}_V} = (< y \mid x >_V \xi \mid \eta)_{\mathcal{H}}.$$

Let us now recall the definitions and properties used in this paper. The notion of a Hilbert C*-module is a generalization of the notion of a Hilbert space. The first use of such objects was made by I. Kaplansky in 1953 ([6]). The research on Hilbert C*-modules began in the seventies (W.L.Paschke, [9]; M.A.Rieffel, [10]). A complex vector space V which is a (right) algebraic module over a C*-algebra $\mathcal{A}$ is a Hilbert $\mathcal{A}$-module if there is a map (inner product) $< .|. >: V \times V \to \mathcal{A}$ with the properties (for all $x, y, z \in V$, $a \in \mathcal{A}$)

$$\begin{aligned} < x+y|z > &= < x|z > + < y|z >, \\ < x|ya > &= < x|y > a, \\ < x|y >^* &= < y|x >, \\ < x|x > &\geq 0, \\ < x|x > &= 0 \Leftrightarrow x = 0 \end{aligned}$$

and such that V is complete with respect to the norm defined by $\| x \| = \sqrt{\|< x \mid x >\|_{\mathcal{A}}}$. For example, any C*-algebra $\mathcal{A}$ is a Hilbert $\mathcal{A}$-module, setting $< a \mid b > = a^* b$.

Two classes of $(\mathcal{A})$-linear operators on V shall be considered in this paper: the C*-algebra $\mathbf{B}_{\mathcal{A}}(V)$ of adjointable (with respect to the Hilbert C*-module inner product) maps and the C*-algebra $\mathbf{K}_{\mathcal{A}}(V)$ of generalized compact operators (the norm-closure of the linear span of all operators $F_{x,y}$, $x, y \in V$, where $F_{x,y}(z) = x < y \mid z >$). $\mathbf{K}_{\mathcal{A}}(V)$ is a closed twosided ideal in $\mathbf{B}_{\mathcal{A}}(V)$.

For more details on Hilbert C*-modules see [8].

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[2]An operator space is a complex vector space V which can be embedded in some $\mathbf{B}(\mathcal{H})$ in a way that $M_n(V)$ (matrices with entries from V) is isometrically embedded in $M_n(\mathbf{B}(\mathcal{H}))$, for all $n \in \mathbf{N}$. A complete isometry is a map $T: X \to Y$ between operator spaces such that all $T_n : M_n(X) \to M_n(Y)$, $T_n[x_{ij}] = [Tx_{ij}]$, are isometries.

Although Hilbert C*-modules are analogues of Hilbert spaces, many Hilbert space properties cannot, in general, be transferred to Hilbert C*-modules, see e.g. [8] or [11]. A special class of Hilbert C*-modules for which most difficulties can be resolved is the class of Hilbert C*-modules over a C*-algebra of (all) compact operators on a Hilbert space, see [1]. Every such module V contains a Hilbert subspace V_e which defines all adjointable operators on V, in the sense that the C*-algebras $\mathbf{B}_{\mathcal{A}}(V)$ and $\mathbf{B}(V_e)$ are isomorphic, as are $\mathbf{K}_{\mathcal{A}}(V)$ and $\mathbf{K}(V_e)$, via the restriction map $T \mapsto T|_{V_e}$. If $\mathcal{A} = \oplus_i \mathbf{K}(\mathcal{H}_i)$ is a general C*-algebra of compact operators and V a Hilbert $\mathcal{A}$-module, then V can be decomposed as $V = \oplus_i V_i$, where $V_i = V\mathbf{K}(\mathcal{H}_i)$. Then $\mathbf{B}_{\mathcal{A}}(V)$ is isomorphic to $\prod_i \mathbf{B}((V_i)_{e_i})$ and $\mathbf{K}_{\mathcal{A}}(V)$ is isomorphic to $\oplus_i \mathbf{K}((V_i)_{e_i})$, where e_i are minimal projections in $\mathbf{K}(\mathcal{H}_i)$.

A known concept in the C*-algebra theory is the multiplier algebra $\mathcal{M}(\mathcal{A})$ of a C*-algebra $\mathcal{A}$, which can be realized as a completion of $\mathcal{A}$ under a certain topology (called strict topology), for details see e.g. [11]. There is also a generalization of this concept for Hilbert C*-modules (for details, see [2]). For a Hilbert $\mathcal{A}$-module V the V-strict topology is defined on any Hilbert $\mathcal{B}$-module W which contains V in such a way that $\mathcal{A}$ is an essential ideal in $\mathcal{B}$ and $V = W\mathcal{A}$ (the set of all products of elements from W and $\mathcal{A}$). A strict completion of a (full[3]) Hilbert $\mathcal{A}$-module V is such a module W which is V-strictly complete. It is proven in [2] that the strict completion of a Hilbert $\mathcal{A}$-module V is the Hilbert $\mathcal{M}(\mathcal{A})$-module $\mathbf{B}_{\mathcal{A}}(\mathcal{A}, V)$ (consisting of all adjointable maps from $\mathcal{A}$ to V).

Throughout this paper, $\mathcal{A}$ shall denote a C*-algebra, $\mathcal{M}(\mathcal{A})$ its multiplier algebra, $\mathcal{H}$ a Hilbert space, V a Hilbert C*-module and $\mathcal{M}(V)$ its strict completion. If $\mathcal{A}$ is represented on $\mathcal{H}$ (resp. if $V \subseteq B(\mathcal{H}, \mathcal{H}_V)$) $\mathcal{A}\mathcal{H}$ (resp. $\phi(V)\mathcal{H}$) denotes the linear span of elements of the form $a\xi$, $a \in \mathcal{A}, \xi \in \mathcal{H}$ (resp. of the form $\phi(x)(\xi) = x \otimes_{\mathcal{A}} \xi$, $x \in V, \xi \in \mathcal{H}$). The inner product of a Hilbert C*-module shall be denoted by $< . \mid . >$ and of a Hilbert space by $(. \mid .)$. $\mathbf{B}(\mathcal{H})$ shall denote the C*-algebra of all bounded linear operators on $\mathcal{H}$. $\mathbf{B}_{\mathcal{A}}(V)$ and $\mathbf{K}_{\mathcal{A}}(V)$ shall denote the C*-algebra of adjointable resp. of generalized compact operators on V. $\cong$ denotes isometric isomorphism.

2. The characterisation Hilbert space for Hilbert C*-modules over C*-algebras of compact operators

Let V be a Hilbert $\mathbf{K}$-module, where $\mathbf{K}$ is the C*-algebra of all compact operators on a fixed Hilbert space $\mathcal{H}$. Let V_e denote the subspace of V

$$V_e = \{xe : x \in V\}$$

[3] V is a full Hilbert $\mathcal{A}$-module if the linear span of all products $< x \mid y >$, $x, y \in V$, is dense in $\mathcal{A}$.

where e is a minimal projection in $\mathbf{K}$, i.e. $e = F_{\xi,\xi}$ for some $\xi \in \mathcal{H}$ of norm one[4]. V_e is a Hilbert space (not only submodule!) in the norm inherited from V and the Hilbert space inner product is given by

$$(xe \mid ye) = tr(e < y \mid x >_V e).$$

For details about V_e, see [1].

PROPOSITION 2.1. *If V is a Hilbert $\mathbf{K}$-module and if $\mathcal{H}_V$ is the corresponding Hilbert space from Blecher's theorem, then*

$$V_e \cong \mathcal{H}_V.$$

PROOF. Let e be the minimal projection $F_{\xi,\xi}$ in $\mathbf{K}$. From Blecher's theorem (ii) it is clear that the set $\phi(V)\mathcal{H}$ is dense in $\mathcal{H}_V$, so it is sufficient to define a linear map $\psi : \mathcal{H}_V \to V_e$ on elements of the form $\phi(x)\eta = x \otimes_{\mathbf{K}} \eta$. Set

$$\psi(x \otimes_{\mathbf{K}} \eta) = xF_{\eta,\xi}.$$

ψ maps $\mathcal{H}_V$ into V_e because $F_{\eta,\xi} = F_{\eta,\xi} F_{\xi,\xi}$. ψ is obviously onto, since $xe \in V_e$ is the image of $x \otimes_{\mathbf{K}} \xi \in \mathcal{H}_V$. Further (note that $F_{\xi,\eta} T F_{\xi',\eta'} = (T\xi' \mid \eta) F_{\xi,\eta'}$ for all $\xi, \xi', \eta, \eta' \in \mathcal{H}$ and $T \in \mathbf{B}(\mathcal{H})$)

$$\begin{aligned}
&(xF_{\eta,\xi} \mid xF_{\eta,\xi})_{V_e} = \\
&tr(e < xF_{\eta,\xi} \mid xF_{\eta,\xi} >_V e) = \\
&tr((< xF_{\eta,\xi} \mid xF_{\eta,\xi} >_V \xi \mid \xi)_{\mathcal{H}} e) = \\
&(< xF_{\eta,\xi} \mid xF_{\eta,\xi} >_V \xi \mid \xi)_{\mathcal{H}}.
\end{aligned}$$

By the expression for the inner product of elementary tensors in $\mathcal{H}_V$, the last quantity is equal to

$$\begin{aligned}
&(xF_{\eta,\xi} \otimes_{\mathbf{K}} \xi \mid xF_{\eta,\xi} \otimes_{\mathbf{K}} \xi)_{\mathcal{H}_V} = \\
&(x \otimes_{\mathbf{K}} F_{\eta,\xi}\xi \mid x \otimes_{\mathbf{K}} F_{\eta,\xi}\xi)_{\mathcal{H}_V} = \\
&(x \otimes_{\mathbf{K}} \eta \mid x \otimes_{\mathbf{K}} \eta)_{\mathcal{H}_V}
\end{aligned}$$

so ψ extends to an isometric, surjective map $\mathcal{H}_V \to V_e$. □

COROLLARY 2.2. *a) If V is a Hilbert $\mathbf{K}$-module, then the C^*-algebra $\mathbf{B}_{\mathbf{K}}(V)$ is isomorphic to $\mathbf{B}(\mathcal{H}_V)$ and $\mathbf{K}_{\mathbf{K}}(V)$ is isomorphic to $\mathbf{K}(\mathcal{H}_V)$.*
b) If V is a Hilbert C^*-module over a general C^*-algebra of compact operators $\mathcal{A} = \oplus_i \mathbf{K}(\mathcal{H}_i)$, let $V_i = V\mathbf{K}(\mathcal{H}_i)$. Then $\mathbf{B}_{\mathcal{A}}(V)$ is isomorphic to $\prod_i \mathbf{B}(\mathcal{H}_{V_i})$ and $\mathbf{K}_{\mathcal{A}}(V)$ is isomorphic to $\oplus_i \mathbf{K}(\mathcal{H}_{V_i})$.

PROOF. This is just the restatement of theorems 5 and 7 (resp. 6 and 9) from [1], using the result from the preceding proposition. □

[4] $F_{\xi,\eta}$ denotes the operator $\theta \mapsto (\theta \mid \eta)\xi$

3. On the strict completion of a Hilbert C*-module

By Blecher's theorem, a Hilbert $\mathcal{A}$-module V (with $\mathcal{A} \subseteq \mathbf{B}(\mathcal{H})$ nondegenerate) can be (completely) isometrically embedded in $B(\mathcal{H}, \mathcal{H}_V)$, so it is natural to ask if the V-strict completion of V can also be realized inside $B(\mathcal{H}, \mathcal{H}_V)$, i.e. if $\mathcal{M}(V) \subseteq B(\mathcal{H}, \mathcal{H}_V)$. Equivalently, the question is if $B(\mathcal{H}, \mathcal{H}_V)$ is V-strictly complete. Recall that the V-strict topology ([2]) on $B(\mathcal{H}, \mathcal{H}_V)$ (which is a Hilbert $\mathbf{B}(\mathcal{H})$-module containing $\phi(V)$ as a Hilbert $\mathcal{A}$-submodule) is defined by the family of seminorms $t \mapsto \| t^*\phi(x) \|$, $x \in V$ and $t \mapsto \| ta \|$, $a \in \mathcal{A}$. We shall further identify V with its ϕ-image in $B(\mathcal{H}, \mathcal{H}_V)$.

Proposition 3.1. *Let V be a Hilbert $\mathcal{A}$-module (with $\mathcal{A} \subseteq \mathbf{B}(\mathcal{H})$ nondegenerate) and $\mathcal{H}_V$ the corresponding Hilbert space from Blecher's theorem. Then $B(\mathcal{H}, \mathcal{H}_V)$ is V-strictly complete.*

Proof. Let $(t_\lambda)_{\lambda\in\Lambda}$ be a V-strictly convergent net in $B(\mathcal{H}, \mathcal{H}_V)$, i.e. the nets $(t_\lambda a)_\lambda$ and $(t_\lambda^* x)_\lambda$ converge in norm in $B(\mathcal{H}, \mathcal{H}_V)$ resp. $B(\mathcal{H})$, for all $a \in \mathcal{A}$ resp. $x \in V$. Denote the (norm-)limits

$$L(a) = \lim_\lambda t_\lambda a, \ a \in \mathcal{A},$$

$$R(x) = \lim_\lambda t_\lambda^* x, \ x \in V.$$

If $\mathcal{A}$ is an unital C*-algebra the net $(t_\lambda)_\lambda$ trivially converges in norm to an operator $t \in B(\mathcal{H}, \mathcal{H}_V)$, for if 1 is the unit element in $\mathcal{A}$ then the net $(t_\lambda)_\lambda = (t_\lambda 1)_\lambda$ converges by presumption to a $t \in B(\mathcal{H}, \mathcal{H}_V)$ and this is the required V-strict limit of $(t_\lambda)_\lambda$.

In the nonunital case, we proceed as follows:

It is easy to check that the above defined maps $L : \mathcal{A} \to B(\mathcal{H}, \mathcal{H}_V)$ and $R : V \to B(\mathcal{H})$ are linear. Further, for all $a \in \mathcal{A}$, $x \in V$

$$L(a)^* x = a^* R(x) :$$

$L(a)^*x = (\lim_\lambda t_\lambda a)^* x = (\lim_\lambda (t_\lambda a)^*) x = (\lim_\lambda a^* t_\lambda^*) x = \lim_\lambda a^* t_\lambda^* x = a^* \lim_\lambda t_\lambda^* x = a^* R(x)$ (since the adjoint map $* : B(\mathcal{H}, \mathcal{H}_V) \to B(\mathcal{H}_V, \mathcal{H})$ is norm-continuous, as are right multiplication $R_x : B(\mathcal{H}_V, \mathcal{H}) \to B(\mathcal{H})$ by a fixed element $x \in V$ and left multiplication $L_a : B(\mathcal{H}) \to B(\mathcal{H})$ by a fixed element $a \in \mathcal{A}$). Set

$$t(a\xi) = L(a)(\xi)$$

for $a \in \mathcal{A}, \xi \in \mathcal{H}$ and

$$t^*(x\xi) = R(x)(\xi)$$

for $x \in V, \xi \in \mathcal{H}$. Extending the above defined maps t and t^* linearly we get a map t defined on the dense subset $\mathcal{A}\mathcal{H}$ of $\mathcal{H}$ (with range in $\mathcal{H}_V$) and a map t^* defined on the dense subset $V\mathcal{H}$ of $\mathcal{H}_V$ (with range in $\mathcal{H}$). Both of these maps are continuous on their domains, check this e.g. for t:

$$\| t(\sum_{i=1}^{n} a_i\xi_i) \| = \| \sum_i (\lim_\lambda t_\lambda a_i)\xi_i \| =$$
$$= \| \sum_i \lim_\lambda t_\lambda a_i \xi_i \| = \| \lim_\lambda t_\lambda (\sum_i a_i \xi_i) \| = \leq$$
$$\leq (\sup_\lambda \| t_\lambda \|) \| \sum_{i=1}^{n} a_i \xi_i \| .$$

(If the net (t_λ) is V-strictly convergent, then it is uniformly bounded by the uniform boundedness principle, because $\mathcal{A}$ is nondegenerately represented on $\mathcal{H}$.)

Accordingly, t and t^* can be extended to bounded linear maps $t : \mathcal{H} \to \mathcal{H}_V$ and $t^* : \mathcal{H}_V \to \mathcal{H}$. These maps are mutually adjoint on the dense subsets $\mathcal{AH}$ resp. $V\mathcal{H}$:

$$\begin{aligned} (t(a\xi) \mid x\eta)_{\mathcal{H}_V} &= (L(a)\xi \mid x\eta) = (\xi \mid L(a)^* x\eta) = (\xi \mid a^* R(x)\eta) \\ &= (a\xi \mid t^*(x\eta))_{\mathcal{H}}, \end{aligned}$$

so t and t^* are mutually adjoint maps.

It is now easy to check that t is the required V-strict limit of $(t_\lambda)_\lambda$, i.e. that $t_\lambda a \to ta$ for all $a \in \mathcal{A}$ and $t_\lambda^* x \to t^* x$ for all $x \in V$. ∎

Corollary 3.2. *For any Hilbert C*-module* $V \subseteq \mathbf{B}(\mathcal{H}, \mathcal{H}_V)$

$$V \subseteq \mathcal{M}(V) \subseteq \mathbf{B}(\mathcal{H}, \mathcal{H}_V)$$

(isometrically).

Finally, note the following fact:

Proposition 3.3. *If V is a Hilbert C*-module, then* $\mathcal{H}_V \mathcal{H}_{\mathcal{M}(V)}$.

Proof. If $\mathcal{A} \subseteq \mathbf{B}(\mathcal{H})$ then $\mathcal{M}(\mathcal{A}) \subseteq \mathbf{B}(\mathcal{H})$. Let $\phi^{\mathcal{M}}$ be the map associated to the Hilbert $\mathcal{M}(\mathcal{A})$-module $\mathcal{M}(V)$ by Blecher's theorem. $\phi(V)\mathcal{H}$ is dense in $\mathcal{H}_V$ and $\phi^{\mathcal{M}}\mathcal{M}\big((V)\big)\mathcal{H}$ is dense in $\mathcal{H}_{\mathcal{M}(V)}$. The map $\phi(x)\xi \mapsto \phi^{\mathcal{M}}(x)\xi$ $(x \in V \subseteq \mathcal{M}(V)), \xi \in \mathcal{H}$ is easily seen to be an isometry from $\phi(V)\mathcal{H}$ into $\phi^{\mathcal{M}}\mathcal{M}\big((V)\big)\mathcal{H}$, so it extends to an isometry from $\mathcal{H}_V$ into $\mathcal{H}_{\mathcal{M}(V)}$. ∎

Remark 3.4. Another use of the Hilbert space $\mathcal{H}_V$ is the possibility of representing $\mathbf{B}_{\mathcal{A}}(V)$ (faithfully and nondegenarately) in $\mathbf{B}(\mathcal{H}_V)$, if V and $\mathcal{A}$ are as in Blecher's theorem. This is a consequence of results from [8] and [3]. Namely, there is a more general construction - the inner tensor product of two Hilbert C*-modules (details in [8]) and in [3] it is shown that the inner tensor product coincides with the Haagerup (module) tensor product. The results on embedding the C*-algebra $\mathbf{B}_{\mathcal{A}}(V)$ in the C*-algebra of the inner tensor product of V with another module, in the special case of tensoring V with $\mathcal{H}^c$ as in Blecher's theorem, yield the embedding $\mathbf{B}_{\mathcal{A}}(V) \subseteq \mathbf{B}(\mathcal{H}_V)$.

Further, it is known ([2]) that $\mathbf{B}_{\mathcal{A}}(V)$ is isomorphic to $\mathbf{B}_{\mathcal{M}(\mathcal{A})}(\mathcal{M}(V))$, so Blecher's theorem and it's corollaries from this paper show that the following isometric embeddings are valid for any Hilbert $\mathcal{A}$-module V with $\mathcal{A}$ faithfully and nondegenerately represented on $\mathcal{H}$ (so $\mathcal{A} \subseteq \mathcal{M}(\mathcal{A}) \subseteq \mathbf{B}(\mathcal{H})$):

$V \subseteq \mathcal{M}(V) \subseteq \mathbf{B}(\mathcal{H}, \mathcal{H}_V)(\subseteq \mathbf{B}(\mathcal{H}, \mathcal{H}_{\mathcal{M}(V)}))$
$\mathbf{K}_{\mathcal{A}}(V) \subseteq \mathbf{B}_{\mathcal{A}}(V) \subseteq \mathbf{B}(\mathcal{H}_V)$
$\mathbf{K}_{\mathcal{A}}(V) \subseteq \mathbf{K}_{\mathcal{M}(A)}(\mathcal{M}(V)) \subseteq \mathbf{B}_{\mathcal{A}}(V) \cong \mathbf{B}_{\mathcal{M}(\mathcal{A})}(\mathcal{M}(V)) \subseteq \mathbf{B}(\mathcal{H}_V)$

Also, the linking algebra of a Hilbert C*-module (defined formally as

$$\mathcal{L} = \begin{bmatrix} \mathbf{K}_{\mathcal{A}}(V) & V \\ V^* & \mathcal{A} \end{bmatrix}$$

and suitably equipped with a C*-algebra structure) can be represented (faithfully and nondegenerately) as a C*-subalgebra of $\mathbf{B}(\mathcal{H}_V \oplus \mathcal{H})$. In short, all important structures related to a Hilbert C*-module V (the algebras of adjointable and of generalized compact operators, the linking algebra, the strict completion) can be concretely represented using the Hilbert space the underlying C*-algebra $\mathcal{A}$ is represented on and Blecher's space $\mathcal{H}_V$.

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