GLASNIK MATEMATIČKI
Vol. 35(55)(2000), 233 – 244

SYMMETRIC BLOCK DESIGNS (61,16,4) ADMITTING AN AUTOMORPHISM OF ORDER 15

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Abstract. There are up to isomorphism and duality exactly three symmetric block designs (61,16,4) admitting an automorphism of order 15, the full orders of their automorphism groups being 270, 90 and 30.

1. Introduction and preliminaries

The aim of this article is to prove the following

Theorem 1.1. *There are, up to isomorphism, five symmetric block designs (61,16,4) admitting an automorphism of order 15. Among them two pairs are dual. The orders of full automorphism groups of the three not isomorphic and not dual designs are 270, 90 and 30.*

This result was obtained by means of combinatorial and group theoretical methods and with help of a computer.

At the beginning we introduce some notation and recall some basic facts concerning symmetric block designs (for details see [1],[2],[6]).

Let $\mathcal{D} = (\mathcal{P}, \mathcal{B}, I)$ be a symmetric (v, k, λ)-design with point set $\mathcal{P}$, line set $\mathcal{B}$ and incidence relation $I \subseteq \mathcal{P} \times \mathcal{B}$. For $x \in \mathcal{B}$, $P \in \mathcal{P}$ denote $\langle x \rangle = \{Q \in \mathcal{P} \mid (Q, x) \in I\}$, $\langle P \rangle = \{y \in \mathcal{B} \mid (P, y) \in I\}$. It is $|\mathcal{P}| = |\mathcal{B}| = v = k(k-1)/\lambda + 1$, and $|\langle x \rangle| = |\langle P \rangle| = k$, $|\langle x \rangle \cap \langle y \rangle| = |\langle P \rangle \cap \langle Q \rangle| = \lambda$, for all $x, y \in \mathcal{B}$, $P, Q \in \mathcal{P}$ with $x \neq y$, $P \neq Q$. In the following we shall use the term *design* for symmetric block design.

For two designs $\mathcal{D}_1$ and $\mathcal{D}_2$ an isomorphism from $\mathcal{D}_1$ onto $\mathcal{D}_2$ is a bijection which maps points onto points and lines onto lines preserving the incidence.

2000 *Mathematics Subject Classification.* 05B05.
Key words and phrases. symmetric block design, orbital structure, indexing, automorphism group.

We denote the set of automorphisms of $\mathcal{D}$ by $Aut\mathcal{D}$. For $x \in \mathcal{B}$, $P \in \mathcal{P}$ and a group $G \leq Aut\mathcal{D}$, we denote by $xG = \{xg | g \in G\}, PG = \{Pg | g \in G\}$ the G-orbits of x and P, respectively. There are as many point orbits as line orbits. Denoting this number by t, we have the partitions:

$$\mathcal{B} = \bigsqcup_{i=0}^{t-1} \mathcal{B}_i, \qquad \mathcal{P} = \bigsqcup_{r=0}^{t-1} \mathcal{P}_r, \tag{1}$$

where $\mathcal{B}_i = x_i G$, $\mathcal{P}_r = P_r G$ for some $x_i \in \mathcal{B}$, $P_r \in \mathcal{P}$, $0 \leq i, r \leq t-1$. Denote $|\mathcal{B}_i| = \Omega_i$, $|\mathcal{P}_r| = \omega_r$. Let $x \in \mathcal{B}_i$, $P \in \mathcal{P}_r$. Then $\gamma_{ir} = |\langle x \rangle \cap \mathcal{P}_r|$, $\Gamma_{ir} = |\langle P \rangle \cap \mathcal{B}_i|$ do not depend on the choice of x and P. The introduced cardinalities satisfy some important relations (s. also [1], [2], [7]):

Lemma 1.2. *It is:*
- (*i*) $\sum_{i=0}^{t-1} \Omega_i = \sum_{r=0}^{t-1} \omega_r = v$,
- (*ii*) $\sum_{r=0}^{t-1} \gamma_{ir} = \sum_{i=0}^{t-1} \Gamma_{ir} = k$,
- (*iii*) $\Omega_i \gamma_{ir} = \omega_r \Gamma_{ir}$,
- (*iv*) $\sum_{r=0}^{t-1} \gamma_{ir} \Gamma_{jr} = \lambda \Omega_j + \delta_{ij} n; \quad \sum_{i=0}^{t-1} \Gamma_{ir} \gamma_{is} = \lambda \omega_s + \delta_{rs} n$,

δ_{ij}, δ_{rs} *being the correspondent Kronecker symbols.*

Because of (iii) we can rewrite (iv) as:

$$\sum_{r=0}^{t-1} \frac{\Omega_j}{\omega_r} \gamma_{ir} \gamma_{jr} = \lambda \Omega_j + \delta_{ij} n; \qquad \sum_{i=0}^{t-1} \frac{\Omega_i}{\omega_r} \gamma_{ir} \gamma_{is} = \lambda \omega_s + \delta_{rs} n. \tag{v}$$

Definition 1.3. *The $t \times t$ matrices (γ_{ir}), (Γ_{ir}) satisfying and Lemma 1.2 are called orbital structures for parameters (v, k, λ) and orbit distrubutions $(\omega_0, \ldots, \omega_{t-1})$, $(\Omega_0, \ldots, \Omega_{t-1})$. We call $\gamma_{i\cdot} \equiv (\gamma_{i0}, \ldots, \gamma_{it-1})$ and $\Gamma_{\cdot r} \equiv (\Gamma_{0r}, \ldots, \Gamma_{t-1r})$ the orbital structures for lines in $\mathcal{B}_i$ and points in $\mathcal{P}_r$, respectively.*

In the following we denote the points of $\mathcal{P}_r$ as $\mathcal{P}_r = (r_0, r_1, \ldots, r_{\omega_r - 1})$ and the lines of $\mathcal{B}_i$ as $\mathcal{B}_i = (\widehat{\imath}_0, \widehat{\imath}_1, \ldots, \widehat{\imath}_{\Omega_i - 1})$. In this context one speaks about r's and $\widehat{\imath}$'s as about global points and lines, which are supplied with indices. Now for each point orbit $\mathcal{P}_r$ the automorphism group G is represented as a permutation group on the indices $0, \ldots, \omega_r - 1$, and analogously for the line orbits.

We shall also use the following results:

Lemma 1.4. *For $\rho \in Aut\mathcal{D}$ denote the sets of points and lines in $\mathcal{D}$ fixed by $\langle \rho \rangle$ by $F_\rho(\mathcal{P})$ and $F_\rho(\mathcal{B})$. By a known result $|F_\rho(\mathcal{P})| = |F_\rho(\mathcal{B})| \equiv F_\rho$, and by [8] $F_\rho \leq k + \sqrt{k - \lambda}$.*

Lemma 1.5. *(s.[4]) Let $\mathcal{D}$ be a (v, k, λ)-design, $\rho \in Aut\mathcal{D}$ and $|\rho| = p > \lambda$, p a prime. If there exists a ρ-fixed line consisting of fixed points only, then*

$k \le F_\rho \le v - (k-\lambda)p$. Otherwise, if each ρ-fixed line contains at least τ full nontrivial ρ-orbits of points, then $F_\rho(\tau p + 1) \le v$.

2. Proof of the theorem

Let $\mathcal{D}$ be a (61,16,4)-design and $\rho \in Aut\mathcal{D}$, $|\rho| = 15$. First we shall determine the action of ρ on $\mathcal{P}$ and $\mathcal{B}$.

Lemma 2.1. *Let $\mathcal{D}$ be a (61,16,4)-design and $\rho \in Aut\mathcal{D}$, $|\rho| = 15$. Then ρ acts on both $\mathcal{P}$ and $\mathcal{B}$ either in five orbits with lengths 1, 15, 15, 15, 15 or in seven orbits with lengths 1, 5, 5, 5, 15, 15, 15.*

Proof. Denote $\sigma = \rho^3$. Then $|\sigma| = 5$. Since $v - (k-\lambda)p = 61 - 12 \cdot 15 < 16$, each σ-fixed line contains, by Lemma 1.5, a nontrivial σ-orbit and $F_\sigma \cdot (5+1) \le 61$. It follows that $F_\sigma \le 10$, which implies $F_\sigma = 6$ or$F_\sigma = 1$. However, for $F_\sigma = 6$ we would have at least two nontrivial σ-orbits in each σ-fixed line and therefore, again by Lemma 1.5, $F_\sigma \cdot (2 \cdot 5 + 1) \le 61$, which implies $F_\sigma = 1$. Thus σ fixes only one point and only one line. As the automorphism ρ permutes the σ-orbits, the ρ-orbits have lengths 1 or 5 or 15. Any element of order 3 in $\langle\rho\rangle$, which fixes a σ-orbit of length 5, fixes each point of such an orbit. But Lemma 1.4 implies that $F_\sigma \le 16 + \sqrt{16-4} < 20$. Thus there are at most three ρ-orbits of length 5. The Lemma is proved. □

The case of ρ-operation in five orbits was solved in another investigation (s.[5]). A computation lasting whole month gave a unique design as solution. Thus we shall restrict us here to the other case, the case of ρ-operation in seven orbits.

Next we determine the orbital structure of $\mathcal{D}$ with respect to ρ. We use the notation of paragraph 1.

Lemma 2.2. *Let $\mathcal{D}$ be a (61,16,4)-design and $\rho \in Aut\mathcal{D}$, $|\rho| = 15$ and let ρ operate in seven orbits. Then there are, up to isomorphism, four possible orbital structures (γ_{ir}) for $\mathcal{D}$ with respect to ρ, listed in Table 1.*

Proof. Such an orbital structure must satisfy the conditions (i)–(v) of Lemma 1.2. By Lemma 2.1 we have $t = 7$, $\Omega_0 = \omega_0 = 1$, $\Omega_j = \omega_s = 5$ for $j, s \in \{1,2,3\}$ and $\Omega_j = \omega_s = 15$ for $j, s \in \{4,5,6,\}$. Here $n = k - \lambda = 16 - 4 = 12$.

Thus we have:

$$\text{(a)} \qquad \sum_{r=0}^{6} \gamma_{ir\cdot} = 16,$$

$$\text{(b)} \qquad \sum_{r=0}^{6} \frac{\Omega_i}{\omega_r} \gamma_{ir}^2 = 4\Omega_i + 12$$

$$\text{(c)} \qquad \sum_{r=0}^{6} \frac{\Omega_j}{\omega_r} \gamma_{ir}\gamma_{jr} = 4\Omega_j, \text{ for } i \neq j, \quad i,j \in \{0,\dots,6\}.$$

The conditions (a) and (b) imply that there are, up to order, eighteen solutions for line orbital structures $\gamma_{i\cdot}$, listed at the beginning of Table 1. The first column denotes the corresponding Ω_j and the second the ordinal number of singular line structure type.

Concerning (c) we can now construct the orbital structures for the given group and parameters. One gets as the only solutions, up to isomorphism, the four structures presented in Table 1. Here the second column denotes the line type ordinal number again.

We proceed by indexing the obtained orbital structures. Referring to paragraph 1, we denote $\mathcal{P}_r = \{r_0, \dots, r_{\omega_r - 1}\}$ and $\mathcal{B}_i = \{\widehat{\imath}_0, \dots, \widehat{\imath}_{\Omega_i - 1}\}$ for $0 \le i, r \le 6$. Now, ρ acts on $\mathcal{P}_r$ as $r_a\rho = r_{a+1}$, and on $\mathcal{B}_i$ as $\widehat{\imath}_a\rho = \widehat{\imath}_{a+1}$, for $a \in \{0, 1, \dots, 15\}$, the sums $a+1$ being modulo ω_r and modulo Ω_i, respectively.

Applying our Algorithm explained in [2] and [4] we get, with help of computer, 9 designs, for each of the Structures 1, 3, 4 three nonisomorphic ones. But some of designs belonging to different orbital structures are isomorphic: Design 5 and 9 with Design 1, Design 4 with 2 and Design 7 with 3. Thus there are, up to isomorphism, five designs satisfying the imposed conditions: Designs 1, 2, 3, 6, 8. Among them, Design 2 is dual with Design 3, and Design 6 with 8. The unique design with five ρ-orbits is also checked to be isomorphic with the selfdual Design 1. Therefore, up to isomorphism and duality, there are exactly three designs:1, 2 and 6, satisfying the conditions of the Theorem.

The results of computation are summarized in Table 2. Each design is presented by its line orbit representatives. Denoting them by $\widehat{x}_0, \dots, \widehat{x}_6$, we get all lines $x_0, \dots, x_{60}$ of the design acting on the representatives with $\langle\rho\rangle$, whereby

$$\rho = 0(1 \dots 5)(6 \dots 10)(11 \dots 15)(16 \dots 30)(31 \dots 45)(46 \dots 60).$$

For the lines this means that

$$\widehat{x}_i\rho^m = \begin{cases} x_i, \text{ for } i = 0,\ m = 0 \\ x_{5(i-1)+m+1}, \text{ for } i \in \{1,2,3\},\ m \in \{0,\dots,6\} \\ x_{15(i-3)+m+1}, \text{ for } i \in \{4,5,6\},\ m \in \{0,\dots,14\}. \end{cases}$$

The isomorphisms among designs are presented as mappings from points and lines of the first design onto points and lines of the second, and dual isomorphism as mappings from lines and points of the first design onto the points and lines of the second.

The automorphism groups of designs 1, 2 and 6 are represented by their consecutive bases, which means in terms of their strong generators: each next generator is the lexicographically first automorphism of design which is not contained in the group generated by previous generators. The generators are given as permutations of the points $0, \ldots, 60$, in their natural ordering. Construction of such a basis can be realized in a very effective manner and enables us to determine the orders of automorphism groups in an easy way (see [3]). Moreover, for each of the three designs we present its point orbit function, which is the mapping of each point to the first point in the natural ordering belonging to the same orbit.

The obtained results prove our Theorem.

TABLE 1

Line orbital structures:

0	0	1	5	5	5	0	0	0
0	1	1	0	0	0	15	0	0
3	2	1	4	1	1	3	3	3
3	3	1	3	3	0	3	3	3
3	4	1	3	0	0	6	3	3
3	5	1	2	2	2	6	3	0
3	6	1	1	1	1	6	6	0
3	7	1	0	0	0	6	6	3
3	8	0	3	2	2	6	3	0
3	9	0	2	2	0	6	6	0
6	10	1	1	1	1	4	4	4
6	11	0	2	2	2	4	4	2
6	12	0	2	2	1	5	4	2
6	13	0	2	2	0	4	4	4
6	14	0	2	1	1	6	3	3
6	15	0	2	1	1	5	5	2
6	16	0	2	1	0	5	4	4
6	17	0	1	1	0	5	5	4

Structure 1

0	0	1	5	5	5	0	0	0
1	4	1	3	0	0	6	3	3
2	4	1	0	3	0	3	6	3
3	4	1	0	0	3	3	3	6
4	15	0	2	1	1	2	5	5
5	15	0	1	2	1	5	2	5
6	15	0	1	1	2	5	5	2

Structure 3

0	0	1	5	5	5	0	0	0
1	9	0	2	2	0	6	6	0
2	9	0	2	2	0	6	0	6
3	9	0	2	2	0	0	6	6
4	10	1	1	1	1	4	4	4
5	13	0	2	0	2	4	4	4
6	13	0	0	2	2	4	4	4

Structure 2

0	0	1	5	5	5	0	0	0
1	6	1	1	1	1	6	6	0
2	6	1	1	1	1	6	0	6
3	6	1	1	1	1	0	6	6
4	13	0	2	2	0	4	4	4
5	13	0	2	0	2	4	4	4
6	13	0	0	2	2	4	4	4

Structure 4

0	1	1	0	0	0	15	0	0
1	5	1	2	2	2	3	6	0
2	5	1	2	2	2	3	0	6
3	7	1	0	0	0	3	6	6
4	13	0	2	2	0	4	4	4
5	13	0	2	0	2	4	4	4
6	13	0	0	2	2	4	4	4

TABLE 2

1.[2]

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	1	6	11	16	17	21	22	26	27	31	33	36	38	41	43
0	1	6	11	18	20	23	25	28	30	46	47	51	52	56	57
0	1	6	11	34	35	39	40	44	45	48	50	53	55	58	60
1	2	8	10	16	20	28	29	31	32	34	38	46	50	58	59
1	3	14	15	16	17	19	23	31	39	40	42	48	56	57	59
6	7	13	15	16	20	28	29	33	41	42	44	48	49	51	55

Automorphism group generators:

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39
40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59
60

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 21 22 23 24
25 26 27 28 29 30 16 17 18 19 20 36 37 38 39 40 41 42 43 44
45 31 32 33 34 35 51 52 53 54 55 56 57 58 59 60 46 47 48 49
50

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 34 35 36 37
38 39 40 41 42 43 44 45 31 32 33 58 59 60 46 47 48 49 50 51
52 53 54 55 56 57 16 17 18 19 20 21 22 23 24 25 26 27 28 29
30

0 1 5 4 3 2 6 10 9 8 7 11 15 14 13 12 17 21 25 29
18 22 26 30 19 23 27 16 20 24 28 38 42 31 35 39 43 32 36 40
44 33 37 41 45 34 47 51 55 59 48 52 56 60 49 53 57 46 50 54
58
0 2 1 5 4 3 7 6 10 9 8 12 11 15 14 13 18 22 26 30
19 23 27 16 20 24 28 17 21 25 29 39 43 32 36 40 44 33 37 41
45 34 38 42 31 35 48 52 56 60 49 53 57 46 50 54 58 47 51 55
59
0 6 7 8 9 10 11 12 13 14 15 1 2 3 4 5 16 17 18 19
20 21 22 23 24 25 26 27 28 29 30 41 42 43 44 45 31 32 33 34
35 36 37 38 39 40 51 52 53 54 55 56 57 58 59 60 46 47 48 49
50

Point orbit function:
0 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 16 16 16 16
16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16
16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16
16
The order of the automorphism group is 270

1. design is selfdual with the mapping:

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 17 28 24 20
16 27 23 19 30 26 22 18 29 25 21 38 34 45 41 37 33 44 40 36
32 43 39 35 31 42 47 58 54 50 46 57 53 49 60 56 52 48 59 55
51
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 20 16 27 23
19 30 26 22 18 29 25 21 17 28 24 44 40 36 32 43 39 35 31 42
38 34 45 41 37 33 50 46 57 53 49 60 56 52 48 59 55 51 47 58
54

2.[2]

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	1	6	11	16	17	21	22	26	27	31	33	36	38	41	43
0	1	6	11	18	20	23	25	28	30	46	47	51	52	56	57
0	1	6	11	34	35	39	40	44	45	48	50	53	55	58	60
1	2	8	10	16	20	28	29	31	32	34	38	46	50	58	59
1	3	14	15	16	17	19	23	31	39	40	42	48	56	57	59
6	7	13	15	16	24	28	30	37	41	43	44	48	50	51	59

Automorphism group generators:

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39
40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59
60
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 21 22 23 24
25 26 27 28 29 30 16 17 18 19 20 36 37 38 39 40 41 42 43 44
45 31 32 33 34 35 51 52 53 54 55 56 57 58 59 60 46 47 48 49
50
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 34 35 36 37
38 39 40 41 42 43 44 45 31 32 33 58 59 60 46 47 48 49 50 51
52 53 54 55 56 57 16 17 18 19 20 21 22 23 24 25 26 27 28 29
30
0 1 5 4 3 2 6 10 9 8 7 11 15 14 13 12 17 21 25 29
18 22 26 30 19 23 27 16 20 24 28 38 42 31 35 39 43 32 36 40
44 33 37 41 45 34 47 51 55 59 48 52 56 60 49 53 57 46 50 54
58
0 2 1 5 4 3 7 6 10 9 8 12 11 15 14 13 18 22 26 30
19 23 27 16 20 24 28 17 21 25 29 39 43 32 36 40 44 33 37 41
45 34 38 42 31 35 48 52 56 60 49 53 57 46 50 54 58 47 51 55
59

Point orbit function:

0 1 1 1 1 1 6 6 6 6 6 11 11 11 11 11 16 16 16 16
16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16
16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16 16
16

The order of the automorphism group is 90

3.[2]

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
0 1 6 11 16 17 21 22 26 27 31 33 36 38 41 43
0 1 6 11 18 20 23 25 28 30 46 47 51 52 56 57
0 1 6 11 34 35 39 40 44 45 48 50 53 55 58 60
1 2 8 10 16 20 28 29 31 32 34 38 46 54 58 60
1 3 14 15 16 17 19 23 31 39 40 42 52 56 58 59
6 7 13 15 16 20 28 29 33 41 42 44 48 50 51 59

The order of the automorphism group is 90

3. design is dually isomorphic to 2. with the mapping:

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 17 28 24 20
16 27 23 19 30 26 22 18 29 25 21 38 34 45 41 37 33 44 40 36
32 43 39 35 31 42 47 58 54 50 46 57 53 49 60 56 52 48 59 55
51
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 20 16 27 23
19 30 26 22 18 29 25 21 17 28 24 44 40 36 32 43 39 35 31 42
38 34 45 41 37 33 50 46 57 53 49 60 56 52 48 59 55 51 47 58
54

4.[3]

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	2	6	8	16	17	21	22	26	27	31	33	36	38	41	43
1	2	6	8	18	20	23	25	28	30	46	47	51	52	56	57
1	2	6	8	34	35	39	40	44	45	48	50	53	55	58	60
0	1	9	11	17	18	20	24	31	33	37	45	47	48	50	54
1	3	14	15	16	20	28	29	31	32	34	38	53	54	56	60
6	7	12	15	17	18	20	24	32	40	41	43	52	53	55	59

The order of the automorphism group is 90

4. design is isomorphic to 2. with the mapping:

0 11 12 13 14 15 3 4 5 1 2 6 7 8 9 10 19 50 39 22
53 42 25 56 45 28 59 33 16 47 36 44 27 58 32 30 46 35 18 49
38 21 52 41 24 55 29 60 34 17 48 37 20 51 40 23 54 43 26 57
31
0 33 39 45 36 42 43 34 40 31 37 38 44 35 41 32 11 2 8 14
5 6 12 3 9 15 1 7 13 4 10 49 60 56 52 48 59 55 51 47
58 54 50 46 57 53 23 24 25 26 27 28 29 30 16 17 18 19 20 21
22

5.[3]

0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	2	6	8	16	17	21	22	26	27	31	33	36	38	41	43
1	2	6	8	18	20	23	25	28	30	46	47	51	52	56	57
1	2	6	8	34	35	39	40	44	45	48	50	53	55	58	60
0	1	9	11	17	18	20	24	31	33	37	45	47	48	50	54
1	3	14	15	16	20	28	29	36	37	39	43	48	49	51	55
6	7	12	15	17	19	20	28	31	35	37	38	53	57	59	60

The order of the automorphism group is 270

5. design is isomorphic to 1. with the mapping:

0 1 3 5 2 4 15 12 14 11 13 6 8 10 7 9 19 39 53 25
45 59 16 36 50 22 42 56 28 33 47 48 20 40 54 26 31 60 17 37
51 23 43 57 29 34 24 44 58 30 35 49 21 41 55 27 32 46 18 38
52
0 31 43 40 37 34 36 33 45 42 39 41 38 35 32 44 11 8 5 12
9 1 13 10 2 14 6 3 15 7 4 30 17 19 21 23 25 27 29 16
18 20 22 24 26 28 58 50 57 49 56 48 55 47 54 46 53 60 52 59
51

6.[3]

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
1 2 6 8 16 17 21 22 26 27 31 33 36 38 41 43
1 2 6 8 18 20 23 25 28 30 46 47 51 52 56 57
1 2 6 8 34 35 39 40 44 45 48 50 53 55 58 60
0 1 9 11 17 18 20 24 31 33 37 45 47 49 50 58
1 3 14 15 16 20 28 29 31 32 34 38 49 53 55 56
6 7 12 15 17 18 20 24 32 40 41 43 48 52 54 55

Automorphism group generators:

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19
20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39
40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59
60
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 21 22 23 24
25 26 27 28 29 30 16 17 18 19 20 36 37 38 39 40 41 42 43 44
45 31 32 33 34 35 51 52 53 54 55 56 57 58 59 60 46 47 48 49
50
0 1 5 4 3 2 7 6 10 9 8 11 15 14 13 12 16 20 24 28
17 21 25 29 18 22 26 30 19 23 27 37 41 45 34 38 42 31 35 39
43 32 36 40 44 33 46 50 54 58 47 51 55 59 48 52 56 60 49 53
57
0 2 1 5 4 3 8 7 6 10 9 12 11 15 14 13 17 21 25 29
18 22 26 30 19 23 27 16 20 24 28 38 42 31 35 39 43 32 36 40
44 33 37 41 45 34 47 51 55 59 48 52 56 60 49 53 57 46 50 54
58

Point orbit function:

0 1 1 1 1 1 6 6 6 6 6 11 11 11 11 11 16 16 16 16
16 16 16 16 16 16 16 16 16 16 16 31 31 31 31 31 31 31 31 31
31 31 31 31 31 31 46 46 46 46 46 46 46 46 46 46 46 46 46 46
46

The order of the automorphism group is 30

7.[4]

0 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
0 1 2 6 7 11 12 16 21 26 31 33 36 38 41 43
0 1 3 6 8 11 13 19 24 29 46 47 51 52 56 57
0 16 21 26 34 35 39 40 44 45 47 50 52 55 57 60
1 2 8 10 17 18 20 24 31 35 43 44 47 48 50 54
1 3 14 15 16 18 22 30 31 32 34 38 47 55 56 58
6 7 13 15 17 18 20 24 38 39 41 45 52 53 55 59

The order of the automorphism group is 90

7. design is isomorphic to 3. with the mapping:

0 31 37 43 34 40 36 42 33 39 45 41 32 38 44 35 3 14 10 1
12 8 4 15 6 2 13 9 5 11 7 55 51 47 58 54 50 46 57 53
49 60 56 52 48 59 16 17 18 19 20 21 22 23 24 25 26 27 28 29
30
0 13 14 15 11 12 1 2 3 4 5 8 9 10 6 7 50 21 34 53
24 37 56 27 40 59 30 43 47 18 31 28 41 60 16 44 48 19 32 51
22 35 54 25 38 57 55 26 39 58 29 42 46 17 45 49 20 33 52 23
36

8.[4]

0 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30
0 1 2 6 7 11 12 16 21 26 31 33 36 38 41 43
0 1 3 6 8 11 13 19 24 29 46 47 51 52 56 57
0 16 21 26 34 35 39 40 44 45 47 50 52 55 57 60
1 2 8 10 17 18 20 24 31 35 43 44 47 48 50 54
1 3 14 15 16 18 22 30 31 32 34 38 47 55 56 58
6 7 13 15 17 19 20 28 34 38 40 41 48 52 54 55

The order of the automorphism group is 30

8. design is dually isomorphic to 6. with the mapping:

0 1 2 3 4 5 7 8 9 10 6 11 12 13 14 15 16 27 23 19
30 26 22 18 29 25 21 17 28 24 20 37 33 44 40 36 32 43 39 35
31 42 38 34 45 41 46 57 53 49 60 56 52 48 59 55 51 47 58 54
50
0 5 1 2 3 4 10 6 7 8 9 15 11 12 13 14 16 27 23 19
30 26 22 18 29 25 21 17 28 24 20 34 45 41 37 33 44 40 36 32
43 39 35 31 42 38 46 57 53 49 60 56 52 48 59 55 51 47 58 54
50

9.[4]

0	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
0	1	2	6	7	11	12	16	21	26	31	33	36	38	41	43
0	1	3	6	8	11	13	19	24	29	46	47	51	52	56	57
0	16	21	26	34	35	39	40	44	45	47	50	52	55	57	60
1	2	8	10	17	18	20	24	31	35	43	44	47	49	50	58
1	3	14	15	16	18	22	30	36	37	39	43	46	50	52	53
6	7	13	15	17	18	20	24	33	34	36	40	53	57	59	60

The order of the automorphism group is 270

9. design is isomorphic to 1. with the mapping:

0	16	19	22	25	28	26	29	17	20	23	21	24	27	30	18	2	10	13	1
9	12	5	8	11	4	7	15	3	6	14	52	60	53	46	54	47	55	48	56
49	57	50	58	51	59	38	36	34	32	45	43	41	39	37	35	33	31	44	42
40																			
0	7	10	8	6	9	1	4	2	5	3	12	15	13	11	14	49	34	25	58
43	19	52	37	28	46	31	22	55	40	16	45	21	54	39	30	48	33	24	57
42	18	51	36	27	60	59	44	20	53	38	29	47	32	23	56	41	17	50	35
26																			

References

[1] A. BEUTELSPACHER, Einführung in die endliche Geometrie I., Bibliographisches Institut, Mannheim 1982

[2] V. ĆEPULIĆ, On symmetric block designs (40,13,4) with automorphisms of order 5, Discrete Mathematics 128 (1994),45–60

[3] V. ĆEPULIĆ, Consecutive bases and automorphism groups, preprint, Zagreb 1990

[4] V. ĆEPULIĆ, On symmetric block designs (40,13,4) with automorphisms of order 13, Glasnik Mat. 31(51) (1996), 11–23

[5] V. ĆEPULIĆ, The unique symmetric block design (61,16,4) admitting an automorphism of order 15 operating standardly, Descrete Mathematics 175 (1997), 259–263

[6] V. ĆEPULIĆ and M. ESSERT, Biplanes (56,11,2) with automorphism group $Z_2 \times Z_2$ fixing some point, J. of Comb. Theory, Series A, 48 (1988),239–246

[7] Z. JANKO and TRAN van TRUNG, Construction of a new symmetric block design for (78,22,6) with the help of tactical decompositions, J. of Comb. Theory, Series A, 40 (1985),451–455

[8] S.S. SEIDEL, A. BLOKHUIS and H.A.WILBRINK, Graphs and association schemes, Algebra and geometry, EUT-Report 83-WSK-02, 1983

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Received: 07.07.99.