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ITERATED RESOLUTIONS

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ABSTRACT. Recently S. Mardešić and the author considered iterated limits in the compact case. Using ANR-resolutions, they also generalized their results to non-compact spaces. This paper gives an analogous polyhedral result in the general case. More precisely, for a given resolution of a topological space, polyhedral resolutions of its terms are constructed in a way that one can organize them naturally to obtain a polyhedral resolution of the same space.

1. INTRODUCTION

It is a well-known fact ([13], [6]) that, in general, a compact Hausdorff space X with $\dim X \leq m$, $m \in \mathbb{N}$, CANNOT be obtained by means of a polyhedral inverse limit $\boldsymbol{p} = (p_\lambda) : X \to \boldsymbol{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$ such that $\dim X_\lambda \leq m$, for all $\lambda \in \Lambda$. (There is an affirmative result in the theory of approximate systems, [8].) As a consequence of this difficulty, an interesting problem arises: Consider an inverse system $\boldsymbol{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$ of compact Hausdorff spaces. Let $\boldsymbol{r}^\lambda = (r_\nu^\lambda) : X_\lambda \to \boldsymbol{Z}^\lambda = (Z_\nu^\lambda, r_{\nu\nu'}^\lambda, N^\lambda)$, $\lambda \in \Lambda$, be arbitrary polyhedral limits. Then, generally, the polyhedra Z_ν^λ, $\lambda \in \Lambda$, $\nu \in N^\lambda$, cannot be organized into an inverse system having the limit space $\lim \boldsymbol{X}$. Assuming the contrary and starting from a compact Hausdorff space X and an inverse system $\boldsymbol{X}$ of compact metric spaces X_λ with $\lim \boldsymbol{X} = X$ (see [6]), one could choose inverse sequences $\boldsymbol{Z}^\lambda$ consisting of compact polyhedra Z_n^λ of minimal dimensions (see [5]), and get a contradiction. (Again, there is a positive solution, up to bonding mappings, in the theory of approximate systems, [12].) Therefore, THE following **problem** arises: Is it possible, for a given $\boldsymbol{X}$, to construct "special" polyhedral (or ANR) limits $\boldsymbol{p}^\lambda : X_\lambda \to \boldsymbol{X}^\lambda$

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and maps $\boldsymbol{p}^{\lambda\lambda'} = (p^{\lambda\lambda'}, p_\nu^{\lambda\lambda'}) : \boldsymbol{X}^{\lambda'} \to \boldsymbol{X}^\lambda$ of the corresponding systems, $\lim \boldsymbol{p}^{\lambda\lambda'} = p_{\lambda\lambda'}$, which can be organized naturally in a way to yield an inverse system $\boldsymbol{Y} = (Y_\mu, q_{\mu\mu'}, M)$ satisfying $\lim \boldsymbol{Y} = \lim \boldsymbol{X}$? Here "naturally" means: $M = \bigcup_{\lambda\in\Lambda}(\{\lambda\} \times N^\lambda)$, $\mu = (\lambda,\nu) \le (\lambda',\nu') = \mu'$ implies $\lambda \le \lambda'$ and $p^{\lambda\lambda'}(\nu) \le \nu'$, $Y_\mu = X_\nu^\lambda$, $q_{\mu\mu'}$ are compositions of some $p_{\nu\nu'}^\lambda$ and $p_\nu^{\lambda\lambda'}$, $\nu \le \nu'$ in N^λ, $\lambda \le \lambda'$ in Λ, while the projections $q_\mu : \lim \boldsymbol{Y} \to Y_\mu$ are compositions of p_λ and p_ν^λ.
In the case of compact Hausdorff spaces, S. Mardešić and the author [11] recently solved the problem in the affirmative. Moreover, using ANR-resolutions, the corresponding construction works generally for arbitrary spaces. (It is well known that inverse limits do not behave properly in the non-compact case, and that their proper substitution are resolutions.) In the general case, a polyhedral solution of the problem remained an open question.

In this paper we have exhibited a construction by induction (quite different from that of [11]), which answers the question in the affirmative. The basic step in this inductive construction is the construction of a special polyhedral resolution of a mapping, when a special kind of a polyhedral resolution of the codomain space is given in advance.

Let us recall some basic notions and facts needed in the sequel. By a space we mean a topological space, and by a mapping a continuous function. $Cov(X)$ denotes the set of all normal coverings of a space X. (These are open coverings which admit a subordinate partition of unity.) If $n \in \mathbb{N}$ and $c = \{\mathcal{U}_1, \cdots, \mathcal{U}_n\} \subseteq Cov(X)$, then $\wedge c$ or $\mathcal{U}_1 \wedge \cdots \wedge \mathcal{U}_n$ denotes the covering of X consisting of all non-empty intersections $\bigcap_i U_i$, where $U_i \in \mathcal{U}_i$. Of course, $\wedge c \in Cov(X)$. If $\mathcal{U}$ is a covering of X and $A \subseteq X$, then $\mathrm{St}(A,\mathcal{U}) \subseteq X$ denotes the union of all members of $\mathcal{U}$ meeting A; $\mathrm{St}\mathcal{U}$ denotes the covering consisting of all $\mathrm{St}(U,\mathcal{U})$, $U \in \mathcal{U}$. Every normal covering $\mathcal{U}$ of X admits a normal covering $\mathcal{U}'$ of X such that $\mathrm{St}\mathcal{U}'$ refines $\mathcal{U}$, $\mathrm{St}\mathcal{U}' \le \mathcal{U}$.
By a polyhedron we mean a triangulable space endowed with the CW-topology. If $\mathcal{U}$ is an open covering of a space X and $|N(\mathcal{U})|$ is the corresponding geometric nerve, then a mapping $p : X \to |N(\mathcal{U})|$ is (strictly) canonical whenever $p^{-1}(\mathrm{St}(U,\mathcal{U})) \subseteq U$ ($p^{-1}(\mathrm{St}(U,\mathcal{U})) = U$), $U \in \mathcal{U}$. Every locally finite partition of unity $\Phi = (\varphi_U, U \in \mathcal{U})$ subordinated to $\mathcal{U}$ determines a canonical mapping $p_\Phi : X \to |N(\mathcal{U})|$. A mapping $f : P \to Q$ of polyhedra is simplicial (PL) provided there exist triangulations K and L of P and Q respectively, such that $f : |K| \to |L|$ maps every closed simplex of K linearly onto (into) a closed simplex of L (see [4]).
Some needed basic definitions and facts on inverse systems, limits and resolutions can be found in [9]. We only recall the definition of a resolution of a space: A map of systems $\boldsymbol{p} = (p_\lambda) : X \to \boldsymbol{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$ is a resolution of X provided the following two conditions are satisfied:

(B1) $(\forall \mathcal{U} \in Cov(X))(\exists \lambda \in \Lambda)(\exists \mathcal{V} \in Cov(X_\lambda)) \quad p_\lambda^{-1}(\mathcal{V}) \le \mathcal{U}$.

(B2) $(\forall\lambda \in \Lambda)(\forall\mathcal{U} \in Cov(X_\lambda))(\exists\lambda' \geq \lambda) \quad p_{\lambda\lambda'}(X_{\lambda'}) \subseteq st(p_\lambda(X), \mathcal{U})$.
If all X_λ are normal, (B2) can be written in the following simpler form:
$(\forall\lambda \in \Lambda)(\forall$ open $U \supseteq Cl(p_\lambda(X))$ in $X_\lambda)(\exists\lambda' \geq \lambda) \quad p_{\lambda\lambda'}(X_{\lambda'}) \subseteq U$.
Finally, an inverse system $\boldsymbol{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$ is said to admit meshes, if there exists a family $\{\mathcal{U}_\lambda \mid \mathcal{U}_\lambda \in Cov(X_\lambda),\ \lambda \in \Lambda\}$ with the following property (see condition (A3) in [16], [15], [14], [10]):
$(\forall\lambda \in \Lambda)(\forall\mathcal{U} \in Cov(X_\lambda))(\exists\lambda' \geq \lambda)(\forall\lambda'' \geq \lambda')\ \mathcal{U}_{\lambda''} \leq p^{-1}_{\lambda\lambda''}(\mathcal{U})$.
A sufficient condition to admit meshes reads as follows (see [14], [10]):
(C) $(\forall\lambda \in \Lambda) \quad cw(X_\lambda) \leq card(\Lambda)$,
where cw denotes the covering weight, i.e., the minimal cardinal of a basis of the family of all normal coverings.

2. Canonical resolutions

It is well known ([7], [4]) that every mapping $f : X \to Y$ admits a polyhedral resolution $(\boldsymbol{p}, \boldsymbol{f}, \boldsymbol{q})$, i.e., $\boldsymbol{p} : X \to \boldsymbol{X}$ and $\boldsymbol{q} : Y \to \boldsymbol{Y}$ are polyhedral resolutions of X and Y, respectively, and $\boldsymbol{f} : \boldsymbol{X} \to \boldsymbol{Y}$ is a map of systems such that $\boldsymbol{f}\boldsymbol{p} = \boldsymbol{q}f$. The known constructions build $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{f}$ simultaneously. It is also known that, in general, $\boldsymbol{q}$ may not exist if $\boldsymbol{p}$ is given in advance (see[16]), while it is not known whether $\boldsymbol{p}$ exists if $\boldsymbol{q}$ is given in advance. (A positive exception is a compact polyhedral resolution $\boldsymbol{q}$ of a compact metric Y, [3]). However, we will show that for a particular type of a polyhedral resolution $\boldsymbol{q}$ given in advance, there exists a polyhedral resolution $(\boldsymbol{p}, \boldsymbol{f}, \boldsymbol{q})$ of f and $\boldsymbol{p}$ is a resolution of the same type. The construction slightly improves the standard ones ([7], Theorem 11; [4], Lemma 4.1, Proposition 4.2 and Theorem 4.5) and makes the key step in solving the problem. It will be convenient to summarize the additional conditions which such a resolution should satisfy.

Definition 1. A resolution $\boldsymbol{p} = (p_\lambda) : X \to \boldsymbol{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$ is said to be *canonical* if it has the following additional properties:
(i) There exists a subset $\Lambda_0 \subseteq \Lambda$ such that

(1) for every $\lambda \in \Lambda_0$, there exist an $n(\lambda) \in \mathbb{N}$ and a $c_\lambda = \{\mathcal{U}_1, \cdots, \mathcal{U}_{n(\lambda)}\} \subseteq Cov(X)$, such that $X_\lambda = |N(\mathcal{U}_\lambda)|$ and $p_\lambda : X \to X_\lambda$ is a canonical mapping with respect to $\mathcal{U}_\lambda$, where $\mathcal{U}_\lambda = \wedge c_\lambda$;

(2) if $\lambda, \lambda' \in \Lambda_0$ and $\lambda \leq \lambda'$, then $n(\lambda) \leq n(\lambda')$, $c_\lambda \subseteq c_{\lambda'} = \{\mathcal{U}'_1, \cdots, \mathcal{U}'_{n(\lambda)}, \cdots, \mathcal{U}'_{n(\lambda')}\}$ and $p_{\lambda\lambda'} : X_{\lambda'} \to X_\lambda$ is the naturally induced simplicial mapping $|N(\mathcal{U}_{\lambda'})| \to |N(\mathcal{U}_\lambda)|$ determined by its values on vertices, i.e., $(U'_1, \cdots, U'_{n(\lambda)}, \cdots, U'_{n(\lambda')}) \mapsto (U_1, \cdots, U_{n(\lambda)})$, which is well defined since each $\mathcal{U}_i$ is a unique $\mathcal{U}'_j$.

(ii) For every $\lambda \in \Lambda \setminus \Lambda_0$, there exists a unique $\lambda_0 \in \Lambda_0$, $\lambda_0 \leq \lambda$, such that

(1) X_λ is the carrier of $p_{\lambda_0}(X)$ with respect to a subdivision $K(\lambda_0, \lambda)$ of $N(\mathcal{U}_{\lambda_0})$;

(2) $p_{\lambda_0\lambda}$ is the inclusion mapping (i.e. the restriction on X_λ of the identity mapping $p_{\lambda_0\lambda_0}$ on X_{λ_0});

(3) $p_\lambda : X \to X_\lambda$ is the restriction of the corresponding mapping p_{λ_0}.
(iii) If $\lambda, \lambda' \in \Lambda \setminus \Lambda_0$ and $\lambda \leq \lambda'$, then
(1) $\lambda_0 \leq \lambda'_0$, where λ_0, λ'_0 are the corresponding indices in Λ_0, and $K(\lambda_0, \lambda') \leq K(\lambda_0, \lambda)$;
(2) $p_{\lambda\lambda'} : X_{\lambda'} \to X_\lambda$ is the restriction of the corresponding mapping $p_{\lambda_0\lambda'_0}$.

REMARK 1. The term *canonical* is convenient and compatible with Definition 4.1 of [4]. Naimely, all p_λ in (ii)(3) are, by (ii)(1), even strictly canonical (see [4], Definition 3.1 and Lemmas 3.3 and 3.4).

LEMMA 1. *Every space X admits a canonical resolution $\boldsymbol{p} : X \to \boldsymbol{X}$.*

PROOF. Lemma 1 follows by results of [7], Theorems 10-11, and [4], Lemma 4.1, Proposition 4.2 and Theorem 4.3 (see also [1], [2] and [15]). However, for the sake of completeness (and because of a few slight changes), let us briefly recall the two main steps in the building of such a special polyhedral resolution. The first step (which assures condition (B1) and a sufficiently large antisymmetric and cofinite indexing set to apply Proposition 4.2 of [4]) follows the first part of the proof of Theorem11 in [7]. The second step (which assures condition (B2) and preserves cofiniteness and strict canonicity - and compactness in the compact case) follows the proof of Proposition 4.2 in [4].

Choose a cofinal subfamily $\mathbb{U} \subseteq Cov(X)$ and for each $\mathcal{U} \in \mathbb{U}$ choose a locally finite partition of unity $\Phi = (\varphi_U, U \in \mathcal{U})$ subordinated to $\mathcal{U}$. It determines a canonical mapping $r_\Phi : X \to |N(\mathcal{U})|$. Let $(C, \leq)$ be the set of all finite subsets of $\mathbb{U}$ ordered by inclusion $\subseteq$. Obviously, $(C, \leq)$ is antisymmetric and cofinite. If $c = \{\mathcal{U}_1, \cdots, \mathcal{U}_n\} \in C$, let $\mathcal{U}_c = \wedge c = \{\bigcap_{i=1}^{n} U_i \neq \emptyset \mid U_i \in \mathcal{U}_i, i = 1, \cdots, n\}$, and let $r_c : X \to Z_c = |N(\mathcal{U}_c)|$ be the canonical mapping determined by the partition of unity $\Phi^1 \cdots \Phi^n = (\varphi^1_{U_1} \cdots \varphi^n_{U_n}, (U_1, \cdots, U_n) \in \mathcal{U}_1 \times \cdots \times \mathcal{U}_n)$ subordinated to $\mathcal{U}_c \in Cov(X)$, where the partitions $\Phi^i = (\varphi^i_{U_i}, U_i \in \mathcal{U}_i)$ are already chosen. If $c \leq c' = \{\mathcal{U}'_1, \cdots, \mathcal{U}'_n, \cdots, \mathcal{U}'_{n'}\}$ (each $\mathcal{U}_i$ is a unique $\mathcal{U}'_j$), let $r_{cc'} : Z_{c'} \to Z_c$ be the naturally induced simplicial mapping determined by its values on the vertices, i.e., $(U'_1, \cdots, U'_n, \cdots, U'_{n'}) \mapsto (U_1, \cdots, U_n)$. Then $\boldsymbol{r} = (r_c) : X \to \boldsymbol{Z} = (Z_c, r_{cc'}, C)$ is a map of the space X to the simplicial (triangulations fixed) polyhedral inverse system $\boldsymbol{Z}$ satisfying condition (B1). Finally, repeating the polyhedra, bonding mappings and projections (see [15], Sec.2.), if it is necessary, we obtain $\boldsymbol{r}^* = (r^*_a) : X \to \boldsymbol{Z}^* = (Z^*_a, r^*_{aa'}, A)$, where $\boldsymbol{Z}^*$ admits meshes and A is also antisymmetric and cofinite. In fact, A is sufficiently large so that $\boldsymbol{Z}^*$ satisfies the stability condition (C) (see [14] or [10]). This $*$-construction is based on the well known Mardešić trick, [9]. More precisely, $(A, \leq) = (F(\mathbb{A}), \subseteq)$, where $F(\mathbb{A}) = \{a \subseteq \mathbb{A} \mid \emptyset \neq a \text{ finite}\}$, $\mathbb{A} = \bigsqcup_{c \in C} (\{c\} \times \mathbb{U}_c)$

and $\mathbb{U}_c \subseteq Cov(Z_c)$ is a cofinal subfamily. The ordered sets C and A are related by an increasing surjection $s : A \to C$ such that $s(\{(c, \mathcal{W}_c)\}) = c$ (see [14], Lemma 1 and Remark 2), while $Z^*_a = Z_{s(a)}$, $r^*_{aa'} = r_{s(a)s(a')}$, $r^*_a = r_{s(a)}$.

In the second step, we first construct (see [4], Lemma 4.1), for every $a \in A$, a family $\{K_{aa'} \mid a' \geq a\}$ of subdivisions $K_{aa'}$ of $N(\mathcal{U}_{s(a)})$, hence, $|K_{aa'}| = Z_{s(a)} = Z^*_a$, such that $K_{aa''} \leq K_{aa'}$ whenever $a' \leq a''$, and that all the carriers $|L_{aa'}|$ of $r^*_a(X) \subseteq Z^*_a$ with respect to $K_{aa'}$, $a \leq a'$, are compatible with the corresponding restrictions of the bonding mappings $r^*_{aa'}$, i.e., $r^*_{aa'}(|L_{a'a''}|) \subseteq |L_{aa''}|$, whenever $a \leq a' \leq a''$. Moreover, these carriers and the corresponding restriction satisfy condition (B2). We also want to retain the basic triangulation $N(\mathcal{U}_{s(a)})$ since it is technically essential for our next construction, which then requires a slight change in defining the indexing set Λ (see the proof of Proposition 4.2 of [4]). Let $\Lambda_0 = \{\lambda = (a, a) \mid a \in A\}$ and let $\Lambda_1 = \underset{a\in A}{\cup}(\{a\} \times A_a) = \{\lambda = (a, a_1) \mid a, a_1 \in A,\ a \leq a_1\}$. Note that $\Lambda_0 \subseteq \Lambda_1$. Define Λ as the disjoint union $\Lambda_0 \sqcup \Lambda_1$ and order it coordinatewise (with respect to the ordering of A) satisfying the following additional condition:If $\lambda_0 = (a, a) \in \Lambda_0$ and $\lambda_1 = (a, a) \in \Lambda_1$, then $\lambda_0 \leq \lambda_1$ and $\lambda_1 \nleq \lambda_0$. Then $(\Lambda, \leq)$ is a partially ordered, unbounded and cofinite set. Observe that $\Lambda_0 \subseteq \Lambda$ is nothing else but A regarded as the subset of Λ preserving its own ordering. Put $K(\lambda, \lambda) = N(\mathcal{U}_{s(a)})$ when $\lambda = (a, a) \in \Lambda_0$, and $K(\lambda, \lambda') = K_{aa'}$ when $\lambda' = (a, a') \in \Lambda \smallsetminus \Lambda_0$. Now if $\lambda = (a, a) \in \Lambda_0$, let X_λ be the whole geometric nerve $\left|N(\mathcal{U}_{s(a)})\right|$, i.e., $X_\lambda = Z^*_a$, and if $\lambda = (a, a_1) \in \Lambda_1 = \Lambda \smallsetminus \Lambda_0$, let $X_\lambda = |L_{aa_1}| \subseteq Z^*_a$. If $\lambda \leq \lambda' = (a', a'_1)$ in Λ, let $p_{\lambda\lambda'} : X_{\lambda'} \to X_\lambda$ be the corresponding restriction mapping of $r^*_{aa'}$. (The special cases $\lambda = \lambda'$ and $\lambda \neq \lambda' \wedge a = a'$ are included since $r^*_{aa} = 1_{Z^*_a}$.) Observe that, for $\lambda \leq \lambda'$ in Λ_0, $p_{\lambda\lambda'} = r^*_{aa'}$ is the naturally induced simplicial mapping $r_{s(a)s(a')} : \left|N(\mathcal{U}_{s(a')})\right| \to \left|N(\mathcal{U}_{s(a)})\right|$. Finally, let $p_\lambda : X \to X_\lambda$, $\lambda = (a, a_1) \in \Lambda$, be the corresponding restriction mapping of r^*_a. In this way we have obtained the desired canonical resolution $\boldsymbol{p} = (p_\lambda) : X \to \boldsymbol{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$ of the space X. Notice that $\boldsymbol{r}^* : X \to \boldsymbol{Z}^*$ is included in $\boldsymbol{p} : X \to \boldsymbol{X}$ as its "basic level" over Λ_0. $\square$

Remark 2. Observe that the canonical resolution $\boldsymbol{p} : X \to \boldsymbol{X}$, constructed in the proof of Lemma 1, has a few additional properties. Namely, the indexing set $(\Lambda, \leq)$ is cofinite and antisymmetric, $\boldsymbol{X}_0 = (X_\lambda, p_{\lambda\lambda'}, \Lambda_0)$ is an inverse system satisfying condition (C) and $\boldsymbol{p}_0 = (p_\lambda) : X \to \boldsymbol{X}_0$ satisfies condition (B1).

Definition 2. A resolution $(\boldsymbol{p}, \boldsymbol{f}, \boldsymbol{q})$ of a mapping $f : X \to Y$ is said to be *canonical* if the resolutions $\boldsymbol{p} : X \to \boldsymbol{X}$ and $\boldsymbol{q} : Y \to \boldsymbol{Y}$ are canonical and all the mappings of $\boldsymbol{f} : \boldsymbol{X} \to \boldsymbol{Y}$ are restrictions of simplicial inclusions.

Let us now state our main lemma.

Lemma 2. *Let $f : X \to Y$ be a mapping and let $\boldsymbol{q} = (q_\mu) : Y \to \boldsymbol{Y} = (Y_\mu, q_{\mu\mu'}, M)$ be a canonical resolution of the space Y. Then there exist* *a canonical resolution* $\boldsymbol{p} = (p_\lambda) : X \to \boldsymbol{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$ *of the space* X *and a map of systems* $\boldsymbol{f} = (f, f_\mu) : \boldsymbol{X} \to \boldsymbol{Y}$ *such that* $(\boldsymbol{p}, \boldsymbol{f}, \boldsymbol{q})$ *is a canonical resolution of the mapping* f.

Before proving Lemma 2, let us recall some details of Lemma 4.1 of [4] and its proof (especially for (v) and (vi)), because of an additional technical condition. When we inductively construct a family $\{K_{aa'} \mid a' \geq a\}$, $a \in A$, (notations from [4]), we take care of all the mappings $p_{a_i a}$, $a_i \leq a$, and all the already constructed families $\{K_{aa'} \mid a' \geq a_i\}$. Assume, in addition, that a PL mapping $g : X_a \to Q$ is given and that there is a family $\{K_{bb'} \mid b' \in B, b' \geq b\}$ of subdivisions $K_{bb'}$ of K_{bb}, $|K_{bb}| = Q$, such that $K_{bb''} \leq K_{bb'}$ whenever $b'' \geq b'$. Moreover, let there exist an increasing injection $t : B \to A$ such that $t(b) = a$. Treating now g as an additional "$p_{a_i a}$", one can construct, in the same way, a desired family $\{K_{aa'} \mid a' \geq a\}$ such that also the restrictions of g preserve the corresponding carriers. This will be needed in the proof of Lemma 2, when we construct the map of systems $\boldsymbol{f}$.

Proof of Lemma 2. Consider the "basic level" of the canonical resolution $\boldsymbol{q} : Y \to \boldsymbol{Y}$, i.e., the restriction $\boldsymbol{q}_0 : Y \to \boldsymbol{Y}_0$ to the subset $M_0 \subseteq M$. Each projection $q_\mu : Y \to Y_\mu = |N(\mathcal{V}_\mu)|$, $\mu \in M_0$, is a canonical mapping determined by a locally finite partitions of unity $\Psi^\mu = (\psi^\mu_V, V \in \mathcal{V}_\mu)$ subordinated to a normal covering $\mathcal{V}_\mu \in Cov(Y)$, while each bonding mapping $q_{\mu\mu'} : Y_{\mu'} \to Y_\mu$, $\mu, \mu' \in M_0$, $\mu \leq \mu'$, is the naturally induced simplicial mapping of the type $|N(\mathcal{V}'_1 \wedge \cdots \wedge \mathcal{V}'_n \wedge \cdots \wedge \mathcal{V}'_{n'})| \to |N(\mathcal{V}_1 \wedge \cdots \wedge \mathcal{V}_n)|$, where each $\mathcal{V}_i$ is some $\mathcal{V}'_j$. Note that each Ψ^μ and f determine a locally finite partition of unity $\Phi^\mu = (\varphi^\mu_V = \psi^\mu_V f, V \in \mathcal{V}_\mu)$, subordinated to $f^{-1}(\mathcal{V}_\mu) \in Cov(X)$. Let $r_\mu : X \to \left|N(f^{-1}(\mathcal{V}_\mu))\right|$ be the canonical mapping determined by Φ^μ, and let $g_\mu : \left|N(f^{-1}(\mathcal{V}_\mu))\right| \hookrightarrow Y_\mu$ be the simplicial inclusion. ($N(f^{-1}(\mathcal{V}_\mu)$ is a subcomplex of $N(\mathcal{V}_\mu)$.) Then $g_\mu r_\mu = q_\mu f$, $\mu \in M_0$. Choose a cofinal subfamily $\mathbb{U} \subseteq Cov(X)$, and for every $\mathcal{U} \in \mathbb{U}$, $\mathcal{U} \neq f^{-1}(\mathcal{V}_\mu)$, $\mu \in M_0$, choose a locally finite partition of unity $\Phi = (\varphi_U, U \in \mathcal{U})$ subordinated to $\mathcal{U}$. Let $r_\Phi : X \to |N(\mathcal{U})|$ be the canonical mapping determined by Φ. We begin now to build a canonical resolution of the space X. Put $\mathbb{U}_0 = \{\mathcal{U} \in \mathbb{U} \mid \mathcal{U} \neq f^{-1}(\mathcal{V}_\mu),$ $\mu \in M_0\} \subseteq \mathbb{U}$ and $\mathbb{U}' = \mathbb{U}_0 \sqcup M_0$. Let $(C, \leq)$ be the set of all finite subsets of $\mathbb{U}'$ ordered by inclusion. Then C is partially ordered, cofinite and directed. We proceed following the construction of a canonical resolution of X described in the proof of Lemma 1. In the first step we obtain $\boldsymbol{r}^* = (r^*_a) : X \to \boldsymbol{Z}^* = (Z^*_a, r^*_{aa'}, A)$ satisfying (B1), where $\boldsymbol{Z}^*$ satisfies condition (C) and A is also antisymmetric and cofinite.

In the second step, the above mentioned additional technical condition appears. Consider a $\mu \in M_0$ and denote $(M_0)_\mu = \{\mu' \in M_0 \mid \mu' \geq \mu\}$. Let $a = \{(\{\mu\}, \mathcal{W}_{\{\mu\}})\} \in A$ such that $Z^*_a = Z_{\{\mu\}} = \left|N(f^{-1}(\mathcal{V}_\mu))\right|$, i.e., $s(a) = \{\mu\}$, where $s : A \to C$ is the increasing surjection from the first step. Let $A_a = \{a' \in A \mid a' \geq a\}$ and let $g^\mu : (M_0)_\mu \to A_a$ be an increasing injection defined by $g^\mu(\mu)=a$ and, for $\mu'>\mu$, $g^\mu(\mu')=a'$ for some $a'>a$, such that $s(a')=\{\mu'\}$. (Recall that M_0 is included in the building of C so that such a function g exists.) Now, when we construct a family $\{K_{aa'}\mid a'\geq a\}$, where $a=g^\mu(\mu)$ and $\mu\in M_0$, we have to take care of the mapping $g_\mu:\left|N(f^{-1}(\mathcal{V}_\mu))\right|=Z_{\{\mu\}}=Z_{s(a)}=Z^*_a\hookrightarrow Y_\mu$ and the already constructed triangulations $K(\mu,\mu')$ of Y_μ, $\mu'\in M\setminus M_0$, as well as of the bonding mappings $r^*_{a_ia}$ and the triangulations $K_{a_ia'}$ of $Z^*_{a_i}$, $a_i\leq a$ and $a'\geq a_i$. This yields a useful property (see $(\star)$ bellow) in defining the desired map of systems. In this way we have enlarged $\boldsymbol{r}^*$ to a canonical resolution $\boldsymbol{p}=(p_\lambda):X\to\boldsymbol{X}=(X_\lambda,p_{\lambda\lambda'},\Lambda)$, where $\boldsymbol{r}^*$ has "survived" as the restriction of $\boldsymbol{p}$ over $\Lambda_0=\{(a,a)\mid a\in A\}\subseteq\Lambda$, and $\boldsymbol{p}$ has the additional properties stated in Remark 2. Moreover, $\boldsymbol{X}$ has the additional property mentioned above:

$(\star)$ If $\mu\in M\setminus M_0$ and $\mu\geq\mu_0\in M_0$ such that $q_{\mu_0\mu}:Y_\mu\hookrightarrow Y_{\mu_0}$ is the inclusion mapping (μ_0 is unique for μ), then there exists a $\lambda\in\Lambda$, $\lambda=(a,a_1)\geq(a,a)\equiv\lambda_0\in\Lambda_0$, where $a=g^{\mu_0}(\mu_0)$, such that $p_{\lambda_0\lambda}:X_\lambda=|L_{aa_1}|\hookrightarrow Z^*_a=Z_{\{\mu_0\}}=X_{\lambda_0}$ is the inclusion mapping and $g_{\mu_0}(X_\lambda)\subseteq Y_\mu$.

Finally, let us define the desired mapping of systems $\boldsymbol{f}=(f,f_\mu):\boldsymbol{X}\to\boldsymbol{Y}$ satisfying $\boldsymbol{f}\boldsymbol{p}=\boldsymbol{q}f$. To define $f:M\to\Lambda$, first consider $M_0\subseteq M$. If $\mu\in M_0$ put $f(\mu)=(a,a)\in\Lambda_0$, where $a=g^\mu(\mu)$. (Notice that f cannot be increasing; see Remark 3 (a) below.) Then let the corresponding mapping f_μ be the simplicial inclusion $g_\mu:X_{f(\mu)}=Z_{\{\mu\}}=Z^*_a=\left|N(f^{-1}(\mathcal{V}_\mu))\right|\hookrightarrow|N(\mathcal{V}_\mu)|=Y_\mu$. If $\mu\in M_1=M\setminus M_0$, let $f(\mu)$ be an index $\lambda=(a,a_1)\in\Lambda$ according to $(\star)$, and let $f_\mu:X_{f(\mu)}\to Y_\mu$ be the restriction of the corresponding mapping $g_{\mu_0}:Z_{\{\mu_0\}}\hookrightarrow Y_{\mu_0}$ to subpolyhedra $X_{f(\mu)}=|L_{aa_1}|\subseteq Z_{\{\mu_0\}}$ and $Y_\mu\subseteq Y_{\mu_0}$. It remains to verify the commutativity conditions, i.e., $f_\mu p_{f(\mu)\lambda}=q_{\mu\mu'}f_{\mu'}p_{f(\mu')\lambda}$ for some $\lambda\geq f(\mu),f(\mu')$, $\mu\leq\mu'$, and $f_\mu p_{f(\mu)}=q_\mu f$, $\mu\in M$. If $\mu\in M_0$, the second equality holds by the definitions of $p_{f(\mu)}$ and f_μ. If $\mu\leq\mu'$ belong to M_0, f_μ and $f_{\mu'}$ are the natural simplicial inclusions, and then the first equality is a consequence of the naturality of the simplicial bonding mappings. (Take a $\lambda=(a,a)\in\Lambda_0$, such that $s(a)=\{\mu,\mu'\}$, i.e., $X_\lambda=Z^*_a=Z_{\{\mu,\mu'\}}=\left|N(f^{-1}(\mathcal{V}_\mu)\wedge f^{-1}(\mathcal{V}_{\mu'}))\right|=\left|N(f^{-1}(\mathcal{V}_\mu\wedge\mathcal{V}_{\mu'})\right|$.) In all other cases, the mappings which appear are the restrictions of the mappings which appear in the case of M_0. Hence, the commutativity follows. This completes the proof of Lemma 2. □

Remark 3. (a) Note that in our construction of $\boldsymbol{f}$, the whole set M_0 is "directly" lifted by f, while in the construction of $\boldsymbol{f}$ in [7], Theorem 11, only the subset of all initial elements of M_0 is "directly" lifted by f (the rest is lifted later - "indirectly"). As a consequence, our $f(M_0)$ consists of (some) initial elements for $(C\leq)$, while in the compared case, it retains the ordering of M_0.

(b) Observe that the constructed function $f:M\to\Lambda$ preserves the special subset M_0, i.e., $f(M_0)\subseteq\Lambda_0$. Furthermore, the restriction function $f|M_0$ is injective, since we consider $f(\mu)\neq f(\mu')$, whenever $\mu\neq\mu'$, even though the case $f^{-1}(\mathcal{V}_\mu) = f^{-1}(\mathcal{V}_{\mu'}) \in Cov(X)$ may occur. (As usually, $Cov(Y)$ and $Cov(X)$ should be treated as families.)

The proof of Lemma 2 yields the following generalization:

LEMMA 3. *Let $(f_j, j \in J)$ be a family of mappings $f_j : X \to Y_j$ and let, for each $j \in J$, $\boldsymbol{q}^j : Y_j \to \boldsymbol{Y}^j$ be a canonical resolution of Y_j. Then there exists a canonical resolution $\boldsymbol{p} : X \to \boldsymbol{X}$ of X and, for each $j \in J$, there exists a map of systems $\boldsymbol{f}^j : \boldsymbol{X} \to \boldsymbol{Y}^j$ such that $(\boldsymbol{p}, \boldsymbol{f}^j, \boldsymbol{q}^j)$ is a canonical resolution of f_j.*

PROOF. Let M_0^j be the special subsets of the corresponding indexing sets M^j of $\boldsymbol{Y}^j$, $j \in J$. Let $\mathbb{U} \subseteq Cov(X)$ be a cofinal subfamily, and let $\mathbb{U}_0 = \{\mathcal{U} \in \mathbb{U} \mid \mathcal{U} \neq (f_j)^{-1}(\mathcal{V}_{\mu_j}),\ j \in J,\ \mu_j \in M_0^j\} \subseteq \mathbb{U}$. Consider the disjoint union $\mathbb{U}' = \mathbb{U}_0 \sqcup (\bigsqcup_{j \in J} M_0^j)$ and proceed as in the proof of Lemma 2. $\square$

REMARK 4. Completing Remark 3(b), we also consider $f^j(\mu_j) \neq f^{j'}(\mu_{j'})$ whenever $j \neq j'$.

DEFINITION 3. Let $f_j : X \to Y_j$ and $\boldsymbol{q}^j : Y_j \to \boldsymbol{Y}^j$, $j \in J$, be as in Lemma 3. Then the above constructed resolutions $\boldsymbol{p} : X \to \boldsymbol{X}$ and $\boldsymbol{f}^j : \boldsymbol{X} \to \boldsymbol{Y}^j$, i.e. $(\boldsymbol{p}, \boldsymbol{f}^j, \boldsymbol{q}^j)$, $j \in J$, are said to be obtained by a *canonical construction* with respect to $(f_j, j \in J)$ and $(\boldsymbol{q}^j, j \in J)$.

The following two lemmas solve the problem of composite mappings in a canonical construction.

LEMMA 4. *Let $f : X \to Y$ and $g : Y \to Z$ be mappings and let $\boldsymbol{r} = (r_\nu) : Z \to \boldsymbol{Z} = (Z_\nu, r_{\nu\nu'}, N)$ be a canonical resolution of Z. Let $\boldsymbol{q} = (q_\mu) : Y \to \boldsymbol{Y} = (Y_\mu, q_{\mu\mu'}, M)$ and $\boldsymbol{g} = (g, g_\nu) : \boldsymbol{Y} \to \boldsymbol{Z}$ be obtained by a canonical construction with respect to g and $\boldsymbol{r}$, and let $\boldsymbol{p} = (p_\lambda) : X \to \boldsymbol{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$ and $\boldsymbol{f} = (f, f_\mu) : \boldsymbol{X} \to \boldsymbol{Y}$ be obtained by a canonical construction with respect to f and $\boldsymbol{q}$. Then there exists a canonical construction with respect to $h \equiv gf : X \to Z$ and $\boldsymbol{r}$ producing the same $\boldsymbol{p} : X \to \boldsymbol{X}$ and $\boldsymbol{h} = \boldsymbol{g}\boldsymbol{f} : \boldsymbol{X} \to \boldsymbol{Y}$.*

PROOF. First note that $(gf)^{-1}(\mathcal{W}) = f^{-1}(g^{-1}(\mathcal{W})) \in Cov(X)$, $\mathcal{W} \in Cov(Z)$. Recall that $g|N_0$ and $f|M_0$ are the inclusion functions and $f(g(N_0)) \subseteq f(M_0) \subseteq \Lambda_0$, where N_0, M_0 and Λ_0 are the special indexing subsets. This implies that a canonical construction with respect to f and $\boldsymbol{q}$ and a canonical construction with respect to gf and $\boldsymbol{r}$ can produce the same indexing set Λ over $Cov(X)$. Let us clarify some key details of the construction.

Consider a canonical construction of $\boldsymbol{p}$ and $\boldsymbol{f}$ with respect to f and $\boldsymbol{q}$. Observe that, if $\mathcal{V}_\mu = g^{-1}(\mathcal{W}_\nu)$, $\nu \in N_0 \subseteq N$, then $f^{-1}(\mathcal{V}_\mu) \in Cov(X)$ has the partition of unity determined by f, g and a given $(\chi_W^\nu, W \in \mathcal{W}_\nu)$. Every other covering $f^{-1}(\mathcal{V}_\mu) \in Cov(X)$, where $\mathcal{V}_\mu \neq g^{-1}(\mathcal{W}_\nu)$, $\nu \in N_0$, has the partition of unity determined by f and a given $(\psi_V^\mu, V \in \mathcal{V}_\mu)$, $\mu \in M_0 \subseteq M$. Finally, a

$\mathcal{U} \in \mathbb{U} \subseteq Cov(X)$ which is not of the form $f^{-1}(\mathcal{V}_\mu)$, has a given partition of unity $(\varphi_U, U \in \mathcal{U})$. Therefore, in a desired canonical construction for $\boldsymbol{p}'(=\boldsymbol{p})$ and $\boldsymbol{h}$ with respect to $h(=gf)$ and $\boldsymbol{r}$, one should choose the partitions of unity $(\varphi_U, U \in \mathcal{U})$ subordinated to normal coverings $\mathcal{U} \in \mathbb{U}'$ of X, such that:

$\varphi_U \equiv \varphi_W^\nu = \chi_W^\nu gf$, $U = (gf)^{-1}(W) \in (gf)^{-1}(\mathcal{W}_\nu) \equiv \mathcal{U}$, $\nu \in N_0$;

$\varphi_U \equiv \varphi_V^\mu = \psi_V^\mu f$, $U = f^{-1}(V) \in f^{-1}(\mathcal{V}_\mu) \equiv \mathcal{U}$, $\mu \in M_0$, where $\mathcal{V}_\mu \neq g^{-1}(\mathcal{W}_\nu)$, $\nu \in N_0$;

φ_U, $U \in \mathcal{U}$, is the same as in the canonical construction of $\boldsymbol{p}$ and $\boldsymbol{f}$ with respect to f and $\boldsymbol{q}$, whenever $\mathcal{U}$ is not of the previous forms.

These data yield the same "first step" with respect to f and $\boldsymbol{q}$ as well as with respect to h and $\boldsymbol{r}$. Then, obviously, the canonical construction can produce $\boldsymbol{p}' = \boldsymbol{p} : X \to \boldsymbol{X}$. Finally, to obtain $\boldsymbol{h} = (h, h_\nu) : \boldsymbol{X} \to \boldsymbol{Z}$ satisfying $\boldsymbol{h} = \boldsymbol{gf}$, one should put $h \equiv fg : N \to \Lambda$, and then $h_\nu \equiv g_\nu f_{g(\nu)} : X_{fg(\nu)} \to Z_\nu$, $\nu \in N$, is well defined. $\square$

LEMMA 5. *Let $f_j : X \to Y_j$ and $g_j : Y_j \to Z$ be mappings satisfying $g_j f_j \equiv h : X \to Z$, for every $j \in \{1, \cdots, k\}$, $k \in \mathbb{N}$. Let $\boldsymbol{r} = (r_\nu) : Z \to \boldsymbol{Z} = (Z_\nu, r_{\nu\nu'}, N)$ be a canonical resolution of the space Z and let, for each $j \in \{1, \cdots, k\}$, $(\boldsymbol{q}^j, \boldsymbol{g}^j, \boldsymbol{r})$ be a canonical resolution of the mapping g_j. Then there is a canonical construction producing canonical resolutions $(\boldsymbol{p}, \boldsymbol{f}^j, \boldsymbol{q}^j)$ and $(\boldsymbol{p}, \boldsymbol{h}, \boldsymbol{r})$ of the mappings f_j and h respectively with the same $\boldsymbol{p} : X \to \boldsymbol{X}$ and satisfying $\boldsymbol{g}^j \boldsymbol{f}^j = \boldsymbol{h}$, $j \in \{1, \cdots, k\}$.*

PROOF. Consider the simplest case $k = 2$, i.e., $g'f' = h = gf : X \to Z$. Clearly, it suffices to solve that case in order to understand the general case. Observe that commutativity implies $h^{-1}(\mathcal{W}) = f'^{-1}(g'^{-1}(\mathcal{W})) = f^{-1}(g^{-1}(\mathcal{W})) \in Cov(X)$, whenever $\mathcal{W} \in Cov(Z)$. Therefore, if $\mathcal{U} = h^{-1}(\mathcal{W}_\nu)$, $\nu \in N_0$, let the corresponding partition of unity be $(\varphi_U, U \in \mathcal{U})$, where $\varphi_U \equiv \varphi_W^\nu = \chi_W^\nu h$, $W \in \mathcal{W}_\nu$, $h^{-1}(W) \neq \emptyset$. Furthermore, if $\mathcal{U} = f^{-1}(\mathcal{V}_\mu)$, $\mu \in M_0$ and $\mathcal{V}_\mu \neq g^{-1}(\mathcal{W}_\nu)$, $\nu \in N_0$, ($\mathcal{U} = f'^{-1}(\mathcal{V}'_{\mu'})$, $\mu' \in M'_0$ and $\mathcal{V}'_{\mu'} \neq g'^{-1}(\mathcal{W}_\nu)$, $\nu \in N_0$), let the corresponding partition of unity be $(\varphi_U, U \in \mathcal{U})$, where $\varphi_U \equiv \varphi_V^\mu = \psi_V^\mu f$, $V \in \mathcal{V}_\mu$, $f^{-1}(V) \neq \varnothing$ ($\varphi_U \equiv \varphi_{V'}^{\mu'} = \psi_{V'}^{\mu'} f'$, $V' \in \mathcal{V}'_{\mu'}$, $f'^{-1}(V') \neq \varnothing$). Finally, if $\mathcal{U} \in \mathbb{U} \subseteq Cov(X)$ is not of the previous forms, take any locally finite partition of unity $(\varphi_U, U \in \mathcal{U})$ subordinated to $\mathcal{U}$. Now the canonical construction proceeds in a unique way (up to the choice of $K_{aa'}$) thus yielding $\boldsymbol{p} : X \to \boldsymbol{X}$ and the "bases" of $\boldsymbol{f} = (f, f_\mu) : \boldsymbol{X} \to \boldsymbol{Y}$ and $\boldsymbol{f}' = (f', f'_{\mu'}) : \boldsymbol{X} \to \boldsymbol{Y}'$, on M_0 and M'_0, for the desired canonical resolutions $(\boldsymbol{p}, \boldsymbol{f}, \boldsymbol{q})$ and $(\boldsymbol{p}, \boldsymbol{f}', \boldsymbol{q}')$ of the mappings f and f', respectively. These "bases" include indices $\mu = g(\nu) \in M_0$ and $\mu' = g'(\nu) \in M'_0$, $\nu \in N_0$, for which $f'(\mu') = f'(g'(\nu)) = f(g(\nu)) = f(\mu) \equiv \lambda \in \Lambda_0$ holds, and mappings f_μ and $f'_{\mu'}$ which are the natural simplicial inclusions of $X_\lambda = \left|N(h^{-1}(\mathcal{W}_\nu))\right|$ into $Y_{g(\nu)} = \left|N(g^{-1}(\mathcal{W}_\nu))\right|$ and $Y'_{g'(\nu)} = \left|N(g'^{-1}(\mathcal{W}_\nu))\right|$, respectively. Clearly, $g_\nu f_\mu = g'_\nu f'_{\mu'} : X_\lambda \hookrightarrow Z_\nu$, since $g'f' = gf : X \to Z$. Also, for $\mu \in M_0 \setminus g(N_0)$

and $\mu' \in M'_0 \setminus g'(N_0)$, the mappings f_μ and $f'_{\mu'}$ are the corresponding natural simplicial inclusions of $X_\lambda = \left|N(f^{-1}(\mathcal{V}_\mu))\right|$ and $X_{\lambda'} = \left|N(f'^{-1}(\mathcal{V}'_{\mu'}))\right|$ into $Y_\mu = |N((\mathcal{V}_\mu))|$ and $Y'_{\mu'} = \left|N((\mathcal{V}'_{\mu'}))\right|$, respectively. Consider now a $\mu = g(\nu) \in M$ and a $\mu' = g'(\nu) \in M'$, where $\nu \in N \setminus N_0$. Let $\nu_0 \in N_0$, $\nu_0 \leq \nu$, $\mu_0 = g(\nu_0) \in M_0$, $\mu_0 \leq \mu$, and $\mu'_0 = g'(\nu_0) \in M'_0$, $\mu'_0 \leq \mu'$, be chosen according to property $(\star)$. Then $Y_\mu \subseteq Y_{\mu_0}$ and $Y'_{\mu'} \subseteq Y'_{\mu'_0}$, while $f(\mu_0) = f'(\mu'_0) \equiv \lambda_0 \in \Lambda$ is already defined. By construction of $\boldsymbol{p}$, there exists a $\lambda_1 \in \Lambda$, $\lambda_1 \geq \lambda_0$, such that $p_{\lambda_0\lambda_1} : X_{\lambda_1} \hookrightarrow X_{\lambda_0}$ is the inclusion mapping (λ_0 and λ_1 have the same first coordinate) and $f_{\mu_0}(X_{\lambda_1}) \subseteq Y_\mu$. Similarly, there exists a $\lambda'_1 \in \Lambda$, $\lambda'_1 \geq \lambda_0$, such that $p_{\lambda_0\lambda'_1} : X_{\lambda'_1} \hookrightarrow X_{\lambda_0}$ is the inclusion mapping and $f'_{\mu'_0}(X_{\lambda'_1}) \subseteq Y'_{\mu'}$. Finally, choose a $\lambda \in \Lambda$, $\lambda \geq \lambda_1, \lambda'_1$, with the same first coordinate. Then $g_{\nu_0} f_{\mu_0} \mid X_\lambda = g'_{\nu_0} f'_{\mu'_0} \mid X_\lambda : X_\lambda \hookrightarrow Z_\nu$. Therefore, in this case we put $f(\mu) = \lambda = f'(\mu')$, and we can define f_μ, $f'_{\mu'}$ as the corresponding restriction mappings of f_{μ_0}, $f'_{\mu'_0}$, respectively. Finally, if $\mu \in M \setminus g(N)$ ($\mu' \in M' \setminus g'(N)$), define $f(\mu) \in \Lambda$ and $f_\mu : X_{f(\mu)} \to Y_\mu$ ($f'(\mu') \in \Lambda$ and $f'_{\mu'} : X_{f'(\mu')} \to Y'_{\mu'}$) as in the proof of Lemma 2. The required commutativity conditions are obviously fulfilled since all the mappings are restrictions of the "first level" inclusions.
So we have obtained the desired canonical resolution $\boldsymbol{p} = (p_\lambda) : X \to \boldsymbol{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$ and mappings of systems $\boldsymbol{f} = (f, f_\mu) : \boldsymbol{X} \to \boldsymbol{Y}$, $\boldsymbol{f}' = (f', f'_{\mu'}) : \boldsymbol{X} \to \boldsymbol{Y}'$ such that $(\boldsymbol{p}, \boldsymbol{f}, \boldsymbol{q})$ and $(\boldsymbol{p}, \boldsymbol{f}', \boldsymbol{q}')$ are canonical resolutions of the mappings $f : X \to Y$ and $f' : X \to Y'$, respectively. Moreover, the construction provides $f'g' = fg : N \to \Lambda$, hence, we may define $h \equiv fg : N \to \Lambda$. Furthermore, for every $\nu \in N$, $g'_\nu f'_{g'(\nu)} = g_\nu f_{g(\nu)}$ holds, and we may define $h_\nu \equiv g_\nu f_{g(\nu)} : X_{h(\nu)} \to Z_\nu$. Consequently, $\boldsymbol{g}'\boldsymbol{f}' = \boldsymbol{g}\boldsymbol{f} = \boldsymbol{h} = (h, h_\nu) : \boldsymbol{X} \to \boldsymbol{Y}$ and $(\boldsymbol{p}, \boldsymbol{h}, \boldsymbol{r})$ is a canonical resolution of the mapping $h = gf = g'f' : X \to Z$. This completes the proof in the case $k = 2$. $\square$

3. Application to inverse systems

In this section we will show how to apply our canonical construction to an inverse system with the purpose of obtaining a "system of canonical resolutions". The first theorem is a generalization of Lemmas 4 and 5 to finite commutative diagrams with a unique maximal element.

Consider an inverse system $\boldsymbol{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$ of spaces and mappings, where Λ is cofinite and antisymmetric. For a fixed $\lambda_0 \in \Lambda$, let $\Delta = (X_\lambda, p_{\lambda\lambda'}, \Lambda^{\lambda_0})$, where $\Lambda^{\lambda_0} = \{\lambda \in \Lambda \mid \lambda \leq \lambda_0\} \subseteq \Lambda$. Then Δ is a finite commutative diagram. Moreover, Δ has the space X_{λ_0} as the unique maximal element (source). Also note that there is at most one mapping connecting a pair of spaces in Δ. For such a diagram Δ, we establish the following theorem:

Theorem 1. *Let for all mappings* $p_{\lambda\lambda'} : X_{\lambda'} \to X_\lambda$ of Δ, $\lambda < \lambda'(< \lambda_0)$, *there exist corresponding canonical resolutions with a unique canonical resolution of each space, satisfying commutativity conditions according to* Δ. *Then there is a canonical construction producing canonical resolutions of all mappings* $p_{\lambda\lambda_0} : X_{\lambda_0} \to X_\lambda$ *of* Δ, $\lambda < \lambda_0$, *with a unique canonical resolution of the source* X_{λ_0} *and satisfying commutativity conditions according to* Δ.

Proof. Observe that such a diagram Δ consists of finitely many finite mapping loops (generated by those of Lemmas 4 and 5), finitely many finite non-looping mapping chains (generated by those of Lemma 4) and finitely many "free" (indecomposable in Δ) mappings, all having the space X_{λ_0} as the domain. We have to construct the desired canonical resolutions of the mappings $p_{\lambda\lambda_0}$, where λ is an immediate predecessor of λ_0 with a unique canonical resolution of X_{λ_0}. In order to do it, we need to "pull-back" (by the mappings $p_{\lambda\lambda_0}$) the given normal coverings with the corresponding partitions of unity of all the immediate predecessors of X_{λ_0}. For the remaining normal coverings of X_{λ_0}, partitions of unity are chosen freely. Then the construction proceeds as in the proof of Lemma 2 (3). However, to be more explicit, the solution of the following simple "general" case will be sufficient to confirm the general case of Δ.

$$
\begin{tikzcd}
 & X \arrow[dl] \arrow[d] \arrow[dr] \arrow[ddl, bend right] \arrow[dd, bend left] \arrow[ddr, bend left] & \\
Y_1 \arrow[dr] & Y_2 \arrow[l] \arrow[d] & Y_3 \arrow[dl] \arrow[d] \\
Z_1 & Z_2 & Z_3
\end{tikzcd}
$$

Assume that all needed is constructed up to "level Y". Let us exhibit a desired canonical construction at the source X. Consider all the "immediate predecessors" of X in Δ, i.e., Z_1, Y_2, and Y_3. The corresponding mappings of Δ, $X \to Z_1$, $X \to Y_2$, $X \to Y_3$, and the emphasized indexing subsets in the assumed canonical resolutions, together with a cofinal $\mathbb{U} \subseteq Cov(X)$, determine the set $(C \leq)$ as before. Furthermore, for all the pull-backed (by $X \to Z_1$, $X \to Y_2$, $X \to Y_3$) coverings in $Cov(X)$, the corresponding partitions of unity are determined by those on the codomain spaces and by the corresponding mappings. For the remaining normal coverings of $\mathbb{U}$ choose arbitrary locally finite partitions of unity on X. In the construction, we should be careful only at the loop consisting of $X \to Y_2 \to Z_2$ and $X \to Y_3 \to Z_2$, but this is solved by Lemma 5. (The canonical resolution of $Y_2 \to Z_2$ is assumed to be the composition of those of $Y_2 \to Y_1$ and $Y_1 \to Z_2$.) The rest of the proof is now a technical routine as before. Of course, by Lemmas 4 and 5, the resolutions of mappings of Δ from X to the spaces which are not its immediate predecessors (Y_1, Z_2 and Z_3) should be the appropriate compositions of assumed and constructed resolutions. This completes the proof of the theorem. $\square$

Consider now an inverse system $\boldsymbol{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$ of spaces X_λ and mappings $p_{\lambda\lambda'} : X_{\lambda'} \to X_\lambda$, $\lambda \leq \lambda'$, where Λ is, in addition, cofinite and partially ordered. (This is not an essential restriction since with every $\boldsymbol{X}$ one can associate, by the "Mardešić trick" [9], a closely related $\boldsymbol{X'}$, made of the same "material", which satisfies those additional conditions.) Since Λ is cofinite, i.e., each λ has at most finitely many predecessors, the construction is by induction on the cardinal $|\lambda| \in \mathbb{N}_0$ of all predecessors of $\lambda \in \Lambda$. For each $\lambda \in \Lambda$ with $|\lambda| = 0$, choose a canonical resolution $\boldsymbol{p}^\lambda = (p^\lambda_\nu) : X_\lambda \to \boldsymbol{X}^\lambda = (X^\lambda_\nu, p^\lambda_{\nu\nu'}, N^\lambda)$ of the space X_λ. Let $n \in \mathbb{N}$. Suppose that, for every $\lambda \in \Lambda$ with $|\lambda| < n$, and for all pairs $\lambda_1 < \lambda_2 (\leq \lambda)$, canonical resolutions $(\boldsymbol{p}^{\lambda_2}, \boldsymbol{p}^{\lambda_1\lambda_2}, \boldsymbol{p}^{\lambda_1})$ of the mappings $p_{\lambda_1\lambda_2}$ are constructed, with a unique canonical resolution $\boldsymbol{p}^\lambda : X_\lambda \to \boldsymbol{X}^\lambda$ of each space X_λ, such that $\boldsymbol{p}^{\lambda_1\lambda_2}\boldsymbol{p}^{\lambda_2\lambda_3} = \boldsymbol{p}^{\lambda_1\lambda_3}$ whenever $\lambda_1 < \lambda_2 < \lambda_3 (\leq \lambda)$. (One may omit all the trivial cases $\lambda_2 = \lambda_1$, since each $\boldsymbol{X}^\lambda$ is unique and $p_{\lambda\lambda}$ is the identity mapping. However, it is convenient to put $\boldsymbol{p}^{\lambda\lambda} = 1_{\boldsymbol{X}^\lambda}$.) Let $\lambda \in \Lambda$ with $|\lambda| = n$, and let $\lambda_1, \cdots, \lambda_n$ be all the predecessors of λ. Then $|\lambda_i| < n$, $i \in \{1, \cdots, n\}$. By the inductive assumption, for all pairs $\lambda_i \leq \lambda_j (< \lambda)$, canonical resolutions $(\boldsymbol{p}^{\lambda_j}, \boldsymbol{p}^{\lambda_i\lambda_j}, \boldsymbol{p}^{\lambda_i})$ of the mappings $p_{\lambda_i\lambda_j}$ are already constructed, with a unique canonical resolution $\boldsymbol{p}^{\lambda_i} : X_{\lambda_i} \to \boldsymbol{X}^{\lambda_i}$ of each space X_{λ_i}, such that $\boldsymbol{p}^{\lambda_i\lambda_j}\boldsymbol{p}^{\lambda_j\lambda_k} = \boldsymbol{p}^{\lambda_i\lambda_k}$ whenever $\lambda_i \leq \lambda_j \leq \lambda_k (< \lambda)$. Apply now Theorem 1 to obtain a canonical resolution $\boldsymbol{p}^\lambda : X_\lambda \to \boldsymbol{X}^\lambda$ of the space X_λ as well as the maps of systems $\boldsymbol{p}^{\lambda_k\lambda} : \boldsymbol{X}^\lambda \to \boldsymbol{X}^{\lambda_k}$ for all immediate predecessors λ_k of λ, such that $(\boldsymbol{p}^\lambda, \boldsymbol{p}^{\lambda_k\lambda}, \boldsymbol{p}^{\lambda_k})$ are canonical resolutions of the mappings $p_{\lambda_k\lambda}$. Finally, if $\lambda_i < \lambda$ is not an immediate predecessor of λ, Lemmas 4 and 5 allow to put $\boldsymbol{p}^{\lambda_i\lambda} \equiv \boldsymbol{p}^{\lambda_i\lambda_k}\boldsymbol{p}^{\lambda_k\lambda}$.

Let us summarize the previous consideration in the following theorem:

THEOREM 2. *Let $\boldsymbol{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$ be an inverse system of topological spaces and mappings, where Λ is, in addition, cofinite and antisymmetric. Then there exists a "system" of canonical resolutions $(\boldsymbol{p}^{\lambda'}, \boldsymbol{p}^{\lambda\lambda'}, \boldsymbol{p}^\lambda)$ of all bonding mappings $p_{\lambda\lambda'} : X_{\lambda'} \to X_\lambda$, $\lambda \leq \lambda'$, with a unique canonical resolution $\boldsymbol{p}^\lambda : X_\lambda \to \boldsymbol{X}^\lambda$ of each space X_λ, $\lambda \in \Lambda$, and satisfying $\boldsymbol{p}^{\lambda\lambda''} = \boldsymbol{p}^{\lambda\lambda'}\boldsymbol{p}^{\lambda'\lambda''}$, whenever $\lambda \leq \lambda' \leq \lambda''$.*

We shall use hereafter the phrase "a system of canonical resolutions" exactly in the sense of Theorem 2. Hence, we may reformulate Theorem 2 as follows:

THEOREM 2′. *Every inverse system $\boldsymbol{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$ of topological spaces and mappings, where Λ is cofinite and antisymmetric, admits a system of canonical resolutions $((\boldsymbol{p}^{\lambda'}, \boldsymbol{p}^{\lambda\lambda'}, \boldsymbol{p}^\lambda), \lambda \leq \lambda')$.*

4. Solution of the problem

We now establish the theorem which solves the stated problem.

Theorem 3. *Every system of canonical resolutions can be naturally organized, without its projections, to obtain an inverse system. More precisely, if* $((\boldsymbol{p}^{\lambda'}, \boldsymbol{p}^{\lambda\lambda'}, \boldsymbol{p}^{\lambda}),\ \lambda \le \lambda')$ *is a system of canonical resolutions over an inverse system* $\boldsymbol{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$, *where* $\boldsymbol{p}^\lambda = (p^\lambda_\nu) : X_\lambda \to \boldsymbol{X}^\lambda = (X^\lambda_\nu, p^\lambda_{\nu\nu'}, N^\lambda)$ *and* $\boldsymbol{p}^{\lambda\lambda'} = (p^{\lambda\lambda'}, p^{\lambda\lambda'}_\nu) : \boldsymbol{X}^{\lambda'} \to \boldsymbol{X}^\lambda$, *then there exists a cofinite inverse system* $\boldsymbol{Y} = (Y_\mu, q_{\mu\mu'}, M)$ *such that*

(*i*) $M = \bigcup\limits_{\lambda\in\Lambda} (\{\lambda\} \times N^\lambda) = \{\mu = (\lambda,\nu) \mid \lambda \in \Lambda,\ \nu \in N^\lambda\}$;

(*ii*) $Y_\mu = X^\lambda_\nu,\ \mu = (\lambda,\nu) \in M$;

(*iii*) $q_{\mu\mu'} : Y_{\mu'} \to Y_\mu,\ \mu \le \mu' = (\lambda',\nu')$, *is the composition of* $p^{\lambda\lambda'}_\nu$ *and* $p^{\lambda'}_{p^{\lambda\lambda'}(\nu),\nu'}$.

Moreover, if the system $\boldsymbol{X}$ *admits a resolution* $\boldsymbol{p} = (p_\lambda) : X \to \boldsymbol{X}$ *of a space* X, *then* $\boldsymbol{q} = (q_\mu = p^\lambda_\nu p_\lambda) : X \to \boldsymbol{Y}$ *is also a resolution of* X.

Proof. We are to order (naturally) the set $M = \bigcup\limits_{\lambda\in\Lambda} (\{\lambda\}\times N^\lambda)$ to obtain a cofinite indexing set $(M, \le)$ for a desired inverse system $\boldsymbol{Y}$. Let us define

$$\mu = (\lambda,\nu) \le (\lambda',\nu') = \mu' \Longleftrightarrow \begin{cases} \lambda = \lambda' \text{ and } \nu \le \nu' \text{ in } N^\lambda \\ \text{or} \\ \lambda \ne \lambda',\ \lambda \le \lambda' \text{ and } (*) \end{cases},$$

where condition $(*)$ means:

$(\forall \nu^* \le \nu \text{ in } N^\lambda)\ p^{\lambda\lambda'}(\nu^*) \le \nu' \text{ in } N^{\lambda'}$ and $p^\lambda_{\nu^*\nu} p^{\lambda\lambda'}_\nu p^{\lambda'}_{p^{\lambda\lambda'}(\nu),\nu'} = p^{\lambda\lambda'}_{\nu^*} p^{\lambda'}_{p^{\lambda\lambda'}(\nu^*),\nu'}$.

Since each set N^λ, $\lambda \in \Lambda$, is cofinite and each $\boldsymbol{p}^{\lambda\lambda'} : \boldsymbol{X}^{\lambda'} \to \boldsymbol{X}^\lambda$, $\lambda \le \lambda'$, is a map of systems, the relation $\le$ on M is well defined. Clearly, $\le$ on M is reflexive. The transitivity requires an easy analysis - first for the special cases $\lambda = \lambda'$, $\lambda' = \lambda''$, and then for the general case. The verification (using commutativity $\boldsymbol{p}^{\lambda\lambda'}\boldsymbol{p}^{\lambda'\lambda''} = \boldsymbol{p}^{\lambda\lambda''}$) is straightforward and we omit it. Furthermore, $(M, \le)$ is cofinite since Λ and all N^λ are cofinite. To verify that $(M, \le)$ is directed and unbounded is also routine. Finally, observe that for every $\mu = (\lambda,\nu) \in M$ and every $\lambda' \ge \lambda$ there exists an $\nu^* \in N^{\lambda'}$ such that, for every $\nu' \in N^{\lambda'}$, $\nu' \ge \nu^*$ implies $(\lambda',\nu') \equiv \mu' \in M$ and $\mu \le \mu'$.

Consider now a triple $\mu \le \mu' \le \mu'' = (\lambda'',\nu'')$ in M and the corresponding bonding mappings, defined by

$q_{\mu\mu'} \equiv p^{\lambda\lambda'}_\nu p^{\lambda'}_{p^{\lambda\lambda'}(\nu),\nu'} : Y_{\mu'} \equiv X^{\lambda'}_{\nu'} \to X^\lambda_\nu \equiv Y_\mu$,

and similarly for $q_{\mu'\mu''}$ and $q_{\mu\mu''}$. Then the commutativity $\boldsymbol{p}^{\lambda\lambda'}\boldsymbol{p}^{\lambda'\lambda''} = \boldsymbol{p}^{\lambda\lambda''}$ in the canonical construction and the definition of the order $\le$ on M, imply $q_{\mu\mu'}q_{\mu'\mu''} = q_{\mu\mu''}$. The verification is straightforward and is omitted. Therefore, $\boldsymbol{Y} = (Y_\mu, q_{\mu\mu'}, M)$ is a cofinite inverse system as we claimed.

Now assume that the inverse system $\boldsymbol{X}$ admits a resolution $\boldsymbol{p} = (p_\lambda) : X \to \boldsymbol{X}$ of a space X. Define, for each $\mu = (\lambda, \nu) \in M$, the mapping $q_\mu = p_\nu^\lambda p_\lambda : X \to Y_\mu$. An easy verification confirms that $q_{\mu\mu'} q_{\mu'} = q_\mu$ holds, whenever $\mu \leq \mu'$. Thus $\boldsymbol{q} = (q_\mu) : X \to \boldsymbol{Y}$ is a map of X to the system $\boldsymbol{Y}$. Furthermore, condition (B1) for $\boldsymbol{q}$ is obviously fulfilled by (B1) of $\boldsymbol{p}$ and by (B1) of each $\boldsymbol{p}^\lambda$, $\lambda \in \Lambda$. It remains to verify condition (B2) for $\boldsymbol{q}$. Let a $\mu = (\lambda, \nu) \in M$ and a $\mathcal{V} \in Cov(Y_\mu)$ be given. We have to prove that there exists a $\mu' \geq \mu$ such that $q_{\mu\mu'}(Y_{\mu'}) \subseteq \mathrm{St}(q_\mu(X), \mathcal{V})$. First choose a $\mathcal{V}' \in Cov(Y_\mu)$ such that $\mathrm{St}\mathcal{V}' \leq \mathcal{V}$. Denote $\mathcal{U} \equiv (p_\nu^\lambda)^{-1}(\mathcal{V}') \in Cov(X_\lambda)$. By (B2) of $\boldsymbol{p}$, there exists a $\lambda' \geq \lambda$ such that $p_{\lambda\lambda'}(X_{\lambda'}) \subseteq \mathrm{St}(p_\lambda(X), \mathcal{U})$. Denote $\mathcal{W} \equiv (p_\nu^{\lambda\lambda'})^{-1}(\mathcal{V}') \in Cov(X^{\lambda'}_{p^{\lambda\lambda'}(\nu)})$. By (B2) of $\boldsymbol{p}^{\lambda'}$, there exists an $\nu^* \in N^{\lambda'}$, $\nu^* \geq p^{\lambda\lambda'}(\nu)$, such $p^{\lambda'}_{p^{\lambda\lambda'}(\nu),\nu'}(X^{\lambda'}_{\nu'}) \subseteq \mathrm{St}(p^{\lambda'}_{p^{\lambda\lambda'}(\nu)}(X_{\lambda'}), \mathcal{W})$ whenever $\nu' \geq \nu^*$ in $N^{\lambda'}$. Choose such an $\nu' \in N^{\lambda'}$ satisfying also $(\lambda', \nu') \equiv \mu' \in M$ and $\mu' \geq \mu$. Then

$$
\begin{aligned}
&q_{\mu\mu'}(Y_{\mu'}) = p_\nu^{\lambda\lambda'}(p^{\lambda'}_{p^{\lambda\lambda'}(\nu),\nu'}(X^{\lambda'}_{\nu'})) \subseteq p_\nu^{\lambda\lambda'}(\mathrm{St}(p^{\lambda'}_{p^{\lambda\lambda'}(\nu)}(X_{\lambda'}), \mathcal{W})) = \\
&p_\nu^{\lambda\lambda'}(\mathrm{St}(p^{\lambda'}_{p^{\lambda\lambda'}(\nu)}(X_{\lambda'}), (p_\nu^{\lambda\lambda'})^{-1}(\mathcal{V}'))) \subseteq \mathrm{St}(p_\nu^{\lambda\lambda'} p^{\lambda'}_{p^{\lambda\lambda'}(\nu)}(X_{\lambda'}), \mathcal{V}') = \\
&\mathrm{St}(p_\nu^\lambda p_{\lambda\lambda'}(X_{\lambda'}), \mathcal{V}') \subseteq \mathrm{St}(p_\nu^\lambda(\mathrm{St}(p_\lambda(X), \mathcal{U})), \mathcal{V}') = \\
&\mathrm{St}(p_\nu^\lambda(\mathrm{St}(p_\lambda(X), (p_\nu^\lambda)^{-1}(\mathcal{V}'))), \mathcal{V}') \subseteq \mathrm{St}(\mathrm{St}(p_\nu^\lambda p_\lambda(X), \mathcal{V}'), \mathcal{V}') = \\
&\mathrm{St}(\mathrm{St}(q_\mu(X), \mathcal{V}'), \mathcal{V}') \subseteq \mathrm{St}(q_\mu(X), \mathcal{V}),
\end{aligned}
$$

where we used the two following facts (general notation):
$f(\mathrm{St}(A, f^{-1}(\mathcal{V}))) \subseteq \mathrm{St}(f(A), \mathcal{V})$ and $\mathrm{St}\mathcal{U}' \leq \mathcal{U} \Rightarrow \mathrm{St}(\mathrm{St}(A, \mathcal{U}'), \mathcal{U}') \subseteq \mathrm{St}(A, \mathcal{U})$.
This completes the proof of the theorem. □

Corollary. *Let $\boldsymbol{p} = (p_\lambda) : X \to \boldsymbol{X} = (X_\lambda, p_{\lambda\lambda'}, \Lambda)$ be an inverse limit of compact Hausdorff spaces, where Λ is cofinite and antisymmetric. Then there exist inverse limits $\boldsymbol{p}^\lambda = (p_\nu^\lambda) : X_\lambda \to \boldsymbol{X}^\lambda = (X_\nu^\lambda, p_{\nu\nu'}^\lambda, N^\lambda)$ of compact polyhedra, $\lambda \in \Lambda$, and maps of systems $\boldsymbol{p}^{\lambda\lambda'} = (p^{\lambda\lambda'}, p_\nu^{\lambda\lambda'}) : \boldsymbol{X}^{\lambda'} \to \boldsymbol{X}^\lambda$, $\lim \boldsymbol{p}^{\lambda\lambda'} = p_{\lambda\lambda'}$, $\lambda \leq \lambda'$, which can be naturally organized to yield the inverse limit $\boldsymbol{q} = (q_\mu) : X \to \boldsymbol{Y} = (Y_\mu, q_{\mu\mu'}, M)$, where $M = \{\mu = (\lambda, \nu) \mid \lambda \in \Lambda, \nu \in N^\lambda\}$, $Y_\mu = X_\nu^\lambda$, $q_{\mu\mu'} = p_\nu^{\lambda\lambda'} p^{\lambda'}_{p^{\lambda\lambda'}(\nu),\nu'}$ and $q_\mu = p_\nu^\lambda p_\lambda$.*

Proof. In canonical constructions of all the resolutions $\boldsymbol{p}^\lambda$ (by Theorems 1 and 2, i.e., Lemmas 1 - 5), one should use only cofinal subfamilies consisting of all finite open coverings. Then the corollary follows by Theorem 3. □

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