## Methodology for Determining of Train Curving Resistances With Respect to Vehicle Mass and Speed

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#### Keywords

Constant resistance Curving resistance Gravitational method Mass Speed

### Ključne riječi

Brzina Gravitaciona metoda Masa Otpor krivine Stalni otpor

Received (primljeno): 2009-04-20 Accepted (prihvaćeno): 2009-10-30

## 1. Introduction

Resistive forces of railway vehicles, especially of locomotives, represent a phenomenon characterized by a complex structure. Numerous theoretical and experimental studies on the resistive force influencing factors have been conducted so far. However, they have not resolved completely the mechanisms of their existence in general, particularly of the curving resistances.

The curving resistances generally depend on the various influencing parameters such as: the curve radius,

the track gauge, the gauge widening, the super elevation, the type of wheel, the type of rail, the wheel set rigidity, the distance between bogies, the number of axles, the distance between axles, the coefficient of friction, the vehicle speed, the vehicle mass, the train arrangement, etc. [1-3].

The best insight into the behavior of the vehicle resistive force phenomenon is obtained by tests on full scale vehicles. Such an approach requires substantial

Original scientific paper

A methodology for determining curving resistances of track vehicles is proposed. The methodology is aimed to gain insight into influence of the arrangement of masses along the train and the train speed on the curving resistance based on test results conducted by using a scaled down train. The utilized scaled down train is a HO scale line freight train in ratio of 1:87 consisting of a EMD SD 35 locomotive and FALNS 121 freight wagons. The considered variants of the train arrangements are as follows: empty train (PPPPPP), one loaded wagon in the front/in the end (TPPPPP and PPPPPT) and two loaded wagons in the front/in the end (TTPPPP and *PPPPTT*). The total train curving resistance  $W_{\rm R}$  is determined based on the decrease of the train steady-state curving speed  $v_{\rm p}$  when compared to the train steady-state speed on the straight track  $v_{\infty}$  under same operating conditions. The curving resistance of a train of arbitrary arrangement is calculated as the difference between the train driving force when driving on a straight, horizontal track  $F_{\rm V}$  (constant resistance  $W_{\rm S}$ ) and the total resistance when curving  $W_{\rm U}$  (driving force  $F_{\rm V}$  in curve). The constant resistances of the locomotive and empty and loaded wagons are determined by application of the gravitational method.

## Metodologija izračuna otpora u zavoju vlaka u ovisnosti od mase i brzine

#### Izvornoznanstveni članak

Metodologija izračuna otpora u zavoju tračničkih vozila osmišljena je za izvođenje eksperimenta na umanjenom modelu teretnog vlaka u HO standardu 1:87, lokomotiva EMD SD 35 i vagoni FALNS 121. Formirani model služi za istraživanje utjecaja vrijednosti i rasporeda mase tereta unutar sastava voza, te brzine gibanja na vrijednost otpora u zavoju. Gravitacijskom metodom se određuju stalni otpori (otpori u ravnini) lokomotive, praznog i natovarenog vagona. Razmatrane kombinacije sastava vlaka su: prazan vlak (*PPPPPP*), jedan natovaren vagon napred nazad (*TPPPPP* i *PPPPT*) i dva natovarena vagona napred nazad (*TTPPPP* i pPPPTT). Otpor u zavoju cijelog vlaka  $W_{\rm R}$  dobije se na osnovu smanjenja brzine gibanja vlaka u zavoju  $v_{\rm R}$  prema brzini gibanja vlaka na pravcu  $v_{\infty}$  pri istim referentnim uvjetima. Otpor u zavoju  $W_{\rm R}$  izračunava se kao razlika vučne sile vlaka na pravcu  $F_{\rm V}$  (stalnog otpora  $W_{\rm S}$ ) i ukupnog otpora cijelog vlaka  $W_{\rm U}$  (vučna sila  $F_{\rm V}$  u zavoju) pri gibanju u zavoju za svaku odabranu kombinaciju sastava vlaka.

### Symbols/Oznake

Symbols, Ozhane	
<ul> <li><i>a</i><sub>11</sub> - acceleration of locomotive, m/s<sup>2</sup></li> <li>- ubrzanje modela lokomotive</li> </ul>	<i>R</i> - curve radius, m - radijus zavoja
<ul> <li><i>a</i><sub>p</sub> - acceleration of empty wagon, m/s<sup>2</sup></li> <li>- ubrzanje praznog vagona</li> </ul>	<ul> <li>v - train speed, m/s</li> <li>- brzina gibanja vlaka</li> </ul>
<ul> <li><i>a</i><sub>T</sub> - acceleration of loaded wagon, m/s<sup>2</sup></li> <li>- ubrzanje natovarenog vagona</li> </ul>	<ul> <li>v<sub>∞</sub> - train steady-state speed, m/s</li> <li>brzina gibanja vlaka po pravcu</li> </ul>
<ul> <li><i>F</i><sub>v</sub> - driving force, N</li> <li>- vučna sila</li> </ul>	<ul> <li>v<sub>R</sub> - train curving speed, m/s</li> <li>brzina gibanja vlaka u zavoju</li> </ul>
<ul> <li><i>f</i><sub>v</sub> - specific driving force, N/kg</li> <li>- specifična vučna sila</li> </ul>	T - time, s - vrijeme
g - gravitation, m/s <sup>2</sup> - gravitacija	<ul> <li>W<sub>0</sub> - other train resistances, N</li> <li>- ostali otpori vožnje</li> </ul>
<ul><li><i>L</i> - railway length, m</li><li>- duljina pruge</li></ul>	W <sub>R</sub> - train curving resistance, N - otpor vožnje u zavoju
$m_{\rm L}$ - locomotive mass, kg - masa lokomotive	<ul> <li>w<sub>R</sub> - specific train curving resistance, N/kg</li> <li>- specifični stalni otpor vožnje u zavoju</li> </ul>
$m_{\rm P}$ - empty wagon mass, kg - masa praznog vagona	<ul> <li>W<sub>s</sub> - train constant resistance, N</li> <li>- stalni otpor vožnje</li> </ul>
$m_{\rm T}$ - loaded wagon mass, kg - masa natovarenog vagona	<ul> <li>w<sub>s</sub> - specific train constant resistance, N/kg</li> <li>- specifični stalni otpor vožnje</li> </ul>
<i>m</i> <sub>T/K</sub> - net load mass, kg - masa tereta (neto)	<ul> <li><i>W</i><sub>SL</sub> - locomotive constant resistance, N</li> <li>- stalni otpor vožnje lokomotive</li> </ul>
$m_{\rm U}$ - total train mass, kg - ukupna masa vlaka	<ul> <li><i>W</i><sub>SP</sub> - constant resistance of empty wagon, N</li> <li>- stalni otpor vožnje praznog vagona</li> </ul>
$m_{\rm v}$ - train mass, kg - masa vlaka	<ul> <li><i>W</i><sub>ST</sub> - constant resistance of loaded wagon, N</li> <li>- stalni otpor vožnje natovarenog vagona</li> </ul>
N - number of wagons, - - broj vagona	$W_{\rm Tr}$ - resistance related to track design characteristics, N - otpor konstrukcijske karakteristike trase
N <sub>p</sub> - number of empty wagons, - - broj praznih vagona	W <sub>U</sub> - total resistance, N - ukupni otpor vožnje
N <sub>T</sub> - number of loaded wagons, - - broj natovarenih vagona	<ul><li>α - track gradient, °</li><li>- nagib pruge</li></ul>

resources in finance and equipment. On the other hand, tests on scaled down vehicles require significantly lower financial resources and provide an easy separate analysis of various influencing factors. In this way, the scaled down vehicle tests can be used in order to provide guidelines for full vehicle tests related to influence of significant factors.

This paper presents methodology for the analysis of influence of operating parameters on the track vehicle curving resistance based on scaled down vehicle (train) tests. The considered operating parameters are the vehicle speed and the size and the arrangement of the load masses along the train. A scaled down train has been chosen, which provides easy and simple variation of load masses along the train, thus providing possibility for conducting large number of experiments.

## 2. Experimental setup description

Experimental research is conducted by the following scaled down models in ratio of 1:87:

**1.** *Train model* – locomotive with six identical freight wagons (Figure 1).



**Figure 1.** Scaled down train model. **Slika 1.** Model kompozicije vlaka.

**2.** Six-axle locomotive model T 155 - EMD SD 35 locomotive model in *HO* scale line, manufactured by *Mehano* Slovenia (Figure 2), which represents scaled down model of *ELECTRO-MOTIVE DIVISION USA* diesel-electric locomotive. The locomotive model mass is 0.502 kg. The model is driven by an 8W/16V DC motor. The motor is placed in the center of mass of the locomotive model and connected to both bogies by cardan shafts.

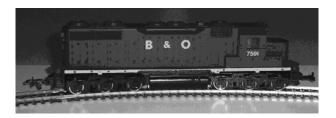


Figure 2. Locomotive model T 155 – EMD SD 35. Slika 2. Model lokomotive T 155 – EMD SD 35.

**3.** Four axle FALNS 121 freigth wagon models - six identical *T* 215 wagon models in *HO* scale line manufactured by *Mehano* Slovenija (Figure 3). The mass of the empty wagon is 0.0835 kg. Each wagon is loaded by a weight having mass of 0.154 kg. Therefore, the total mass of loaded wagon is equal to 0.2375 kg.



Figure 3. FALNS 121 freight wagon model. Slika 3. Model vagona tip FALNS 121.

4. Experimental track with variable gradient  $\alpha$  for determining of constant resistive forces of locomotive and wagon model by gravitational method (Figure 4).

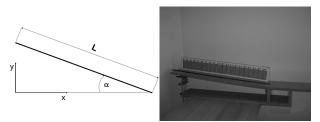


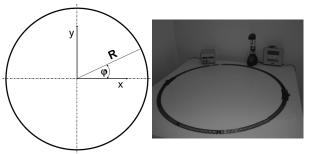
Figure 4. Experimental track with variable gradient. Slika 4. Eksperimentalni nagib

5. Horizontal straight experimental track - radius R 000 ( $R=\infty$ ), length L = 1 m, HO scale line (Figure 5).



**Figure 5.** Horizontal straight experimental track. **Slika 5.** Eksperimentalna trasa pruge na pravcu.

**6.** *Horizontal curved experimental track* - radius *R* 457.2 mm /360 ° (full circle), *HO* scale line (Figure 6).



**Figure 6.** Horizontal curved experimental track with radius *R* 457.2 mm /360 °.

Slika 6. Eksperimentalna trasa s R 457.2 mm /360 °.

## 3. Aim of experiment

The train curving resistance  $W_{R}$  on a specified track with radius *R* depends on the train arrangement, mass, and speed:

$$W_{\rm R} = f(R, m_{\rm II}, v)$$
.

The total train mass  $m_{\rm U}$  includes the locomotive mass  $m_{\rm L}$ , and the train mass  $m_{\rm V}$ . The train mass is determined by the number of freight wagons N and their load: empty wagon mass  $m_{\rm P}$  or loaded wagon mass  $m_{\rm T}$ . The curving resistance of the train  $W_{\rm R}$  is then given as:

$$W_{\rm R} = f(R, m_{\rm P}, m_{\rm T}, N, v).$$

The experiments are aimed to provide data of train speed v and train resistances (total curving resistance  $W_{\rm U}$ , constant resistance  $W_{\rm s}$ , and curving resistance  $W_{\rm R}$ ) for the train of specified mass and arrangement. Knowing the train constant resistances  $W_{\rm s}$  (can be obtained e.g. by application of gravitational method [4]) and the train steady-state speed, the curving resistance  $W_{\rm R}$  can be easily calculated as demonstrated in Section 7.

Therefore, in each experiment the train of chosen arrangement is driven along the track with specified radius R at constant (steady-state) speed v and constant driving motor power P = const. The steady-state speed is

reconstructed based on measurement of time T required for the train to drive along specified part of the track of length L.

#### Train parameters:

a) Train mass

The train consists of the locomotive and six freight wagons. Total train mass  $m_{\rm U}$  corresponds to the sum of the locomotive mass  $m_{\rm L}$  and the mass of wagons  $m_{\rm V}$ . Mass of wagons includes mass of empty  $m_{\rm p}$  and loaded  $m_{\rm T}$  wagons. Mass of the loaded wagon (mass of empty wagon plus mass of load (load weight) is constant during experiment. The train mass is varied by changing number of loaded wagons (0, 1, or 2).

b) Arrangement of wagons

For the chosen freight train with specified mass, various arrangements of the loaded freight wagons are chosen.

c) Number of wagons

The train with N identical four-wheel freight wagons is formed. The train consists either of  $N_p$  empty wagons or combination of  $N_p$  empty and  $N_T$  loaded wagons.

*d) Train speed* 

Constant power driving condition along track length L and during time T is maintained by varying the train speed v and the driving force  $F_v$ . The minimum train speed is defined as the minimum speed at which the train moves with a constant speed and the motor voltage is stable. The maximum speed is defined as speed at which train derailment will not occur.

Accordingly, the following terms are valid:

1. Train steady-state speed:

$$v = \frac{L}{T}.$$
 (1)

2. Mass of train loaded wagons:

$$m_{\rm T} = m_{\rm P} + m_{\rm T/K}.$$

where  $m_{T/K}$  is net mass of load which is transported by a wagon.

3. Mass of train wagons:

 $mV = NP \cdot mP + NT \cdot mT. \tag{3}$ 

4. Total mass of train (locomotive + empty and loaded freight wagons):

$$m_{\rm II} = m_{\rm L} + m_{\rm V}. \tag{4}$$

5. Number of train wagons:

$$N = N_{\rm p} + N_{\rm T}.$$
(5)

6. Arrangement of train wagons. Two variants of the arrangements are proposed:

- all empty wagons denoted as P and

– various arrangements of empty P and loaded wagons denoted as T.

Parameters of track:

- a) Curve radius R of horizontal track H:  $R 000 - \text{curve radius } R = \infty$ , straight line and R 457.2 - curve radius R = 457.2 mm.
- b) Track gradient  $\alpha$ .

## 4. Design of experiment

The experiments are aimed to determine curving resistance  $W_{\rm R}$  of a train with various axle loads driving along straight or curved track.

#### 4.1. Plan for determining of train resistances

The total resistance  $W_{\rm U}$  is equal to a sum of the train constants resistance  $W_{\rm S}$ , the resistances related to the geometric and design characteristics of the track  $W_{\rm Tr}$  and other resistances  $W_{\rm O}$ :

$$W_{\rm U} = W_{\rm S} + W_{\rm Tr} + W_{\rm O}.$$
 (6)

Generally, the methods used to determine driving resistance imply the total driving resistance, but not individual contributions. The resistance considered here relates to the total resistance subtracted by the resistance related to the geometric and design characteristics of the track. Therefore, it is necessary to determine the individual contributions.

By using scaled down model based experiment approach, the influence of other resistances  $W_0 = 0$  (wind resistance, initial motion resistance and acceleration resistance) and  $W_{Tr} = W_R$  can be excluded easily. Then, the total resistance may be regarded as a sum of constant resistances and resistances related to geometric and design characteristics of the track [4]. The resistance related to geometric and design characteristics of the track considers only curving resistance, which implies:

$$W_{\rm U} = W_{\rm S} + W_{\rm Tr} = W_{\rm S} + W_{\rm R}.$$
 (7)

The train constant resistance  $W_{\rm S}$  is equal to the sum of the locomotive constant resistance  $W_{\rm SL}$  and the constant resistance of the wagons  $W_{\rm SV}$ , i.e. empty wagons  $W_{\rm SP}$  and loaded wagons  $W_{\rm ST}$ :

$$W_{\rm S} = W_{\rm SL} + W_{\rm SV} = W_{\rm SL} + N_{\rm P} \cdot W_{\rm SP} + N_{\rm T} \cdot W_{\rm ST}$$
(8)

The total train resistance  $W_{\rm II}$  then reads:

$$W_{\rm U} = W_{\rm SL} + N_{\rm P} \cdot W_{\rm SP} + N_{\rm T} \cdot W_{\rm ST} + W_{\rm R} \,.$$
 (9)

The train curving resistance  $W_{\rm R}$  can be found by rearranging Eq. (7):

$$W_{\rm R} = W_{\rm U} - W_{\rm S} = W_{\rm U} - W_{\rm SL} - N_{\rm P} \cdot W_{\rm SP} - N_{\rm T} \cdot W_{\rm ST}$$
 (10)

#### 4.2. Plan for arrangement of masses along train

A train consisting of a locomotive and six freight wagons is considered. The length of the chosen train is the maximum allowable length limited by the half of the circumference of the experimental curved track. In this way, the correction of the curving resistance is not required, which considers the length of train and the length of curve  $(L_V/L_R)$  [4]. The considered variants of the freight train arrangements are given in Table 1.

**Table 1.** Variants of freight train arrangements (P – empty wagons, T – loaded wagons).

**Tablica 1.** Raspored praznih P i natovarenih T vagona u sastavu vlaka vučenog lokomotivom

LOKOMOTIVA // VAGONI				IV	V	VI
	Ρ	Р	Ρ	Ρ	Ρ	Ρ
	Т	Ρ	Ρ	Ρ	Ρ	Р
	Ρ	Р	Р	Ρ	Ρ	Т
	Т	Т	Ρ	Ρ	Ρ	Ρ
	Ρ	Р	Р	Р	Т	Т

The proposed arrangement variants are chosen in order to give insight into the curving resistance behavior in terms of (i) the arrangement of train masses for extreme cases (loaded wagon at the very end or in the very front of the train) and (ii) the magnitude of wagon masses (empty, loaded, or fully loaded wagon). Other possible variants are not proposed to be covered by experiments, because they have been found to be potentially irrelevant for the analysis and because they would unnecessarily largely increase the final number of experiments.

## 4.3. Plan for choice of DC motor armature voltage and vehicle speed

The locomotive is driven by a DC electric motor. The motor has linear characteristics. The motor speed is adjusted by means of armature voltage controller. The train speed on straight or curved experimental track is varied by changing the motor armature voltage. Armature voltage of 4V has been found to be the minimum value at which a steady-state motion of the train through a curve is achieved. Note that the driving resistance is higher when the train drives through curve when compared to the driving along a straight line. If the armature voltage is lower than 4 V, a steady-state motion of the train is not achieved characterized by periodical stop-and-go motion due to weak contact between the wheel and the rail. In addition, during stop intervals the motor experiences short circuit conditions hence resulting in the heating of the motor and the power source. The maximum armature voltage at which a steady-state motion is achieved is equal to 12 V. If the voltage is higher than 12 V, the train motion includes additional oscillatory and parasitic motions and there exist a possibility of the train derailment.

The train model speeds at the minimum and the maximum armature voltage (4 and 12 V) are equal to approximately 0,10 and 0,58 m/s, respectively. When referred to the full scale train, the scaled down train model speeds correspond to 31,32 and 181,65 km/h, respectively. These values define a speed range of practical meaning for the research of train resistances.

In order to provide comparable experiment operating conditions on straight and curved track with respect to the value of train speed, the value of voltage (reference voltage) is additionally adjusted prior to each experiment. The voltage is directly measured on empty experimental track and precisely adjusted by power source potentiometer. This procedure is repeated for each value of the voltage considered (4, 5, 6, 7, 8, 9, 10, 11, 12 V).

#### 4.4. Measuring equipment and apparatus

The train speed along the experimental track is obtained by off-line processing of the train motion records. The experimental track comprises markers with 5 mm resolution. The train motion is recorded by using *SONY TRV 19* digital camcorder with framerate equal to 25 frames per second. The time during train motion is measured by using a clock with resolution of 0,01 seconds. The clock includes:

MICROCHIP 16 F 877 microcontroller.

Two OMRON photoelectric sensors with sensitivity range from 100 to 300 mm.

Ampermeter with measurement range of 1,5 A and accuracy class 0,1.

A precise battery powered programmable DC power source is used in experiments. The main features are: fine voltage control in the range of 1,5...25 V DC and maximum current of 1,5 A.

## 5. Determining of constant resistances

According to expression (8), the train constant resistance is defined as the sum of constant resistances related to the locomotive  $W_{\rm SL}$  and the wagons: empty wagons  $W_{\rm SP}$  and/or loaded wagons  $W_{\rm ST}$ . The constant resistances are determined by separate measurements by application of the gravitational method to the train driving on track with gradient [5] as follows:

1. Locomotive constant resistance:

$$W_{\rm SL} = m_{\rm L} \cdot (g \cdot tg\alpha - a_{\rm L}) \tag{11}$$

2. Constant resistance of empty wagon  $W_{\rm SP}$ :

$$W_{\rm sp} = m_{\rm p} \cdot (g \cdot tg\alpha - a_{\rm p}) \tag{12}$$

3. Constant resistance of loaded wagon  $W_{st}$ :

$$W_{\rm ST} = m_{\rm T} \cdot (g \cdot tg\alpha - a_{\rm T}) \tag{13}$$

The final results are fitted by polynomial regression curves of appropriate order using least-square optimization method [5].

# 6. Determining of driving force on straight line

The experiments are aimed to obtain dependences of driving force vs. train steady-state speed ( $F_v$  vs.  $v_{\infty}$ ) for trains of various arrangements (see Table 1) driving on the horizontal, straight experimental track under operating conditions of stable and constant driving motor supply voltage. Ten repeats for each variant of the train arrangement are proposed to be conducted for the purpose of statistical analysis.

During steady-state motion of a train (*a*=0) along a horizontal, straight track, the locomotive driving force is equal to the constant resistance  $F_v = W_s$ . Since the train consists of a locomotive and various numbers of empty and loaded wagons, the driving force reads:

$$F_{\rm V} = W_{\rm S} = W_{\rm SL} + N_{\rm P} \cdot W_{\rm SP} + N_{\rm T} \cdot W_{\rm ST.} \tag{14}$$

The single constant resistances ( $W_{\rm SL}$ ,  $W_{\rm SP}$ , and  $W_{\rm ST}$ ) are obtained by using gravitational method as explained in Section 5.

The specific driving force  $f_{\rm V}$  on straight track is given as:

$$f_{\rm V} = w_{\rm S} = \frac{F_{\rm V}}{m_{\rm U}} = \frac{W_{\rm S}}{m_{\rm U}} \,. \tag{15}$$

For example, for a train consisting of one loaded and five empty wagons, i.e. for a train of *TPPPPP* arrangement, the driving force reads:

$$F_{\rm V} = W_{\rm S} = W_{\rm SL} + 5 \cdot W_{\rm SP} + 1 \cdot W_{\rm ST}$$

and the corresponding specific force  $f_{V}$  is:

$$f_{\rm V} = \frac{W_{\rm SL} + 5 \cdot W_{\rm SP} + 1 \cdot W_{\rm ST}}{m_{\rm L} + 5 \cdot m_{\rm P} + 1 \cdot m_{\rm T}}.$$

### 7. Determining of curving resistance

The train curving resistances  $W_{\rm R}$  is based on comparison of experimentally obtained results of total resistances for the train driving along straight  $W_{\rm s}$  and curved track  $W_{\rm U}$  under the same operating conditions. The operating conditions are related to the driving DC motor armature voltage, which needs to have the same value disregarding the type of the track (straight or curved). This can be achieved by fine adjustments prior to each experiment as explained in Subsection 4.3 [6]. It is proposed to make thirty repeats of experiments for each train arrangement in order to provide enough data for a quality statistical analysis.

The train curving resistance  $W_{\rm R}$  is equal to the difference between the driving forces on the straight  $F_{\rm v}$  and the curved track  $W_{\rm u}$ :

$$W_{\rm R} = F_{\rm V} - W_{\rm U} = W_{\rm S} - W_{\rm U} =$$
  
=  $(W_{\rm SL} + N_{\rm p} \cdot W_{\rm SP} + N_{\rm T} \cdot W_{\rm ST})_{\infty} -$   
 $- (W_{\rm SL} + N_{\rm p} \cdot W_{\rm SP} + N_{\rm T} \cdot W_{\rm ST})_{\rm R}$  (16)

and the specific train curving resistance  $w_{\rm R}$  is:

$$w_{\rm R} = \frac{W_{\rm R}}{m_{\rm U}} = \frac{F_{\rm V} - W_{\rm U}}{m_{\rm U}} = \frac{W_{\rm S} - W_{\rm U}}{m_{\rm U}} \,. \tag{17}$$

The curving resistance  $W_{\rm R}$  is determined based on the decrease of the magnitude of the train curving speed  $v_{\rm R}$  when compared to the train speed on straight track  $v_{\infty}$ . The considered influencing parameters are the train arrangement (see Table 1) and the curving speed  $v_{\rm R}$ . The curving resistances  $W_{\rm R}$  for considered train arrangements are as follows:

1. Six empty wagons (PPPPPP):

$$W_{\rm R(6P)} = W_{\rm S(6P)} - W_{\rm U(6P)} = F_{\rm V(6P)} - (W_{\rm SL} + 6 \cdot W_{\rm SP})$$
(18)

2. One loaded wagon and five empty wagons *(TPPPPP)*:

$$W_{\text{R(T5P)}} = F_{\text{V(T5P)}} - (W_{\text{SL}} + 1 \cdot W_{\text{ST}} + 5 \cdot W_{\text{SP}})$$
(19)

3. Five empty wagons and one loaded wagon (*PPPPPT*):

$$W_{\rm R(SPT)} = F_{\rm V(SPT)} - (W_{\rm SL} + 5 \cdot W_{\rm SP} + 1 \cdot W_{\rm ST})$$
 (20)

4. Two loaded and four empty wagons (TTPPPP):

$$W_{\rm R(2T4P)} = F_{\rm V(2T4P)} - (W_{\rm SL} + 2 \cdot W_{\rm ST} + 4 \cdot W_{\rm SP})$$
(21)

5. Four empty and two loaded wagons (PPPPTT):

$$W_{\rm R(4P2T)} = F_{\rm V(4P2T)} - (W_{\rm SL} + 4 \cdot W_{\rm SP} + 2 \cdot W_{\rm SP}).$$
 (22)

The obtained values of the train total curving resistance  $W_{\rm U}$  and the train curving resistance  $W_{\rm R}$  are presented graphically.

In the particular case the track radius is R 457,2 mm (see Figure 6). The steady-state train curving speed  $v_{\rm R}$  can be calculated as:

$$v_{\rm R} = \frac{L}{T},\tag{23}$$

where L is the experimental track length and T is the time required for the train to drive along the track of length L. The length of the experimental track is:

$$L = 2 \cdot R \cdot \pi \cdot n, \tag{24}$$

where *n* is the number of completed laps.

For the particular curve radius R = 457,2 mm and the chosen number of completed laps n = 2, the total length of the distance traveled is  $L = 2 \cdot 0,4572 \cdot \pi \cdot 2 = 5,745$  m. Thus, the steady-state curving speed  $v_{\rm R}$  is equal to:

$$v_{\rm R} = \frac{5,745}{T}$$
 (25)

### 8. Conclusions

This paper presents a methodology for determining the train curving resistances with respect to train mass and speed. A detailed description of experiments is included that can be utilized in order to obtain experimental recordings required for curving resistance calculation by using the proposed methodology. The influence of all operating parameters relevant for the calculation and analysis of train curving resistances is analyzed. The main conclusions are as follows:

The train curving resistance  $W_{\rm R}$  is obtained based on the decrease of the train curving speed magnitude  $v_{\rm R}$ when compared to the speed on straight, horizontal track  $v_{\rm m}$  under the same operation conditions.

The curving resistance of a train of arbitrary arrangement is calculated as the difference between the train driving force when driving on a straight, horizontal track  $F_v$  (constant resistance  $W_s$ ) and the total resistance when curving  $W_{\rm II}$  (driving force  $F_v$  in curve) as:

$$W_{\rm R} = F_{\rm V} - W_{\rm U} = W_{\rm S} - W_{\rm U}$$

The specific curving resistance of the train  $w_{\rm R}$  is given as:

$$w_{\rm R} = \frac{W_{\rm R}}{M_{\rm U}} = \frac{F_{\rm V} - W_{\rm U}}{M_{\rm U}} = \frac{W_{\rm S} - W_{\rm U}}{M_{\rm U}}.$$

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