Ciprian Necula*

UDK 336.761.5
Original scientific paper
Izvorni znanstveni rad

A COPULA-GARCH MODEL

ABSTRACT

In the present study we develop a new two-dimensional Copula-GARCH model. This type of two-dimensional process is characterized by a dependency structure modeled using a copula function. For the marginal densities we employ a GARCH(1,1) model with innovations drawn from a t-Student distribution. The model can be easily extended by using more sophisticated processes for the marginal densities. The static specification of the model assumes that the dependency structure of the two data series does not vary in time implying that the parameters of the copula function are constant. On the other hand, the dynamic specification models explicitly the dynamics of these parameters. We econometrically estimate the parameters of the two specifications using various copula functions, focusing on the mixture between the Gumbel and Clayton copulas. We employ daily index returns from two emerging and two developed financial markets. The main finding is that including a varying dependency structure improves the goodness-of-fit of the Copula-GARCH model.[1]

Keywords: *copula functions, multidimensional GARCH, volatility, dependency structure*

JEL Classification: C51, F37, G17

Acknowledgements: I would like to thank Professor Moisă Altăr for useful comments and suggestions. Financial Support from PN II PCE Grant 1863/2009 entitled "Modeling the Influence of the Trinomial Uncertainty-Volatility-Risk on the Dynamics of Complex Financial-Economic Systems" and financed by The Romanian National University Research Council (CNCSIS) is gratefully acknowledged.

1. INTRODUCTION

The linear correlation coefficient is central to the modern financial modeling. Unfortunately, when the return distribution deviates from the elliptic class which includes the Gaussian, the correlation coefficient is not able to correctly capture the dependency structure between the assets. Several studies have empirically proved that asset returns are heavy tailed and successfully fitted to the data leptokurtic distributions like the Generalized Hyperbolic Distributions. For example, Necula (2009a) estimated the parameters of the Generalized Hyperbolic Distribution for the returns of several Eastern European emerging markets and concluded that the estimated GH distribution represents a good approximation (at least up to the 4th order term) of the empirical distribution quantified using nonparametric kernel

* Senior Lecturer, DOFIN, Faculty of Finance and Banking, Bucharest University of Economics, Bucharest, Romania; Researcher, Center for Advanced Research in Finance and Banking (CARFIB), Bucharest University of Economics, Bucharest; Researcher, Centrul de Analiză şi Prognoză Economico-Financiară (CAPEF); email: ciprian.necula@fin.ase.ro; home page: www.dofin.ase.ro/cipnec/

[1] Paper received 8 October 2009.

methods. Moreover, the linear correlation coefficient cannot capture a non-linear dependency structure in the data (Blyth, 1996; Shaw, 1997). Copulas represent a way to surpass the deficiencies of the linear correlation coefficient. The theory of copulas dates back to Sklar (1959), but its application in financial modeling is far more recent. Nelsen (1999) provide an introduction to copula theory, while Cherubini *et al.* (2004) provide a discussion of copula methods for financial applications.

Volatility clustering is a phenomenon well documented in the financial literature. Since the pioneering work of Engle (1982) and Bollerslev (1986) manifold variations of the one-dimensional GARCH volatility model have been developed. However, in order to quantify more exactly the market risk one has to account both for the volatility of the assets and for the dependency structure between them. The Copula-GARCH is a multidimensional GARCH process that models the dependency structure using a copula function. Jondeau and Rockinger (2006) developed a two-dimensional Copula-GARCH model using the Planket copula, the Gaussian copula and the t-copula. Since the Planket copula and the Gaussian copula does not account for tail dependence the authors concluded that dependency should be modeled with t-copulas. Unfortunately, this kind of model can not be extended easily to three or more dimensions since the number of parameters of the t-copula increases with a square law. Patton (2006a, 2006b) employed the Clayton copula, and Hu (2006) used the Gumbel copula. The Gumbel copula captures dependence only in the upper tail, while the Clayton copula models the dependence only in the lower tail. However, a series of studies pointed out that the asset returns are characterized by dependency both in the lower and in the upper tails. One can capture such a dependence structure with the t-copula or with a Gumbel-Clayton mixture. For example, Necula (2009b) assessed the dependency structure between stock indexes in several Eastern European markets by econometrically estimating the parameters of various parametric copula functions and concluded that the mixture between a Clayton copula and a Gumbel copula and the t-copula are the most appropriate copula functions to capture the dependency structure of two financial return series. The advantage of the Gumbel-Clayton mixture is that the number of parameters remains constant as the dimension increases, while the number of the parameters of the t-copula increases with a power law.

In the present paper we develop a new two-dimensional Copula-GARCH model. To account for heavy tails we model the marginal densities using a GARCH(1,1) process with innovations drawn from a t-Student distribution. The static specification of the model assumes that the dependency structure of the two data series does not vary in time implying that the parameters of the copula function are constant. On the other hand, the dynamic specification models explicitly the dynamics of these parameters. We econometrically estimate the two specifications using various copula functions, focusing on the mixture between the Gumbel and Clayton copulas. Models based on this kind of mixtures capture dependence both in the lower and in the upper tails and can be easily extended to more dimensions.

The rest of the paper is organized in three sections. In the second section we introduce the static and the dynamic specifications of the model. In the third section we analyze the results of the econometrical estimation of the two specifications. The final section concludes.

2. THE COPULA-GARCH MODEL

As it is well-known, a copula represents the cumulative distribution function (cdf) of a multidimensional distribution with uniform marginal distributions. Sklar (1959) proved that a copula function represents the connection between a bi-dimensional distribution and its two marginal distributions, capturing the dependency structure. More precisely, if F is the cdf of the bi-dimensional distribution and F_1 and F_2 are the cdfs of the marginal distributions, there is a unique copula C such that:

$$F(x_1, x_2) = C(F_1(x_1), F_2(x_2)), \qquad (1)$$

Also, if the cdfs for the bi-dimensional and for the marginal distributions are known, the associated copula function is given by (Sklar, 1959):

$$C(u_1, u_2) = F\left(F_1^{-1}(u_1), F_2^{-1}(u_2)\right), \qquad (2)$$

An important class of copula functions consists of Archimedean copulas. An Archimedean copula is given by:

$$C(u_1, u_2) = \psi^{-1}(\psi(u_1) + \psi(u_2)), \qquad (3)$$

where the generator function ψ has the following properties: $\psi(1) = 0$, $\psi' > 0$, and $\psi'' < 0$.

The most commonly used copulas in finance are the product copula (i.e. the copula that models independence), the Gaussian copula, the t-copula, and three Archimedean copulas: Frank, Gumbel and Clayton. As we have already mentioned, financial series usually have dependence both in the lower tail and the upper tails. Such a dependence structure can be modeled using the t-copula or with a Gumbel-Clayton mixture, the latter having the advantage of being more parsimonious as the dimensions increases.

2.1 THE STATIC SPECIFICATION

Let x_{1t}, x_{2t} the two asset returns series. The static specification of the Copula-GARCH(1,1) is given by the following characteristics:

- t
he dynamics of x_1 is described by a GARCH(1,1) process with leptokurtic innovations:

$$\begin{cases} x_{1t} = \mu_1 + \varepsilon_{1t} \\ \varepsilon_{1t} = \sigma_{1t} z_{1t} \\ z_{1t} \quad iid \quad t(\nu_1) \\ \sigma_{1t}^2 = \omega_1 + \alpha_1 \varepsilon_{1t-1}^2 + \beta_1 \sigma_{1t-1}^2 \end{cases}, \qquad (4a)$$

- t
he dynamics of x_2 is described by a GARCH(1,1) process with leptokurtic innovations:

$$\begin{cases} x_{2t} = \mu_2 + \varepsilon_{2t} \\ \varepsilon_{2t} = \sigma_{2t} z_{2t} \\ z_{2t} \quad iid \quad t(\nu_2) \\ \sigma_{2t}^2 = \omega_2 + \alpha_2 \varepsilon_{2t-1}^2 + \beta_2 \sigma_{2t-1}^2 \end{cases}, \qquad (4b)$$

- t
he dependency structure between the innovations z_{1t}, z_{2t} is modeled by a copula function C_{θ_t}, characterized by the vector of parameters θ_t;
- t
he dependency structure *does not vary in time:* $\theta_t = \theta$.

To model the dependency structure we employ the following copula functions:

- t

he Gaussian copula ($\theta := \rho$):

$$C_{\rho}^{Gauss}(u_1,u_2)=\phi_{2,\rho}\left(\phi^{-1}(u_1),\phi^{-1}(u_2)\right), \tag{5a}$$

where ϕ is the cdf of the standard normal distribution, and $\phi_{2,\rho}$ the cdf of the bi-dimensional normal distribution with correlation ρ.

❖ The t-copula ($\theta := (\nu,\rho)$):

$$C_{\nu,\rho}^{t}(u_1,u_2)=t_{2,\nu,\rho}\left(t_{\nu}^{-1}(u_1),t_{\nu}^{-1}(u_2)\right), \tag{5b}$$

where t_ν is the cdf of the t distribution with ν degrees of freedom, and $t_{2,\nu,\rho}$ the cdf of the bi-dimensional t distribution with correlation ρ.

❖ Frank copula ($\theta := \alpha$, $\alpha \neq 0$):

$$C_{\alpha}^{F}(u_1,u_2)=-\frac{1}{\alpha}\ln\left(1+\frac{\left(e^{-\alpha u_1}-1\right)\left(e^{-\alpha u_2}-1\right)}{e^{-\alpha}-1}\right), \tag{5c}$$

❖ Clayton copula ($\theta := \alpha$, $\alpha > 0$):

$$C_{\alpha}^{C}(u_1,u_2)=\max\left(\left[u_1^{-\alpha}+u_2^{-\alpha}-1\right]^{-\frac{1}{\alpha}},0\right), \tag{5d}$$

❖ Gumbel copula ($\theta := \alpha$, $\alpha > 1$):

$$C_{\alpha}^{G}(u_1,u_2)=\exp\left(-\left[(-\ln u_1)^{\alpha}+(-\ln u_2)^{\alpha}\right]^{\frac{1}{\alpha}}\right), \tag{5e}$$

❖ Gumbel-Clayton mixture ($\theta := (\omega,\alpha_1,\alpha_2)$):

$$C_{\omega,\alpha_1,\alpha_2}^{mix}(u_1,u_2)=\omega C_{\alpha_1}^{C}(u_1,u_2)+(1-\omega)C_{\alpha_2}^{G}(u_1,u_2), \tag{5f}$$

As was already mentioned, the main advantage of the Archimedean copulas (Frank, Clayton, Gumbel and the G-C mixture) resides in the fact that the number of the parameters is constant no matter the dimension of the model.

2.2 THE DYNAMIC SPECIFICATION

The dynamic specification of the Copula-GARCH(1,1) is given by the following characteristics:

❖ The dynamics of x_1 is described by a GARCH(1,1) process with leptokurtic innovations given by equation (4a);

❖ The dynamics of x_2 is described by a GARCH(1,1) process with leptokurtic innovations given by equation (4b);

❖ The dependency structure between the innovations z_{1t}, z_{2t} is modeled by a copula function C_{θ_t}, characterized by the vector of parameters θ_t;

❖ The dependency structure *may vary in time*, the parameters of the copula function having a dynamics given by:

$$\theta_t = f(\theta_{t-1}, z_{1t-1}, z_{2t-1}). \tag{6a}$$

Although several alternatives were tested for the dynamics of the copula parameters, we arrived at the conclusion that the most appropriate specification is of the form:

$$\theta_t = T(a_1 + a_2\theta_{t-1} + a_3 z_{1t-1} z_{2t-1}), \tag{6b}$$

where $T(\cdot)$ is a proper transform than ensures that the parameter is inside the existence interval of the copula function.

The model can be extended to encompass a more general specification of the dynamics of the copula parameters:

$$\theta_t = T\left(a_0 + \sum_{i=1}^{p} a_{1i}\theta_{t-i} + \sum_{i=1}^{q} a_{2i} z_{1t-i}^2 + \sum_{i=1}^{r} a_{3i} z_{2t-i}^2 + \sum_{i=1}^{s} a_{4i} z_{1t-i} z_{2t-i}\right), \tag{6c}$$

In the dynamic specification of the model we employ only the three copula functions that fit best the data from Eastern European markets (Necula, 2009b): Frank copula, t-copula and Gumbel-Clayton copula. The specification of the dynamics of the parameters of these copula functions is as follows:

- Frank copula C_{α_t}:

$$\alpha_t = T(a_0 + a_1 z_{1t-1} z_{2t-1}), \tag{7}$$

where $T(x) = x$;

- Gumbel-Clayton mixture:

$$C_{\omega_t, \alpha C_t, \alpha G_t} = \omega_t C_{\alpha C_t}^{Clayton} + (1-\omega_t) C_{\alpha G_t}^{Gumbel}, \tag{8a}$$

$$\begin{cases} \theta_t = ct = \hat{\theta} \\ \alpha G_t = ct = \hat{\alpha} G \\ \alpha C_t = T(a_0 + a_1 z_{1t-1} z_{2t-1}) \end{cases}, \tag{8b}$$

where $T(x) = x^2$, and $\hat{\theta}, \hat{\alpha}G$ are the estimated values obtained in the static specification of the model; therefore, in this case, only the parameter of the Clayton copula is allowed to vary in time.

- t-copula C_{ν_t, ρ_t}:

$$\begin{cases} \nu_t = ct = \hat{\nu} \\ \rho_t = T(a_0 + a_1 z_{1t-1} z_{2t-1}) \end{cases}, \tag{9}$$

where $T(x) = \tanh\left(\frac{x}{2}\right)$, $\hat{\nu}$ is the estimated value obtained in the static specification of the model, and only the dynamics of the correlation parameter is studied.

In the following section we econometrically estimate the two specifications of the model and analyze whether the dynamic specification is statistically superior to the static one.

3. ESTIMATION RESULTS

The data used in the study consists of daily returns between January 1998 and March 2009 for stock indexes from two Eastern European emerging markets, Czech Republic

(PX50) and Hungary (BUX) and from two developed financial markets, Germany (DAX) and USA (SP500).

The methodology for estimating the Copula-GARCH model for the pair PX50 and BUX, and the pair DAX and SP500 consists of the following steps:

- ❖ estimating the GARCH(1,1) model for each of the two data series using Maximum Likelihood Estimation, and obtaining the residuals;
- ❖ estimating the parameters of the copula function by maximizing the likelihood function, $L = \sum_{t=1}^{T} \ln c\left(F_1\left(\hat{z}_{1t}\right), F_2\left(\hat{z}_{2t}\right); \theta_t\right)$, where $c(u_1, u_2) = \frac{\partial^2 C}{\partial u_1 \partial u_2}(u_1, u_2)$ is the so-called copula density, $\hat{z}_{1t}, \hat{z}_{2t}$ the two residual series, and F_1, F_2 are the cdfs of the residuals.

Therefore, we estimated the parameters of the Copula-GARCH by using the Inference Functions for Marginals (IFM) method (Yan, 2006). This method provides consistent estimators for the parameters of the copula and it is less computing intensive than the Exact Maximum Likelihood (EML). The econometric methods and techniques employed in the study have been implemented in Maple.

First we estimate the static specification. The estimated parameters of the copula functions of the residuals series are presented in Table 1 and in Table 2[2].

Table 1

Estimated parameters for BUX - PX50 GARCH residuals

Copula	Parameters			AIC	GoF statistics	
					KS	AD
Frank	4.3459*** (0.2115)			-412.22	0.0319	3.1472
Clayton	0.8743*** (0.0471)			-315.06	0.0672	8.2341
Gumbel	1.5658*** (0.0343)			-366.85	0.0521	5.7926
G-C mixture	0.3702*** (0.0665)	1.2421*** (0.1951)	1.6852*** (0.0651)	-403.22	0.0424	3.6671
Gaussian	0.5652*** (0.0162)			-398.12	0.0551	7.1321
t	8		0.5806*** (0.0197)	-418.39	0.0411	3.8231

*standard errors in parenthesis; *** denotes statistical significance at 1%; AIC is the Akaike Information Criterion statistic; KS and AD are the Kolmogorov-Smirnov and Anderson-Darling goodness of fit tests statistics*

The parameters of the copula functions are highly statistically significant in both cases. To better assess the performance of each specific copula function we implemented the Kolmogorov-Smirnov test and Anderson-Darling test for copula goodness-of-fit (Fermanian, 2005).

[2] The estimation results of the GARCH models can be provided upon request.

Table 2

Estimated parameters for DAX - SP500 GARCH residuals

Copula	Parameters			AIC	GoF statistics	
					KS	AD
Frank		3.9444*** (0.2283)		-320.42	0.0447	2.7880
Clayton		0.7727*** (0.0611)		-308.59	0.0657	7.7741
Gumbel		1.6054*** (0.0427)		-310.94	0.0427	2.7381
G-C mixture	0.2149*** (0.0575)	1.2269*** (0.6935)	1.8933*** (0.0718)	-330.22	0.0212	0.5426
Gaussian		0.5574*** (0.0202)		-315.23	0.0557	5.1065
t		5	0.5628*** (0.0234)	-348.13	0.0344	1.5933

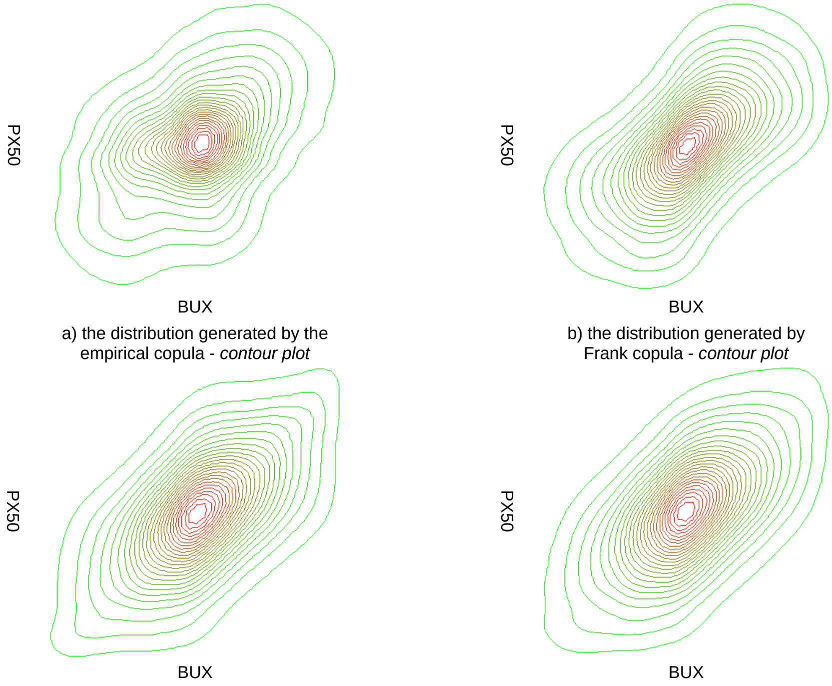
*standard errors in parenthesis; *** denotes statistical significance at 1%; AIC is the Akaike Information Criterion statistic; KS and AD are the Kolmogorov-Smirnov and Anderson-Darling goodness of fit tests statistics*

More specifically, this kind of copula goodness-of-fit tests are based on the assumption that, under the null of a correctly specified copula function, an appropriate transformation of the residuals is $\chi^2(2)$ distributed, fact that can be tested using the standard Kolmogorov-Smirnov and Anderson-Darling nonparametric tests for equality between two one-dimensional distributions. Using the Akaike Information Criterion (AIC) and the two goodness-of-fit statistics we can conclude that, in the case of the two returns pairs analyzed in the study, the Frank copula, the t-copula and the Gumbel-Clayton mixture copula are the most appropriate to model the dependency structure of the normalized residuals from the two one-dimensional GARCH(1,1) processes.

Figure 1 depicts the estimated bi-dimensional distribution for BUX-PX50 GARCH residuals using the best three estimated parametrical copulas. For comparison, the bi-dimensional distribution computed using the so-called empirical copula is also depicted. The empirical copula (Deheuvels, 1979) was estimated using non-parametric econometric techniques (Gijbels and Mielniczuk, 1990; Fermanian and Scaillet, 2003). The kernel empirical copula ($\hat{C}$) is given by $\hat{C}(u_1,u_2)=\frac{1}{T}\sum_{i=1}^{T} G_{u_1,h}\left(\frac{u_1-\hat{F}_1(x_1^i)}{h}\right) G_{u_2,h}\left(\frac{u_2-\hat{F}_2(x_2^i)}{h}\right)$, where $G_{u,h}(\cdot)$ is the Gaussian kernel with bandwidth h, and $\hat{F}_1,\hat{F}_2$ are the empirical cdfs of the two marginal distributions, estimated by non-parametric one-dimensional kernel methods. The length of the bandwidth was chosen according to the well-known "rule of thumb" of Silverman (1986).

Figure 1

Estimated bi-dimensional distributions for BUX-PX50 GARCH residuals



a) the distribution generated by the empirical copula - *contour plot*

b) the distribution generated by Frank copula - *contour plot*

c) the distribution generated by the G-C mixture copula - *contour plot*

d) the distribution generated by the t- copula - *contour plot*

Next we estimate the dynamic specification of the model. In this specification we only analyze the three copula functions that best fitted the static specification: the Frank copula (eq. 7), the Gumbel-Clayton mixture (eq. 8b), and the t-copula (eq. 9). The results are presented in Table 3 and Table 4.

Table 3

Estimated parameters of the dynamics equation for BUX - PX50 pair

Copula	**Parameters**		**AIC dynamic specification**	**AIC static specification**
	a_0	a_1		
Frank	4.0652*** (0.2091)	0.4326*** (0.1640)	-415.23	-412.22
G-C mixture	1.0321*** (0.0872)	0.2341*** (0.0649)	-413.41	-403.22
t	1.2623*** (0.0549)	0.1065*** (0.0421)	-422.74	-418.39

*standard errors in parenthesis; *** denotes statistical significance at 1%; AIC is the Akaike Information Criterion statistic*

Table 4
Estimated parameters of the dynamics equation for DAX-SP500 pair

Copula	Parameters		AIC dynamic specification	AIC static specification
	a_0	a_1		
Frank	3.6736*** (0.2265)	0.6870*** (0.1914)	-310.29	-300.42
G-C mixture	0.4564*** (0.0069)	0.6484*** (0.1008)	-361.56	-330.22
t	1.2244*** (0.0679)	0.1438*** (0.0464)	-352.86	-348.13

*standard errors in parenthesis; *** denotes statistical significance at 1%; AIC is the Akaike Information Criterion statistic*

The parameters of the dynamic equation of the parameters of the copula functions are highly statistically significant in both cases. The Akaike Information Criterion statistic implies that the dynamic specification fits the data better than the static one.

4. CONCLUDING REMARKS

In the present paper we developed a new two-dimensional Copula-GARCH model. To account for heavy tails we modeled the marginal densities using a GARCH(1,1) process with innovations drawn from a t-Student distribution, but the model can be easily extended to employ more sophisticated leptokurtic distributions such as α- stable distributions or Generalized Hyperbolic Distributions. The static specification of the model assumes that the dependency structure of the two data series does not vary in time implying that the parameters of the copula function are constant. On the other hand, the dynamic specification models explicitly the dynamics of these parameters.

We econometrically estimated the two specifications using various copula functions, focusing on the mixture between the Gumbel and Clayton copulas. Models based on this kind of mixtures capture tail dependence and can be easily extended to more dimensions. The estimation of the copula function parameters was performed using the Inference Functions for Marginals (IFM) method. For the static specification, according to the Kolmogorov-Smirnov and Anderson-Darling „goodness-of-fit" tests, the mixture between the Clayton copula and the Gumbel copula, the Frank copula, as well as the t-Student copula are the appropriate copula functions to capture the dependency structure of the two normalized residuals series. For the dynamic Copula-GARCH model we analyzed various specifications of the dynamics of copula parameters opting for the parsimonious ones. According to the Akaike Information Criterion the dynamic Copula-GARCH model outperforms the static one.

This result implies that including a varying dependency structure may improve the estimation of market risk using Monte Carlo method due to improvements in the consistency of the simulations of future evolution paths of the two prices. As further research we intend to develop a framework for market risk assessment of a portfolio under the assumption that the returns of the assets follow the dynamic specification of the Copula-GARCH model.

REFERENCES

Blyth, S. (1996), "Out of line", *Risk*, 9, 10, 82-84.

Bollerslev, T. (1986), "Generalized Autoregressive Conditional Heteroskedasticity," *Journal of Econometrics,* 31:307–327.

Cherubini, U., Luciano, E.; and W. Vecchiato, (2004), *Copula Methods in Finance*, Wiley.

Deheuvels, P. (1979), "La fonction de dependance empirique et ses proprietes. Un test non paramtrique d'independence," *Acad. Roy. Belg. Bull. Cl. Sci.,* 65:274-292.

Engle, R. (1982), "Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation," *Econometrica,* 50:987–1007.

Fermanian, J. D. (2005), "Goodness-of-fit tests for copulas," *Journal of Multivariate Analysis* 95:119-152.

Fermanian, J. D. and O. Scaillet (2003), "Nonparametric estimation of copulas for time series", *Journal of Risk*, 5:25–54.

Gijbels, I., and J. Mielniczuk, (1990), "Estimating the density of a copula function," *Comm. Statist. Theory Methods*, 19:445-464.

Hu, L. (2006), "Dependence Patterns across Financial Markets," *Applied Financial Economics,* 10:717-729.

Jondeau, E., and M. Rockinger, (2006), "The Copula-GARCH model of conditional dependencies," *Journal of International Money and Finance,* 25:827-853.

Necula, C. (2009a), "Modeling Heavy Tailed Stock Index Returns Using the Generalized Hyperbolic Distribution"*, Romanian Journal of Economic Forecasting,* 10(2):118-131.

Necula, C. (2009b), "Modeling the Dependency Structure of Stock Index Returns Using a Copula Function Approach"*, Romanian Journal of Economic Forecasting,* forthcoming.

Nelsen, R.B. (1999), *An Introduction to Copulas*. Springer-Verlag.

Patton, A. (2006a), "Modeling Asymmetric Exchange Rate Dependence," *International Economic Review,* 47:527-556.

Patton, A. (2006b), "Estimation of Multivariate Models for Time Series of Possibly Different Lengths," *Journal of Applied Econometrics,* 21:147-173.

Shaw, J. (1997), "Beyond VAR and Stress Testing", in: S. Grayling (Ed), *VaR: Understanding and Applying Value at Risk*, London: Risk Publications, 211-224.

Silverman, B.W. (1986), *Density Estimation for Statistics and Data Analysis*, Chapman and Hall.

Sklar, A. (1959), "Fonctions de répartition à n dimensions et leurs marges," *Publications de l'Institut de Statistique de l'Université de Paris* 8:229–231.

Yan, J. (2006), "Multivariate Modeling with Copulas and Engineering Applications", in: H. Pham (Ed.), *Handbook of Engineering Statistics*, Springer, 973-990.

COPULA-GARCH MODEL

SAŽETAK

U ovom smo istraživanju razvili novi dvodimenzionalni Copula-GARCH model. Ovu vrstu dvodimenzionalnih procesa karakterizira zavisna struktura stvorena koristeći spojnu funkciju (kopulu). Za marginalne gustoće koristili smo GARCH(1,1) model s inovacijama preuzetim iz t-Student distribucije. Model se može lako proširiti koristeći sofisticiranije procese za marginalne gustoće. Statička specifikacija modela pretpostavlja da zavisna struktura dva niza podataka ne varira u vremenu te tako podrazumijeva da su parametri spojne funkcije konstantni. S druge strane, dinamička specifikacija eksplicitno određuje dinamiku ovih parametara. Ekonometrijski procjenjujemo parametre dvije specifikacije koristeći razne spojne funkcije, uz naglasak na mješavinu između Gumbelove i Claytonove kopule. Koristili smo dnevne indekse zarade s dva razvijena i dva financijska tržišta u razvoju. Glavni nalaz upućuje na to da uključivanje promjenjive zavisne strukture poboljšava sukladnost distribucije Copula-GARCH modela.

Ključne riječi: *spojne funkcije, multidimenzionalni GARCH, volatilnost, zavisna struktura*