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LOCAL ENTROPY GENERATION DURING STEADY HEAT CONDUCTION THROUGH A PLANE WALL

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Preliminary notes

A non-dimensional analytical model of local and total entropy generation during steady heat conduction through a plane wall is developed in the paper. Relevant non-dimensional variables used in the model are the variable x/δ and the ratio of boundary thermodynamic temperatures T_{s1}/T_{s2}. The influence of these variables on the value of the local entropy generation is quantified first, and then the values of partial derivatives of local and total entropy generation with respect to the mentioned variables are quantified. The analytical expression of the relative local entropy generation (with respect to total entropy generation) is also given. The boundary values of the given variables are also quantified for all considered cases. The results of calculations are presented in the appropriate diagrams.

Keywords: *analytical model, entropy generation, plane wall, steady heat conduction*

Lokalna entropijska produkcija pri stacionarnom provođenju topline kroz ravnu stijenku

Prethodno priopćenje

U radu je razvijen bezdimenzijski analitički model lokalne i ukupne entropijske produkcije pri stacionarnom provođenju topline kroz ravnu stijenku. Kao relevantne bezdimenzijske varijable u modelu se javljaju varijabla x/δ i omjer graničnih (rubnih) temperatura T_{s1}/T_{s2}, pa je prvo kvantificiran njihov utjecaj na vrijednost lokalne entropijske produkcije. Nadalje je kvantificirana vrijednost parcijalnih derivacija lokalne (ukupne) entropijske produkcije o navedenim varijablama. Dan je i analitički izraz normirane (relativne) lokalne entropijske produkcije prema ukupnoj entropijskoj produkciji. Kvantificirane su i vrijednosti za sve razmatrane slučajeve za granične vrijednosti navedenih varijabli. Rezultati proračuna prikazani su u odgovarajućim dijagramima.

Ključne riječi: *analitički model, entropijska produkcija, ravna stijenka, stacionarno provođenje topline*

1 Introduction
Uvod

In many formulations of entropy analysis of different processes, the following terms are used: entropy change, entropy transfer and entropy generation. In this paper, the entropy generation is analytically quantified on one example of irreversible process.

Heat transfer between two systems of different temperatures is one of typical irreversible processes. The most interpreted model is that in which both systems have constant temperatures and the total entropy generation is given as a function of these temperatures and the exchanged heat. In these models, the way of the heat exchange from one system to another is not presented, and furthermore, the way in which the entropy increase is generated is not described. If the considered problem is steady heat conduction through a plane wall, which is often the case in practice, it can be presumed that one system has a constant temperature of one wall, the other system has a constant temperature of the other wall, and between the systems there is a plane wall of the given thickness, the thermal conductivity and the wall area. During the steady heat conduction through the plane wall, the temperature field is a linear function and the local entropy production occurs at the infinite temperature difference, between two differential layers of the plane wall. The sum (integral) of these differential changes draws upon the total entropy generation which is presented in this paper.

2 Mathematical model
Matematički model

As expressed in the introduction, the model can be considered as a closed system, and the entropy equation can be written as:

$$\dot{S}_{\text{gen}} = \Delta\dot{S}_{\text{iz.syst}} = \dot{S}_2 - \dot{S}_1 \quad (1)$$

The isolated system consists of a plane wall of thickness δ, thermal conductivity λ and of the wall area A which is perpendicular to the direction of the heat transfer rate. The temperature of one boundary surface is constant and equal to T_{s1}, and of the other is equal to T_{s2}, and these are also constant temperatures of the system 1 and the system 2 which are separated by the plane wall (Fig. 1). The next assumption is that the whole plane wall is in ideal contact with the system 1 and with the system 2.

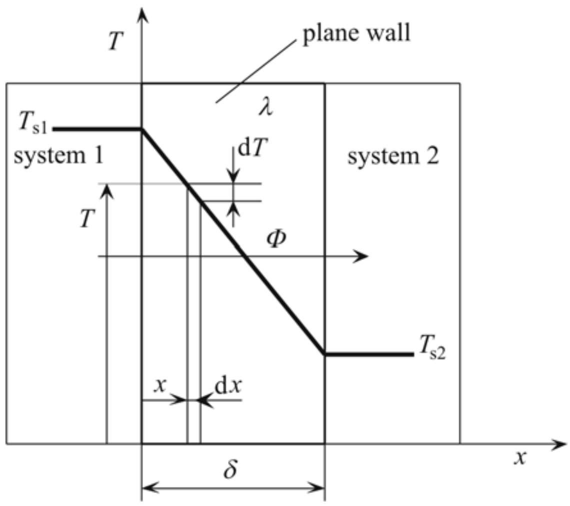

Figure 1 *Temperature distribution in the plane wall which is in the ideal contact with the system 1 and system 2*
Slika 1. *Temperaturna distribucija u ravnoj stijenci koja je u idealnom kontaktu sa sustavom 1 i sustavom 2*

According to [1], the expression for the local entropy generation in the case of one-dimensional steady heat conduction through the plane wall is:

$$\dot{S}_{\mathrm{gen}}''' = \lambda \cdot \frac{\left(\frac{\mathrm{d}T}{\mathrm{d}x}\right)^2}{T^2}. \quad (2)$$

The equation for the steady temperature field through the plane wall, according to [2] is

$$T = T(x) = C_1 \cdot x + C_2. \quad (3)$$

After the operations of deriving and integration are done, the integral form of the local entropy generation follows

$$\dot{S}_{\mathrm{gen}} = \lambda \cdot A \cdot C_1^2 \cdot \int_0^x \frac{\mathrm{d}x}{(C_1 \cdot x + C_2)^2}. \quad (4)$$

When the integration is performed and the boundaries are inserted, the following is obtained

$$\dot{S}_{\mathrm{gen}} = \frac{\lambda \cdot A \cdot C_1}{C_2} \cdot \left(1 - \frac{C_2}{C_1 \cdot x + C_2}\right). \quad (5)$$

The constants C_1 and C_2 can be expressed with the given boundary temperatures of the walls, and they are, according to [2]

$$C_1 = -\frac{T_{\mathrm{s1}} - T_{\mathrm{s2}}}{\delta}, \quad (6a)$$

$$C_2 = T_{\mathrm{s1}}. \quad (6b)$$

By inserting the equations (6a) and (6b) into the equation (5), and by introducing the variable u

$$u = \frac{T_{s1}}{T_{\mathrm{s2}}} \quad (7)$$

the expression for the local entropy generation in the non-dimensional form is

$$\frac{\dot{S}_{\mathrm{gen}}}{\frac{\lambda \cdot A}{\delta}} = \frac{(u-1)^2}{u} \cdot \frac{\frac{x}{\delta}}{u - (u-1) \cdot \frac{x}{\delta}}. \quad (8)$$

From the equation (8), it can be seen that the non-dimensional local entropy generation depends on the temperature ratio u, and the non-dimensional variable x/δ. For $x=0$, the value of the local entropy generation is equal to zero, while for $x/\delta = 1{,}0$, for the given u, the maximal value of the local entropy generation is obtained, which is

$$\left(\frac{\dot{S}_{\mathrm{gen}}}{\frac{\lambda \cdot A}{\delta}}\right)_{\max} = \frac{(u-1)^2}{u}. \quad (9)$$

The dimensional form of the maximal entropy generation is obtained if the expression (7) is inserted into the equation (9)

$$\dot{S}_{\mathrm{gen,max}} = \frac{\lambda \cdot A}{\delta} \cdot \frac{(T_{\mathrm{s1}} - T_{\mathrm{s2}})^2}{T_{\mathrm{s1}} \cdot T_{\mathrm{s2}}}. \quad (10)$$

For the steady heat conduction, the heat transfer rate is equal to

$$\Phi = \frac{\lambda \cdot A}{\delta} \cdot (T_{\mathrm{s1}} - T_{\mathrm{s2}}) \quad (11)$$

and the equation (10) can be rewritten in the following form:

$$\dot{S}_{\mathrm{gen,max}} = \Phi \cdot \frac{T_{\mathrm{s1}} - T_{\mathrm{S2}}}{T_{\mathrm{s1}} \cdot T_{\mathrm{s2}}} = -\frac{\Phi}{T_{\mathrm{s1}}} + \frac{\Phi}{T_{\mathrm{s2}}}. \quad (12)$$

The equation (12) presents the entropy generation between two systems of different but constant temperatures, and it is well known and given in many reference books on thermodynamics, like in [3]. The location of the entropy generation cannot be seen in this expression, because it is assumed that between the two systems there is a wall of infinite small thickness, of negligible mass and negligible physical properties. Thereby, there is a temperature jump from the system 1 to the system 2, and accordingly the entropy is generated on the contact wall which is presented in Fig. 2.

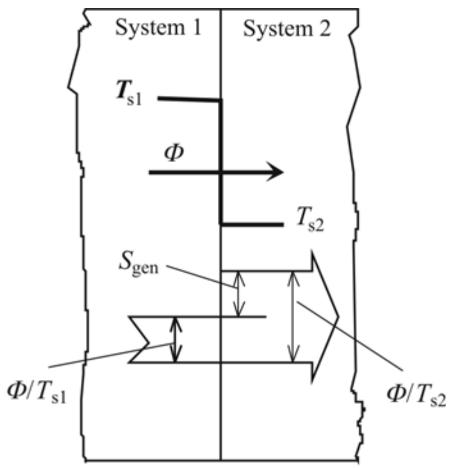

Figure 2 *Qualitative view of entropy generation between two systems with negligible thickness of intermediate wall*
Slika 2. *Kvalitativni prikaz entropijske produkcije između dvaju tijela uz zanemarenje razdijelne stijenke*

It is interesting to determine the limit of the equation (8) when $u \to \infty$. The limit of the local entropy generation follows from the expression:

$$\lim_{u\to\infty}\left(\frac{\dot{S}_{\mathrm{gen}}}{\frac{\lambda \cdot A}{\delta}}\right) = \lim_{u\to\infty}\left(\frac{(u-1)^2}{u} \cdot \frac{\frac{x}{\delta}}{u - (u-1) \cdot \frac{x}{\delta}}\right) = \frac{\frac{x}{\delta}}{1 - \frac{x}{\delta}}, \quad (13)$$

which shows that the local entropy generation depends on the value of non-dimensional variable x/δ.

By dividing the equation (8) with the equation (9), the expression for the relative entropy generation is obtained

$$\frac{\dot{S}_{\mathrm{gen}}}{\dot{S}_{\mathrm{gen,max}}} = \frac{\frac{x}{\delta}}{u - (u-1) \cdot \frac{x}{\delta}}. \quad (14)$$

For $x/\delta = 0$, the value of the above expression is equal to zero, and for $x/\delta = 1$, the value is equal to 1. If $u \rightarrow \infty$, which value does the above expression tend to? If $(u-1)x/\delta < u$, that means that for $u \rightarrow \infty$, the above expression tends to zero.

3
The diagrams and the interpretation of the results
Dijagrami i interpretacija rezultata

The diagram in Fig. 3 shows the equation (8). The values $0 \leq x/\delta \leq 1$ are shown in the x-axis and the non-dimensional entropy generation $\dot{S}_{\text{gen}}/(\lambda \cdot A/\delta)$ in the y-axis. Parametric curves are shown for the values of u = 1, 2, 3, 4 and 5.

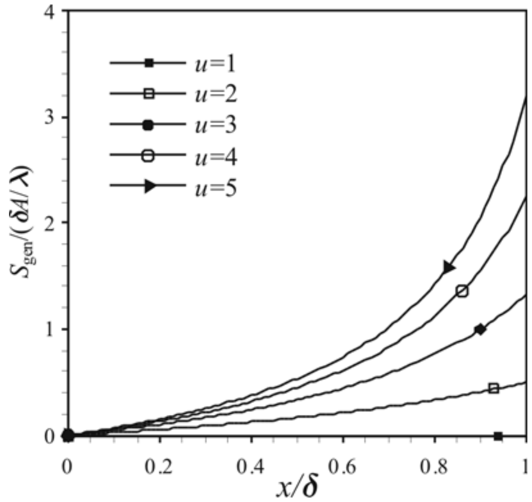

Figure 3 *Non-dimensional entropy generation as a function of non-dimensional variable x/δ for the values of u=1, 2, 3, 4 and 5.*
Slika 3. *Bezdimenzijska entropijska produkcija u ovisnosti od bezdimenzijske varijable x/δ za vrijednosti u=1, 2, 3, 4 i 5.*

The diagram shows the increase of the local entropy generation with the increase of the variable x/δ, and with the increase of the value of u.

The influence of the above values on the local entropy generation can be analytically shown if the equation (13) is derived with respect to variables x/δ and u.

$$\frac{\mathrm{d}}{\mathrm{d}\dfrac{x}{\delta}}\left(\frac{\dot{S}_{\text{gen}}}{\dfrac{\lambda \cdot A}{\delta}}\right)_u = \left(\frac{u-1}{u-(u-1)\cdot\dfrac{x}{\delta}}\right)^2 . \tag{15}$$

The diagram in Fig. 4 shows the equation (15) from which it can be seen that the rate of change of the local entropy generation with the variable x/δ continuously increases with the increase of x/δ, and with the increase of the value of u.

For the values of $x/\delta = 0$, and $x/\delta = 1{,}0$ (the boundaries of the interval), the values of the rate of change of the local entropy generation are defined with the following equations:

$$\frac{\mathrm{d}}{\mathrm{d}\dfrac{x}{\delta}}\left(\frac{\dot{S}_{\text{gen}}}{\dfrac{\lambda \cdot A}{\delta}}\right)_{x/\delta=0} = \left(\frac{u-1}{u}\right)^2 , \tag{16}$$

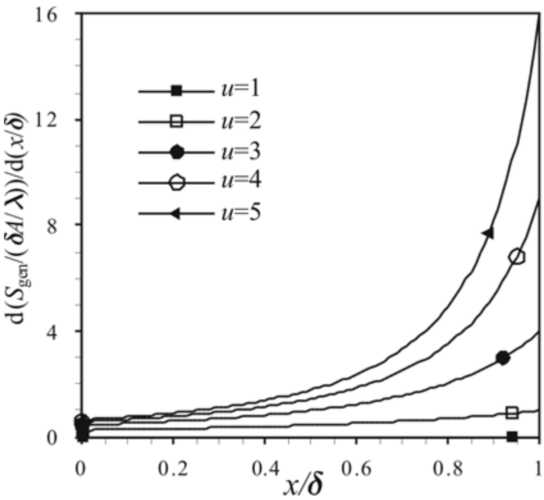

Figure 4 *The rate of change of local entropy generation with the variable x/δ*
Slika 4. *Brzina promjene lokalne entropije po varijabli x/δ*

$$\frac{\mathrm{d}}{\mathrm{d}\dfrac{x}{\delta}}\left(\frac{\dot{S}_{\text{gen}}}{\dfrac{\lambda \cdot A}{\delta}}\right)_{x/\delta=1} = (u-1)^2 , \tag{17}$$

so that for the given value of u, the value of the rate of change of the local entropy generation changes within the values which are defined with the above equations (16) – (17).

If the value of $u \rightarrow \infty$, the value of the rate of change of the entropy generation is

$$\frac{\mathrm{d}}{\mathrm{d}\dfrac{x}{\delta}}\left(\frac{\dot{S}_{\text{gen}}}{\dfrac{\lambda \cdot A}{\delta}}\right)_{u\rightarrow\infty} = \frac{1}{\left(1-\dfrac{x}{\delta}\right)^2} , \tag{18}$$

which means that the value of the above equation is equal to one, for $x/\delta = 0$ and the value tends to infinity for $x/\delta = 1$. It can be concluded that the variable x/δ has a significant influence on the rate of change of the local entropy generation.

By partially deriving the equation (8) with respect to variable u, at x/δ = const, it is possible to get the function of the rate of change of the local entropy generation according to the next equation:

$$\frac{\mathrm{d}}{\mathrm{d}u}\left(\frac{\dot{S}_{\text{gen}}}{\dfrac{\lambda \cdot A}{\delta}}\right)_{x/\delta=\text{const}} = \frac{u-1}{u^2}\cdot\frac{\dfrac{x}{\delta}\cdot\left(\dfrac{x}{\delta}+2u-u\cdot\dfrac{x}{\delta}\right)}{\left(u-(u-1)\cdot\dfrac{x}{\delta}\right)^2} . \tag{19}$$

Fig. 5 shows the above equation.

Fig. 5 shows a very interesting pattern of behaviour, which is manifested in the fact that every parameter curve x/δ = const first increases with the increase of u, reaches maximum (local extreme) and then decreases. The way to get explicitly the local extreme is by using the numerical procedure. The analytical way to determine the local extreme is not possible because of the complexity of the function, which is not given in the paper. Only the parametric curve $x/\delta = 1{,}0$ continuously increases and

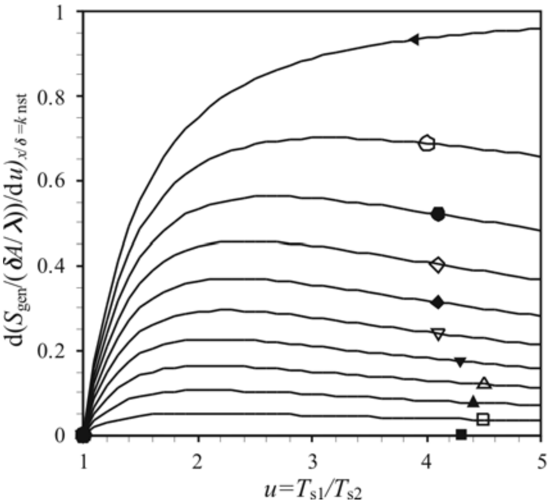

Figure 5 *The influence of the value of u on the rate of change of the local entropy generation*
Slika 5. *Utjecaj iznosa u na brzinu promjene lokalne entropijske produkcije*

reaches the value equal to one when $u \to \infty$. These maximal values and the corresponding stationary points are shown in the diagram in Fig. 6.

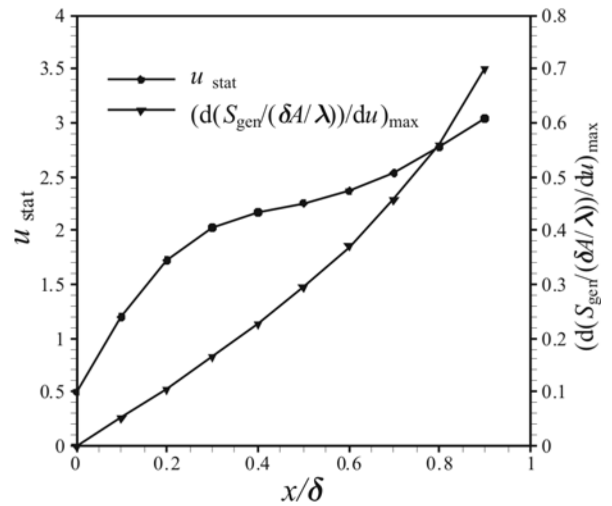

Figure 6 *Maximal rates of changes of local entropy generation (the right hand side axis) and corresponding* u_{stat} *(left hand side axis) as a function of the variable x/δ*
Slika 6. *Maksimalne brzine lokalne entropijske produkcije, (desna ordinatna os) i pripadajući* u_{stat} *(lijeva ordinatna os) u ovisnosti o omjeru x/δ*

The diagram in Fig. 6 shows the continuous increase of the maximal value of the rate of change of local entropy generation (line with triangles) in the interval 0 to 1,0, and also the continuous increase of the stationary values u_{stat}. In the interval $0,1 \le x/\delta \le 0,6$, the increase of the stationary values u_{stat} is very small. Outside the interval, stationary values u_{stat} begin to rapidly increase towards infinity, which is not shown in the diagram in Fig. 6.

Other parametric curves x/δ = const acquire a value equal to zero when $u \to \infty$. It can be easily shown in the following way

$$\lim_{u\to\infty}\left[\frac{u-1}{u^2}\cdot\frac{\frac{x}{\delta}\left(\frac{x}{\delta}+2u-u\cdot\frac{x}{\delta}\right)}{\left(u-(u-1)\cdot\frac{x}{\delta}\right)^2}\right]_{x/\delta=1,0}=\lim_{u\to\infty}\frac{u^2-1}{u^2}=1,0. \qquad (20)$$

$$\lim_{u\to\infty}\left[\frac{u-1}{u^2}\cdot\frac{\frac{x}{\delta}\cdot\left(\frac{x}{\delta}+2u-u\cdot\frac{x}{\delta}\right)}{\left(u-(u-1)\cdot\frac{x}{\delta}\right)^2}\right]_{0\le x/\delta<1,0}=0. \qquad (21)$$

That means that all parametric curves $0 \le x/\delta < 1,0$ would drop down to zero in the diagram in Fig. 5, when $u \to \infty$, only the curve $x/\delta = 1,0$ would, in this case, reach the value one.

From the analysis of the influence of the variables u and x/δ, it can be concluded that the change of the variable x/δ has a much stronger influence on the rate of change of the local entropy generation than the variable u.

For $x/\delta = 1,0$ and given u, the value of the maximal entropy generation is attained, and it is defined by the equation (9). It is clear that for $u = 1$, for each x/δ, the value of the local entropy generation is equal to zero, because it is a case of heat transfer at diminishing temperature difference (reversible process of heat transfer). With the increase of the value u, the value of the local entropy generation also increases, but if $u \to \infty$, the value of the local entropy generation is defined by the equation (13), and it is shown in Fig. 7.

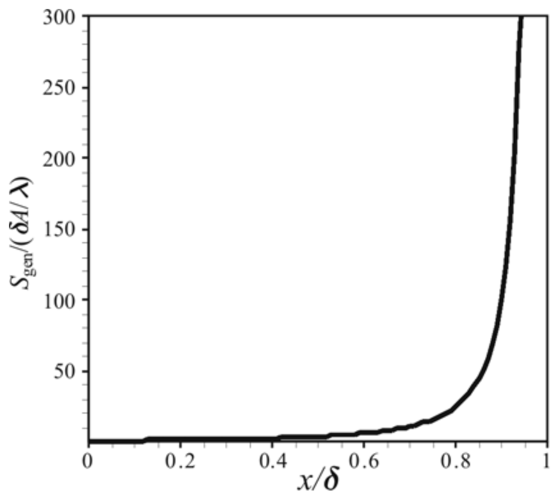

Figure 7 *Local entropy generation as a function of x/δ for u → ∞*
Slika7. *Lokalna entropijska produkcija u ovisnosti x/δ za u → ∞*

From the above diagram, it is obvious that for $u \to \infty$, the local entropy generation slowly increases with the increase of variable x/δ. Only for $0,9 \le x/\delta \le 1,0$ the value of the local entropy generation is increased from 9,0 to ∞, which can be proven by deriving the equation (13) with respect to variable x/δ, and therefore:

$$\frac{\mathrm{d}}{\mathrm{d}\left(\frac{x}{\delta}\right)}\cdot\left(\frac{\frac{x}{\delta}}{1-\frac{x}{\delta}}\right)=\frac{1}{\left(1-\frac{x}{\delta}\right)^2}, \qquad (13)$$

from which it follows that for $x/\delta = 1,0$, the value of the first derivative tends to infinity, i.e. for $x/\delta = 1,0$ the equation (11) has a vertical asymptote, as can be also seen in Fig. 7. The relative entropy generation, the equation (14), is shown in Fig. 8.

The diagram in Fig. 8 shows that the relative entropy generation increases, for each u, from 0 to 1 with the

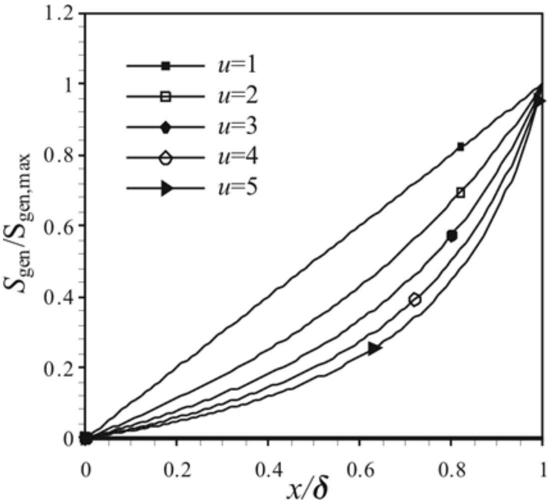

Figure 8 *Relative local entropy generation*
Slika 8. *Normirana (relativna) lokalna entropijska produkcija*

increase of variable x/δ from 0 to 1. With the increase of value u, the values of the relative local entropy generation, for the same x/δ, are decreasing. Furthermore, it can be concluded that with the increase of values of u, the curves u_i = const are closer to each other, i.e. the difference between the values of the relative entropy generation becomes smaller and smaller and, for $u \to \infty$, the value of the relative local entropy generation tends to zero.

4 Conclusion
Zaključak

The acquired analytical model of the non-dimensional local entropy generation at steady heat conduction through the plane wall has crystallized two significant variables: non-dimensional variable x/δ and non-dimensional temperature ratio of the boundary temperatures of the plane wall $u = T_{s1}/T_{s2}$, which is explicitly shown in the equation (8), i.e. as a diagram in Fig. 3, which clearly shows that the local entropy generation continuously increases with the increase of these variables. The local entropy generation occurs when $u \to \infty$, the equation (18), which is shown in the diagram (Fig. 7), where it is indicated that the local entropy generation increases slowly up to $0 \le x/\delta \le 0{,}9$; and for $0{,}9 < x/\delta \le 1{,}0$, the value of the local entropy generation is changed from 10,0 to ∞. For u and $x/\delta = 1{,}0$, the maximal entropy generation is achieved, which also determines the local entropy generation, the equation (14), which is shown in the diagram (Fig. 8), which indicates that the relative entropy generation for each u is from 0 to 1, and it decreases if u increases.

By partial derivation of the equation of the local entropy generation for the variables x/δ and u, it is possible to quantify the rate of change of these variables on the rate of change of the local entropy generation. The rate of change of the local entropy generation continuously increases with the rate of change of x/δ (the equation (15), i.e. diagram in Fig. 4). However, with the rate of change u, the rate of change of the local entropy generation for each parameter value reaches maximum (local extreme), Fig. 5, after which all parameter curves u = const tend to 0, when u_1 tends to infinity. There is monotonous increase of the rate of the local entropy generation only for parameter curve $u = 1{,}0$, which tends to 1 when u_1 tends to infinity. If the diagrams (Figures 4 and 5) are compared, it can be concluded that the rate of the local entropy generation is influenced more by the rate of change of x/δ, than by the rate of change of u_1.

Finally, it can be concluded that the necessary entropy generation is analytically quantified in this paper, and therefore, according to Gouy-Stodole 4th theorem, the total work loss for the model of steady heat conduction through the plane wall.

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Symbols
Simboli

A – area, m^2
C_1 – constant, K/m
C_2 – constant, K
δ – the thickness of the plane wall, m
Φ – the rate of heat transfer, W
λ – the thermal conductivity of the plane wall, W/(m·K)
$\dot{S}$ – entropy, W/K
T – thermodynamic temperature, K
$u = T_{s1}/T_{s2}$ – the ratio of the boundary thermodynamic temperatures of the plane wall
x – local variable, m

Note:
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