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OPTIMIZATION OF LAMINATES SUBJECTED TO FAILURE CRITERION

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The paper is aimed on laminate optimization subjected to maximum strain criterion. The optimization problem is based on the use of continuous design variables. The thicknesses of layers with the known orientation are used as design variables. The optimization problem with strain constraints are formulated to minimize the laminate weight. The design of the final thickness is rounded off to integer multiples of the commercially available layer thickness.

Key words: laminate, optimization, failure criterion

Optimizacija laminata ovisno o kriteriju razaranja. Cilj članka je optimizacija laminata ovisno o kriteriju maksimalnog naprezanja (kriterija razaranja). Problem optimizacije je temeljen rabljenjem plana varijabli. Debljine vrsta laminata sa poznavanjem orijentacije je rabljena kao plan varijabli. Optimizacija problema s ograničenom deformacijom je temeljena kao minimalizacija težine laminata. Završne debljine su zaokružene na raznovrsne trgovačke debljine laminata.

Ključne riječi: laminat, optimizacija, kriterij razaranja

INTRODUCTION

The unidirectional fibrous composite materials are the directional dependence of the strength on a macroscopic scale. Composite layers are much stronger in the fiber direction than in the direction perpendicular to the fibers. For loads that are primarily parallel to the fibers, either in the tension or compression, the material strength is generally governed by the failure of the fibers. For loads transverse to the fibers, failure is controlled by the failure of the much weaker matrix material [1].

CLASSICAL LAMINATION THEORY

Similarly to the Euler-Bernoulli beam theory and the plate theory, the classical lamination theory (CLT) can be applied to thin laminates only (the span a and $b > 10 \times$ thickness t) with small displacements w in the transverse direction ($w < t$).

In CLT displacements are considered to be continuous throughout the total thickness of the laminate and the constitutive equations are linear.

The strains relations in the condensed form can be noted as

$$\{\varepsilon(x)\} = \{\bar{\varepsilon}(x,y)\} + z\{\kappa(x,y)\}, \tag{1}$$

where $\bar{\varepsilon}$ is the vector of mid-plane strains and is the vector of curvatures that are constant throughout the thickness.

In modelling a laminate it is assumed that each individual layer of the laminate behaves as a linear elastic material. All layers are bonded together with a perfect bond and each lamina of a composite material behaves macroscopically as a homogeneous orthotropic material.

The constitutive equations can be written in the condensed hyper-matrix form

$$\begin{pmatrix} \{N\} \\ \{M\} \end{pmatrix} = \begin{pmatrix} [A][B] \\ [\mathrm{B}][D] \end{pmatrix} \begin{pmatrix} \{\bar{\varepsilon}\} \\ \{\kappa\} \end{pmatrix} \tag{2}$$

with

$$\{[A],[B],[D],\} = \int_{-h/2}^{+h/2} [E(z)]\{1, z, z^2\} dz. \tag{3}$$

The elasticity matrix can be expressed as

$$[E] = \sum_{n=1}^{N} \frac{{}^n h}{h} [{}^n E],\ [{}^n E] = [\bar{T}({}^n\theta)]^T [{}^n E_L \mathrm{I} \bar{T}({}^n\theta)],$$

$$[\bar{T}({}^n\theta)] = ([T(\theta)]^T)^{-1} \tag{4}$$

where T is the transformation matrix [2].

STRENGTH DESIGN WITH THICKNESS DESIGN VARIABLES

The optimization process is applied to the approximate problem represented by the polynomial approxi-

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mation. The coefficients of the polynomial function are determined by the least squares regression.

For regression analysis the singular value decomposition is used. When the objective function and constraints are approximated and their gradients with respect to the design variables are calculated based on chosen approximation, it is possible to solve the approximate optimization problem. The algorithm SLP (Sequential Linear Programming) method was used for solving the approximate optimization problem. The iterative process of SLP within each optimization loop is shown below:

1. $p=0, X^p=X^m$
2. $p=p+1$
3. Linearize the problem at by creating a first order Taylor Series expansion of the objective function and retained constraints:
$$F(X)=F(X^{p-1})+\nabla F(X^{p-1})(X-X^{p-1})$$
$$g(X)=g(X^{p-1})+\nabla g(X^{p-1})(X-X^{p-1})$$
4. Use this approximation of optimization instead of the original nonlinear functions:
Minimize: $F(X)$
Subject to: $g(X)\leq 0$ and $\overline{X}_i^L \leq X_i \leq \overline{X}_i^U$
5. Find an improved design X^p (using the modified feasible direction method)
6. Check feasibility and convergence. If both of them are satisfying, go to 7. Otherwise, go to step 2.
7. $X^{m+1}=X^p$

Using the SLP method the solving process is iterated until convergence is achieved. Convergence or termination checks are performed at the end of each optimization loop in general optimization. The optimization process continues until either convergence or termination occurs.

It is important to distinguish between the fibre failure (FF) and the inter-fibre failure (IFF). In the case of shear plane stress, the IFF criteria discriminates three different modes [3, 4]. The IFF Mode A is when perpendicular transversal cracks appear in the lamina under transverse tensile stress with or without in-plane shear stress. The IFF Mode B denotes perpendicular transversal cracks, but in this case they appear under in-plane shear stress with small transverse compression stress. The IFF Mode C indicates the start of oblique cracks when the material is under significant transversal compression.

The FF and the three IFF modes yield separate failure indices. The failure index for FF is defined as

$$I_{FF}=\begin{cases}\sigma_1/X_t \;\; of \;\; \sigma_1>0\\ -\sigma_1/X_c \;\; of \;\; \sigma_1<0\end{cases} \tag{5}$$

For IFF with positive transverse stress, Mode A is active. The failure index in this case is defined as

$$I_{IFF,A}=\sqrt{\left(\frac{\tau_{12}}{S}\right)^2+\left(1-p_{6t}\frac{Y_t}{S}\right)^2\left(\frac{\sigma_2}{Y_t}\right)^2}+p_{6t}\frac{\sigma_2}{S}$$

$$if \;\; \sigma_2\geq 0 \tag{6}$$

where $p_{6t}=0{,}3$.

Under negative transverse stress, either Mode B or Mode C is active, depending on the relationship between in-plane shear stress and transversal shear stress. The failure indices are defined as

$$I_{IFF,B}=\frac{1}{S}\left[\sqrt{\tau_{12}^2+(p_{6c}\sigma_2)^2}+p_{6c}\sigma_2\right]$$

$$if\begin{cases}\sigma_2<0\\ \dfrac{\sigma_2}{\tau_{12}}\leq\dfrac{F_{2A}}{F_{6A}}\end{cases} \tag{7}$$

and

$$I_{IFF,C}=\frac{Y_c}{\sigma_2}\left[\sqrt{\left(\frac{\tau_{12}}{2(1+p_{2c})S}\right)^2+\left(\frac{\sigma_2}{Y_c}\right)^2}\right]$$

$$if\begin{cases}\sigma_2<0\\ \dfrac{\sigma_2}{\tau_{12}}\geq\dfrac{F_{2A}}{F_{6A}}\end{cases} \tag{8}$$

where, $p_{6c}=0{,}2$.

The limit between Mode B and Mode C is defined by the relation F_{2A}/F_{6A}, where

$$I_{2A}=\frac{S}{2p_{6c}}\left[\sqrt{1+2p_{6c}\frac{Y_c}{S}}-1\right] \tag{9}$$

$$F_{6A}=S\sqrt{1+2p_{2c}} \quad p_{2c}=p_{6c}\frac{F_{2A}}{S} \tag{10}$$

We will consider the optimal design of a symmetric balanced laminate with fixed orientation angles [2]. Because of the laminate symmetry, only the thicknesses $t_k, k=1,\ldots,I,$ of one-half of the total number of layers, $I=N/2$, are used as design variables. The laminate is considered to be under the action of combined uniform in-plane stress resultants N_x and N_y.

The optimization problem is formulated in the following form:

$$\text{minimize } W=\sum_{k=1}^{I}2\rho_k t_k \tag{11}$$

subject to

$$g_{kj}=(P_j^{(k)}\varepsilon_{1k}+Q_j^{(k)}\varepsilon_{2k}+R_j^{(k)}\gamma_{12k})-1\leq 0 \tag{12}$$

for $k=1,\ldots,I,\quad j=1,\ldots,J,$

where and are the density and the thickness, respectively, of the k-th layer, $P_j^{(k)}$, $Q_j^{(k)}$, $R_j^{(k)}$, are coefficients that define the j-th boundary of a failure envelope for each layer in the strain space, and the $\varepsilon_{1k},\varepsilon_{2k},\varepsilon_{12k}$ are the strains in the principal material direction in the k-th layer. For a maximum strain criterion, which puts bounds on the values of the strains in the principal material directions, the failure envelope has four facets with P and Q defined as an inverse of the normal failure strains in the longitudinal and transverse directions to the fibers, once in tension and once in compression. The coefficient R is the inverse of the shear failure strain for positive shear and for negative shear. The nonlinear programming problem is transformed to a linear by the help of sequential linear programming. The strain constraint of Equation (12) is a nonlinear function of the thickness variables and, therefore, is linearized as:

$$g_{kjL}(t_k) = g_{kj}(t_{k0}) + \sum_{i=1}^{I}(t_i - t_{0i}) \left(P_j^{(k)} \frac{\partial \varepsilon_{1k}}{\partial t_i} + Q_j^{(k)} \frac{\partial \varepsilon_{2k}}{\partial t_i} + R_j^{(k)} \frac{\partial \gamma_{12k}}{\partial t_i} \right) \tag{13}$$

where $\partial\varepsilon_{1k} / \partial t_i, \partial\varepsilon_{2k} / \partial t_i, \partial\gamma_{12k} / \partial t_i$ are the derivatives of the principal material direction strains in the kth layer with respect to the thickness of the ith layer. For a specified in-plane loading, the derivative of the laminate strains with respect to the thickness variables can be determined by differentiating in-plane part of stress-strain relation:

$$\frac{\partial\{N\}}{\partial t_1} = \frac{\partial[A]}{\partial t_1}\{\bar{\varepsilon}\} + [A]\frac{\partial\{\bar{\varepsilon}\}}{\partial t_1} = 0 \tag{14}$$

The derivatives of the mid-plane strains are:

$$\frac{\partial\{\bar{\varepsilon}\}}{\partial t_1} = -[A]^{-1}[\bar{Q}]_{(i)}\{\bar{\varepsilon}\} \tag{15}$$

$$\frac{\partial[A]}{\partial t_1} = [\bar{Q}]_{(i)} \tag{16}$$

The derivatives of the strains in the fiber and transverse to the fiber are calculated from:

$$\frac{\partial\{\bar{\varepsilon}_L\}}{\partial t_1} = -[T]\frac{\partial\{\bar{\varepsilon}\}}{\partial t_1}, \tag{17}$$

where A and T are matrix of in/plane stiffness and transformation matrix, respectively.

The linear approximations to the strain constraints can be constructed using Equation (13) at any step of the sequential linearizations.

NUMERICAL EXAMPLE

Design of thicknesses $[t_1, t_2, t_3, t_4]_S$ of a laminate with orientation of angle $[0/45/-45/90]_S$ under loading N_x = 725,6 kN/m, N_y = 181,4 kN/m. Properties of the layers of AS4/3501-6 Carbon/Epoxy material [3] are:
E_1 = 132,8 GPa, E_2 = 10,6 GPa, ν_{xy} = 0,3, G_{12} = 5,9 GPa.

The maximum strain failure limits for the material are: $\varepsilon_1^t = \varepsilon_1^c = 0{,}0115, \varepsilon_2^t = \varepsilon_2^c = 0{,}00535, \gamma_{12}^s = 0{,}02.$

RESULTS AND DISCUSSION

The optimization process was solved with the help of Sequential Linear Programming method. Inside the SLP method there was used the Modified Feasible Direction (MFD) algorithm. The MFD has the own iterative process within each optimization loop. There were used the Kuhn-Tucker conditions for constrained problem. The objective function value in the first design step is 8,4 N. The general optimization process was stopped after 44 design sets. The objective function value after optimization process is 7,7574 N. In the Figure 1 there are shown changes of the stresses σ_1 during the optimization process. In the Figure 2 there are shown changes of the stresses σ_2 during the optimization process. In the Figure 3 there are shown changes of design variables during the optimization process.

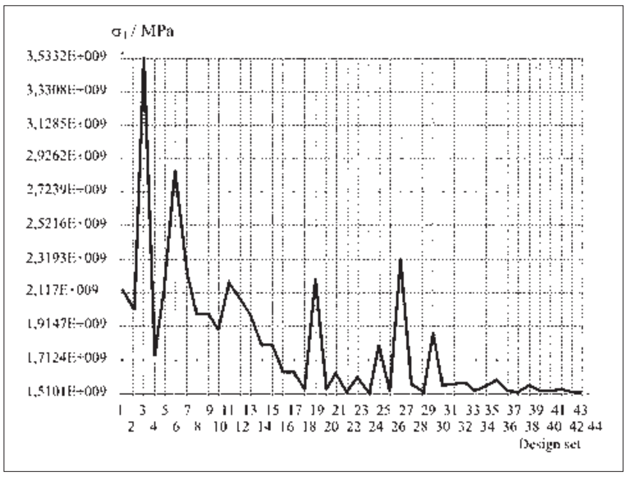

Figure 1 The changes of σ_1 during optimization process

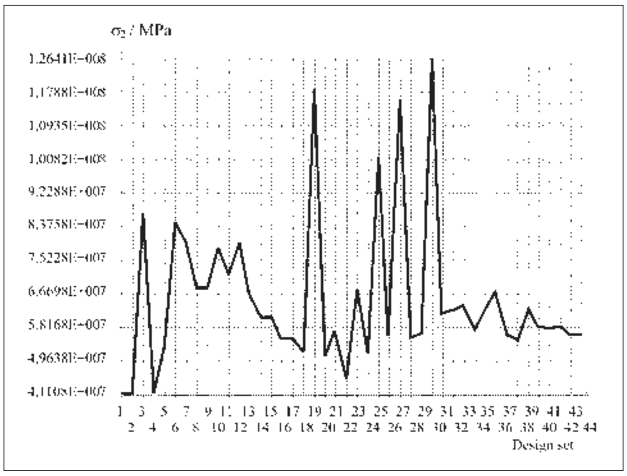

Figure 2 The changes of σ_2 during optimization process

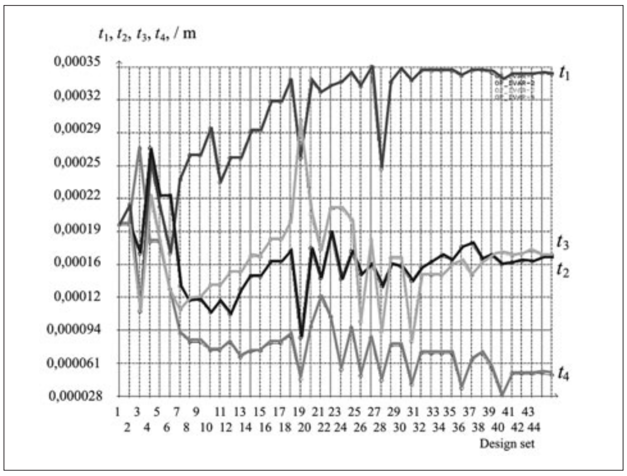

Figure 3 Design variables during the optimization process

CONCLUSION

The paper deals with laminate thickness optimization subjected to maximum strain criterion. Finding a strength design of laminate thickness is the following process.

First, the design was formulated in terms specific to the problem. For the modelling and analysis, the classical laminate theory and Bernoulli/Kirchhoff hypothesis for plates were used [5].

Second, the problem was established as a mathematical optimization problem. In the frame of numerical optimization we made the minimization of weight subject to strain constraints. Design variables were thicknesses of layers of the laminate plate.

Finally, the problem was solved using SLP method in the numerical program. Maximum number of iterations of SLP was 100.

The total thickness of the laminate is 1,4776 mm. The design can be rounded off to 1,5 mm.

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Note: The responsible translator for English language is Martina Pl'uchtová, Košice, Slovakia.