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System Architecture for 3D Gravity Modelling

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Ključne riječi: gravimetrijsko modeliranje, kompleksni modeli, arhitektura softvera.

Abstract

A flexible software architecture for gravity modelling is established and the advantage is discussed of having several alternative programs to handle complex 3D models. The flexible architecture consists of four parts, implemented in a distributed computer environment: the three-dimensional model builders and visualizers (GOCAD software, version 7.0), the model representation translators (GOCAD software or GEOMOD software), the forward simulation algorithms of gravimetric data (applying Talwani-Ewing and Götze-Lahmeyer methods in the finite-element representation class), and the inversion (model updating) scheme manager based on the Cordell-Henderson inversion procedure.

A good software architecture should at least keep the model building and updating software separate from the forward simulation software. Inversion schemes can then be realized by communication between the two parts of software.

Several synthetic cases are shown to demonstrate the use and the capability of the architecture and methods applied. The gravity fields of complex 3D models, i.e. overhanging and non-overhanging salt domes, are simulated. The gravimetric anomalies for both cases have very similar shapes. Gravity modelling can distinguish between these, because the existing mass differences result in anomaly differences both for surface profiles and X-sections. The capability of the inversion procedure is also shown in the discussed synthetic case. The inversion manager is able to create the global structural forms represented as a horizon with constant density contrast (a two-layer model) from residual gravity anomalies.

Sažetak

Razvijena je elastična arhitektura softvera za gravimetrijsko modeliranje, te su izložene prednosti programa koji mogu obradivati kompleksne 3D modele. Softver se sastoji od četiri dijela, koja su ugrađena u odgovarajuće kompjutorsko sučelje: programa za oblikovanje i prikazivanje trodimenzionalnih modela (GOCAD softver, verzija 7.0), programa za formatiranje i izmjenu podataka između različitih softvera (GOCAD softver ili GEOMOD softver), programa s ugrađenim algoritmima za računanje gravimetrijskog djelovanja (Talwani-Ewing i Götze-Lahmeyer metode u području metoda konačnih elemenata) i programa za gravimetrijsku inverziju temeljenog na Cordell-Hendersonovom postupku inverzije.

Smatra se boljom arhitekturom držati odvojenim softver za oblikovanje i izmjenu modela od softvera za računanje gravimetrijskog djelovanja, a postupak inverzije se ostvaruje razmjenom podataka između ta dva dijela.

Na sintetskim primjerima izložena je upotreba i mogućnosti razvijenog softvera i primijenjenih metoda. Prikazana su teoretska gravimetrijska polja solnih doma, koje predstavljaju složene geološke modele. Gravimetrijske anomalije ovih modela imaju vrlo slične obrise, te je jedino gravimetrijskim modeliranjem moguće međusobno razlučiti ove modele zbog različitog rasporeda masa, koje uzrokuju različite vrijednosti anomalija, kako za površinske profile, tako i za X-sekciju. Mogućnosti postupka inverzije su prikazane na priloženom sintetskom primjeru. Postupkom inverzije dobivena je struktura po granici s razlikom gustoće, odnosno za jednostavni dvoslojni model, na temelju rezidualne gravimetrijske anomalije.

1. INTRODUCTION

Gravimetry is applied in hydrocarbon, groundwater, mineral and geotechnical studies. In hydrocarbon studies, gravity modelling is often used on a regional scale to delineate the basin and the main structural characteristics associated with the targets. On a local scale it is used in structural interpretation to define oil and gas structures. However, in detailed reservoir and production modelling gravimetry is not fully utilised, although a lot of different data sets obtained by surface and borehole measurements can be used to more precisely constrain the interpreted structures (Fig. 1).

In order to use all available data sets it is important to have modelling software which should be at least three-dimensional to accurately describe the subsurface properties in space and enable realistic simulations to be formulated. It should be able to process complex and detailed three-dimensional models, as for surface modelling. Therefore for borehole modelling, applying gravimetric algorithms to borehole measurements, can aid calculation of the effects of both surface and borehole locations. The latter are very important in the production phase to constrain structural interpretations of reservoirs on a local scale.

It must be emphasized that high-quality gravity data acquired by high-precision instruments should be avail-

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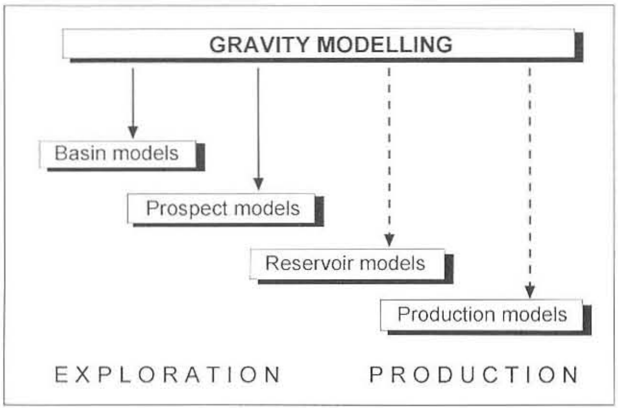

Fig. 1 The effect of gravity modelling on building of the different geological models in practice from exploration to production.

able for successful gravity modelling. On the local scale high-resolution gravity data should be used for gravity modelling of small complex structures. Data spacing depends on the model element size, but it also affects the precision of the regional-residual separation, which is a very critical step in gravity interpretation and significantly controls the final results.

This paper discusses a flexible software architecture and the advantage of having several programs to handle complex three-dimensional models.

2. GRAVITY MODELLING

The problem of gravity interpretation involves the solutions of both the forward and inverse problem. The direct problem or the forward modelling deals with the theoretical calculation of the gravity anomalies for a given model. The inverse problem or gravity inversion deals with the calculation of structural forms for given gravity anomalies. Both problems can be solved by using two different approaches, the space domain approach and the frequency domain or spectral approach.

The "Space Domain Methods" using a volume integral, are frequently applied for the forward calculation of the gravitational attraction of arbitrary shaped bodies. In other terms, these are analytical techniques to calculate the gravity fields of complicated models whose geometry is approximated by a certain number of small elementary bodies, such as horizontal laminas, prisms, or polyhedrons, that have analytical solutions (Fig. 2).

TALWANI & EWING (1960) suggested a method to calculate the gravity anomaly of a three-dimensional body by the numerical integration of a set of horizontal polygonal laminae that approximate the shape of the body. NAGY (1966) stated a formula to calculate the gravitational attraction of a vertical rectangular prism. An iterative method for calculation of the gravitational attraction, where the causative body is approximated by vertical prisms, was suggested by CORDELL & HENDERSON (1968). The gravity effect of each prism element is approximated by the vertical right-cylinder source formula and the vertical-line-source formula. BOTEZATU et al. (1971) suggested the method for calculation of the gravity effect by dividing the body into small-sized cubes. Also MUFTI (1973, 1975) presented approximation formulas for a cube's gravity field and a gravity modelling scheme which uses cubes as building blocks. The method based on the exact calculations of the gravity and magnetic anomalies of polygonal prisms was presented by PLOUFF (1976). BHATTACHARYYA (1978) considered computer modelling in gravity and magnetic interpretation using the solution of the direct and inverse problems. Three-dimensional interactive modelling in gravity and magnetics was suggested by GÖTZE & LAHMEYER (1988). They presented the interactive modelling program where the body is constructed from polyhedra of suitable geometry. The gravity effect of the homogeneous polyhedron is calculated by transforming the volume integral into a sum of line integrals.

The methods which use Fourier analysis belong to a second group - "Wave Number Domain Methods". These methods convert the gravity anomaly into Fourier series and provide for gravity inversion, commonly using the Fast Fourier Transform. Development of these methods started with PARKER (1973) and OLDENBURGH (1974), who used Fourier Transform Analysis of the gravity anomaly. There are a large number of methods belonging to this group, but only a few will be mentioned here. GRANSER (1987) used Fast Fourier Transform and an exponential density-depth function. LINES et al. (1988) carried out a cooperative inversion of geophysical data. CHENOT & DEBEGLIA (1990) presented the method of depth-mapping inversion of gravity and magnetic fields. GUSPI (1992) proposed one FFT gravity inversion method based on Parker-Oldenburgh formulas.

Gravity inversion using the Fourier transform or other techniques leads to smoothing of the data because successful convergence requires removal of the high-frequency components. Therefore it can be successfully applied in the gravity modelling of large areas (i.e. basin structures) but not in the cases of small complex

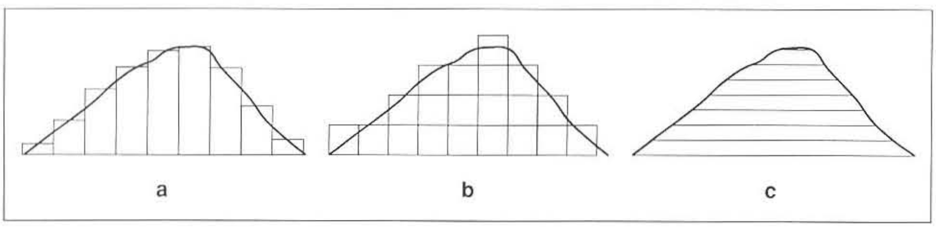

Fig. 2 Cross section of the body represented by vertical prisms (a), cubic blocks (b), and contours (c).

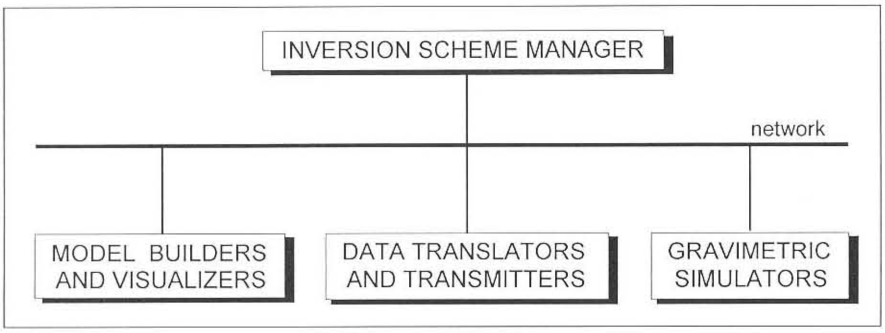

Fig. 3 Software architecture.

structures (i.e. overhanging salt dome, overthrust, etc.). Gravity inversion can be very useful in generating an overview of the study area and thereby enabling separation of prospective zones and direction of further research activity.

The forward modelling can be applied in the cases of small complex bodies and it allows more detailed interpretation of the gravity data. Besides, the interpreter can be more involved in the interpretation process applying *a priori* knowledge derived from the data acquired by other methods (seismic methods, the data from bore holes).

3. ARCHITECTURE

Different 3D models (velocity, geological, reservoir) that are produced by various model builders, are not always accessible by gravimetric routines. Typically, models have to be exchanged and reformatted before the algorithms will work. A good open architecture can keep the model building and updating software separate from the forward simulation software and can realize inversion schemes by communication between the two worlds. This means that at any time, models of various related properties can be checked with different constraining data sets. It is also clear that the architecture applied is very flexible enabling a relatively rapid easy change of the implementations on the subsystems very quickly.

The flexible architecture, consisting of four parts, can be implemented in a distributed computer environment (Fig. 3):

- the three-dimensional model builders and visualizers,
- the model representation translators,
- the forward simulation algorithms of gravimetric data,
- the inversion (model updating) scheme manager.

3.1. 3D MODEL BUILDERS AND VISUALIZERS

Two representations are typically used in 3D model builder programs, i.e. grid or finite element representations (FEM) and boundary representations (BREP), each of them has advantages for specific visualization and simulation algorithms. In this work, the connection has been established between a three-dimensional BREP modeller (GOCAD) and several gravimetric programs operating on different FEM representations of the three-dimensional model.

GOCAD is three-dimensional modelling software based on the "Discrete Smooth Interpolation Method" which is the centre of a new approach based on decomposition of lines into segments, surface into triangular facets and volume into tetrahedrons (MALLET, 1989). GOCAD can be used very efficiently for modelling surfaces defined by various type data, and for exporting the data for the calculation processes. In the export process a transformation is needed in some cases. It is especially important that GOCAD can easily handle complex structures such as overhanging salt domes and overthrusts. It also enables the easy calculation of synthetic gravimetric data for such geological models.

3.2. MODEL REPRESENTATION TRANSLATORS

Several transformation functions were needed and implemented in the distributed computer environment to exchange data from the model builder to the different gravimetric simulation routines. The internal GOCAD model representation, i.e. triangulated surface (BREP) was transformed into two finite element (FEM) representations, being horizontal contourline slabs, and vertical triangular prisms. This is realized using available transformation programs in GOCAD or external programs such as GEOMOD. The output is used by forward and inverse modelling software.

3.3. FORWARD GRAVIMETRIC SIMULATION ALGORITHMS

Two forward calculation algorithms have been implemented, Talwani-Ewing (using horizontal slabs) and Götze-Lahmeyer (using vertical prisms).

In the Talwani-Ewing method the three-dimensional body is first represented by contours that are then replaced by horizontal irregular n-sided polygonal laminae. If the number of sides n is sufficiently large, the polygon can approximate the contour line as closely as desired. The gravity anomaly produced by each lamina can be calculated analytically at any external point. A contour line on the surface of the arbitrary shaped body at depth z below station P (Fig. 4) is replaced by an n-sided polygonal lamina of infinitesimal thickness dz (TALWANI & EWING, 1960). The gravity anomaly at

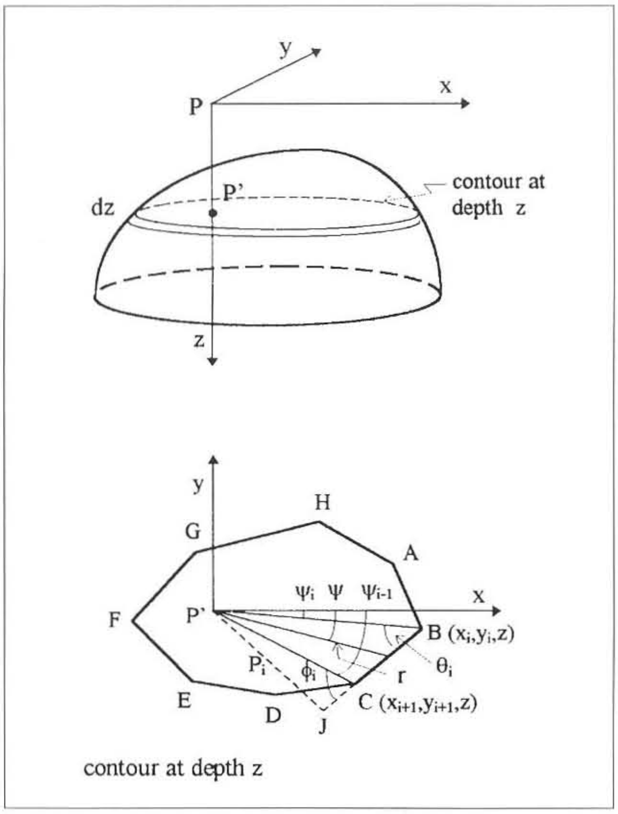

Fig. 4 Geometric elements of the polygonal lamina used to calculate the gravity effect (after TALWANI & EWING, 1960).

station P, which is the vertical component of the gravitational attraction, produced by mentioned polygonal lamina can be expressed as:

$$g_z = V\, dz \tag{1}$$

Term V is the gravity anomaly caused by the polygonal lamina per unit thickness. V is expressed by the surface integral, but it can be reduced on two line integrals (both along the boundary of the polygonal lamina) and can be given by the expression:

$$V = K\rho \left[\int d\psi - \int z/\sqrt{r^2 + z^2}\, d\psi \right] \tag{2}$$

where K is the universal gravitational constant, ρ is the volume density of the lamina and z, y and r are the cylindrical coordinates (Fig. 4). P' is the projection of the station P on the plane of the polygonal lamina.

After several transformations and substitutions the final expression for a gravity anomaly caused by polygonal lamina is:

$$V = K\rho \left[\sum_{i=1}^{n} \left\{ \psi_{i+1} - \psi_i - \arcsin \frac{z \cos \theta_i}{\sqrt{p_i^2 + z^2}} + \arcsin \frac{z \cos Ø_i}{\sqrt{p_i^2 + z^2}} \right\} \right] \tag{3}$$

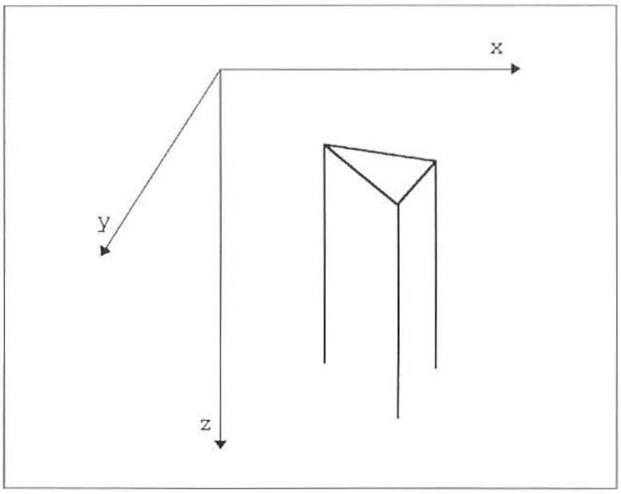

Fig. 5 Geometry of the elementary body used to calculate the gravity effect (after GÖTZE & LAHMEYER, 1988).

where p_i=PΔJ. All terms (p_i, y_i, y_{i+1}, $\cos q_i$ and $\cos y_i$) can be expressed in x_i and y_i coordinates of the polygonal points.

The total anomaly g_z produced by whole body is expressed as:

$$g_z \text{ total} = \int_{z_{bottom}}^{z_{top}} dz \tag{4}$$

Using the afore mentioned expressions the authors gave the expression suitable for programming, where all terms are given in x, y and z coordinates of the polygonal points.

The Götze-Lahmeyer method to calculate the attraction of a homogeneous polyhedron at station P is based on the evaluation of the volume integral. The gravity anomaly g(P), after derivation of the potential with respect to the vertical component z, is given by expression:

$$\frac{\partial U}{\partial z}(P) = g(P) = K\rho \iiint_{Poly.} \frac{\partial}{\partial z} \left(\frac{1}{r} \right) dv \tag{5}$$

where U(P) is the potential at station P, r is distance between P and dm (dm=ρdv, dv=ρdxdydz), and K is the gravitational constant. Corresponding to the relevant theories of vector analysis, an equation is established where the surface integral has to be calculated for the whole polyhedron surface. A transformation of the coordinates system (into the surface oriented coordinate system) is carried out to simplify the mathematical expressions in the evaluation of the surface integral. Finally a conversion of the surface integral into a linear integral via polygon, which limits the surface, is carried out.

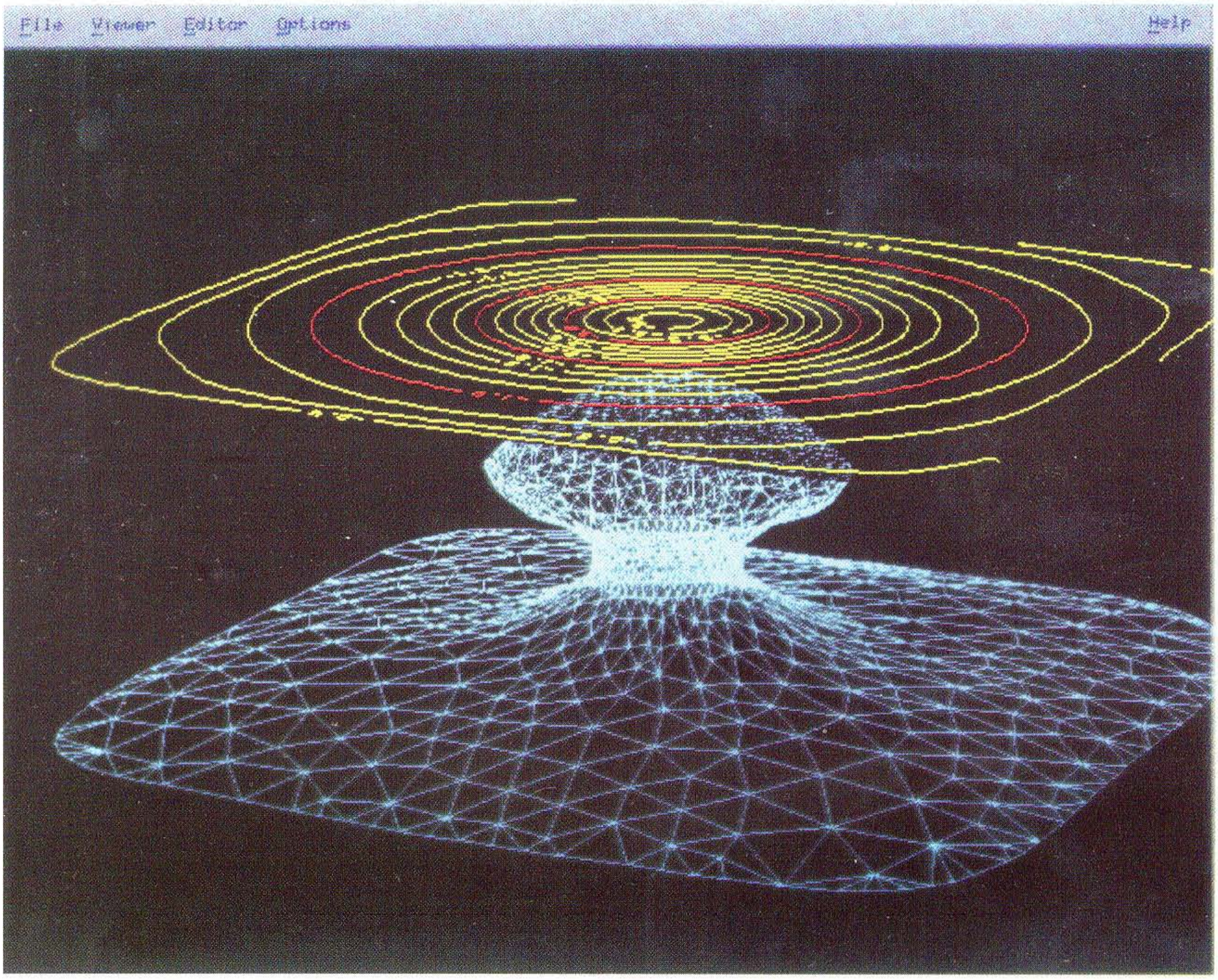

Fig. 6 Overhanging salt dome with gravity field at surface (a), which is detached as a map (b). Contour interval 0.2 x 10^{-5}m/s^2, density contrast $\Delta\rho$=-400 kg/m^3.

The authors also produced a final formula to calculate the gravity anomaly produced by the polyhedron, which is suitable for programming. A subroutine has been published, written in FORTRAN, to calculate the gravity effect of a vertical prism, infinite in the z direction with a triangle as the upper boundary. This elementary body, whose shape (geometry) is presented in Fig. 5, can be used to construct more complicated bodies. In this case the gravity effect of the entire body would be expressed as a sum of the gravity effects of the elementary bodies. This subroutine was the second routine implemented for the gravity forward modelling.

The option, which uses the Götze-Lahmeyer method can be used to calculate the gravity effect of noncomplex (simple) structures. For computation of the gravity effect of complex structures, as for instance a salt dome (overhanging) or overthrust, the option using the Talwani-Ewing method should be used.

3.4. INVERSION SCHEME MANAGER

The inverse scheme manager should support the inversion strategies. One approach is by a trial and error process, another by automatic (constrained) inversion.

The Cordell-Henderson gravity inversion scheme is useful to produce the first model, which is constrained here on a simple two-layer model. It is an iterative scheme based on the standard trial and error approach. The scheme consists in general, of three aspects:

- construction of a simple start model,
- calculation of the gravity anomaly for a given model, and
- modification of the model based on differences between calculated gravity anomaly and observed gravity anomaly.

In the first step the gravity anomaly is digitized on a rectangular grid. The interpreted body is represented by rectangular vertical prisms in a regular grid with a flat top and bottom. The thickness of the prism element below the q-th grid point (station) $t_{1,q}$ for the starter model is established by means of the Bouguer slab formula:

$$t_{1,q} = B\, g_{obs,q} \qquad B = \frac{1}{2\pi K\rho} \tag{6}$$

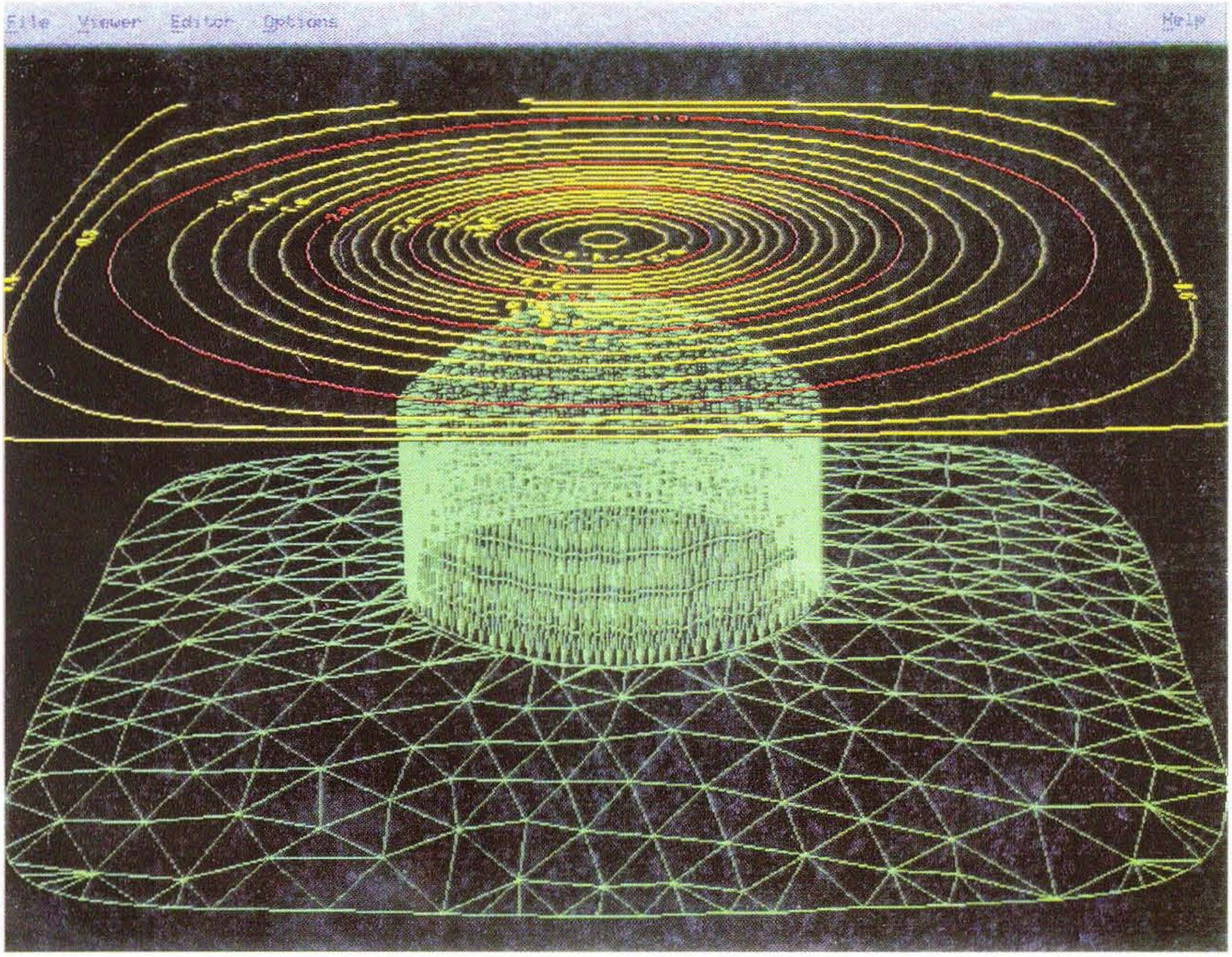

Fig. 7 Cylindrical salt dome with gravity field at surface (a), which is detached as a map (b). Contour interval $0.2 \times 10^{-5} m/s^2$, density contrast $\Delta\rho=-400\ kg/m^3$.

where K is the gravitational constant, ρ is the density, $g_{obs,q}$ is the observed gravity at q-th station. The authors suggested the following relationship for the next modification or improvement of the model:

$$t_{n+1,q} = t_{n,q} \left(\frac{g_{obs,q}}{g_{calc,n,q}} \right) \tag{7}$$

where n is number of iteration and $g_{calc,n,q}$ is calculated gravity value below q-th station at n-th iteration.

The prisms are then approximated by vertical cylinders since the formula calculating the gravity effect of the vertical cylinder is much simpler and consumes less calculation time. The goodness of fit is expressed in terms of the root-mean-square error (RMS error). However, also the largest error can be used.

The inversion scheme described is used for obtaining the initial model of a two-layer structure. The result of more complex models (based on other data) can be verified by using forward modelling techniques and adjusted by using the trial and error approach.

4. CASES

Two synthetic cases (overhanging and non-overhanging salt dome) are shown as an example of the use of architecture and the capability to simulate the gravimetric fields for complex models (Figs. 6 and 7). The models and the gravimetric fields at the surface are shown using GOCAD software for both cases. The effect of the difference in shape of the bodies is clearly visible (Figs. 6 and 7). The gravity anomalies at the centres of the salt domes are $-3.6 \times 10^{-5} m/s^2$ for the overhanging and $-4.6 \times 10^{-5} m/s^2$ for the cylindrical dome, respectively.

Both surface gravimetric anomalies have concentric shapes, but on the surface gravity profiles (Fig. 8) the differences in gravity attraction are evident, although they decrease with the increase of the distance from the centre of domes. Calculated gravimetric data for X-sections just outside the cylindrical and overhanging salt domes are shown in Figs. 9 and 10. For cylindrical body, the isolines are almost vertical for the first 500 metres, and for overhanging structure, the gradient increases faster with depth. The X-section of differences in gravity attraction between domes are shown

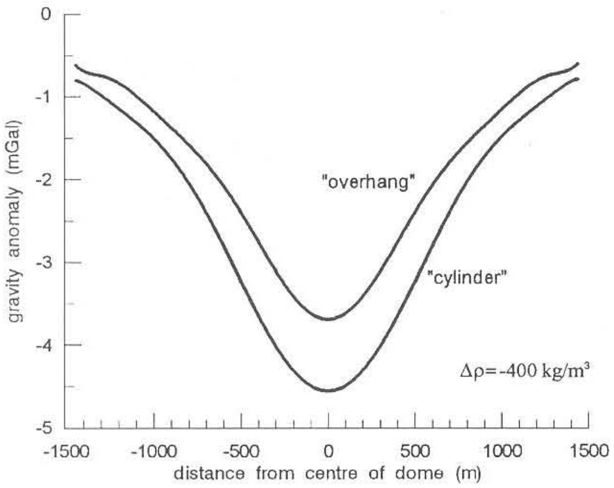

Fig. 8 Gravity anomalies at the surface profile.

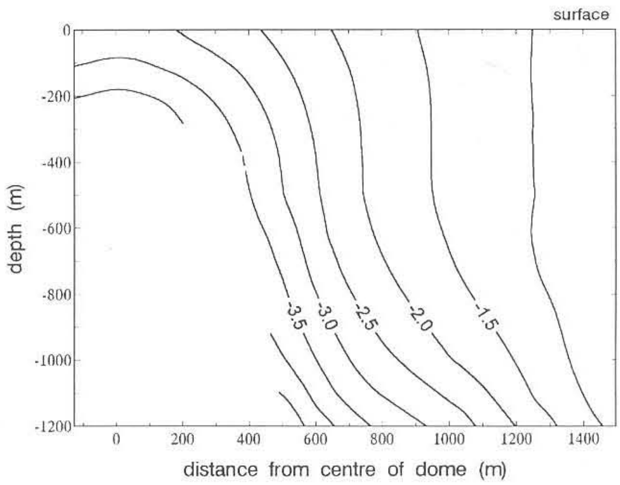

Fig. 9 Gravity field for X-section of cylindrical salt dome. Contour interval 0.5 x 10^{-5}m/s^2, density contrast Δρ=-300 kg/m^3.

in Fig. 11, where the values also decrease with the increase of distance from the centre of domes.

With respect to the previous examples it can be concluded that it is impossible on the basis of anomaly shape to make any conclusions about source body. But gravity modelling can distinguish between cases because the existing mass differences result in clear anomaly differences both for surface profiles and for X-sections, although the possibility of the precise separation of the domes decreases with the increasing distance from the dome centre. Therefore in a practical sense, a salt dome can be defined more precisely if bore hole and gravity measurements are situated closer to the dome centre.

The best illustration of the previous statement is the following example. If automatic gravity inversion is applied on the gravity field produced by an overhanging dome, satisfactory results can not be obtained. The automatic gravity inversion procedure can not process complex structure, which has few z-values for the same x and y coordinates. The result is unstable inversion, which means that a satisfactory model cannot be produced. The conclusion is that such a structure has a complex shape, which can be established by forward modelling. It is obvious, that gravity modelling can distinguish these cases. Other complex structures, such as reservoirs fissured by faults or other structural features will also benefit from these kinds of analysis.

Another synthetic case can be described to illustrate the capability of the inversion procedure. The inversion manager gives the structural forms of the horizon with constant density contrast bounding a single layer from residual gravity anomalies using a regular grid. It must be noticed that the residual-regional separation should be carried out, meaning that the regional gravity effect must be removed from observed gravity anomalies by using appropriate methods. The gravity field at the surface produces the structure shown in Fig. 12. Satisfactory interpretation of the gravity field can be attained in the case of more complex geological models, as well.

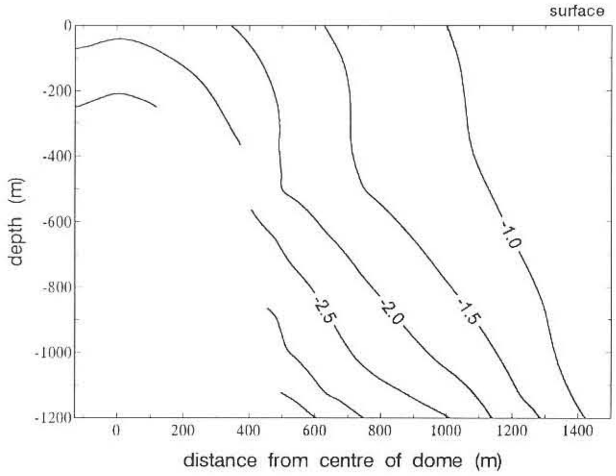

Fig. 10 Gravity field for X-section of overhanging salt dome. Contour interval 0.5 x 10^{-5}m/s^2, density contrast Δρ=-300 kg/m^3.

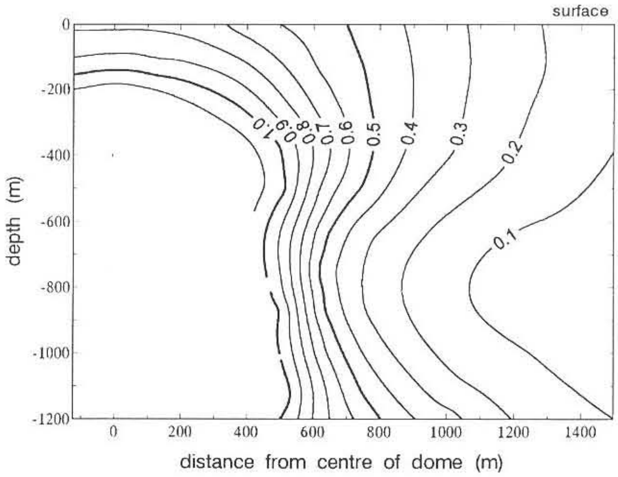

Fig. 11 Difference in gravity fields of overhanging and cylindrical salt domes. Contour interval 0.1 x 10^{-5}m/s^2.

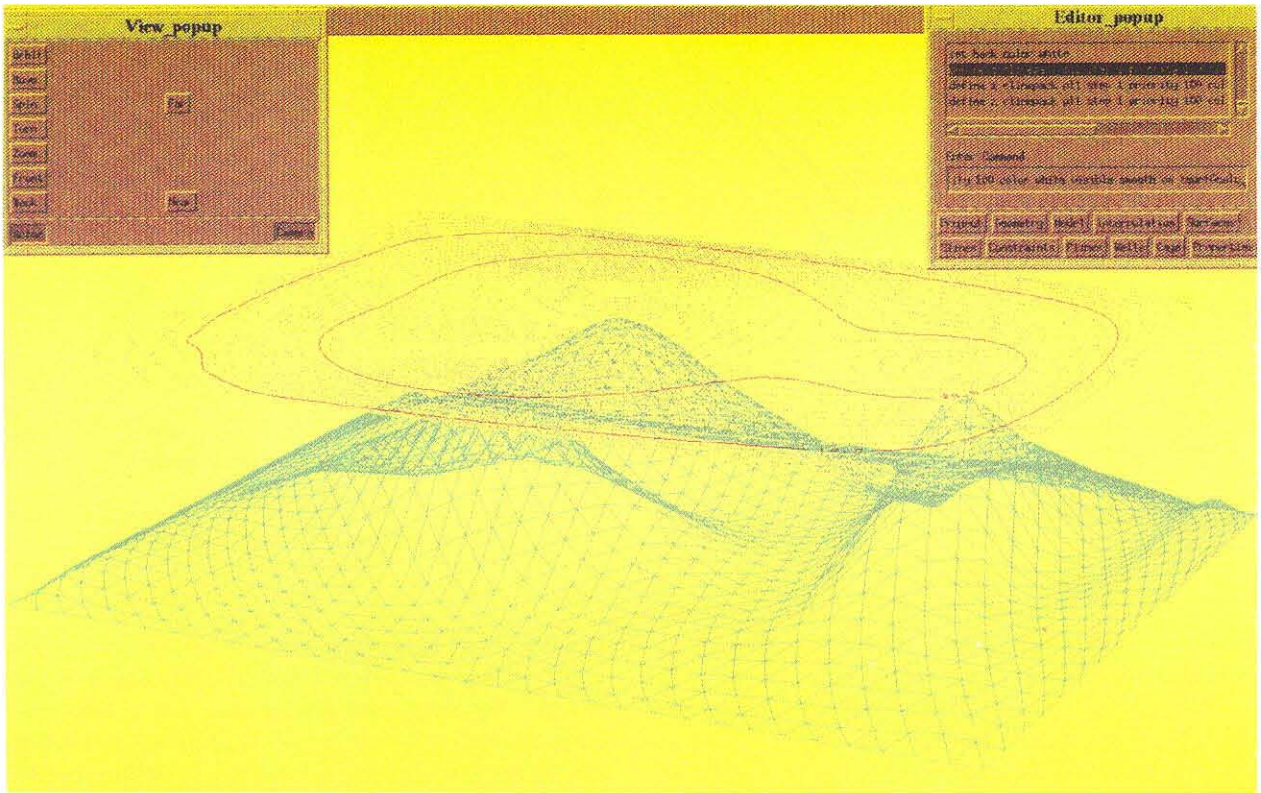

Fig. 12 The structure obtained from a given gravity field (gravity inversion).

5. CONCLUSIONS

Two conclusions can be drawn. Firstly, the application of gravimetric data to constrain 3D simple and complex (structural and reservoir) models in the appraisal and development phases is possible and should be incorporated in daily practice. Secondly, the described architecture, separating the model building and visualization subsystems from the algorithmic subsystems for forward and inverse simulation will support this in an open environment. The communication between two environments consists of a set of transformation routines for various three-dimensional model representations. It will allow the easy integration of other simulation software, such as magnetic, seismic and so on.

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