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Effect of Glueline Shape on Strength of Mortise and Tenon Joint

Utjecaj oblika lijepljene površine na čvrstoću spoja rupe i čepa

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ABSTRACT • *Tenon joints are widely used in constructions of skeletal furniture and numerous laboratory experiments were conducted with the aim of determining the rigidity and strength of such joints. In industrial practice, the sloppily applied glue often covers only flat, rectangular planes of the tenon. In this situation, it can be intuitively predicted that such joints are characterised by low strength but the precise assessment of this difference, requires carrying out appropriate experiments and numerical calculations. Therefore, the aim of the performed investigations was: to define the strength of a tenon joint in the construction of a chair with a connecting piece, to determine the distribution of shear and normal stresses in the glue bond and to ascertain the influence of the glueline on this strength. The finite element mesh for the tenon joint was developed by using orthotropic, 20-node block elements, whereas the glue bond was modelled using isotropic elements 0.1 mm thick. The performed investigations showed that the shape of the glueline exerts a strong influence on the strength of the tenon joint. The pressure of the tenon on the mortise via the layer of the glue bond changes the form of deformations. Non-dilatational deformations, which generate shear stresses of values exceeding the ultimate strength, develop in rectangular bonds, whereas in superellipse bonds non-dilatational deformations clearly restrict the pressure of the tenon on the mortise and, by doing so, decrease considerably the level of dangerous shear stresses.*

***Key words**: glueline, mortise and tenon joint, numerical analysis, wood*

SAŽETAK • *Zaobljeni čepovi i rupe česti su elementi okvirnih konstrukcija namještaja s kojima su obavljeni brojni laboratorijski eksperimenti s ciljem određivanja krutosti i čvrstoće takvih spojeva. U industrijskoj praksi nepravilno izvedenim postupkom lijepljenja često je moguće pokrivati samo ravne pravokutne površine čepa. U takvom se slučaju može intuitivno predvidjeti da će takvi spojevi imati manju čvrstoću, no za preciznu potvrdu navedenih sumnji trebaju se provesti odgovarajući eksperimenti i računalne simulacije. Stoga je cilj ovog rada bio definirati čvrstoću spoja čepa i rupe u konstrukciji noge i poveznika stolice, odrediti posmična i normalna naprezanja u lijepljenom spoju i utvrditi utjecaj oblika lijepljene površine na čvrstoću. Mreža konačnih elemenata za spoj čepa i rupe bila je izvedena ortotropnim 20-čvornim elementima, pri čemu je sloj ljepila modeliran kao izotropni element debljine 0,1 mm. Provedena su istraživanja pokazala da oblik sloja ljepila ima velik utjecaj na čvrstoću spoja čepa i rupe. Pod utjecajem tlaka u sljubu čepa i rupe putem sloja ljepila mijenja se oblik deformacije. Nerastezljiva deformacija koja stvara posmična naprezanja čija vrijednost prelazi graničnu čvrstoću razvija se u pravokutnom obliku površine lijepljenog spoja pri čemu u zaobljenom dijelu lijepljenog spoja čepa i rupe nerastezljiva deformacija vidno smanjuje tlak kojim čep djeluje na rupu i čime se znatno smanjuje stupanj neželjenih posmičnih naprezanja.*

***Ključne riječi**: lijepljena površina, spoj čepa i rupe, numerička analiza, drvo*

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1 INTRODUCTION
1. UVOD

Skeletal furniture represents the oldest wooden constructions manufactured by connecting individual elements with one another by means of shape joints and glue. Changes in production technologies as well as types of the applied materials have not altered the fact that tenon joints still remain the most frequently applied and continue to be considered as the strongest connections used in furniture industry. Despite practical and sound knowledge about the strength and dimensional proportions of such joints, attempts are still being made to find solutions that would improve the behaviour of loaded constructional wood elements and joints and, at the same time, new methods of stress control are being developed. In an attempt to develop the optimal shapes of shape-adhesive joints, (Nakai and Takemura, 1995) elaborated mathematical formulas which describe correctly the rigidity of tenons in rectangular and superellipse cross sections. In other studies, the same researchers (Nakai and Takemura, 1996) analysed the distribution of shear stresses in the tenon and indicated the need to avoid torsional loads, which could cause cracks and tearing of wood tissue. Also (Hill and Eckelman, 1973) investigated the problems related to deformability and bending stresses. (Eckelman, 1970) and (Smardzewski, 1990) developed computer programs dedicated to the analysis of furniture rigidity and strength in semi-rigid joints, including tenon joints. However, in the above-mentioned studies, the researchers failed to take into account glue bonds and their impact on the rigidity and strength of the analysed joints. Several other studies were also devoted to problems connected with the distribution of shear stresses in glue bonds of shape-adhesive joints (Haberzak, 1975; Matsui, 1991; Pelicane, 1994; Smardzewski, 1996; Smardzewski, 1998; Smardzewski, 2002) in which researchers analysed the character of stress distributions in tests of tensile, bending and torsional loads. Mathematical models describing these distributions were also elaborated. However, in the case of tenon joints, all these investigations failed to assess how the shape of the glueline affects the strength of the joint. In the case of careful application of the glue, both on the tenon and mortise, the glue bond should have a shape of a thin-walled superellipse sticking well to the tenon and joint on their entire circumference. Unfortunately, in industrial conditions, it is not very uncommon that carelessly applied glue covers only flat, rectangular surfaces of the tenon and in the contact with the mortise it forms two parallel bonds which are not connected with each other. Intuitively, it can be predicted that such joints are characterised by poorer strength but the precise assessment of this difference requires carrying out appropriate experiments and numerical calculations. Numerical modelling provides a rational alternative to expensive and time-consuming laboratory experiments, especially in the situation when such investigations involve the determination of deformations and stresses in thin layers of glue bonds difficult for experimental analyses. That is why, the author decided to use in this study the method of finite elements for the analysis of the state of stresses in the glueline.

The objective of the performed investigations was to define the strength of a tenon joint in the construction of a chair with a connecting piece, to determine the distribution of shear and normal stresses in the glue bond and to ascertain the impact of the glueline shape on this strength.

2 MATERIALS AND METHODS
2. MATERIJALI I METODE

A chair construction with a connecting bar was selected for investigations, choosing the connection of the stretcher with the back leg as the construction node (Fig. 1)

Bearing in mind the perpendicular position of the connecting bar and the leg in the global coordinate system and the different longitudinal course of wood fibres resulting from this fact, two separate local systems were applied. For the connecting bar, it was assumed that wood fibres would run parallel to the Y axis and the radial-tangential plane would be situated on the XZ plane of the global system of coordinates. In the case of the leg, fibres would be oriented towards the Z axis, while the radial-tangential plane would be situated on the XY plane. A gap 0.1 mm thick along the entire circumference was placed between the tenon and the mortise in which a superellipse glue bond was formed. Bearing in mind the symmetry of the examined construction, only one of its side frames was subjected to strength analysis during which the front edge of the seat was loaded with a concentrated force of 800 N value (Fig.2).

The value, direction and sense of this force corresponded to extreme conditions of this piece of furni-

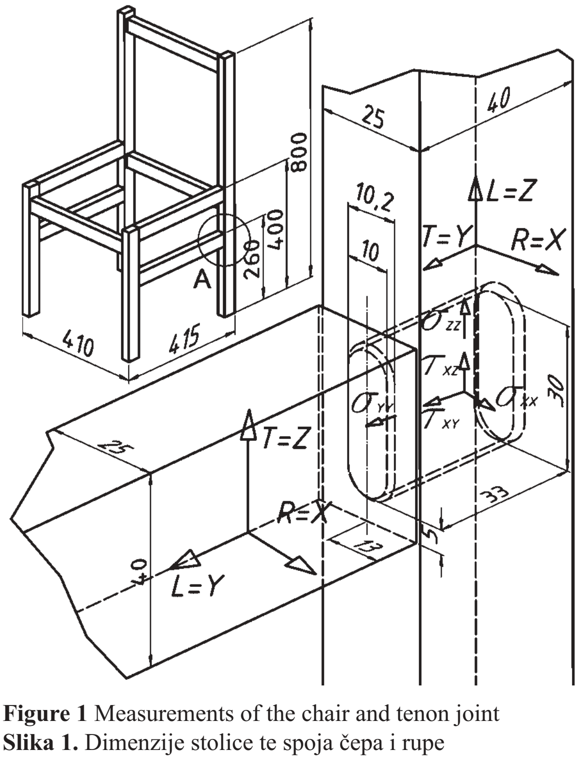

Figure 1 Measurements of the chair and tenon joint
Slika 1. Dimenzije stolice te spoja čepa i rupe

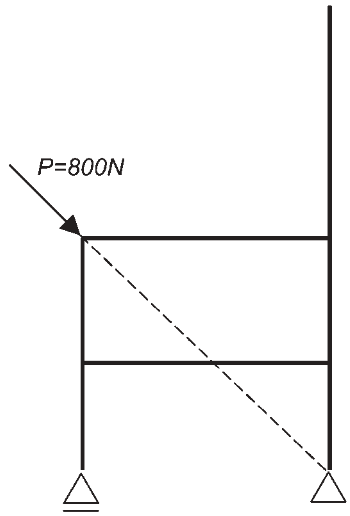

Figure 2 Static diagram of the chair construction loading
Slika 2. Dijagram statičkog opterećenja stolice

ture by a user weighing about 160 kg. In the course of the static analysis of the chair side frame, appropriate concentrated forces were transferred onto the construction node which corresponded to internal bending moments, shear forces and normal forces (Fig. 3).

The finite element mesh for the tenon joint was developed in the environment of the Algor® program (Fig. 4). For this purpose, for the modelling of the joint wooden parts, the author used orthotropic, 20-node block elements, whereas the glue bond was modelled using isotropic elements 0.1 mm thick.

When selecting experimental material from which the model of the chair frame was prepared, numerical calculations were carried out for beech wood (*Fagus sylvatica* L.). Its elastic properties are presented in Tab. 1 on the basis of investigations carried out by Hearmon (1948), Bodig and Goodman (1973). Taking into account different orientation of individual wood anatomical directions in individual joint elements, the Table 1 gives values corresponding to two local coordinate systems. Another material component, commonly used during furniture assembly, is polyvinyl acetate (PVAC) glue. In order to determine the modulus of lin-

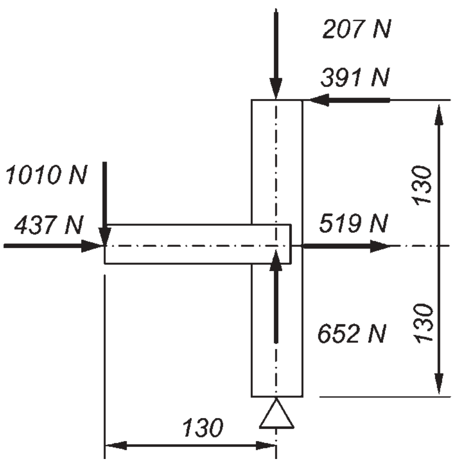

Figure 3 Loading diagram of the back leg joint with the connecting bar
Slika 3. Dijagram opterećenja stražnje noge i okvirnice

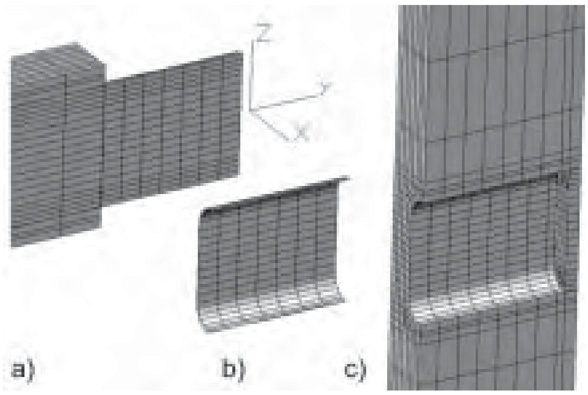

Figure 4 Numeric model of the tenon joint: a) tenon, b) glueline, c) mortise
Slika 4. Numerički model spoja čepa i rupe: a) čep, b) sloj ljepila, c) rupa

ear elasticity of this glue, the author used both results of his own investigations as well as literature data.

Table 1 Elastic properties of beech (*Fagus sylvatica* L.) wood used in experiments according to Hearmon (1948), Bodig and Goodman (1973)
Tablica 1. Elastične vrijednosti za bukovinu (*Fagus sylvatica* L.) korištenu u istraživanju (Hearmon, 1948; Bodig and Goodman, 1973)

Element / *Element*		**Value** *Vrijednost*
Leg / *Noga*	Connecting bar / *Okvirnica*	
Wood density / *Gustoća drva* g/cm^3		0,75
Modulus of elasticity / *modul elastičnosti*, MPa		
$E_L = E_Z$	$E_L = E_Y$	13969,98
$E_R = E_X$	$E_R = E_X$	2284,97
$E_T = E_Y$	$E_T = E_Z$	1160,06
Rigidity modulus / *modul krutosti*, MPa		
$G_{LT} = G_{ZY}$	$G_{LT} = G_{ZY}$	1082,72
$G_{LR} = G_{ZX}$	$G_{LR} = G_{YX}$	1645,18
$G_{RT} = G_{XY}$	$G_{RT} = G_{XZ}$	471,06
Poisson's ratio / *Poissonov koeficijent*		
$\upsilon_{LR} = \upsilon_{ZX}$	$\upsilon_{LR} = \upsilon_{YX}$	0,450
$\upsilon_{LT} = \upsilon_{ZY}$	$\upsilon_{LT} = \upsilon_{ZY}$	0,510
$\upsilon_{RT} = \upsilon_{XY}$	$\upsilon_{RT} = \upsilon_{XZ}$	0,750
$\upsilon_{TR} = \upsilon_{YX}$	$\upsilon_{TR} = \upsilon_{ZX}$	0,360
$\upsilon_{RL} = \upsilon_{XZ}$	$\upsilon_{RL} = \upsilon_{XY}$	0,075
$\upsilon_{TL} = \upsilon_{YZ}$	$\upsilon_{TL} = \upsilon_{YZ}$	0,044

According to Wilczynski (1998), modulus of linear elasticity of the PVAC glue determined in the torsional test of prismatic glued samples amounted to 358 MPa. Dziegielewski (1990) in their indirect investigations, determined the linear elasticity modulus of the PVAC glue at the level of 28450-33450 MPa. The value similar to that reported by Wilczynski (1998), equalling 465.74 MPa was obtained by Smardzewski (1998) investigating uniform, oar-shaped samples subjected to stretching. Taking into consideration the results of direct laboratory tests, the value equalling 465.74 MPa was selected for further investigations from the result by which glue linear elasticity modulus was determined.

Numerical calculations comprised two models of joints which differed with regard to the glueline shape. The first model was represented by the joint in which the glue bond - in the form of two cuboids measuring

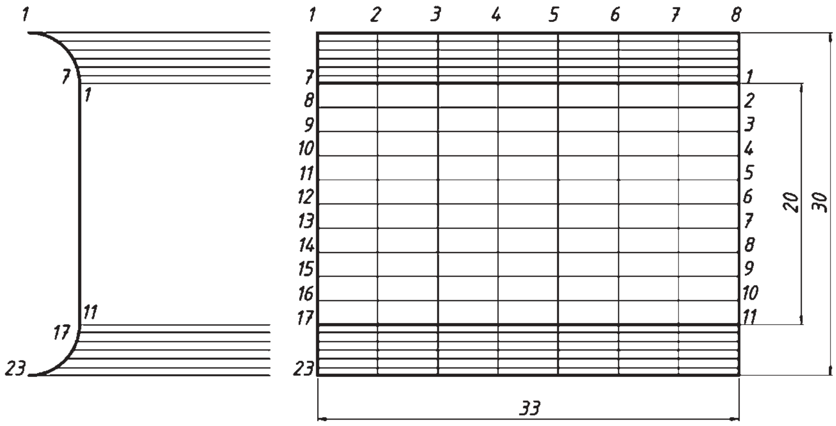

Figure 5 Measurements of glue bond and designations of stress measurement points
Slika 5. Veličine nanosa ljepila i oznake mjernih točaka naprezanja

0.1x20x33 mm – was developed on the opposite flat surfaces of the tenon. In Figure 5, numbers 1 to 11 in the vertical plane and 1 to 8 in the horizontal plane designate nodes of the finite element network. In the case of the second model, the superellipse glue bond 0.1 mm thick in the joint was developed along the entire surface of the tenon. Here, the vertical numbering from 1 to 23 also included parts of the bond solidified on the cylindrical parts of the tenon (Fig. 5). During numerical calculations, the change of the glueline shape and thickness caused by the pressure of the tenon on the mortise was determined as well as changes of the reduced, shear and normal stresses in the network nodes on the surface of the glue bond.

3 RESULTS AND DISCUSSION
3. REZULTAT I DISKUSIJA

The analysis of deformed networks of the numerical model revealed that the joint tenon underwent both rotation and bending. The result of this deformation was an apparent non-proportionality of linear and non-dilatational stresses of the glue bond along its edges. In the case of rectangular bond, non-dilatational stresses are dominant and the greatest ones occur in corners and along the edge adjacent to the tenon base (Fig. 6a).

These deformations developed as a result of the bending of the tenon, where the farthest fibres underwent extreme elongation or shortening. In the remaining portions of the tenon, linear deformations were not significant enough to exert influence on bond deformations. Therefore, in this part the glueline was subjected to proportional torsion between two rigid adherents. When examining the deformations of the superellipse glue bond, it was found that both non-dilatational and linear deformations occurred in it. Figure 6b shows the change of the glueline thickness caused by the pressure of the tenon on the mortise. Both at the front and at the back of the tenon, as a result of pressure, the glue bond reduced its thickness from 0.1 mm to 0.087 or 0.099 mm and as a result of tension it increased its thickness to 0.102 or 0.103 mm. The pressure of the tenon on the mortise via the glue line reduced significantly non-dilatational deformations and consequently reduced the value of the developing stresses.

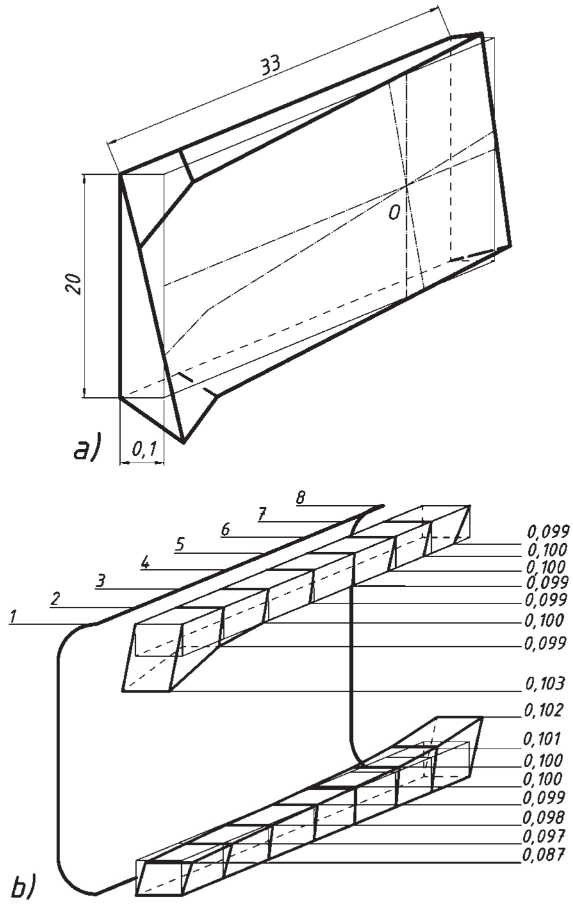

Figure 6 Glueline deformations: a) rectangular bond, b) superellipse bond
Slika 6. Deformacija površine ljepila: a) pravokutna površina, b) zaobljena površina

The observed non-proportionality of non-dilatational deformations in the glue bond causes migration of its permanent centre and unequal distribution of stresses. By assigning the complex state of stress which develops in the glue bond a uniaxial state characterised by the reduced stress σ_R according to the formula:

$$\sigma_{R_{\max}} = \frac{1}{\sqrt{2}}\Big((\sigma_{XX} - \sigma_{YY})^2 + (\sigma_{XX} - \sigma_{ZZ})^2 + (\sigma_{ZZ} - \sigma_{YY})^2 + 6\big(\tau_{XY}^2 + \tau_{ZY}^2 + \tau_{XZ}^2\big)\Big)^{0.5} \quad (1)$$

where:
σ_{ij} - normal stress for selected direction,
τ_{ij} - shear stress for selected plane,

it was determined that, in the case of rectangular bonds, the greatest reduced stresses according to Von Mises occurred in the left bottom corner of the bond reaching the value of 112.84 MPa (Fig. 7a). At the same time, it should be emphasised that stresses in the remaining corners were from 1.33 to 2.62 times smaller. Appropriate normal and shear stresses constitute components of the reduced stresses. Figure 7b shows the distribution of shear stresses τ_{XZ}, τ_{XY}. These stresses are responsible for the shear of the glue bond, and hence for the strength of the entire tenon joint. The highest shear stresses, similar to the greatest reduced stresses, developed in the left bottom corner of the glue bond. Their value amounted to 59.89 MPa. In the remaining corners, stresses were found to be by 30 % to 50 % of the value of maximal stresses. Bearing in mind the technical shear strength of the PVAC glue bonds which ranges from 17 MPa to 23 MPa, the destruction process is initiated in the glue bond corners. This, obviously, does not mean that once the shear stresses reach the critical values in these particularly critical points of the glueline, the entire join must necessarily be destroyed. The fact of a high concentration of reduced or shear stresses in the corners of the glue bond causes only decohesion in several places on a small surface. At a short distance from these corners the level of stresses declines dramatically below the level of permissible values. By the calculation of mean values of the reduced and shear stresses for the entire surface of the glue bond, the following values were obtained: σ_R=32.46 MPa and τ=16.28 MPa, respectively. Therefore, it can be said that in conditions of extremely unfavourable operational loading of the chair construction, shear stresses balanced at the limit of the employed glue shear strength.

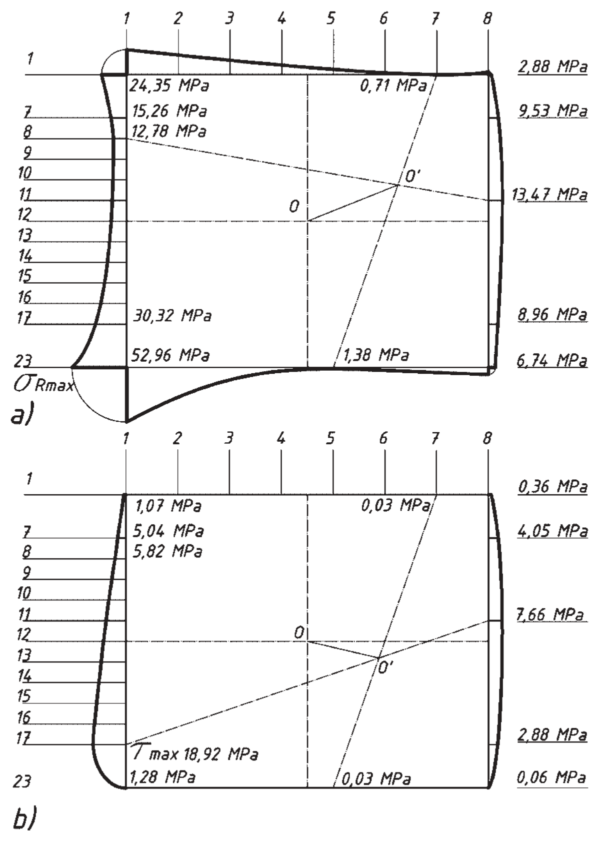

Figure 7 Stress distribution in the rectangular glue bond: a) reduced stresses according to Von Mises, b) shear stresses τ_{xz}, τ_{xy}

Slika 7. Raspodjela naprezanja u pravokutnom sljubu: a) smanjena naprezanja prema von Misesu, b) posmična naprezanja τ_{xz}, τ_{xy}

Additionally, in the case of the experimental glue bonds, it was also found that the permanent centre of the bond was not situated in its geometrical centre. By linking opposite sides of the bond with intervals in points characterised by the smallest values of the reduced and shear stresses (Fig. 6), the true position of the permanent centre "O" was obtained. This point moved to the right on the bond horizontal axis of symmetry. The displacement of the permanent centre was caused, primarily, by the bending of the tenon.

According to von Mises it was found that the greatest reduced stresses for superellipse glue bonds also occurred in the left bottom corner of the unwound glue bond (i.e. under the tenon axis) and reached the value of 52.96 MPa (Fig. 8a). It should be emphasised that stresses in the corners of the opposite edge were by 7.8 to 18.3 times smaller. (Fig. 8b) also shows the distribution of shear stresses τ_{XZ}, τ_{XY}. The greatest of them, reaching the value of 18.92 MPa, were generated at the point where the shape of the glue bond changed from rectangular into round (node number 17). It is also interesting to observe that the stresses on the horizontal edges of the glue bond were negligibly small.

After calculating the mean value of reduced and shear stresses on the entire glue bond surface, the following values were obtained: σ_R=14.31 MPa and τ=4.35 MPa. It is therefore evident that the shear stresses, at the selected unfavourable operational load of the construction of the examined piece of furniture, constitutes only 25% of the value of the permissible stresses corresponding to the glue shearing strength limit. Slightly higher mean values of reduced stresses were caused by the presence of normal stresses, σ_{ZZ} and σ_{YY}. During the bending of the joint and torsion of the glueline, the tenon was subjected to deflection and torsion and pressed on the bottom and top portions of the glue bond causing very high normal stresses σ_{ZZ} inside it. This is why small and safe shear and reduced stresses were generated in the glueline which resulted in a smaller material effort in comparison with the rectangular glue bond.

A regularity also observed in the case of the superellipse glue bond was the fact that its permanent centre was not situated in its geometrical centre. Linking opposite sides of the bond with intervals in points characterised by the smallest and the greatest values of the reduced and shear stresses, another position of the point of the permanent centre was obtained (Fig. 8). This point moved to the right but its precise position was difficult to establish precisely.

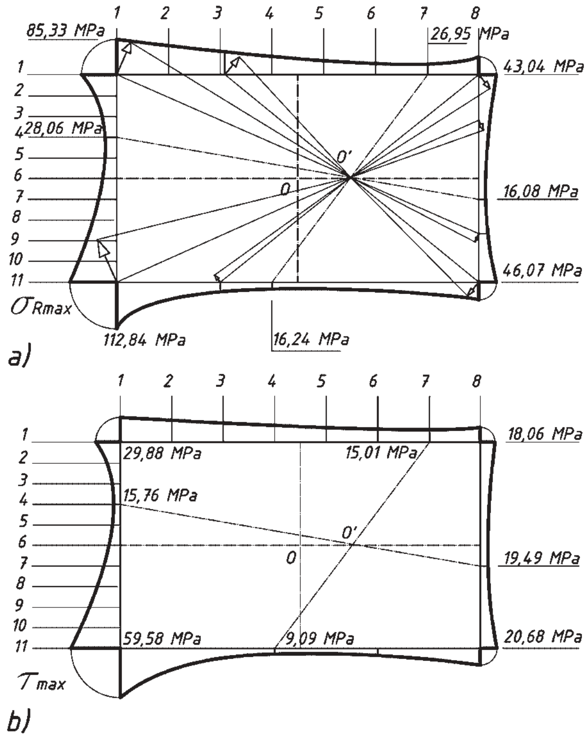

Figure 8 Stress distribution in the superellipse glue bond: a) reduced stresses according to von Mises, b) shear stresses τ_{xz}, τ_{xy}

Slika 8. Raspodjela naprezanja u zaobljenom sljubu: a) smanjena naprezanja prema von Misesu, b) posmična naprezanja τ_{xz}, τ_{xy}

4 CONCLUSION
4. ZAKLJUČAK

The performed investigations revealed that the shape of the glueline exerted a definite influence on the strength of the examined tenon joint. The pressure of the tenon on the mortise via a layer of glue bond changed the form and size of its stresses. In the case of rectangular bonds, only non-dilatational stresses developed which generated shear stresses whose values exceeded the limiting strength. In the case of superellipse glue bonds, non-dilatational stresses clearly restricted the pressure of the tenon on the mortise reducing significantly the level of dangerous shear stresses.

Bearing in mind the results of the performed numerical calculations, the following conclusions were drawn:

1. Properly executed and well solidified superellipse glue bonds, in comparison with rectangular bonds, increase four-fold the strength of the tenon joint at the identical operational loads.
2. In the case of rectangular gluelines, shear stresses are of destructive nature and are distributed non-linearly along all its edges.
3. In superellipse glue bonds, the greatest shear stresses occur on one edge adjoining the tenon offset.
4. As the result of the pressure of the tenon on the mortise via the superellipse glue bond, the highest normal stresses are generated which reduce significantly shear stresses.
5. In industrial practice, it is important to apply glue on the entire surface of the tenon and mortise.

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