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STRUCTURAL ANALYSIS AND OPTIMIZATION OF CONCRETE SPHERICAL AND GROINED SHELLS

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Subject review

An understanding of successful shells of the past combined with modern structural analysis methods can aid engineers in designing efficient thin shell concrete structures. This paper presents structural analyses and the optimization study of several notable concrete shells around the world. The finite element analysis using Sofistik software was found to give results close to those from shell theory and to be more powerful when complicated real-life examples were presented. In structural optimization of the concrete spherical and groined shells an attempt is made to reduce overall tensile stress, deflection and reinforcement to the system while changing the limits of the structure's shape. The structural optimization study of these shells shows that a distributed concrete thickness reduces shell stresses, deflections and reinforcements.

Keywords: *concrete shells, groined shell, spherical shells, structural analysis, structural optimization*

Statička analiza i optimizacija betonskih sfernih i svodastih ljusaka

Pregledni članak

Razumijevanje ranijih uspješnih ljusaka kombinirano sa suvremenim metodama proračuna konstrukcija može pomoći inženjerima u projektiranju efikasnih tankih betonskih ljuskastih konstrukcija. U ovom radu je prikazana statička analiza i optimizacija nekoliko značajnih betonskih ljusaka u svijetu. Analiza pomoću metode konačnih elemenata uporabom programa Sofistik dala je rezultate bliske onima prema teoriji ljusaka te se pokazala moćnijom za prikazane složene izvedene primjere. Pri optimizaciji konstrukcije betonskih sfernih i svodastih ljusaka nastojalo se reducirati ukupno vlačno naprezanje, progib i armaturu u sustavu uz mijenjanje granica oblika konstrukcije. Optimizacija konstrukcije ovih ljusaka pokazala je da raspodijeljena debljina betona reducira naprezanja, progibe i armaturu ljuske.

Ključne riječi: *betonske ljuske, optimizacija konstrukcije, sferne ljuske, statička analiza, svodasta ljuska*

1 Introduction
Uvod

Analytical procedures of thin shell analysis generally require the solution of ordinary or partial differential equations, which are not usually obtainable for complex structures. Hence, numerical methods are relied upon, such as the finite element method, for acceptable solutions. Engineering judgment and experience may be guides as reliable as analytical procedures based on simplifying assumptions. Factors such as size, geometry of the shell, especially the type and amount of curvature, boundary conditions, and load distribution must be considered collectively by the designer in the choice of analytical procedures.

The widespread use of large-scale computer programs for structural analysis has reduced reliance on classical methods, provided insight into previously unsolved problems and held out the hope for automatic optimized design.

Design by membrane theory provided essentially a correct basis for design in the hands of experienced shell designers; but bending theory gotten from computer-based numerical solutions gives the inexperienced engineer the opportunity to explore the behaviour of various different forms.

2 Comparison between shell theory and finite-element analysis for a spherical dome
Usporedba teorije ljusaka i metode konačnih elemenata za sfernu kupolu

A comparison of a spherical dome was made between the Sofistik finite element analysis results and shell theory equations using an example found in [1].

The rigidly supported spherical dome of Fig. 1 is analyzed for uniform gravity load over the dome surface. The parameters of the problem are: r = 28,80 m (94,5 ft), constant shell thickness h = 10,2 cm (4 in), α = 28°, parallel circle radius at springing r_0 = rsinα =13,49 m (44,25 ft), Poisson's ratio v = 1/6 = 0,167, q = 2,55 kN/m^2 (dead load) + 1,76 kN/m^2 (live load) = 4,31 kN/m^2 (90 psf). Concrete class is C30/37, concrete strength is f_{ck} = 30 N/mm^2 and modulus of elasticity E = 32000 N/mm^2.

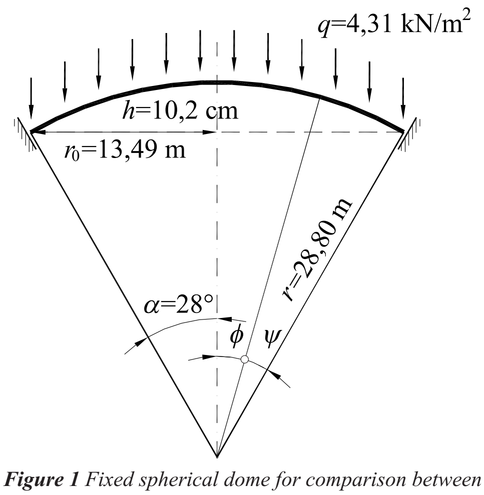

Figure 1 *Fixed spherical dome for comparison between finite element analysis and analytic example*
Slika 1. *Sferna kupola na upetim osloncima za usporedbu između metode konačnih elemenata i analitičkog primjera*

The forces were computed on the basis of the membrane theory, where meridional force is

$$N_{\phi}^{'} = -\frac{qr}{(1+\cos\phi)} \quad (1)$$

and hoop force is

$$N_{\theta}^{'} = -qr\left(\cos\phi - \frac{1}{1+\cos\phi}\right) \quad (2)$$

as listed in Tab. 1.

The equations for displacements of the dome edge due to membrane forces ("errors") and due to edge forces X_1=H=1 and X_2=M_α=1 ("corrections") derived from a bending theory can be found in [1]. Also, the size of the correction forces (X_1, X_2) required was computed by setting up the two equations of compatibility at the dome support. Forces over the dome given in Tab. 1 were obtained by combining membrane values with those due to X_1 and X_2.

In Sofistik, finite element analysis (FEA) program, the geometry for the dome was generated using Sofiplus program based on Autocad [2].

A four-noded quadrilateral plate element (quad) was used in meshing the dome. A rough mesh (6014 elements) was applied for initial comparison and then refined until the forces converged to the values found using analytical example. The bottom edge of the dome was completely constrained. A fine mesh (12028 elements) was used in the final analysis as shown in Fig. 2.

Tab. 1 lists the meridional force N, the hoop force N_θ and the meridional moment M that were calculated by Sofistik and those calculated using shell theory. These forces and moments were obtained at the bottom edge and at the apex of the spherical dome.

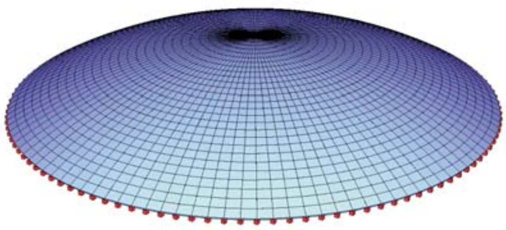

Figure 2 *Fine mesh of a spherical dome on fixed supports in Sofistik*
Slika 2. *Gusta mreža sferne kupole na upetim osloncima iz Sofistika*

Tab. 1 also shows that the classical theory developed by Geckeler gives results close to those found from the numerical analysis. The edge value of N_θ from the membrane theory changed drastically due to the boundary conditions, but the meridional force N did not change very much. It should be noted that the bending moments are generally small and restricted to a narrow zone at the edge of the dome. At the apex of the shell the finite element results are close to those from membrane theory or classical theory. Comparing the calculations using shell theory with the Sofistik results, the finite element model of the spherical dome is found to be valid and more complicated examples can be explored.

3
Analyzed spherical and groined shells
Analizirane sferne i svodaste ljuske

Figure 3 *Kresge-MIT Auditorium, Boston, USA (Saariner, 1954)*
Slika 3. *Kresge-MIT auditorij, Boston, SAD (Saariner, 1954)*

Figure 4 *CNIT Exhibition Hall, Paris, France (Esquillan, 1958)*
Slika 4. *CNIT izložbena dvorana , Pariz, Francuska (Esquillan, 1958)*

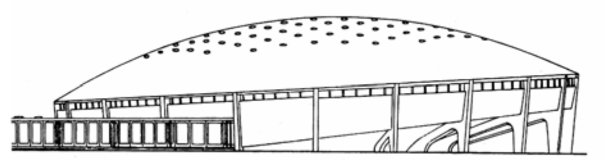

Figure 5 *Ehima Public Hall, Matsuyama, Japan (Tange&Tsuboi)*
Slika 5. *Ehima javna dvorana, Matsuyama, Japan (Tange&Tsuboi)*

Using the Sofistik finite element program that solves large-scale structural analysis problems, several thin shell structures were examined. Figs 3-6 show some of the remarkable early shells for the Kresge – MIT auditorium in Boston, CNIT Exhibition Hall in Paris, Ehima Public Hall in Matsuyama and Het Evoluon in Eindhoven.

Tab. 2 lists the presented shell structures. Columns 1-4 in Tab. 2 list the name, date of completion, location, and type of these shells. Columns 5-8 list the dimensions.

Table 1 *Comparison of shell theory results and finite element analysis results*
Tablica 1. *Usporedba rezultata teorije ljusaka i MKE za sfernu kupolu na upetim osloncima*

Value	Shell theory		Sofistik, FEA		Shell	Sofistik, FEA	
	Membrane	Classical	rough mesh	fine mesh	theory	rough mesh	fine mesh
ψ	0° (edge)				28° (apex)		
N_ϕ /kN/m	-65,67	-62,60	-63,40	-63,00	-62,02	-63,90	-62,60
N_θ /kN/m	-43,78	-11,38	-10,46	-10,51	-62,02	-63,20	-62,50
M_ϕ/kNm/m	0	-1,16	-1,10	-1,18	0	0,08	0,04

Table 2 *Notable shell structures*
Tablica 2. *Značajne ljuskaste konstrukcije*

Name of construction	Year	Location	Type	Radius R/m	Length L/m	Rise d/m	Thickness h/cm	Significance	Construction System
Kresge auditorium, MIT	1954	Boston, USA	1/8 sphere, triangular plan	34	49	-	8,9	Supported on three points	Cast in place concrete
CNIT Exhibition Hall	1958	Paris, France	Groined vault, triangular plan	100-200	205,5 (218)	46,3	12	Two-layer shell spread apart by vertical walls (box cross section) to prevent buckling	Cast in place concrete
Ehima Public Hall	-	Matsuyama, Japan	Shallow spherical shell	-	49,35	7,0	8,0	Spherical inclined dome, edge ring supported by 20 columns	Cast in place concrete
Het Evoluon	1966	Eindhoven, Netherlands	Spherical calotte	-	77,0	9,0	-	Prestressed edge ring	Precast concrete

Figure 6 *Het Evoluon, Eindhoven, Netherlands (Kalff, 1966)*
Slika 6. *Het Evoluon izložbena dvorana, Eindhoven, Nizozemska(Kalff)*

Columns 9-10 indicate the significance of each shell and construction system. These structures were built before the use of computers.

3.1
Kresge Auditorium
Kresge auditorij

This building, designed by a noted modernist architect, Eero Saarinen, consists of a one-eighth spherical segment dome-shaped concrete roof enclosing a triangular area approximately 49 m (160 ft) on a side. The dome is entirely supported on three points at the vertices of the triangle. The total mass of the roof is approximately 1500 tons, and the thickness of the roof shell is 8,9 cm (3,5 in) which is increased near the edge beams up to 14 cm. The 8,9 cm (3,5 in.) concrete shell is covered with 5,1 cm (2 in) of glass fiberboard and a second nonstructural layer of lightweight concrete 5,1 cm (2 in) thick. Additions had to be made to this structure, since Saarinen's sculptural cutting of the shell created severe edge disturbances to the membrane stresses in the shell that had to be counteracted by an edge beam 45,7 cm (18 in) high. There were also large stresses created at the three points of support. These were reinforced with tapered H-shaped steel ribs, which in turn were connected to a steel hinge allowing for movement. In the end, after the formwork was removed it was discovered that the edges were deflecting an unacceptable amount (clearly well over 12,7 cm (5 in)) due to uncontrolled creep. Additional supports were added in the form of (4-by-9-in) steel tubes spaced at 3,35 m (11 ft), which were also used to support the window wall [3].

The problems with this building did not end with the resolution of the structural problems. The shell was difficult and unusual to construct, and significant difficulties were encountered in concrete placement (poor consolidation), protection of the reinforcing steel (inadequate concrete cover) and above all in the waterproofing the roof of the building. The satisfactory resolution of these problems had to wait until decades after the commissioning of the building and through several trials of different roofing procedures.

The original neoprene roofing was later replaced with lead-coated copper roofing and then copper roofing. The repair of the construction was costly and forced the closure of the building for a few months.

3.2
CNIT Exhibition Hall
Izložbena hala CNIT

The famous CNIT exhibition hall at La Defense in Paris, designed by a French engineer, Nicolas Esquillan, is supposed to be the largest shell roof in the world with a span of 206 m. The immense CNIT three-point supported shell (Fig. 4) shows an example where the original form was chosen to achieve a simple picture of structural behaviour, in this case as three arches [1]. It consists of six intersecting double-shell parabolic vaults approximated to a triangular groined vault with three horizontal ridges and very slight circumferential curvatures below the ridges so that the loads would be transmitted as directly as possible to the three supports instead of having to be transmitted partly by edge beams as in the Kresge Auditorium shell [4]. However, it would be simpler to design the full dome than a triangular piece cut out of it due to the instability of the free edge, which creates a potential for buckling. This problem was prevented using a two-layer shell spread apart by vertical walls. The overall depth of the system is 1,9 m at the crown and 2,7 m at the spring line. The thickness of each layer is 6 cm at the crown and 12 cm at the spring line. The interior precast cross walls are 6 cm thick. This was built during the years 1957/58, before the use of computers [5].

3.3
Ehima Public Hall
Javna dvorana Ehima

The Ehima Public Hall in Matsuyama, Japan, designed by Japanese engineers, Tange and Tsuboi, is a shallow spherical inclined shell supported by 20 columns. A ring is provided around the base between columns. The thickness of the shell is 8 cm with a diameter of 49,35 m and a rise of 7 m at the crown [6].

3.4 Het Evoluon
Kongresna dvorana Het Evoluon

The Het Evoluon in Eindhoven was the last major project of the Netherlands designer Louis Kalff. The building is unique due to its resemblance to a landed flying saucer, which makes it look very futuristic. The dome has a diameter of 77 m and rests on 12 V-shaped columns. The overall height of the building is 30 m [6].

4 Structural optimization study
Optimizacija konstrukcije

Our optimization studies on these structures fall into three categories: (1) A thickness optimization study that compares a shell with uniform thickness to one in which the thickness is optimally distributed over the area, (2) a size optimization study that examines the size of the edge beam, and (3) a material optimization study. The maximum compressive and tensile stresses and maximum deflections are evaluated. The maximum compressive stresses are not discussed but shown to be well within the limits of concrete strength except for the original design of the Kresge auditorium (Tab. 3). However, during repair of the Kresge auditorium the strength of the concrete was found to be well above the required strength, ranging from 31 to 38 MPa (4500 to 5500 psi) [7].

The concrete material properties assume a unit weight of 25 kN/m^3, a Young's Modulus of 36 GPa (C45/55) and a Poisson's ratio of 0,2. The reinforcing steel material properties assume a yield strength of 500 MPa and a Young's Modulus of 200 GPa. The load on the structure is its self weight and snow load of 1,25 kN/m^2 uniformly distributed on the horizontal projection.

In Sofistik FEA program, the Sofiplus was used as pre-processing tool for model building and mesh generation. The quadrilateral or triangular element (in the case of MIT auditorium) was used in meshing.

4.1 Thickness optimization
Optimizacija debljine

The distribution of thickness was obtained via free optimization. Free optimization refers to determination of the thickness of the shell free hand, that is, without a computer algorithm.

It is assumed that each element of the mesh for Kresge auditorium has the same initial thickness equal to 8 cm. The program Bemess in Sofistik carried out the task of reinforcement design according to Eurocode 2, Part 1. The criterion for thickness optimization was the punching design performed by Bemess. The optimum solution of the design task shows a clear distribution of larger thickness around the supports equal to 30 cm (Fig. 7). The colors represent the gradation of thickness.

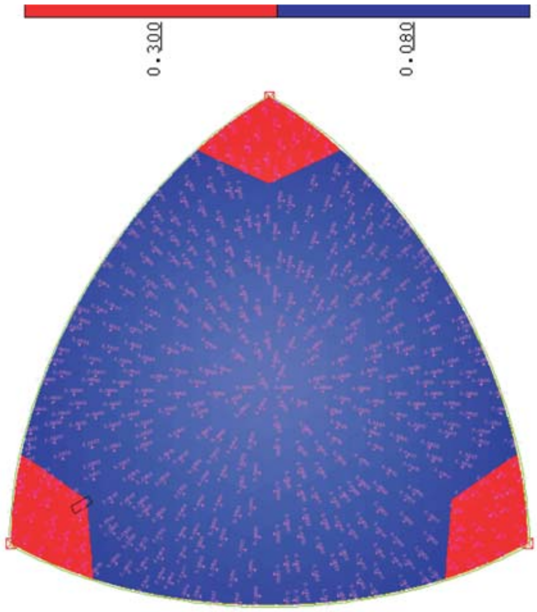

***Figure** 7 Structural optimization design for Kresge auditorium*
***Slika** 7. Optimizacija konstrukcije Kresge auditorija*

The deflections and maximum tensile stresses of the shell with an optimized thickness distribution (design 2) are compared to the same shell with a uniform thickness of 8,9 cm (design 1) and another distributed thickness shell with edge beam of uniformly varying cross section (design 3) (Tab. 3).

In the original design 1 the concrete shell is reinforced with a stiffening beam (20×45 cm) around the perimeter of the building, and the concrete class is C30/37. In the design 2 the concrete strength of the distributed thickness shell and 20×45 cm edge beam is C40/50. The design 3 comprises distributed thickness shell and (30×30 cm to 30×70 cm) edge beam with higher concrete strength C45/55.

The maximum tensile principal stresses (loadcase dead load) are equal to 14,43 MPa, 3,89 MPa, and 2,77 MPa for the design 1, design 2 and design 3, respectively (Tab. 3). These maximum tension stresses occur in the region of

***Table 3** Principal stresses, displacements and reinforcements for three shell designs*
***Tablica 3.** Glavna naprezanja, pomaci i armatura za tri rješenja ljuske*

Design	Loading	Top principal stress/MPa		Bottom principal stress/MPa		Displacement/mm		Reinforcement/cm^2/m	
		Min.	Max.	Min.	Max.	Min.	Max.	Top	Bottom
1	Dead load	-19,26	**14,44**	-23,20	12,78	-298,86	133,31	32,87	39,58
	Snow load	-9,71	7,41	-11,55	6,53	-153,22	68,43	Punching failure	
2	Dead load	-5,89	3,57	-13,01	**3,89**	-50,34	15,10	20,56	49,75
	Snow load	-4,78	4,13	-8,09	3,87	-48,76	16,58		
3	Dead load	-4,81	2,48	-8,15	**2,77**	-36,57	9,96	7,33	13,37
	Snow load	-3,60	3,05	-4,79	3,52	-40,64	12,72		

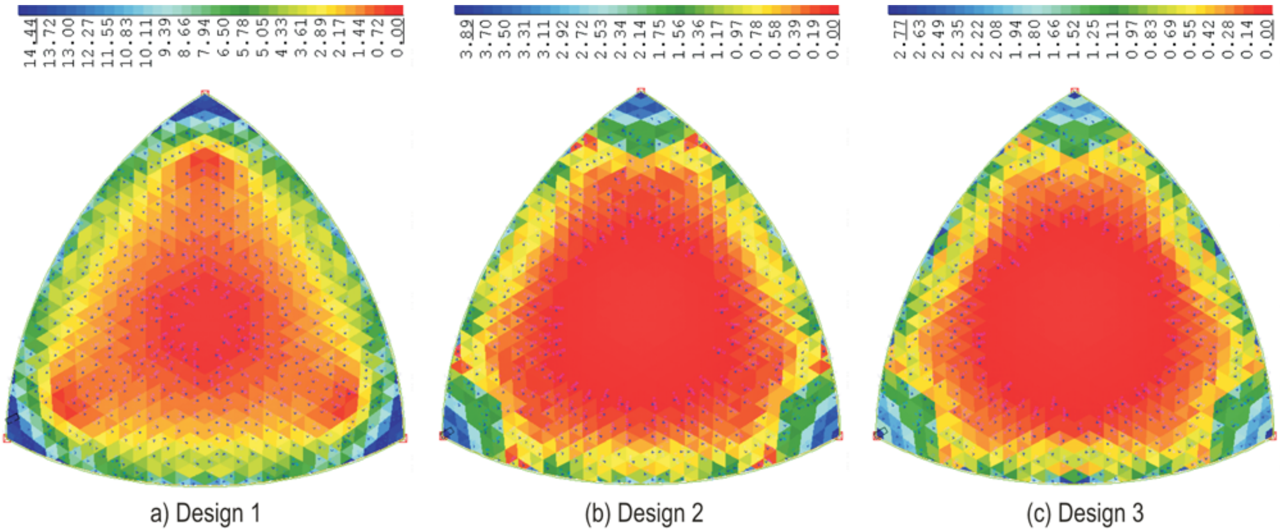

Figure 8 *Principal tension stresses in the Kresge shell (for dead load)*
Slika 8. *Glavna vlačna naprezanja u ljusci Kresge (za vlastitu težinu)*

the supports, gradually decreasing to zero at the center of the shell. Note that positive stresses indicate tension, negative stresses compression.

For a comparison of stresses between the three shell designs, we examine the amount of tensile area in each shell with an understanding that tension is undesirable in thin shell concrete structures. Fig. 8 shows the areas of tension maximum principal stresses for the three shell designs. The figures clearly show that the design 3 develops less tensile area and smaller maximum tension stresses, and thus is a more efficient design.

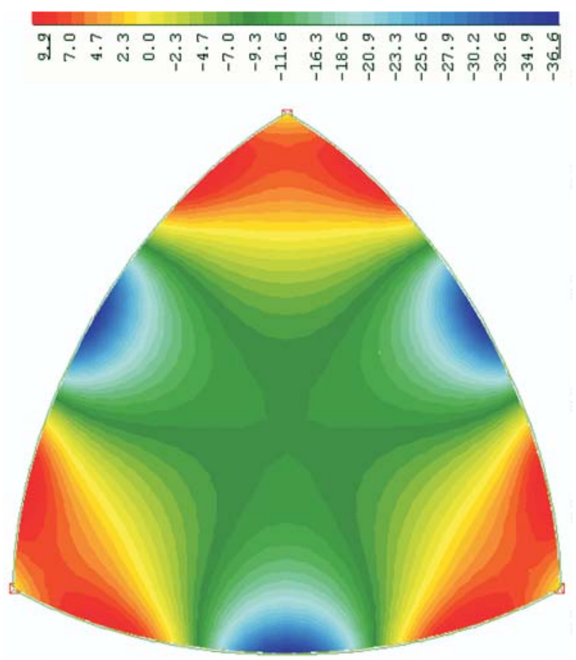

Figure 9 *Downward deflections in the Kresge shell (for dead load)*
Slika 9. *Vertikalni pomaci u ljusci Kresge (za vlastitu težinu)*

The deflections of the shell reflect stiffness and safety. Heinz Isler, respected designer of thin shell concrete structures, set a maximum accepted deflection to span ratio (Δ/L) equal to 1/300 for his shells [8, 9]. We can use this value as a reference for examining the maximum downward deflection in this shell, which occurs at the high point of the side span L (Fig. 9). The Δ/L values are equal to 1/164, 1/973, and 1/1340 for the design 1, design 2 and design 3, respectively. These ratios, except for design 1, are significantly smaller than that imposed by Isler. In addition, Tab. 3 shows that the deflections for the distributed thickness shells (design 2 and 3) are much smaller than the uniform thickness shell (design 1). The negative sign indicates downward displacement in Tab. 3.

Examination of the contours of maximum downward displacement shows that the uniform thickness of 8,9 cm shell has not only a larger maximum, but that maximum extends over a greater area than the other two shells.

4.2
Size and material optimization
Optimizacija dimenzija i materijala

The effects of the beam size and material properties for the Kresge shell were also obtained via free optimization. The design variables that are changed are the height of the edge beam, and the concrete strength. The edge beam of uniformly varying cross section (height varies from 30 cm at apex to 70 cm at supports) in design 3 enhances the shell stiffness, reducing maximum (principal) tensile stresses and deflection, and thereby reducing reinforcements (Tab. 3). Also, the higher concrete strength of C45/55 reduces the deflection and the amount of reinforcements.

The optimization study has been extended to the same type of CNIT shell structure but decreased in size. The analysis was performed for a one-layer shell with span L=80 m. The span to thickness ratio (L/h) is equal to 800 for this one-layer shell which is almost half the L/h value of 1713 for the 206 m span two-layer CNIT shell. However, when the ratio is increased some problems like buckling can easily appear in the structure. Furthermore, the structural behaviour is more complex because of the growing problem of buckling, displacements, strains and stresses. Structural optimization of this groined vault type dome shell results in a distribution of thickness throughout the shell. Figure 10 shows an optimum design with a clear distribution of larger thickness around the supports and a nice gradation of thickness towards the center to minimize stresses and deflections. The thickness is increased stepwise from 10 to 40 cm by increments of 2,5 cm.

The maximum tensile (principal) stresses are equal to 9,83 MPa and 10,43 MPa for dead load and snow load respectively (Fig. 11). Hence, the tensile stresses are

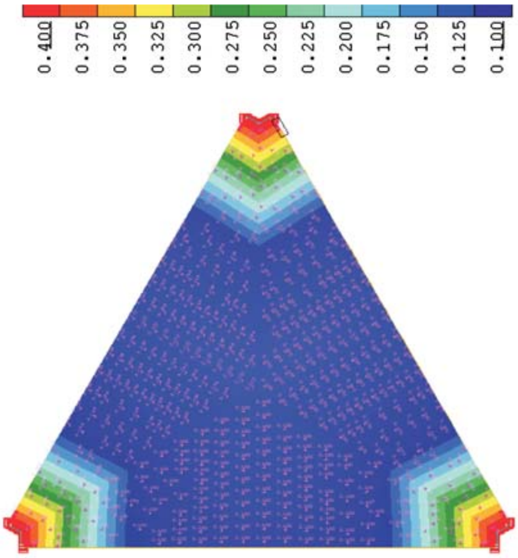

Figure 10 *Structural optimization design for the smaller CNIT shell*
Slika 10. *Optimizacija konstrukcije manje ljuske CNIT*

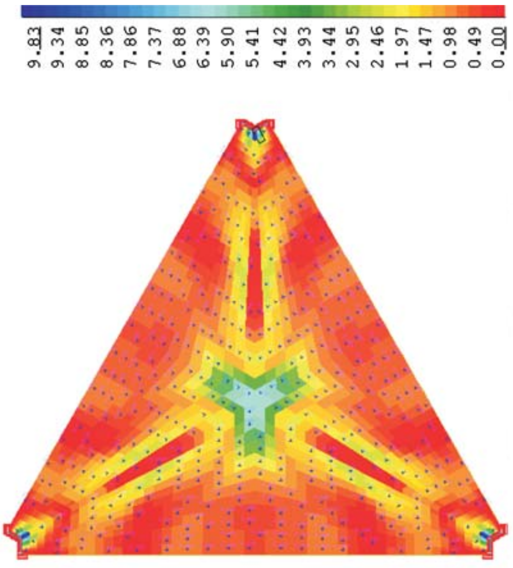

Figure 11 *Principal tension stresses in the smaller CNIT shell (for dead load)*
Slika 11. *Glavna vlačna naprezanja u manjoj ljusci CNIT (za vlastitu težinu)*

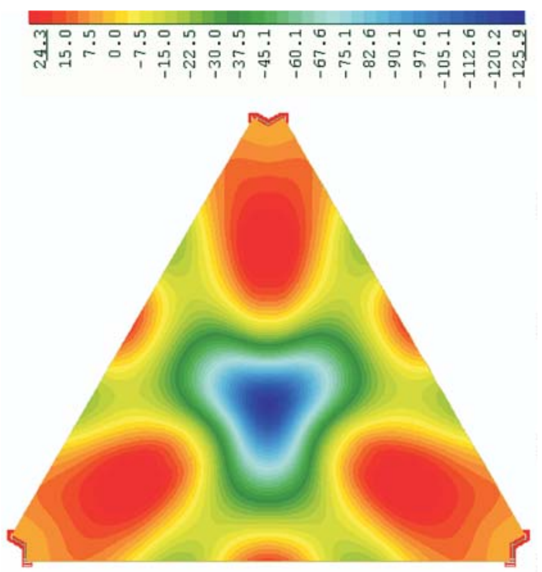

Figure 12 *Downward deflections in the smaller CNIT shell (for dead load)*
Slika 12. *Vertikalni pomaci u manjoj ljusci CNIT (za vlastitu težinu)*

greater than the tensile capacity of even the stronger concrete. For instance, in structures built by Candela [8], who is another respected designer of thin concrete shells, the tensile stresses are typically below the tensile strength of concrete. But in any case Candela included reinforcing steel throughout for incidental stresses that may arise from creep, shrinkage, and temperature effects.

The maximum downward deflection in the smaller CNIT structure occurs at the top of the shell as shown in Fig. 12. The Δ/L value is equal to 1/636 for the 80 m span shell which is smaller than deflection to span ratio of 1/300 imposed by Isler (see item 4.1).

Structural optimization of Evoluon results in a shell with uniform thickness of 8 cm, reinforced with meridional and hoop ribs. The ribbed model built in Sofistik has a 20×30 cm ring at the top of the dome around 6,70 m diameter skylight. Radiating off of the ring beam at the top of the dome are 30×60 cm ribs at 7°. Added are two hoop 30×60 cm ribs that are located at 6 m, and 12,2 m from the edge ring. The edge ring at the junction of the upper and lower shell is 40×60 cm with a 77 m diameter. A 60×80 cm bottom ring is supported by 80×80 cm V-shaped columns. The lower shell also has two hoop 30×60 cm ribs that are located at 6 m, and 15,4 m from the edge ring (Fig. 13).

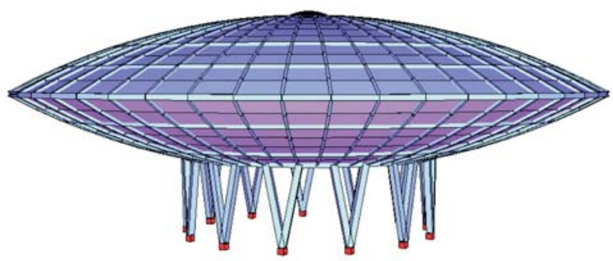

Figure 13 *Ribbed model of Evoluon shell built in Sofistik*
Slika 13. *Rebrasti model ljuske Evoluon iz Sofistika*

The tensile principal stresses are shown in Fig. 14. The maximum tensile (principal) stresses are equal to 14,20 MPa and 3,21 MPa for dead load and snow load respectively (dark blue).

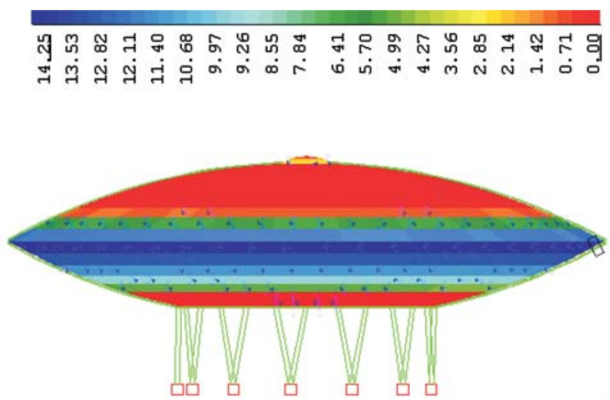

Figure 14 *Principal tension stresses in the ribbed Evoluon shell (for dead load)*
Slika 14. *Glavna vlačna naprezanja u rebrastoj ljusci Evoluon (za vlastitu težinu)*

The maximum downward deflection in ribbed Evoluon shell occurs at the apex of the dome (Fig. 15). The Δ/L value is equal to 1/635 which is smaller than 1/300 limit used by Isler.

Table 4 *Effect of rise d on maximum tensile principal stress, displacement and reinforcement for Ehima dome*
Tablica 4. *Utjecaj uzvišenja d na glavna vlačna naprezanja, pomake i armaturu za kupolu Ehima*

Rise d/m	Loading	Max. tensile principal stress/MPa	Max. downward displacement/mm	Reinforcement/cm^2/m	
				Top	Bottom
7	Dead load	2,80	5,60	6,35	4,35
	Snow load	1,62	3,21		
8	Dead load	2,46	4,46	5,64	3,85
	Snow load	1,39	2,54		
9	Dead load	2,21	3,75	5,13	3,47
	Snow load	1,22	2,09		

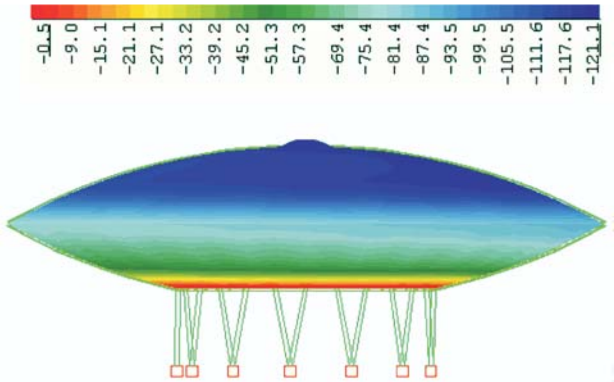

Figure 15 *Downward displacements in the ribbed Evoluon shell (for dead load)*
Slika 15. *Vertikalni pomaci u rebrastoj ljusci Evoluon (za vlastitu težinu)*

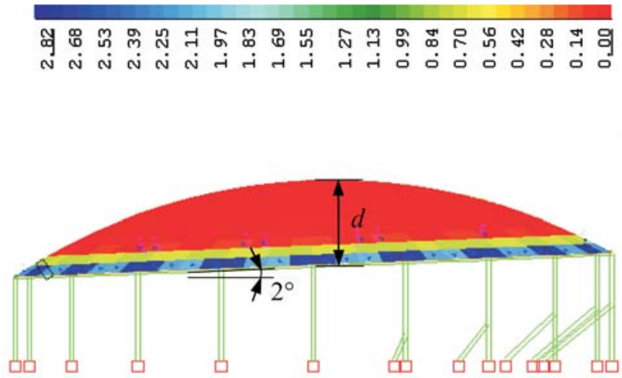

Figure 16 *Areas of tension (based on principal stress) in the Ehima dome (for dead load)*
Slika 16. *Vlačna područja (na osnovi glavnih naprezanja) u kupoli Ehima (za vlastitu težinu)*

4.3 Shape optimization
Optimizacija oblika

The effects of the shell rise are also obtained via free optimization. The values for diameter (span) and thickness of Ehime shell are kept constant in the shape optimization designs. The dimensions of the ring and columns are assumed equal to 40×60 cm and 50×50 cm, respectively. The slope of the shell is set equal to 2°. The only variable that is changed is rise *d*. The rise *d* varies from 7, 8 and 9 meters in this study.

Tab. 4 gives the effects of *d* on the maximum tensile (principal) stress, downward displacement and reinforcement for a spherical shell with a uniform thickness of 8 cm. It is seen that the maximum tensile principal stress, downward deflection and reinforcement decrease with *d*. Increasing the rise by cca 30 % decreases the maximum tensile principal stresses, the deflections and the reinforcements by 23 %, 34 % and 20 %, respectively.

The shell with d=7 m as in original design for Ehime shell has Δ/L value equal to 1/8812 which is significantly smaller than 1/300 imposed by Isler.

If the larger distance between the supporting columns had been assumed, the shell would have greater maximum tensile (principal) stress, downward deflection and reinforcement.

Also, our structural optimization study of spherical domes has shown that as the ring size (stiffness) increases, it "keeps" more of the tension, and in the limit case where the area of ring is equal to ∞, the system becomes a fixed dome with the small maximum tensile principal stress and reinforcement. In the other direction as the ring size (stiffness) decreases, the dome maximum tensile principal stress and reinforcement increase substantially.

Fig. 16 shows the areas of maximum tensile principal stress in the Ehime dome. The red color represents the area with no tension. It can be seen that bending moments and tensile principal stresses are restricted to a narrow zone at the edge of the dome. This area becomes smaller as the ring size increases.

5 Conclusion
Zaključak

The finite element analysis in this paper demonstrates that structural optimization leads to a more efficient design of the analyzed concrete shells and that such a tool is useful for designing thin shell concrete structures.

In the past, the designer relied on calculations and methods that were limited by hand calculations. Comparison of the results between shell theory and the finite element model of the spherical dome shows that finite element results are reliable and they can be used in analysis and design of these structures with confidence.

A detailed structural optimization study of Kresge Auditorium and CNIT shows that varying the thickness of the shell, with the largest thickness at the supports, leads to the most effective design in terms of reduced tensile stresses, reduced deflections, and most efficient use of material. We can conclude that a distributed thickness is appropriate for these designs. Our results also indicate that the Kresge shell could have been designed with the edge beam of varying cross section and higher concrete strength than the documented C30/37 (4000 psi) strength in order to reduce the excessive deflections. Structural optimization results of Evoluon indicate that the shell could have been designed as a ribbed model that would be less thick to reduce the weight. The shape optimization study of Ehima shell shows that the shell rise could have been larger than 7 m original design (flatter shell). If the larger distance between the supporting columns had been assumed, the shell would have greater tensile stress, deflection and reinforcement. Note that the structural optimization of these structures could be conducted using the most sophisticated structural optimization techniques available today.

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