Uroš Klanšek[1]
Mirko Pšunder[2]

COST OPTIMIZATION OF TIME SCHEDULES FOR PROJECT MANAGEMENT[3]

ABSTRACT

The paper presents the cost optimization of the time schedules for project management. The nonlinear programming (NLP) model for the cost optimization of the time schedules under the generalized precedence relations between the project activities was developed and applied. The existing NLP optimization models have focused on the cost optimal solution of the project scheduling problems which include simplifying assumptions regarding the precedence relationships among the project activities. In this way, this research work aims to propose the NLP optimization model for making optimal time-cost decisions applicable to actual projects in project management. The generalized reduced-gradient method was used for the NLP optimization. The obtained results include the minimum total cost project schedules and the optimal project time-cost curves. The proposed optimization approach enables the insight into the interdependence between the project duration and the total project cost. The decision-maker can more effectively estimate the effect of the project deadline on a total project cost before the submission of a tender. An application example and an example of the time-cost trade-off analysis are presented in the paper to demonstrate the advantages of the proposed approach.

JEL: *M11, O21, O22, C61*

Keywords: *project management, planning models, project analysis, optimization techniques*

1. INTRODUCTION

Since the development of the critical path method (CPM) in the late 1950s, the cost effective scheduling has received substantial attention from numerous researchers because of its practical relevance in project management. Namely, execution of each project activity, in normal duration, requires employment of certain resources and direct cost. Completing the activity in its reduced duration (i.e. activity crashing) requires additional resources at extra cost. On the other hand, the activity crashing leads to decreasing project duration and indirect cost. Keeping the project costs within the budget and not to exceed the project duration may be identified as project critical success factors, see e.g. Indihar-Štemberger et al. (2009).

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[1] Assistant Professor, University of Maribor, Faculty of Civil Engineering, Smetanova 17, SI-2000 Maribor, Slovenia. Scientific affiliation: construction management and economics, optimization, operational research. Phone: + 386 2 22 94 378. Fax: + 386 2 25 24 179. E-mail: uros.klansek@uni-mb.si.

[2] Full Professor, University of Maribor, Faculty of Civil Engineering, Smetanova 17, SI-2000 Maribor, Slovenia. Scientific affiliation: construction management and economics. Phone: + 386 2 22 94 351. Fax: + 386 2 25 24 179. E-mail: mirko.psunder@uni-mb.si.

In this way, the cost effective time schedules in project management are traditionally achieved in a time-consuming cost-duration analysis of various alternatives for start times and durations of the project activities. However, doubt always exists as to whether or not the obtained project schedule is optimal. In order to surmount the mentioned disadvantages of the traditional approach, various different optimization methods have been proposed for the cost optimization of the project schedules. A number of authors have conducted the research works that are focused on optimal project scheduling using classical approximate heuristic methods. For instance, Feng et al. (1997), Li et al. (1999), Hegazy (1999), Leu and Yang (1999) and Senouci and Eldin (2004) proposed the models for optimization of the project schedules using the genetic algorithms (GA). Shtub et al. (1996) and Azaron et al. (2007) developed the simulated annealing (SA) optimization model formulations. Gagnon et al. (2002) proposed the tabu search (TS) optimization model to minimize the cost of the project. Adeli and Karim (1997) presented the neural network (NN) model formulation for the cost optimal project scheduling. Xiong and Kuang (2008) introduced an optimization model for the cost optimization of the project schedules using the ant colony optimization (ACO). Yang (2007) proposed the particle swarm optimization (PSO) model formulation for the cost optimal scheduling in project management.

Considering the exact mathematical programming methods, the cost optimization of the project schedules has been handled mainly by different linear programming (LP) methods, see e.g. Demeulemeester et al. (1998); Achuthan and Hardjawidjaja (2001); Möhring et al. (2001); Vanhoucke et al. (2002), Sakellaropoulos and Chassiakos (2004). Since the LP optimization methods can handle only linear relations between the variables, the nonlinear terms of the optimization models have been formulated as the discrete relationships between the variables or they have been approximated with (piece-wise) linear functions.

However, even the earliest studies in this field have recognized the nonlinear nature of the project cost-duration relationships. Furthermore, the production costs oftentimes appear in practice as nonlinear functions of different parameters, see e.g. Klanšek and Kravanja (2006) and Rep et al. (2008). Therefore, the nonlinear programming (NLP) optimization techniques have been proposed to solve the project scheduling optimization problems with continuous nonlinear cost functions, see e.g. Kapur (1973); Deckro et al. (1995), Deckro and Hebert (2002), and Turnquist and Nozick (2004). Nevertheless, in most of the published research works the cost optimization of the project time schedules was performed considering only the finish-to-start precedence relationships between the project activities.

On the other hand, the aim of this paper is to develop the model for the cost optimization of the time schedules considering realistic project characteristics such as nonlinear nature of the cost-duration relationships and generalized activity precedence relations. The NLP model for the cost optimization of the time schedules under the generalized precedence relations between the project activities was developed and applied. In this way, this research work aims to propose the NLP optimization model for making optimal time-cost decisions applicable to actual projects in project management. The generalized reduced-gradient method by Drud (1994) was used for the NLP optimization. The obtained results include the minimum total cost project schedules and the optimal project time-cost curves. An application example and an example of the time-cost trade-off analysis are presented in the paper to demonstrate the advantages of the proposed approach.

2. NLP OPTIMIZATION PROBLEM FORMULATION

The general NLP optimization problem may be formulated in the following form:

$$\text{Minimize } z = f(\mathbf{x}),$$
$$\text{subject to:}$$
$$\mathbf{h}(\mathbf{x}) = \mathbf{0}, \qquad \text{(NLP)}$$
$$\mathbf{g}(\mathbf{x}) \leq \mathbf{0},$$
$$\mathbf{x} \in \mathbf{X} = \{\mathbf{x} \mid \mathbf{x} \in \mathbf{R}^n, \mathbf{x}^{LO} \leq \mathbf{x} \leq \mathbf{x}^{UP}\},$$

where $\mathbf{x}$ is a vector of the continuous variables, defined within the compact set $\mathbf{X}$. Functions $f(\mathbf{x})$, $\mathbf{h}(\mathbf{x})$ and $\mathbf{g}(\mathbf{x})$ are the (non)linear functions involved in the objective function z, the equality and inequality constraints, respectively. All the functions $f(\mathbf{x})$, $\mathbf{h}(\mathbf{x})$ and $\mathbf{g}(\mathbf{x})$ must be continuous and differentiable. In the context of the project scheduling optimization problem, the continuous variables define schedule parameters such as activity durations, start times, direct costs, etc. The objective function determines the total project cost. Equality and inequality constraints and the bounds of the continuous variables represent a rigorous system of the generalized precedence relationship constraints, the activity duration constraints and the project duration constraints of the project scheduling optimization problem.

3. NLP OPTIMIZATION MODEL FORMULATION

The cost optimization of the time schedules was performed by the NLP approach. In this way, the proposed NLP optimization model formulation consists of the total cost objective function, the generalized precedence relationship constraints, the activity duration constraints and the project duration constraints. The following total project cost objective function is defined for the cost optimization of the project time schedules:

$$C_T = \sum_{i \in I} C_i(D_i) + C_I(D_P) + P(D_L) - B(D_E), \qquad (1)$$

where objective variable C_T represents the total project cost, set I comprises the project activities i, $i \in I$, $C_i(D_i)$ denotes the direct cost-duration functions of the project activities i, $i \in I$, $C_I(D_P)$ is the project indirect cost-duration function, $P(D_L)$ is the penalty-duration function and $B(D_E)$ is the bonus-duration function. The variables D_i, D_P, D_L and D_E denote the durations of the project activities i, $i \in I$, the actual project duration, the amount of time - the project is late, and the amount of time - the project is early, respectively.

The project activity direct cost-duration functions can be defined from the resources allocated to it, e.g. see Klanšek and Kravanja (2006). The project indirect cost depends upon the actual project duration and it usually includes the initial project costs, the business operating and overhead costs, and the machinery/equipment operating costs. The project penalty cost depends upon the amount of time - the project is late, while the project award bonus depends upon the amount of the time - the project is early. Both the penalty cost for late project completion and the award bonus for early project completion are usually determined from the project contract. The total project cost objective function is subjected to the rigorous system of the generalized precedence relationship constraints, the activity duration constraints and the project duration constraints. The project activities are mutually linked by the generalized precedence relationships, see Figure 1.

Figure 1

Precedence relationships between the project activities
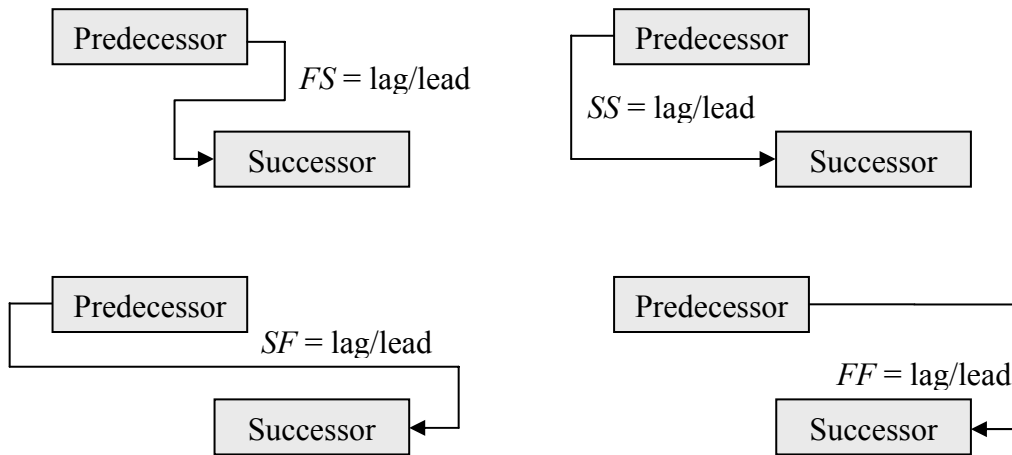
In this way, each project activity i, $i \in I$, is connected to its succeeding activities j, $j \in J(i)$ by fulfilling at least one of the following generalized precedence relationship constraints:

Finish-to-Start (*FS*): $\qquad S_i + D_i + L_{i,j} \leq S_j,$ $\qquad\qquad\qquad\qquad\qquad$ (2)

Start-to-Start (*SS*): $\qquad S_i + L_{i,j} \leq S_j,$ $\qquad\qquad\qquad\qquad\qquad\qquad$ (3)

Start-to-Finish (*SF*): $\qquad S_i + L_{i,j} \leq S_j + D_j,$ $\qquad\qquad\qquad\qquad\qquad$ (4)

Finish-to-Finish (*FF*): $S_i + D_i + L_{i,j} \leq S_j + D_j,$ $\qquad\qquad\qquad\qquad$ (5)

where S_i is the start time of activity i, $i \in I$, D_i is the activity duration, $L_{i,j}$ is the lag/lead time between the activity i, $i \in I$, and the succeeding activity j, $j \in J(i)$, and S_j is the start time of the succeeding activity j, $j \in J(i)$. The actual project duration D_P is determined within the optimization model in the following form:

$$D_P = S_{i\omega} + D_{i\omega} - S_{i\alpha},$$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (6)

where $S_{i\omega}$ and $D_{i\omega}$ represent the start time and the duration of the last project activity $i\omega$, $i\omega \in I$, and $S_{i\alpha}$ denotes the start time of the first project activity $i\alpha$, $i\alpha \in I$. Since the project activities must be executed between the project start and the finishing times, the following constraint is set to limit the completion times of the project activities:

$$S_i + D_i - S_{i\alpha} \leq D_P.$$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (7)

The relationship between the actual project duration D_P, the amount of time - the project is late D_L, the amount of time - the project is early D_E and the target project duration D_T, is formulated as follows:

$$D_P - D_L + D_E = D_T.$$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (8)

Only one of the variables D_L and D_E can, at the most, take a nonzero value in any project scheduling solution. Accordingly, these two variables are additionally constrained by the following equation:

$$D_L \, D_E = 0. \tag{9}$$

The nomenclature of the proposed NLP optimization model formulation is given in Appendix.

4. AN APPLICATION EXAMPLE

The example from construction project management practice is considered to demonstrate the applicability of the proposed NLP optimization model. The considered project scheduling optimization problem was originally presented and solved by Sakellaropoulos and Chassiakos (2004) using the mixed-integer linear programming (MILP) approach. The example project refers to the upgrading of an existing two-lane undivided highway to a four-lane divided motorway with controlled traffic access. The example project consists of 29 activities. The precedence relationships and the lag/lead times between the succeeding project activities are presented in Table 1. While the lag times are defined with positive numbers, the lead times are determined with negative numbers.

Table 1

Precedence relationships and lag/lead times between the example project activities

Activity		Succeeding activity	Precedence relationship	Lag/lead time
ID	Description	ID		[day]
Service road A:				
1.	Rock excavation	2.	Finish-to-Start	−3
		3.	Finish-to-Start	0
		6.	Finish-to-Start	0
2.	Embankment construction	3.	Finish-to-Start	0
		7.	Finish-to-Start	0
3.	Sub-base and base layers	4.	Finish-to-Start	0
		8.	Finish-to-Start	0
4.	Asphalt layer	5.	Start-to-Start	1
		9.	Finish-to-Start	0
5.	Temporary marking and signing	10.	Finish-to-Start	0
		11.	Finish-to-Start	0
Service road B:				
6.	Earth and semi-rock excavation	7.	Finish-to-Start	−1
7.	Embankment construction	8.	Finish-to-Start	0
8.	Sub-base and base layers	9.	Finish-to-Start	0
9.	Asphalt layer	10.	Finish-to-Finish	1
10.	Temporary marking and signing	11.	Finish-to-Start	0
Main road:				
11.	Traffic diversion	12.	Finish-to-Start	0
12.	Rock excavation	13.	Start-to-Start	2
		15.	Finish-to-Start	−4
13.	Earth and semi-rock excavation, existing pavement removal	14.	Start-to-Start	2
14.	Sub-grade stabilisation, retaining wall/culvert construction	15.	Finish-to-Start	−2
15.	Embankment construction	16.	Finish-to-Start	−6

	17.	Start-to-Start	4
	18.	Finish-to-Start	4
	19.	Finish-to-Start	0
16. Drainage pipe construction	−	−	−
17. Drainage layer	20.	Start-to-Start	3
18. Planting at roadway verges	−	−	−
19. Electrical installations at roadway verges	−	−	−
20. Ditches	21.	Start-to-Start	2
21. Sub-base layer	22.	Start-to-Start	2
22. Base layer	23.	Finish-to-Start	−9
23. Median island (New Jersey)	24.	Start-to-Start	6
	25.	Finish-to-Start	−4
24. Electrical installations in median island	−	−	−
25. Asphalt layer #1	26.	Start-to-Start	4
26. Asphalt layer #2	27.	Finish-to-Start	0
27. Friction course overlay	28.	Finish-to-Start	−3
28. Final marking and signing	29.	Finish-to-Start	0
29. Traffic restoration	−	−	−

The alternative cost-duration options and the direct cost-duration functions for the project activities are presented in Table 2. The continuous direct cost-duration functions are developed for each project activity on the basis of the alternative cost-duration options by using the curve-fitting calculations. While the first option refers to the normal activity duration and cost, the last option represents the crashed activity duration and cost. Moreover, the first option for duration of each activity defines the upper bound on the activity duration, while the last option denotes the lower bound on the activity duration.

Table 2

Alternative cost-duration options and direct cost-duration functions of the activities

Activity ID	Option 1		Option 2		Option 3		Direct cost-duration function
	Duration	Cost	Duration	Cost	Duration	Cost	
1.	5	2,030	4	2,300	−	−	$-270.00\,D_1 + 3{,}380.00$
2.	8	1,020	7	1,280	6	1,510	$-15.00\,D_2{}^2 - 35.00\,D_2 + 2{,}260.00$
3.	8	1,700	7	1,850	6	2,090	$45.00\,D_3{}^2 - 825.00\,D_3 + 5{,}420.00$
4.	4	590	3	730	−	−	$-140.00\,D_4 + 1{,}150.00$
5.	2	90	−	−	−	−	90.00
6.	4	910	3	1,100	−	−	$-190.00\,D_6 + 1{,}670.00$
7.	2	250	−	−	−	−	250.00
8.	7	1,490	6	1,650	5	1,830	$10.00\,D_8{}^2 - 290.00\,D_8 + 3{,}030.00$
9.	4	520	3	750	−	−	$-230.00\,D_9 + 1{,}440.00$
10.	2	90	−	−	−	−	90.00
11.	1	50	−	−	−	−	50.00
12.	8	3,260	7	3,580	6	3,710	$-95.00\,D_{12}{}^2 + 1105.00\,D_{12} + 500.00$
13.	5	1,140	4	1,400	3	1,720	$30.00\,D_{13}{}^2 - 530.00\,D_{13} + 3{,}040.00$
14.	4	300	3	450	−	−	$-150.00\,D_{14} + 900.00$
15.	8	1,020	6	1,300	5	1,430	$-3.\overline{33}\,D_{15}{}^2 - 93.\overline{33}\,D_{15} + 1{,}980.00$
16.	9	790	8	900	6	1,180	$10.00\,D_{16}{}^2 - 280.00\,D_{16} + 2{,}500.00$
17.	13	3,340	12	3,750	11	4,060	$-50.00\,D_{17}{}^2 + 840.00\,D_{17} + 870.00$
18.	9	470	8	650	7	830	$-180.00\,D_{18} + 2{,}090.00$
19.	6	460	5	600	4	810	$35.00\,D_{19}{}^2 - 525.00\,D_{19} + 2{,}350.00$

20.	6	1,280	5	1,430	–	–	$-150.00\,D_{20}+2,180.00$
21.	14	1,090	12	1,320	10	1,560	$1.25\,D_{21}^{2}-147.50\,D_{21}+2,910.00$
22.	14	900	11	1,140	9	1,400	$10.00\,D_{22}^{2}-330.00\,D_{22}+3,560.00$
23.	14	2,220	12	2,510	11	2,690	$11.66\,D_{23}^{2}-448.33\,D_{23}+6,210.00$
24.	3	230	–	–	–	–	230.00
25.	6	1,590	5	1,790	4	1,990	$-200.00\,D_{25}+2,790.00$
26.	10	2,630	9	2,930	8	3,240	$5.00\,D_{26}^{2}-395.00\,D_{26}+6,080.00$
27.	8	2,060	7	2,450	6	2,660	$-90.00\,D_{27}^{2}+960.00\,D_{27}+140.00$
28.	10	320	9	440	8	610	$25.00\,D_{28}^{2}-595.00\,D_{28}+3,770.00$
29.	1	50	–	–	–	–	50.00

Notes: D_1 to D_{29} denote the durations of the project activities (labelled 1 to 29) in days. The activity direct costs are given in cost units.
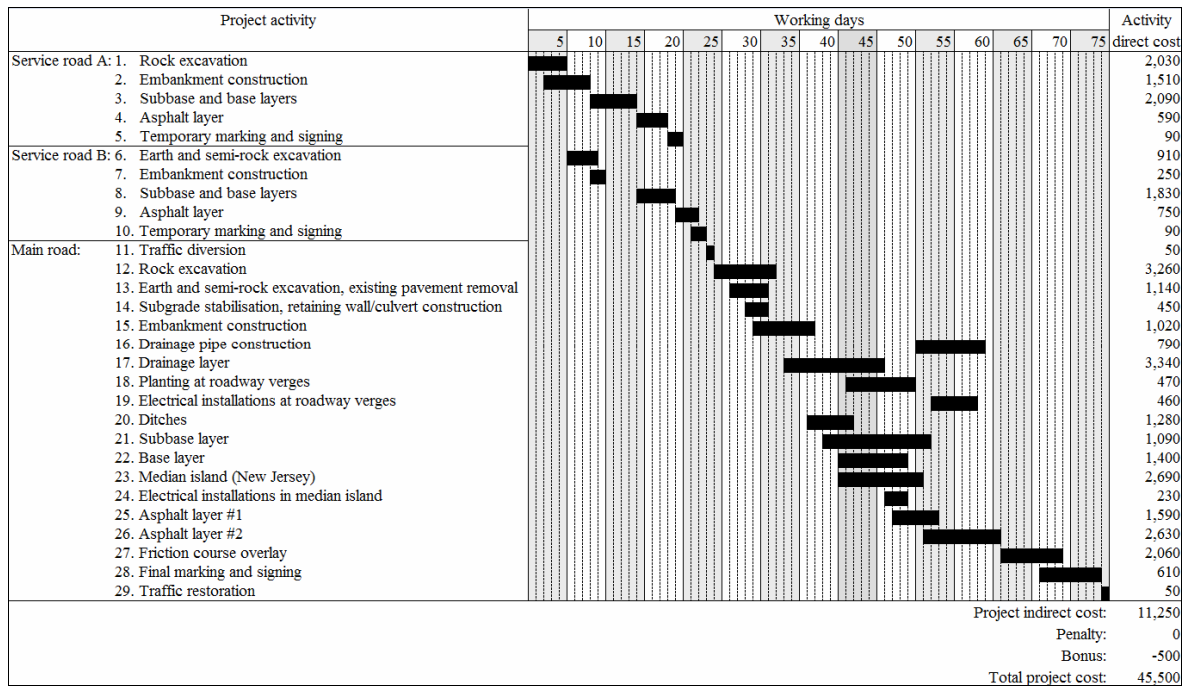
The indirect project cost is 150 cost units per day. The per-period penalty for late project completion is 200 cost units per day, while the per-period bonus for early project completion is 100 cost units per day. The target project duration is fixed at 80 days as suggested by Sakellaropoulos and Chassiakos (2004). The objective of the optimization was to find the project time schedule with the optimal start times and the optimal durations of the activities with respect to the minimum total project cost. The proposed NLP optimization model formulation was applied. A high-level language GAMS (General Algebraic Modelling System) by Brooke *et al*. (1988) was used for modelling and for the data inputs/outputs. CONOPT (Generalized reduced-gradient method) by Drud (1994) was used for the optimization. The considered project scheduling optimization problem was solved using the personal computer, Intel Core2 Duo T8100, 2.10 GHz, 4 GB RAM DDR2 and 250 GB SATA hard disc. The convergence of the optimal NLP solution was achieved in much less than a second.

Since the NLP denotes the continuous optimization technique, the cost optimization of the project time schedule was performed in two successive steps. In the first step, the ordinary NLP optimization was performed to calculate the optimal continuous variables (e.g. activity start times, activity durations, etc.) inside their upper and lower bounds. In the second step, the calculation was repeated/checked for fixed and rounded variables (from in the first stage obtained continuous values to their nearest discrete values).

The optimization of the example project time schedule gained the minimum total project cost of 45,500 cost units and the optimal actual project duration of 75 days. The minimum total project cost of 45,500 cost units was obtained at optimum combination between the 75-day actual project duration and the 80-day target project duration. In this way, the minimum total project cost consists of 46,000 cost units of the project direct + indirect costs, which is reduced for the award bonus of 500 units gained on account of the 5-day early-completion period. Both the minimum total project cost and the optimum actual project duration were identical to those reported by Sakellaropoulos and Chassiakos (2004). Since the considered project scheduling problem represents the convex optimization problem (i.e. all the functions within the model are convex), for which both the MILP methods and the gradient-based algorithms usually easily find the global optimum, the obtained minimum cost solution of 45,500 cost units was expected. The obtained optimum time schedule of the example project is presented in Figure 2.

Figure 2

Optimal time schedule of the example project

Project activity	Working days	Activity direct cost
Service road A: 1. Rock excavation		2,030
2. Embankment construction		1,510
3. Subbase and base layers		2,090
4. Asphalt layer		590
5. Temporary marking and signing		90
Service road B: 6. Earth and semi-rock excavation		910
7. Embankment construction		250
8. Subbase and base layers		1,830
9. Asphalt layer		750
10. Temporary marking and signing		90
Main road: 11. Traffic diversion		50
12. Rock excavation		3,260
13. Earth and semi-rock excavation, existing pavement removal		1,140
14. Subgrade stabilisation, retaining wall/culvert construction		450
15. Embankment construction		1,020
16. Drainage pipe construction		790
17. Drainage layer		3,340
18. Planting at roadway verges		470
19. Electrical installations at roadway verges		460
20. Ditches		1,280
21. Subbase layer		1,090
22. Base layer		1,400
23. Median island (New Jersey)		2,690
24. Electrical installations in median island		230
25. Asphalt layer #1		1,590
26. Asphalt layer #2		2,630
27. Friction course overlay		2,060
28. Final marking and signing		610
29. Traffic restoration		50
	Project indirect cost:	11,250
	Penalty:	0
	Bonus:	-500
	Total project cost:	45,500

The comparison between the MILP optimization model presented by Sakellaropoulos and Chassiakos (2004) and the NLP model proposed in this paper shows differences in the model sizes. While the MILP optimization model, with respect to the above-demonstrated example, comprised 218 (in)equality constraints and 132 variables, the NLP model included 97 (in)equality constraints and 91 variables. Although a precise comparison between the sizes of both models cannot be performed in details, one can notice that the proposed NLP model required smaller model size formulation for the same project scheduling optimization problem than the MILP model.

The main reason for such differences lies in the fact, that the reformulation of the original nonlinear optimization problem into the linear optimization problem often requires large size problem formulation. The MILP optimization model handles the discrete variables explicitly, but the non-linearities, within the MILP model, must be formulated as the discrete relationships between the variables or they must be approximated with (piece-wise) linear functions. On the other hand, the NLP model enables the optimization of the continuous variables, while the non-linearities are handled explicitly. The NLP optimization model is simpler and requires lower computational effort than the MILP model. The result accuracy of both optimization models is the same. In this way, the obtained results demonstrate the applicability of the proposed NLP model for solving practical nonlinear project scheduling optimization problems.

5.   AN EXAMPLE OF THE TIME-COST TRADE-OFF ANALYSIS

In order to better demonstrate the advantages of the proposed approach, the paper also presents the example of the complete time-cost trade-off analysis of the project obtained, using the NLP optimization. The considered project scheduling optimization problem is a modified time-cost trade-off problem for a small building project presented by Yang (2005). The example project consists of 7 activities. The precedence relationships and the lag times

between the succeeding project activities are presented in Table 3. The crash/normal points and the direct cost-duration functions of the project activities are presented in Table 4.

Table 3

Precedence relationships and lag times between the project activities

Activity		Succeeding activity	Precedence relationship	Lag time
ID	Description	ID		[day]
1.	Underground service	2.	Start-to-Start	2
2.	Concrete works	3.	Finish-to-Start	3
3.	Exterior walls	4.	Finish-to-Start	0
4.	Roof construction	5.	Finish-to-Start	0
		6.	Finish-to-Start	0
5.	Floor finish	7.	Finish-to-Start	0
6.	Ceiling	7.	Finish-to-Finish	6
7.	Finish work	–	–	–

Table 4

Crash/normal points and direct cost-duration functions of the project activities

Activity		Duration [days]		Direct cost [$]		Direct cost-duration function
ID	Description	Crash	Normal	Crash	Normal	
1.	Underground service	3	6	4,500.00	1,500.00	$250\,D_1^2 - 3{,}250\,D_1 + 12{,}000$
2.	Concrete works	10	12	7,000.00	5,000.00	$-10{,}969.6299\,\ln(D_2) + 32{,}258.5063$
3.	Exterior walls	8	12	3,600.00	2,000.00	$11{,}664\,\exp(-0.1469\,D_3)$
4.	Roof construction	6	8	3,100.00	2,000.00	$-550\,D_4 + 6{,}400$
5.	Floor finish	3	4	3,000.00	2,000.00	$-1{,}000\,D_5 + 6{,}000$
6.	Ceiling	4	6	4,000.00	2,500.00	$-750\,D_6 + 7{,}000$
7.	Finish work	10	14	2,800.00	1,000.00	$75\,D_7^2 - 2{,}250\,D_7 + 17{,}800$
	Project	42	55	28,000.00	16,000.00	

Notes: D_1 to D_7 denote the durations of the project activities (labelled 1 to 7) in days.

The daily indirect project cost is $200.00. While the per-period penalty for late project completion is set to be $400.00/day, the per-period bonus for early project completion is determined to be $300.00/day. The resulting NLP optimization model formulation for the adopted project scheduling optimization problem included 25 (in)equality constraints and 25 variables.

The main task of the project time-cost trade-off analysis was to find the optimal project duration and its corresponding minimum total project cost time schedule. The second task was to find the optimal time-cost curves of the project which are drawn by the minimum total project cost value at each feasible combination between the actual project duration and the target project duration.

The proposed NLP optimization model was run repetitively (one time for each combination) for various values of actual and target project durations. This repetitive procedure of the NLP optimization achieved the optimal solutions for 196 combinations between the actual project duration and the target project duration. The total solver time required for each obtained optimal NLP solution of the project scheduling problem was much less than a second.

The optimal time-cost curves of the project were developed from the minimum total project costs obtained for all 196 feasible combinations between the actual project duration and the target project duration. Since the amount of time - the project is late/early is calculated

as a difference between the actual and the target project duration (which are both fixed in each run), the NLP optimization gained 14 different optimal project time schedules for actual project durations defined from the 55-day normal project duration to the 42-day crashing project duration. While the obtained optimal solutions for the project time schedules are presented in Table 5, the gained optimal time-cost curves of the project are given in Figure 3.
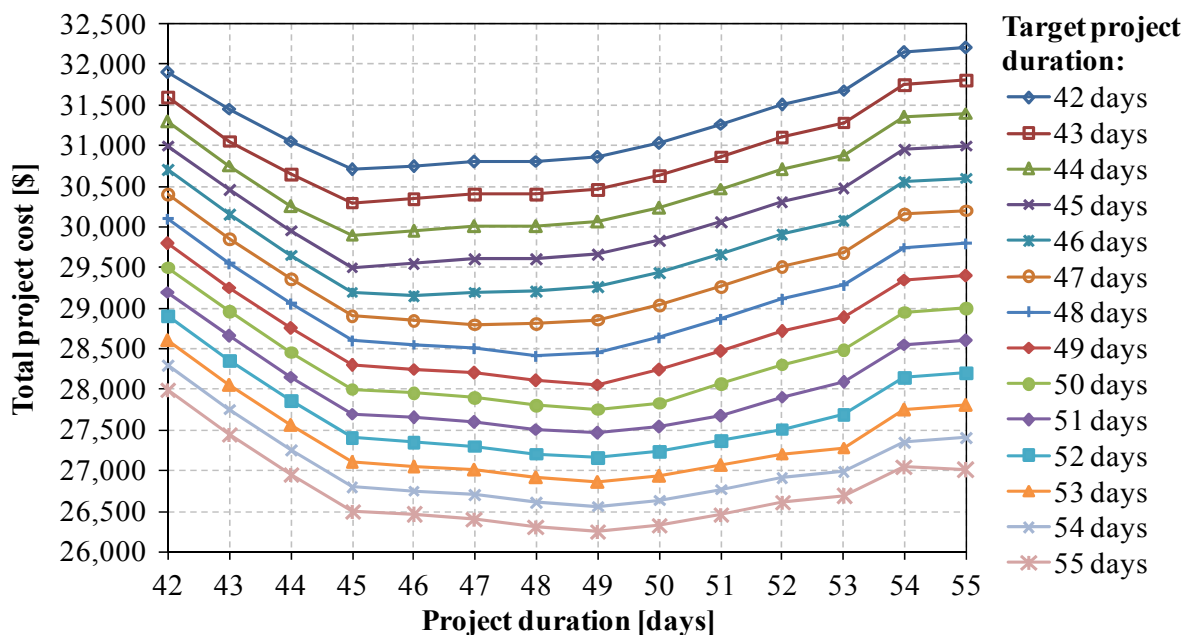
Table 5

Optimal solutions for the project time schedules

Activity			Project duration [days]													
ID Description			42	43	44	45	46	47	48	49	50	51	52	53	54	55
1. Underground service	Start	[day]	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	Duration [days]		6	6	6	6	6	6	6	6	6	6	6	6	6	6
2. Concrete works	Start	[day]	3	3	3	3	3	3	3	3	3	3	3	3	3	3
	Duration [days]		10	11	11	12	12	12	12	12	12	12	12	12	12	12
3. Exterior walls	Start	[day]	16	17	17	18	18	18	18	18	18	18	18	18	18	18
	Duration [days]		8	8	8	8	8	8	9	9	10	11	9	10	12	12
4. Roof construction	Start	[day]	24	25	25	26	26	26	27	27	28	29	27	28	30	30
	Duration [days]		6	6	6	6	7	8	6	7	7	7	8	8	7	8
5. Floor finish	Start	[day]	30	31	31	32	33	34	33	34	35	36	35	36	37	38
	Duration [days]		3	3	4	4	4	4	4	4	4	4	4	4	4	4
6. Ceiling	Start	[day]	31	31	33	34	33	34	36	34	38	38	36	38	38	38
	Duration [days]		6	6	6	6	6	6	6	6	6	6	6	6	6	6
7. Finish work	Start	[day]	33	34	35	36	37	38	37	38	39	40	39	40	41	42
	Duration [days]		10	10	10	10	10	10	12	12	12	12	14	14	14	14

Figure 3

Optimal time-cost curves of the project

The analysis of the optimal project time schedule presented in Table 5 indicates that the NLP optimization extended the actual project duration rather than the amount of time - the project was early. This is because the optimal scheduling of the activities for such actual project duration resulted in bigger cost savings than the alternative (i.e. earlier project completion with an extra award bonus). Further analysis of the optimal time-cost curves of the project demonstrates that the 55-day target project duration is an optimal one for the contractor, while the 42-day target project duration represents the optimal solution for the investor.

Namely, the 55-day target project duration enables the contractor to complete the project 6 days before deadline while keeping the total project cost on the minimum value of $26,258.02. On the contrary, the 42-day target project duration represents the optimal solution for the investor since it encourages the contractor to complete the project earlier than in 49 days (which is optimal project duration for the contractor). If the 42-day target project duration is specified in the contract, the contractor will naturally seek the minimum total cost of $30,700.00 and will want to complete the project within 45 days, see Figure 3. In this way, Table 5 and Figure 3 present the complete project time-cost trade-off analysis of the considered small building project obtained, using the NLP optimization approach. The optimal time-cost curves of the project may be used for evaluation of the optimal project duration that will lead to the minimum total project cost.

6.   DISCUSSION

The discussion in this section is addressed to the advantages and the limitations of the proposed NLP optimization model. In particular, the limitations of the proposed optimization model may be found in deterministic time and cost estimates. Contrary to stochastic approach, the deterministic approach does not consider uncertainty within the project execution. In cases when a considerable amount of risk is associated with the realization of the project activities, e.g. such as complex logistic projects (Kamnik et al., 2010), the stochastic consideration of the project parameters may estimate the execution of the project more realistically than the deterministic approach.

However, the deterministic approach to project management problems is widely used in practice on the account of the serviceability nature of its results. The application of modern activity-based costing systems (see e.g. Knežević and Mizdarković, 2010) into the deterministic project scheduling can significantly improve the performance of project management. Since the managers' perception and activities within the information management area are mainly cost oriented (Žabjek et al., 2008), the deterministic cost optimization of project time schedule using the proposed model provides the decision-maker valuable and easily explainable results when the estimation of the project input parameters is performed by experienced practitioner.

The proposed NLP optimization model enables the project time scheduling to be performed simultaneously with the total project cost minimization. Contrary to the heuristic optimization models, which calculate the approximate solutions, the proposed NLP optimization model yields the exact solution for the project scheduling problem. Since the NLP denotes the continuous optimization technique, the rounding of the continuous solution into an integer solution may be exposed as the limitation of the proposed model. On the other hand, the main advantages of the presented optimization technique are the model and application simplicity. Moreover, solving the project scheduling optimization problem, using the presented NLP optimization model, avoids the need for (piece-wise) linear or discrete approximation of the nonlinear expressions, which has been the traditional approach proposed for solving this optimization problem, using the LP optimization techniques. Since the state-

of-the-art NLP optimization methods can efficiently solve comprehensive and highly nonlinear optimization problems, the applicability of the proposed NLP optimization model is not limited to weakly nonlinear project scheduling optimization problems.

CONCLUSION

This paper presents the cost optimization of time schedules for project management. The NLP optimization model for the cost optimization of the project time schedules under the generalized precedence relations between the project activities was developed and applied. The input data of the NLP optimization model include: the project network with defined preceding and succeeding activities, the precedence relationships and the lag/lead times between the activities, the normal/crash points and the direct cost-duration functions of the activities, the project indirect cost-duration function, the penalty-duration function and the bonus-duration function.

The nonlinear continuous total project cost objective function was subjected to the rigorous system of the generalized precedence relationship constraints, the activity duration constraints and the project duration constraints. For specified input data, the NLP optimization model yields the minimum total project cost and the project time schedule with the optimal start times and the optimal durations of the project activities.

The optimization was carried out in a calculating process, where the start times and the durations of the project activities were considered simultaneously in order to obtain the minimum of the nonlinear total project cost. The obtained results of the optimization include the minimum total cost project schedules and the optimal project time-cost curves. The optimal time-cost curves of the project may be used for evaluation of the optimal project duration that will lead to the minimum total project cost.

Solving the project scheduling optimization problem, using the proposed NLP optimization model, avoids the need for (piece-wise) linear approximation of the nonlinear expressions, which has been the traditional approach proposed for solving this optimization problem, using the LP optimization models. The proposed NLP optimization model yields the exact solution of the project scheduling optimization problem.

The existing NLP optimization models have focused on the cost optimal solution of the project scheduling problems which include simplifying assumptions regarding the precedence relationships among the project activities. On the other hand, the present work aims to incorporate generalized precedence relationships between project activities and to propose the NLP optimization model for making optimal project time-cost decisions applicable to actual projects.

Since the proposed optimization approach enables the insight into the interdependence between the project duration and the total project cost, the decision-maker can more effectively estimate the effect of the project deadline on a total project cost before the submission of a tender. In this way, the proposed NLP optimization model is intended to be of considerable value to practitioners in project management. The application example and the example of the time-cost trade-off analysis were presented in the paper to demonstrate the advantages of the proposed approach.

The present study will serve as the basis for further research on the cost optimization of the schedules for project management. Further research effort will be focused on explicit nonlinear-discrete optimization of the project schedules, the multi-project scheduling and the resource-constrained scheduling.

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APPENDIX

NLP optimization model formulation nomenclature

A.1. Indices
i project activity, $i \in I$
j succeeding project activity, $j \in J(i)$

A.2. Constants
D_T target project duration
$L_{i,j}$ lag/lead time between the activities

A.3. Variables
B project award bonus
C_i activity direct cost
C_I indirect project cost
C_T total project cost
D_i activity duration
D_E amount of time - the project is early

D_L amount of time - the project is late
D_P actual project duration
P project penalty cost
S_i activity start time

A.4.   Symbols
α first project activity
ω last project activity

TROŠKOVNA OPTIMIZACIJA TERMINSKIH PLANOVA ZA VODENJE PROJEKATA

SAŽETAK

U radu je prikazana troškovna optimizacija terminskih planova za vodenje projekata. Za troškovnu optimizaciju terminskih planova s generaliziranim vremenskim vezama između projektnih aktivnosti je razvijen i upotrebljen model nelinearnog programiranja (NLP). Postojeći optimizacijski modeli NLP su usredotočeni na rješavanje problema troškovne optimizacije projektnih planova koji uključuju pojednostavljene pretpostavke glede vremenskih veza između projektnih aktivnosti. Na taj način je cilj predstavljenog istraživačkog rada predložiti aplikativni optimizacijski model NLP za učinkovito donošenje odluka pri vodenju realnih projekata. Za optimizaciju je upotrebljena generalizirana metoda reduciranoga gradienta. Postignuti rezultati sadrže projektne planove s minimalnim ukupnim troškovima i optimalne krivulje ovisnosti ukupnih troškova projekta od dužine trajanja projekta. Budući da predloženi optimizacijski pristup omogućuje uvid u odnos između dužine trajanja projekta i ukupnih troškova projekta, donositelj odluka može bolje procijeniti učinak roka za dovršetak projekta na ukupne troškove projekta prije dostavljanja same ponude. Za prikaz prednosti predloženog pristupa su u radu pokazani primjer upotrebe predstavljenoga modela i primjer analize ovisnosti ukupnih troškova projekta od dužine trajanja projekta.

JEL: *M11, O21, O22, C61*

Ključne riječi: *vođenje projekta, modeli za planiranje, analiza projekta, optimizacijske tehnike*