PURCHASING POWER PARITY AS A LONG-RUN EQUILIBRIUM: CO-INTEGRATION TEST IN THE CASE OF CROATIA (1991-1996)*

Andreja Pufnik**

1 INTRODUCTION

The theory of purchasing power parity is one of the most important and most controversial theories, whose aim is to define a long-run equilibrium exchange rate. The principle of purchasing power parity is important not only because it is the basic element of many exchange rate models, but also because of its implications for managing economic policy. Namely, the aim of theoretical efforts is to find suitable methods, one of which is purchasing power parity, which economic policy makers can rely upon in evaluating the size of discrepancy of the short-run equilibrium exchange rate (market exchange rate) from its long-run equilibrium value. This has proven to be a difficult task.

The modern notion of purchasing power parity is tied to the contribution of the Swedish economist Gustav Cassel who has formalized and empirically tested the theory in the first half of the century. The idea upon which the theory rested is that the value of a currency and the demand for it are determined on the basis of the amount of goods and services which can be bought

* *This paper was originally published in Economic trends and Economic Policy (Privredna kretanja I ekonomska politika) 1997, No. 55,57-85.*

** *Andreja Pufnik, Croatian National Bank.*

for the currency unit inside the country, i.e. on the basis of its domestic purchasing power, which is inversely proportional to the price level of goods and services. When we apply this idea to two countries, the value of one country's currency in relation to the other country's currency is the exchange rate, which is therefore determined on the basis of the relation of the domestic purchasing powers of the two currencies, i.e. on the basis of the relation of price levels in these countries.

Thus we arrive to the ***absolute*** version of the theory of purchasing power parity, according to which the equilibrium exchange rate between currencies of two countries is determined on the basis of price levels in these countries (Dornbusch, 1987, p. 1076):

$$(1.1)\quad e = P/P^* = \frac{\text{the price of a basket of goods in domestic currency}}{\text{the price of the same basket in foreign currency}},$$

where e is the exchange rate (number of units of domestic currency for a unit of foreign currency), and P and P* are domestic and foreign price levels expressed in the currencies of these countries.

To achieve such purchasing power parity it is therefore necessary to form such nominal exchange rate that will equalize the price (expressed in a common currency) of a particular basket of goods of the same kind in the two countries, and thereby equalize the real purchasing power of currencies. It is therefore, an essential characteristic of the absolute purchasing power parity that the real equilibrium exchange rate is equal to one. It has been shown that the principle of absolute purchasing power parity is not realized frequently because of the existence of transaction costs, information costs, duties and other trade barriers, limited competition etc. These factors are limiting the quick reaction of consumers and companies to international differences of prices of particular goods and prevent equalizing of absolute price levels expressed in a common currency. In addition to this, the basket of goods and the system of weights vary from one country to another and there is a number of goods that are not traded in international trade ("nontradables"), which also influences the divergence of the short-run equilibrium exchange rate from the exchange rate according to the purchasing power parity.

If the factors which cause the non-realization of the absolute purchasing power parity are constant in time, then the relative purchasing power parity can be the case (Dornbusch, 1987, p. 1076):

$$(1.2)\quad e = \Theta P/P^*\ ,$$

where Θ is a constant, which reflects the existence of different factors preventing the realization of absolute purchasing power parity. Therefore, for the realization of the relative purchasing power parity it is necessary to equalize the rate of change of the bilateral nominal exchange rate to the difference between inflation rates in the two countries. In that case the exchange rates are formed in such manner that the real purchasing power of currencies remains unchanged. The real exchange rate will therefore be constant. Because of real changes in economies (for example production growth, changes in consumers' preferences, technological progress) and the ensuing change of relative prices, the relative version of the theory of purchasing power parity will frequently remain unrealized.[1]

Empirical research showed that in the short-run[2] there are actually significant deviations of the market exchange rate from the equilibrium exchange rate in terms of purchasing power parity, which reflects in the real exchange rate changes. Thus the purchasing power parity is most frequently interpreted as a long-run equilibrium principle. For the exchange rate and the relation of domestic and foreign prices to be in a long-run relation it is not necessary for the exchange rate to be constant at every moment, but it can fluctuate around a long-run mean. Then the expected real exchange rate value is constant rather than the actual one.

The research undertaken in the late eighties[3] showed that many macroeconomic time series, including the series of exchange rates and prices, are a non-stationary processes, i.e. that they include a trend component. These investigations put into doubt the credibility of previous results that were based on a regression analysis without the previous verification whether these variables are stationary or not. Namely, the standard regression analysis provides meaningful results only when it is applied to stationary series; otherwise, we are facing the problem of the so- called spurious regressions.[4]

---

[1] *See Balassa (1964) and Detken (1995).*

[2] *See Isard (1977), Kravis and Lipsey (1978), Webster (1987), Milone (1986), Frenkel (1981), Hakkio (1984), and Krugman (1978).*

[3] *See e.g. Engle and Granger (1987), Corbae and Ouliaris (1988), Enders (1988), Taylor (1988), and Taylor and McMahon (1988).*

[4] *Spurious regression is a term coined by Granger and Newbold and is used for results of regressions (carried out with the method of the ordinary least squares) of two non-stationary economic series with a marked trend (Griffiths et al., 1993; p. 696). The indicators of representability of the regression model on the basis of original (non-stationary) series, if we disregard the Durbin-Watson indicator, at first sight are good - a high coefficient of determination R^2 and a significant t-statistics, but, in essence, they have no real value. Common t-tests in such cases can lead us to wrong conclusions, because they too often lead to the rejection of the null hypothesis about*

An essential characteristic of stationary series is their mean reversion, a final variance that does not change during the course of time, and a "limited memory", by which it is meant that the consequences of today's shocks will disappear in the future. Therefore, if the stochastic process[5] {Xt} is to be stationary, the following three conditions must be met (Charemza and Deadman, 1992, p. 118):

$E(X_t) = \text{constant} = \mu$,
$Var(X_t) = \text{constant} = \sigma^2$,
$Cov(X_t X_{t+j}) = \sigma_j$.

If one or several of the above mentioned conditions are not met, then the process is non-stationary. The resolving of the non-stationary problem, that is the removing of the trend component, is usually carried out by a procedure of differencing reiterated until the series becomes stationary. In relation to the procedure of differencing, it would be suitable to introduce the notion of integrated time series. Following Engle and Granger (1987), we are defining such series in the following way:

Definition 1:

A non-stationary series, which can be transformed to a stationary series by differencing d times is integrated of order d, and is usually denoted by $x_t \sim I(d)$. If the series is stationary, the differencing is unnecessary, and the series is denoted by $x_t \sim I(0)$. If we have two series x_{1t} and x_{2t} and $x_{1t} \sim I(0)$, while $x_{2t} \sim I(1)$, then $(x_{1t} + x_{2t}) \sim I(1)$. Also, if $x_t \sim I(d)$ then $(\alpha + \beta x_t) \sim I(d)$, where α and β are constants.

However, the investigations in which the non-stationary problem was solved by differencing also contain some shortcomings. The procedure of differencing the time series can disrupt their stable economic relation, i.e. their long-run characteristics could be lost[6]. This problem can be resolved by the method of co-integration.

the nonexistence of relation and the acceptance of the hypothesis about the significance of a relation which is only apparent. Granger and Newbold are suggesting that in cases when R^2 is high and the DW statistics is low the relation is assessed on the basis of the first differences of the variables, rather than on the basis of their levels, for, in that case, we will not estimate too frequently their apparent relation as a significant one.

[5] *By stochastic process we understand a family of random variables.*

[6] *This is why we have to take into consideration the levels instead of the differences of variables if we wish to carry out the research of their long-run relations (Griffiths et al., 1993).*

2
THE THEORETICAL BASIS OF THE CO-INTEGRATION METHOD

2.1 The Long-run and the Co-integration Method

In their seminal paper in this field "Co-integration and Error Correction: Representation, Estimation and Testing" Engle and Granger (1987) have noticed and explained the important relation between the non-stationary processes and the concept of long-run equilibrium. In this work, they introduced the concept of co-integration of time series. It is said that two or several non-stationary variables are co-integrated if the deviations from their common long-run path do not change over time, so that the following definition is generally valid for two variables (according to Engle and Granger, 1987):

Definition 2:

Time series x_t and y_t are co-integrated of order d,b[7] (i) if both series are integrated of order d and (ii) if there is a vector $\alpha(\neq 0)$ such that the linear combination $\alpha_1 x_t + \alpha_2 y_t$ is integrated of order d-b, b>0. The vector $\alpha = [\alpha_1, \alpha_2]$ is called a cointegrating vector.

We arrive to the concept of co-integration and long-run equilibrium in the following way[8]. Let us assume that the long-run equilibrium relation between the observed variables has the form $y_t=\beta x_t$, i.e. that $y_t-\beta x_t=0$. In this case, ε_t in the equation below will indicate how distant y_t and x_t are from the equilibrium and it can be called an equilibrium error.

If y_t and x_t are co-integrated and if the error ε_t is stationary with the mean equal to zero, then:

---

[7] *This is usually written as x_t, $y_t \sim CI(d,b)$.*

[8] *The explanation is based on the case when two time series are integrated of the first order. Although the concept of co-integration of time series is quite general for its explanation, usually the simplest case is used - when both variables are an integrated processes of the first order. In that case, their linear combination:*

$y_t-\beta x_t = \varepsilon_t$

is also I(1). However, it is possible for ε_t to be stationary, that is I(0). For that to happen, it is necessary for the existing trends in series y_t and x_t to annul each other. Therefore, a long-run co-integrated relation between two time series exists if both are I(1), but their linear combination is I(0).

(2.1) $y_t = \beta x_t + \varepsilon_t$.

Since $\varepsilon_t \sim (0,\sigma^2)$ is stationary, the variables y_t and x_t are in a long-run equilibrium relation. In that case (when y_t and x_t are co-integrated), by estimating the above equation with the method of ordinary least squares we are getting a super consistent estimator of the parameter β[9] that describes the long-run equilibrium relation between the variables.

Therefore, the most interesting case in empirical analysis is when the series transformed with the cointegrating vector become stationary. This happens when $b = d$[10], so that the linear combination of variables is stationary. In this case, the components of the vector of co-integration can be identified with the parameters of the long-run relation of variables.

The essence of co-integration, therefore, lies in the following: while two or more time series are non-stationary, their stationary linear combination can exist. Thus the testing of the existence of co-integration between variables is carried out by testing whether the residuals from the co-integration equation, obtained by the method of ordinary least squares, are stationary.

Figure 1 and 2 show the relation between two variables xt and yt that are non-stationary, since the positive trend is obviously present in both. However, the variables in Figure 1 are drifting apart, thus the difference between them is not stationary. Although not necessarily, the variables in this figure can be of the same order of integration (probably x_t is of the first order and y_t of the second order of integration).

On the other hand, the variables in Figure 2 move in time in the same direction. On the basis of the figure one can assume that the observed variables are of the same order of integration. The fact that the differences between x_t and y_t do not have a clear tendency of increase or decrease indicates that these differences (or, more generally, the linear combination of x_t and y_t) are very probably stationary. Figure 2, therefore, illustrates the basic idea of co-integration, according to which it is necessary, if there is a ***long-run*** relation between two (or several) non-stationary variables, that the deviations from this relation are stationary.

---

[9] *This estimator converges toward the real parameter faster than the estimator in the case when the variables are not integrated.*

[10] $x_t, y_t \sim CI(d,d)$

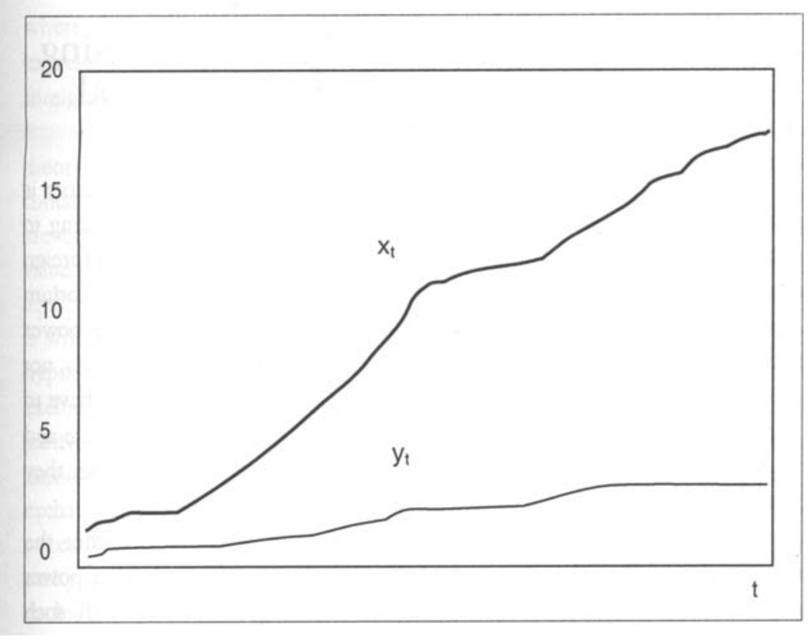

Figure 1

ILLUSTRATION OF SERIES THAT DIVERGE WITH TIME

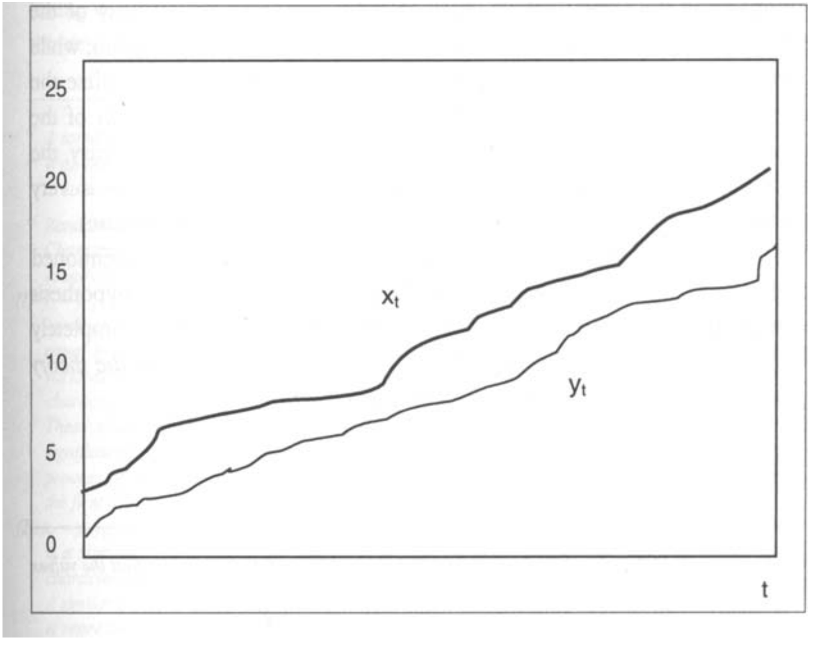

Figure 2

ILLUSTRATION OF SERIES DRIFTING TOGETHER

Source: Charemza and Deadman (1992), p. 145.

2.2 Testing the theory of purchasing power parity with the method of co-integration

In recent years, the theory of purchasing power parity is being studied with the application of co-integration analysis, according to which, in spite of their short-run dynamics, the series of domestic and foreign exchange rates and prices can converge toward a long-run equilibrium relation. According to this new approach, for the theory of purchasing power parity to prove as suitable for explaining fluctuations of exchange rates, not all restrictions specified by the classic (strict) version of the theory[11] have to be satisfied. The emphasis is placed now on whether the exchange rate and series of domestic and foreign prices have a common trend, i.e. whether they are co-integrated.

The co-integration is tested by examining whether the deviations from the relation imposed by the theory of purchasing power parity have a tendency to return to some specific fixed mean. If such tendency exists, the variables are in a stable long-run relation, in spite of short-run divergence. The results of testing the theory by the method of co-integration are very diverse (part of them confirms the validity of the theory of purchasing power parity as a long-run equilibrium condition, while a part does not). This results suggest that it is not good to generalize the results of any research, since they largely depend on the selection of the countries that are being analyzed, on the selection of a standard country, the length of the time span, on the chosen price index and, what is very important, on the specification of the model (uni-, bi- or multi-variate).

The models are differentiated into the three mentioned groups according to the level of restriction in the process of the hypothesis formulation. For the criteria of differentiation to be completely understandable it is necessary to define a ***general model of testing the theory of purchasing power parity*** (Cheung and Lai, 1993):

$$e_t = \alpha_0+\alpha_1 P_t-\alpha_2 P_t^*+\xi_t \ , \tag{2.2}$$

---

[11] *The classic or strict version of the theory of parity of purchasing power requires that the values of coefficients in equation (2.2) be as follows:*

$\alpha_0 = 0,\ \alpha_1 = \alpha_2 = 1$.

This means that there is a proportional relation between the exchange rate and prices, and only random deviations are permitted.

where e_t denotes the nominal exchange rate (the price of foreign currency expressed in domestic currency units), P_t the domestic price level and P_t^* the foreign price level. All variables are expressed in a logarithmic form.

The ***univariate*** model of testing the acceptability of the theory of purchasing power parity is the most restrictive approach, since it contains restrictions according to which the coefficients next to price indexes are equal (symmetry restriction between domestic and foreign) and their value is equal to one (the restriction of equal proportionality), that is $\alpha_0=0$ and $\alpha_1=\alpha_2=1$[12]. This method tests whether the real exchange rate is stationary, that is whether it has a constant mean[13], which is necessary for accepting the hypothesis of validity of the theory of purchasing power parity. If the real exchange rate (that is the short-run divergence from the purchasing power parity) is showing, contrary to this requirement, the characteristics of the random walk[14], then there will be a tendency of drifting apart between the exchange rate series and the series of ratios between the domestic and foreign prices. The relation, which is described by the theory of purchasing power parity, will not be realized even in a long time period, because the real exchange rate does not have a constant expected value and can diverge limitlessly from a stable long-run mean. In other words, the real exchange rate is non-stationary. A significant part of research where univariate models of testing the long-run purchasing power parity in the world[15] are applied led to the rejection of the

---

[12] *A somewhat less restrictive version permits that α_0 is not equal to zero.*

[13] *With the mean equal to zero in the absolute version of the theory of purchasing power parity, or to some constant in the relative version.*

[14] *Random walk is a special case of a non-stationary stochastic process (Griffits et al., 1993; Charemza and Deadman, 1992). This is an autoregressive process of the first order, which is usually represented by the following equation:*

(1) $y_t = y_{t-1}+\varepsilon_t$,

in which ε_t is a series of identically distributed independent random variables with the mean equal to zero. The ε_t is therefore a stochastic process that we call a white noise process. A variance of the process of the random walk is a function of time. A series which shows the characteristics of the random walk slowly moves up or down as a result of random shocks. These characteristics are indicating that in longer time periods a process can have values significantly different from the mean, so consequently we call such series a stochastic trend. The process of the random walk is an example of an integrated process of the first order, because the first difference of the series, that is

(2) $x_t = y_t - y_{t-1} = \varepsilon_t$

is a stationary process. It has been shown that many economic and financial series show the characteristics of random walk.

A similar example of non-stationary stochastic process is called random walk with a drift and is represented by the equation:

(3) $y_t = \mu y_{t-1}+\varepsilon_t$, $\mu \neq 0$

[15] *See e.g. Corbae and Ouliaris (1988), Fisher and Park (1991), and Henricsson and Lundbaeck (1995).*

hypothesis of the validity of the theory, as it has been shown in the majority of cases that real exchange rates are non-stationary.

The ***bivariate*** model of testing the validity of the theory of purchasing power parity is less restrictive. The symmetry limit ($\alpha_1=\alpha_2$) is retained, but the principle of equal proportionality is relaxed. The testing is done by the Engle-Granger two-step method, which is a suitable test of co-integration for only two time series. In these models it is permitted that the coefficient next to the price ratio at home and abroad differs from the one of the value required by the classic (strict) version of the theory of purchasing power parity. The motivation to permit the coefficient next to the price ratio to be different than one is based on the wish to include into the model the transportation costs and errors occurred in the process of measuring the prices[16], the reason why the observed series are an imperfect replacement for the theoretical variables. The equilibrium nominal exchange rate in this case will be proportional to the ratio of domestic to foreign prices, but not necessarily equally proportional. The results of bivariate tests carried out in the world often confirm[17] the validity of the theory of purchasing power parity as a long-run equilibrium condition, but there are also many unfavorable[18] results.

The introduction of the symmetry limit and equal proportionality can lead to the bias of the test of the real exchange rate and to the unjustified rejection of the co-integration hypothesis (i.e. acceptance of the null hypothesis of the nonexistence of co-integration). This happens because the linear combination of non-stationary series is, generally, also non-stationary, except in the case of correct co-integration coefficients. Consequently, the results indicating the nonexistence of co-integration can also be interpreted as a rejection of the given restrictions related to the equilibrium relation, rather than the rejection of the equilibrium relation itself. This suggests the caution necessary in interpreting the results of the long-run purchasing power parity testing based on the behavior of time series of the real exchange rate. Namely, if the results show that the real exchange rate is not stationary, this can mean that the conditions of symmetry and equal proportionality are not fulfilled, while the results can still be consistent with long-run purchasing power parity that includes errors made in measuring the prices and/or transportation costs.

---

[16] *An illustration of a model of error in measuring prices and a model of transportation costs can be found in Taylor (1988).*

[17] *For example see Taylor and McMahon (1988), Kim (1990), Tang and Butiong (1994), and Christev and Noorbakhsh (1994).*

[18] *For example see Taylor (1988), McNown and Wallace (1990), Bahmani-Oskooee (1993), and Thacker (1995).*

In the ***multivariate*** model of the validity testing of the theory of purchasing power parity as a long-run equilibrium condition all restrictions related to the co-integration coefficient are relaxed. The restrictions are relaxed because the countries are using different weights for price index construction. Consequently, the coefficients in a co-integration regression can differ from one country to another. It is only required that the coefficients have a correct sign (positive). These tests focus on the existence of any linear combination of variables, and are performed using the Johansen test of co-integration, or VAR model[19]. Research in the world [20] has shown that in this case the co-integration between the exchange rate and domestic and foreign prices is confirmed more often than in earlier tests. The advocates of the application of this model believe that, having in mind the unstable nature of non-stationary series, the existence of a stationary relation between the variables is more important than the observed deviations from the value of coefficients that are required by the strict version of the theory of purchasing power parity[21].

2.2.1 Testing the order of integration

Prior to our testing of the existence of co-integration between the nominal exchange rates and prices, we must examine the characteristics of time series of the mentioned variables, because only the non-stationary variables of the same order of integration d[22] can form a co-integrated long-run system.

The testing for the order of integration[23] of a variable y_t (or, in our case, e_t and p_t) can be started by testing whether it is a non-stationary variable of the first order of integration, i.e. whether it has the characteristics of the process of the random walk:

$$y_t = y_{t-1} + \varepsilon_t \quad , \tag{2.3}$$

---

[19] *By VAR (Vector autoregression) modeling is meant an autoregressive representation of a multidimensional time series.*

[20] *Liu (1992), Cheung and Lai (1993), Chen (1995), and MacDonald (1993).*

[21] *Liu (1992).*

[22] *d = the number of differencing necessary for the variables to be transformed into a stationary series.*

[23] *The explanation of the procedure of testing the order of integration and the existence of co-integration is based on Charemza and Deadman (1992), and Griffiths et al. (1993).*

where ε_t is a series of identically distributed random variables with the mean equal to zero. At first sight, it would seem that the simplest thing to do is to test, on the basis of Student-t statistics, the hypotheses according to which the parameter ρ in the autoregressive equation

$$y_t = \rho y_{t-1} + \varepsilon_t \tag{2.4}$$

is equal to one. In case this hypothesis is accepted, the variable has the characteristics of the process of the random walk. However, this is not the method recommended in this case. Estimate of the parameter ρ by the ordinary least squares method in the autoregressive equation (2.4) can be largely biased, because very little is known about the distribution of Student-t statistics when the variable is non-stationary.

Dickey and Fuller[24] suggested a simple method for testing the order of integration of the variable y_t in the equation (2.4). The method is called Dickey-Fuller test (DF test). The DF test is based on the estimation of the equation (2.4) of the equivalent regression equation:

$$\Delta y_t = \delta y_{t-1} + \varepsilon_t \ , \tag{2.5}$$

which can also be written in the following way:

$$y_t = (1+\delta) y_{t-1} + \varepsilon_t \ , \tag{2.6}$$

where $\rho = 1+\delta$. In essence, the DF test is the testing of whether the estimate (obtained by the ordinary least squares method) of the parameter δ from the equation (2.5) is significantly negative. Namely, in the case when the parameter δ is less than zero it follows that the parameter ρ is less than one and, consequently, the time series is integrated of order zero, i.e. it is stationary. On the other hand, if the parameter δ is equal to zero then the parameter ρ is equal to one, and consequently the observed series has the characteristics of the process of the random walk. This process, as we already observed, is a non-stationary stochastic process of the first order of integration.

To test the hypotheses, it is necessary to know the distribution of statistics used in the test and the associated critical region. In the equation (2.5)

---

[24] *Dickey, D.A. and W.A. Fuller (1979), "Distributions of the estimators for autoregressive time series with unit root", **Journal of the American Statistical Association**, 74, 427-431.*

the statistics (the ratio of the estimate of parameter δ to its standard error) does not have the usual Student-t distribution. Namely, if it is shown that y_t is a variable of the first order of integration, then the equation (2.5) is a regression of the variable of zero order of integration to a variable of the first order of integration. In this case, the t-ratio does not have an asymptotically normal distribution, but a distribution, which is negatively skewed.

In this paper, we have used the tables of critical values for Student-t statistics distribution published in the book by Charemza, W.W. and D.F. Deadman (1992). These tables differ from the tables prepared by e.g. Dickey and Fuller or MacKinnon. They are not constructed analytically but through simulation, because the distribution of Student-t statistics in the case of regression of a variable, which is I(0) on the variable I(1), is not precisely known. This is why the critical values are subject to some errors and depend on the structure of the model used for the simulation.

For a given level of significance (1%, 5% or 10%) the empirical test value - DF statistics (t-statistics calculated in the usual manner) is compared with the lower (t_L) and upper (t_U) critical value. If the calculated t-statistics is less than the lower critical value for a certain number of observation (n), the null hypothesis should be rejected and the alternative hypothesis of the stationarity of variable y_t accepted.

If the calculated DF statistics is higher than the upper critical value, the null hypothesis cannot be rejected. In the case when the value of DF statistics is between the upper and lower critical value, the test does not lead to a decision, therefore we do not know whether to reject or accept the null hypothesis.

In the case when the results of testing on the basis of the equation (2.5) show that we cannot reject the null hypothesis, the variable y_t can be integrated of order higher than zero, or it may not be integrated at all. The next step would be to test whether the order of integration is equal to one. If y_t~I(1) then Δy_t~I(0). Therefore, we are repeating the test using Δy_t instead of y_t. The DF equation now is:

$$(2.7)\quad \Delta\Delta y_t = \delta\Delta y_{t-1} + \varepsilon_t \ .$$

Again we want to test whether δ is negative and significant. If we reject the null hypothesis and accept the alternative $\delta<0$, the series Δy_t is stationary and y_t~I(1). If we cannot reject the null hypothesis, we will test whether y_t~I(2), i.e. we will carry out the DF regression in which $\Delta\Delta\Delta y_t$ is the variable on the left-hand side of the equation and $\Delta\delta y_{t-1}$ is on the right-hand side. We can continue this procedure until we determine the order of integration of the variable y_t, or until we determine that the variable y_t cannot be transformed into a stationary series by differencing.

The testing of the order of integration of the variable y_t is done, therefore, by a successive carrying out of the DF test on the variables y_t, Δy_t, $\Delta\Delta y_t$ etc., until we reach a stationary series, or until we determine that we cannot obtain a stationarity of the time series by differencing. In practice, it is not common for the economic series to be of the integration order over two.

The shortcoming of the DF test is that it does not take into consideration the possible autocorrelation of errors ε_t that are the reason why the estimation of the equation (2.5) by the method of the ordinary least squares is not efficient. Dickey and Fuller[25] give a simple solution to the problem of autocorrelation of errors that consists in the using of left-hand side variables with a time lag on the right-hand side of the equation so as to remove the autocorrelation of errors. The test is called the Augmented Dickey-Fuller test, or ADF test. It is generally considered to be one of the most efficient tests among the simpler tests of integration. This is one of the most widespread tests of the existence of a unit root. That is why the ADF test is used in the empirical part of this research.

The equation (2.5) is extended by the introduction of additional exogenous variables with the aim of neutralizing the autocorrelation of errors. Lags of the variable on the left-hand side of the equation are used as additional explanatory variables. The ADF equivalent of the DF equation (2.5) is:

$$(2.8)\quad \Delta y_t = \delta y_{t-1} + \sum_{i=1}^{k} \delta_i \Delta y_{t-i} + \varepsilon_t$$

The number of lags k should be relatively small in order to preserve the degrees of freedom, but large enough to remove the autocorrelation of errors. For example, if for k=2 the Durbin-Watson autocorrelation statistics is low and indicates an autocorrelation of the first order, it is useful to increase k in the hope that we will remove the autocorrelation. The procedure of testing is the same as in the DF test.

---

[25] *Dickey, D.A. and W.A. Fuller (1981), "Likelihood ratio statistics for autoregressive time series with a unit root",* ***Econometrica****, 49, 1057-1072.*

2.2.2 Testing the existence of co-integration

If it is shown that the variables are integrated of the same order d, there is a possibility that they are co-integrated. The testing of the existence of co-integration is carried out in the same way as the testing of the order of integration. The difference is that we are now interested in the order of integration of the linear combination of the observed variables which represents the deviation from a long-run equilibrium. The DF and ADF tests are also used in testing to determine whether the equilibrium error, which we shall denote with ε_t, is stationary. If the variables are co-integrated, then the equilibrium error is stationary, or of null order of integration $\varepsilon_t \sim I(0)$. If, on the other hand, the observed variables are not co-integrated, then the equilibrium error ε_t is a non-stationary process. As opposed to the testing of integration, the distribution of Student-t statistics and the critical values in the co-integration test depend on the number of estimated co-integration parameters in the co-integration regression. The co-integration equation in our case has the following form:

$$e_t = \alpha_0 + \alpha_1 p_t + \varepsilon_t \tag{2.9}$$

where e_t is the logarithm of the nominal exchange rate, p_t is the logarithm of the ratio of prices at home and abroad, α_0 is a constant, and $_1\alpha$ is an unknown co-integration parameter which has to be estimated, most often by the method of the ordinary least squares.

In the co-integration testing the residuals obtained on the basis of the co-integration equation (2.9) are included into the DF (2.5) or ADF (2.8) equation. In this way, we are getting the following equations:

$$\Delta \hat{\varepsilon}_t = \delta\, \hat{\varepsilon}_{t-1} + \xi_t \quad \text{DF equation} \tag{2.10}$$

$$\Delta \hat{\varepsilon}_t = \delta\, \hat{\varepsilon}_{t-1} + \Sigma\delta_i\, \hat{\varepsilon}_{t-i} + \xi_t \quad \text{ADF equation} \tag{2.11}$$

where $\hat{\varepsilon}_t$ is the estimate of the equilibrium error obtained by the method of the ordinary least squares.

We shall reject the null hypothesis according to which the co-integration has not been realized, as it is done in the integration testing, if Student t-value (DF or ADF statistics) for parameter δ is under the lower limit, and we shall not reject it if it is above the upper limit. There is also an inconclusive range between the upper and lower limit. In the case when the t-ratio is inside this area, we are not sure whether to reject or accept the null-hypothesis.

If we determine that the observed variables are co-integrated, then we are accepting that the parameter α_1 from the co-integration equation is a valid estimate of the long-run equilibrium relation between the variables. However the t-statistics of parameter α_1 does not, unfortunately, indicate the precision of the estimator in the co-integration equation and cannot be used in conclusions about the population parameter, because both variables are integrated. But, as was mentioned, the estimator from the co-integration equation obtained by the method of the smallest squares is "super consistent", because it converges towards the real parameter faster then the estimator in the usual case when variables are not integrated.

3
EMPIRICAL RESULTS OF THE CO-INTEGRATION ANALYSIS OF LONG-RUN PURCHASING POWER PARITY IN THE CASE OF CROATIA

The goal of this part of the study is to test the order of integration and the possible co-integration of variables included in the model of purchasing power parity. If we determine that the time series are non-stationary and of the same order of integration and that their linear combination (divergence from long-run equilibrium) is stationary, we shall conclude that the exchange rates and prices are in the long-run co-integrative relation.

The seasonally unadjusted monthly series of nominal exchange rates for marks, liras and dollars (the price of foreign currency expressed in kuna - HRK), and monthly series of domestic and foreign retail prices in the period from December 1991 (when our country became monetarily independent) to September 1996, are analyzed in the study. The bilateral exchange rates, which are preferred by most authors because of the arbitrariness of the choice of weights in calculating the effective exchange rate which can influence the results of validity tests of the theory of purchasing power parity, were used. The values of observed series are expressed in the form of base indexes[26], where December 1991 was used as the base period. In that month, Croatia became monetarily independent. The Croatian National Bank published for the first time in that month an exchange rate list. We

---

[26] *The series of base indexes of nominal exchange rates were obtained on the basis of the midpoint exchange rate of NBH (Croatian National Bank), on the last day of the month. Reports of the Central Bureau of Statistics and the publication "Main Economic Indicators" are sources used for calculating the base price indexes.*

believe that there was an intention for the exchange rates published at that moment to be as close as possible to the "equilibrium" rates, however having in mind at the same time the arbitrariness of that choice.

For the standard countries we selected our most important foreign trade partners Germany and Italy, as well as the United States, the most frequently used as the comparison country in the previously analyzed empirical tests of long-run purchasing power parity.

The estimate was made using a bivariate co-integration model of testing the theory of purchasing power parity of the form:

(3.1) $e_t = \alpha_0 + \alpha_1(P_t/P_t^*) + \varepsilon_t$,

or

(3.2) $e_t = \alpha_0 + \alpha_1 p_t + \varepsilon_t$,

where et is the logarithm of the nominal exchange rate (the price of the foreign currency expressed in HRK), P_t is the logarithm of the domestic price level, P_t^* is the logarithm of the foreign price level and p_t is the ratio of logarithms of domestic and foreign price levels. As already said, the bivariate model retains the symmetry limit, i.e. the equality of both coefficients next to the price indexes, and discards the principle of equal proportionality, according to which the coefficient α_1 has to be equal to one. This is a less restrictive model for testing the theory of purchasing power parity in the long run, because it takes in account the errors made in the measuring of prices and/or transportation costs. The validity of the theory of purchasing power parity in the less restrictive form is a necessary condition for the real exchange rate stationarity. Therefore, if it can be shown that the exchange rate and prices in the bivariate model of the purchasing power parity are not in a long-run co-integrative relation, it can be automatically concluded that the real exchange rate is also not a stationary process with zero mean (in the absolute version of the classical theory of purchasing power parity) or with the some constant mean (in the relative version).

The analysis is based on the Engle-Granger two-step method for testing the co-integration. The first step examines the order of integration of variables, and the second tests whether the linear combination of observed variables is stationary, that is whether the variables are co-integrated. The Augmented Dickey-Fuller test (ADF test) of the existence of a unit root with four lags was used in both steps.[27]

[27] *For k=4 the DW statistics shows that the problem of autocorrelation of residuals does not exist.*

The results of the analysis of the order of integration of time series of nominal exchange rates and prices are shown in the following tables:

Table 1

THE RESULTS OF TESTING THE EXISTENCE OF A UNIT ROOT IN SERIES OF NOMINAL EXCHANGE RATES

Comparison Country	Levels ADF statistics	First differences ADF statistics	Second differences ADF statistics
Germany	-0.20	-1.50	-4.75
Italy	-0.01	-1.70	-5.08
USA	-0.08	-1.44	-4.73

Table 2

RESULTS OF TESTING THE EXISTENCE OF A UNIT ROOT IN RELATIVE PRICE SERIES

Comparison Country	Levels ADF statistics	First differences ADF statistics	Second differences ADF statistics
Germany	-0.06	-1.49	-3.24
Italy	-0.09	-1.48	-3.21
USA	-0.10	-1.50	-3.21

The calculated empirical test values from the above tables are compared with the following critical values:

Table 3

CRITICAL VALUES FOR THE SAMPLE SIZE n=50 AND NUMBER OF ESTIMATED PARAMETERS m=0

Level of significance	Lower limit	Upper limit
1%	-2.74	-2.43
5%	-2.00	-1.86
10%	-1.66	-1.52

Source: Charemza and Deadman (1992), p. 319-324.

On the basis of comparison of the ADF statistics for the levels and first differences of time series of exchange rates and the relative prices with the critical values we can conclude that the null hypothesis of non-stationarity cannot be rejected in any case.[28] Since we cannot accept the hypothesis of stationarity for any series of levels and first differences of variables of nominal exchange rates and price ratios, the further analysis should test whether second differences of series of nominal exchange rates and price ratios are stationary, that is whether the level of integration of variables is of the second order I(2). The test is performed on the basis of the following equations:

$$\Delta\Delta\Delta e_t = \delta\Delta\Delta e_{t-1} + \sum_{i=1}^{k} \delta_i \Delta\Delta\Delta e_{t-i} + \varepsilon_t \tag{3.3}$$

and

$$\Delta\Delta\Delta p_t = \delta\Delta\Delta p_{t-1} + \sum_{i=1}^{k} \delta_i \Delta\Delta\Delta p_{t-i} + \varepsilon_t \tag{3.4}$$

In all cases the calculated ADF test value was under the lower critical value (significantly negative) already on the level of significance of 1%. That is why we accepted the alternative hypothesis, according to which the second differences of variables are stationary, which means that the series of levels of nominal exchange rates and price ratios can be made stationary by the use of second differences. Since the levels of variables are integrated of the same (second) order, we can make the second step in the co-integration analysis using the Engle-Granger method.

Our goal is now to determine whether the variables are in a long-run co-integration relation. In the case that such relation exists, their linear combination (equilibrium error ε_t) must be stationary, i.e. of zero order of integration.

With the method of the ordinary least squares we obtained the following estimates of parameters α_0 and α_1 in the co-integration equations:

$$\underset{}{e_t^{GER}} = \underset{(0.071259)}{0.604} + \underset{(0.01191)}{0.881} p_t^{GER} + \varepsilon_t^{GER}, \tag{3.5}$$

$$e_t^{ITA} = \underset{(0.083919)}{0.936} + \underset{(0.009942)}{0.814} p_t^{ITA} + \varepsilon_t^{ITA}, \tag{3.6}$$

---

[28] *Except in the case of first differences of the time series of the lira on the level of 10% significance.*

(3.7) $e_t^{USA} = 0.682 + 0.873 p_t^{USA} + \varepsilon_t^{USA}$,
(0.090750) (0.010702)

where GER, ITA and USA are Germany, Italy and the United States, while the values in parenthesis are the standard errors. After that, we tested whether the estimated residuals (equilibrium errors) from the above co-integration equations are stationary. Using the ADF test we obtained the following empirical test values:

Table 4

RESULTS OF TESTING THE EXISTENCE OF A UNIT ROOT IN TIME SERIES OF EQUILIBRIUM ERRORS (RESIDUALS)

Country	Level ADF statistics
Germany	-1.98
Italy	-1.20
USA	-2.27

In order to determine whether the equilibrium errors are a stationary processes, that is the variables of the zero order of integration I(0), the above statistics are compared with the following critical values:

Table 5

CRITICAL VALUES FOR THE SAMPLE SIZE n=50 AND NUMBER OF ESTIMATED PARAMETERS m=1

Country	Level ADF statistics
Germany	-1.98
Italy	-1.20
USA	-2.27

Source: Charemza and Deadman (1992), p. 319-324.

Since the calculated ADF statistics are not significantly negative in any case, which is necessary for accepting the alternative hypothesis of stationarity of equilibrium errors, we can conclude that the linear combination of the nominal exchange rate and relative prices is non-stationary. The observed variables are not, therefore, in a long-run co-integration relation, because the short-run deviations from the long-run equilibrium relation do not fluctuate around a particular constant value. There exists, therefore, a tendency of a permanent divergence of the series of nominal exchange rates and relative prices in all cases included in the research. Finally, we can also conclude that, since the variables are not co-integrated, the necessary condition of stationarity of the real exchange rate is also unfulfilled.

4
CONCLUSION

It is already known that the theory of purchasing power parity cannot be used by the economic policy agents as a precise criterion in the assessment of the size of the deviation of the market exchange rate from its long-run equilibrium value, because the size of the necessary adjustment of the exchange rate, which we obtain by the already well-established calculation of the index of the real effective exchange rate, depends for example on an arbitrarily chosen basic period, weights of countries in effective exchange rate indexes, the selected price index etc. In addition to that, the calculation neglects the influence of structural changes on the changes in the long-run equilibrium exchange rate, and thus it is difficult to determine to what degree a real appreciation of 20%, for instance, represents a deviation from the "real" long-run exchange rate value. It could be the case that the currency was undervalued in the base period, or the structural changes (e.g. the increase of preferences in favor of the country's export goods) has caused the increase of the equilibrium relative prices. All this makes it harder to conclude whether the currency is overvalued by 20%, 10% or is not overvalued at all. In numerous papers it is pointed out that the theory of purchasing power parity can be used only approximately in the assessment of the size of the market exchange rate deviation from its long-run equilibrium level. Artus (1978, p. 297) sums up: "The theory of purchasing power parity is a useful, although a crude measure which can help the economic policy makers to keep the exchange rate at a reasonable level and to prevent it to be overvalued or undervalued by ten, twenty or thirty percent, as it frequently happened in the past. Even the countries that are letting their exchange rate to be determined to a large degree by a free interaction of market forces, most probably because of their lesser dependence on foreign trade and a stricter monetary policy, need a measuring stick which can signal the situations in which the market forces 'pulled' the exchange rate too far from its sustainable value in the long-run. The method seems adequate for these purposes, although the results should be interpreted taking into account different possible sources of bias. Finally, it is always good to compare the results obtained by the method of purchasing power parity with the stance of the balance of payments of some country. For example, it would be absurd to maintain that the exchange rate is adequate on the basis of indicators of purchasing power parity if, at the same time, there is a considerable and constant worsening in the balance of payment that cannot be explained by cyclic or other temporary factors."

The results of co-integration analysis suggest that the principle of purchasing power parity in the case of Croatia is not realized even in a long run. Consequently, its use in the exchange rate modeling is very questionable. Furthermore, it has been shown that the economic policy makers cannot resort to it as an adequate concept of equilibrium exchange rate that would serve in the assessment of the size of the market exchange rate deviation from its long-run equilibrium value.

The results of research in the world which confirm the long-run validity of the theory of the purchasing power parity suggest that when the variables are co-integrated, there is a built-in adjustment process that in the long run prevents relation errors to become bigger and bigger. In this case, the economic policy measures trying to "pull" the exchange rate from its equilibrium value in a short-run will not be effective the disequilibrium in the balance of payments in a long-run, which can happen in the cases when the purchasing power parity as a long-run equilibrium condition is not realized. In such cases, and according to the results of this research this applies also to Croatia: the interventions by the economic policy-makers aiming to bring the market exchange rate closer to its value according to the principle of long-run purchasing power parity are undesirable as they can lead to the occurrence of the long-run imbalances.

In the case of Croatia we must bear in mind the possible sources of bias which have led to the rejection of the hypothesis of the existence of a long-run equilibrium relation between the exchange rates and relative prices as assumed by the theory of purchasing power parity. Namely, the results largely depend on the selection of the standard country, the time period, the price index, the base period, the table of critical values etc. Apart from this, the time period since the monetary independence of Croatia to date probably is not long enough to confirm the thesis that deviations from the relation between the nominal exchange rate and prices tend to revert to a the long-run mean as required by the theory of purchasing power parity. Additionally, in this relatively short period there were many structural changes because of the transition from a socialist to a traditional economic system; the economy overcame the war, hyperinflation, and many changes to the economic laws.

The Engle-Granger method of testing co-integration also has some weaknesses. In this method, the DF and ADF tests of stationarity of time series in which non-stationarity is a null hypothesis are most frequently used. The Monte Carlo studies showed that on the basis of the results of this method the null hypothesis is accepted as a rule, except in cases when there are strong proofs against it. Therefore, the methodology itself is very often not strong enough to indicate the co-integration in cases when it actually exists. Furthermore, according to the asymptotic theory, the results of the co-integration test should not be sensitive to the selection of the explanatory variable in the co-integration equation. However, this has shown to be incorrect in the case of the insufficiently long series of data. Besides, the possible errors in the estimation of the co-integration equation are reflected in the results of the test of stationarity of equilibrium errors (residuals).

LITERATURE

Artus, J.R., 1978, "Methods of Assessing the Long-Run Equilibrium Value of an Exchange rate", *Journal of International Economics*, 8(3), 277-299.

Babić, M., 1993, *Međunarodna ekonomija (International economics)*, Zagreb: Mate.

Bahmani-Oskooee, M., 1993, "Purchasing Power Parity Based on Effective Exchange Rate and Cointegration: 25 LDSs' Experience with its Absolute Formulation, *World Development*, 21(6), 1023-1031.

Balassa, B., 1964, "The Purchasing Power Parity Doctrine: A Reappraisal", *Journal of Political Economy*, 72(6), 584,596.

Butiong, R. Q. and M. Tang, 1994, "Purchasing Power Parity in Asian Developing Countries: A Co-Integration Test", *Asian Development Band Statistical Report Series*, 17, 1-19.

Charemza, W. W. and Deadman D. F., 1992, *New Directions in Econometric Practice: General to Specific Modeling, Cointegration and Vector Autoregression*, Edward Elgar, Aldershot.

Chen, B., 1995, "Long-Run Purchasing Power Parity: Evidence from some European Monetary System Countries", *Applied Economics*, 27(4), 377-383.

Cheung, Y. W. and K. S. Lai, 1993, "Long-Run Purchasing Power Parity during the Recent Float", *Journal of International Economics*, 34(1/2), 181-192.

Clarc, P. et al., 1994, *Exchange Rates and Economic Fundamentals: A Framework for Analysis*, IMF Occasional Paper 115, Washington D.C.

Corbae, D. and S. Ouliaris, 1988, "Cointegration and Tests of Purchasing Power Parity", *The Review of Economics and Statistics*, 79(3), 508-511.

Detken, C., 1995, "When is a Currency Fundamentally Correct?", *Economic and Financial Prospects*, 2, 1-4.

Diebold, F.X., 1988, *Empirical Modeling of Exchange Rate Dynamics*, Springer-Verlag, Berlin, Heidelberg, New York.

Dornbusch, R., 1980, *Open Economy Macroeconomics*, Basic Books Inc. Publishers, New York.

Dornbusch, R., 1987, "Purchasing Power Parity", in: J. Eatwell, M. Milgate and P. Newmand (Eds.), *The New Palgrave Dictionary of Economics*, y. 3, Basingstoke.

Enders, W., 1988, "ARIMA and Cointegration Tests of PPP under Fixed and Flexible Exchange Rate Regimes", *The Review of Economics and Statistics*, 70(3), 504-508.

Engle, R.F. and C.W.J. Granger, 1987, "Co-Integration and Error Correction: Representation, Estimation and Testing", *Econometrica*, 55(2), 251-276.

Frenkel, J.A., 1978, "Purchasing Power Parity: Doctrinal Perspective and Evidence from the 1920s", *Journal of International Economics*, 8(3), 169-191.

Frenkel, J.A., 1981, "The Collapse of Purchasing Power Parities during the 1979's", *European Economic Review*, 16, 145-165.

Granger, C.W.J., 1986, "Developments in the Study of Cointegrated Economic Variables", *Oxford Bulletin of Economics and Statistics*, 48(3), 213-228.

Griffiths, W.E. et al., 1993, *Learning and Practicing Econometrics*, John Wiley & Sons, Inc.

Hakkio, C.S., 1984, "A Re-Examination of Purchasing Power Parity, A Multi-Country and Multi-Period Study", *Journal of International Economics*, 17, 265-277.

Hall, R.E. and J.B. Taylor, 1986, *Macroeconomics: Theory, Performance and Policy*, W.W. Norton&Company, New York, London.

Henricsson, R. and E. Lundbaeck, 1995, "Testing the Presence and the Absence of Purchasing Power Parity: Results for Fixed and Flexible Regimes", *Applied Economics*, 27, 635-641.

Isard, P., 1977, "How Far Can We Push the 'Law of one Price'", *American Economic Review*, 67(5), 942-948.

Kim, Y., 1990, "Purchasing Power Parity in the Long Run: A Cointegration Approach", *Journal of Money, Credit and Banking*, 22(4), 491-503.

Kravis, I.B. and R.E. Lipsey, 1978, "Price Behavior in the Light of Balance of Payments Theories", *Journal of International Economics*, 8(3), 193,246.

Krugman, P.R., 1978, "Purchasing power Parity and Exchange Rates: Another Look at the Evidence", *Journal of International Economics*, 8(3), 397-407.

Liu, P.C., 1992, "Purchasing Power Parity in Latin America: A Co-Integration Analysis", *Weltwirtschaftliches Archiv*, 123, 662-679.

MacDonald, R., 1993, "Long Run Purchasing Power Parity: Is It for Real?", *The Review of Economics and Statistics*, 75(4), 690-695.

McNown, R. and M.S. Wallace, 1990, "Cointegration Tests of Purchasing Power Parity among Four Industrial Countries: Results for Fixed and Flexible Rated", *Applied Economics*, 22, 1729-1737.

Milone, L.M., 1986, "The Law of one Price: Further Empirical Evidence Concerning Italy and the United Kingdom", *Applied Economics*, 18(6), 645-661.

New Palgrave: Dictionary of Modern Economics, 1987, The Macmillan Press, London, New York, Tokyo.

Noorbakhsh, A. and A. Christev, 1994, Price Parities and Foreign Exchange Rate Determination during Transition in Eastern European Countries: A Cointegration Appraisal of PPP Theory, unpublished study.

Officer, L.H., 1976, "The Purchasing-Power-Parity Theory of Exchange Rates: A Review Article", *IMF Staff Papers*, 23(1), 1-61.

Ohno, K., 1990, "Estimating Yen/Dollar and Mark/Dollar Purchasing Power Parities", *IMF Staff Papers*, 37(3), 700-725.

Patel, J., 1990, "Purchasing Power Parity as a Long-Run Relation", *Journal of Applied Econometrics*, 5(4), 367-379.

Taylor, M.P. and P.C. McMahon, 1988, "Long-Run Purchasing Power Parity in the 1920s", *European Economic Review*, 32, 179-197.

Taylor, M.P., 1988, "An Empirical Examination of Long-Run Purchasing Power Parity Using Cointegration Techniques", *Applied Economics*, 20, 1369-1381.

Thacker, N., 1995, "Does PPP Hold in the Transition Economies? The case of Poland and Hungary?" *Applied Economics*, 27(6), 477-481.

Webster, A., 1987, "Purchasing Power Parity as a Theory of International Arbitrage in Manufactured Goods: an Empirical View of UK/US Prices in the 1970s", *Applied Economics*, 19(11), 1433-1456.