THE EFFECT OF LARGE EDDIES ON THE CONVECTIVE HEAT/MASS TRANSFER OVER COMPLEX TERRAIN: ADVANCED THEORY AND ITS VALIDATION AGAINST EXPERIMENTAL AND LES DATA

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Abstract: This paper presents a new theory of the convective heat/mass transfer. It focuses on (i) advanced treatment of turbulent mixing caused by large-scale semi-organised eddies overlooked in the classical theory and (ii) interactions between large eddies and surface roughness elements up to very high obstacles such as buildings, rocks and hills. Large-scale structures in the shear-free convective boundary layers consist of strong plumes and wider but weaker downdraughts. Close to the surface they cause local "convective winds" blowing towards the plume axes. The latter generate turbulence, in addition to its generation by the buoyancy forces, and strongly enhance turbulent fluxes of heat and other scalars. This mechanism is especially important over very rough surfaces. The proposed advanced model is validated against data from measurements over different sites and also through large-eddy simulation of convective boundary layers (CBLs) over a range of surfaces from very smooth to extremely rough. Excellent correspondence between model results, field observations and large-eddy simulations is achieved. The obtained resistance and heat/mass transfer laws are recommended for practical use in meso-scale, weather-prediction, climate and other environmental models.

Keywords *– convection, semi-organised eddies, surface fluxes*

1. ADVANCED LARGE-EDDY SHEAR MODEL

Large eddies in the CBL are characterised by the velocity scale $W_* = (F_{bs} h)^{1/3}$, where F_{bs} is the buoyancy flux at the surface and h is the CBL depth. They live much longer than the CBL overturning time, $h / W_* \sim 10^3$ s. Accordingly the large-scale convergence flow field near the surface is treated as a quasi-steady internal boundary layer (IBL), in which the smaller-scale turbulence is in local equilibrium. Thus principal length scales characterising the free CBL are (i) its depth h; (ii) the IBL depth; (iii) the "constant-flux" layer depth, estimated as one tenth of h_I ; (iv) the depth h_{SL} of the "surface layer" with pronounced vertical increments in the convective wind speed and potential temperature or buoyancy (known to be an order of magnitude larger than the large-eddy MO lengths $L_* = U_*^3 / F_{bs}$, where U_* is the "minimum friction velocity" caused by "convective wind"); (v) the depth h_{SSL} of the "surface shear layer" (SSL), estimated as one tenth of L_* , so $h_{SSL} \sim 10^{-1} L_*$, $L_* \sim 10^{-3} h$; (vi) typical height of roughness elements, estimated as $h_0 \sim 25\, z_{0u}$; (vii) horizontal scale X of the IBL, estimated as one half the distance between plume structures; so $X \sim \frac{1}{2} h$.

Of these scales, h is the largest (in the atmosphere, of order 10^3 m). The depth of the surface layer is much smaller: $h_{SL} \sim 10\, L_* = 10 \left(U_* / W_*\right)^3 h \sim 10^{-2} h$. The IBL depth is only an order of magnitude smaller that h (then $h_I \sim 10^{-1} h \sim 10^2 L_*$, which is why the "constant flux layer" and the "surface layer" coincide). The fact that the surface layer occupies only the lower 10% of the IBL essentially simplifies our analysis and allows modelling the near-surface part of the IBL through the MO similarity theory, or alternatively, through the eddy-viscosity/conductivity/diffusivity ($K_{\{M,H,D\}}$) model with the IBL-flow velocity U and the large-eddy friction velocity U_* substituted for the ordinary wind velocity u and friction velocity u_*:

$$K_{\{M,H,D\}} = z\left[k_{\{u,T,q\}} U_* + \frac{W_*}{C_{\{U,\Theta,Q\}}} \left(\frac{z}{h}\right)^{1/3} \right]. \tag{1}$$

Here, $k_{\{u,T,q\}}$ are von Karman constants and $C_{\{U,\Theta,Q\}}$ are three other empirical constants. The IBL is similar to a stagnation point boundary layer near x=0 (defined as the midway point between two vertically rising plumes), where large scale velocity above the IBL, U_I, behaves as $U_I \sim W_* x / X$. In our approximate analysis the average value $< U_I > \sim W_*$ is taken for the advective velocity over the length scale X.

In the bulk of the IBL the velocity perturbation $\tilde{u}\,(x,z) = U - U_I$ is determined by the following equation:

$$U_I(x) \frac{\partial \tilde{u}}{\partial x} \sim \frac{\partial}{\partial z}\left(\frac{W_* z^{4/3}}{h^{1/3}} \frac{\partial \tilde{u}}{\partial z} \right). \tag{2}$$

Its approximate solution, taking U_{I} as a constant, shows that $\tilde{u}$ matches smoothly the surface layer solution Eq. (6)-(7) as $z / h_I \to 0$ and $z / L_* \to \infty$; and similarly for the temperature and humidity perturbations $\theta - \theta_I$ and $q - q_I$.

The IBL thickness $h_I(x)$ increases from x=0 (the stagnation zone) in the same way as a plume thickness grows in a convective boundary layer, namely

$$W_* \frac{dh_I}{dx} \sim 0.24 W_c (z = h_I) = 0.24 W_* \left(\frac{h_I}{h}\right)^{1/3},$$
$$\text{so that } h_I \sim 10^{-1} \frac{x^{3/2}}{h^{1/2}} \text{ and } < h_I > \sim 10^{-1} h. \tag{3}$$

Here, the Prandtl velocity scale $W_c = (F_{bs} z)^{1/3}$ characterises the vertical velocity variance $\sigma_w(z = h_I)$; the coefficient 0.24 is taken after lab experiments of Deardorff et al (1980).

In the surface layer, where $z \le 10 L_*$, the "mean" profiles are determined by the approximately depth-constant local turbulent fluxes:

$$\tau = U_*^2 = K_M \frac{\partial U}{\partial z}, \quad F_\theta = F_{\theta s} = K_H \frac{\partial \theta}{\partial z}, \quad F_q = F_{qs} = K_D \frac{\partial q}{\partial z}. \tag{4}$$

Equations (1), (4) are solved with boundary conditions formulated in terms of aerodynamic surface potential temperature θ_0 and specific humidity q_0,

$$U = 0, \ \theta = \theta_0, \ q = q_0 \quad \text{at} \quad z = z_{0u}, \tag{5}$$

where z_{0u} is the roughness length for momentum, to obtain

$$U = \frac{U_*}{k_u}\left\{ \ln\frac{z}{z_{0u}} - 3 \ln \frac{1 + (k_u C_U)^{-1} (z / L_*)^{1/3}}{1 + (k_u C_U)^{-1} (z_{0u} / L_*)^{1/3}} \right\}, \tag{6}$$

$$\left\{\frac{k_T U_*(\theta-\theta_0)}{-F_{\theta s}}, \frac{k_q U_*(q-q_0)}{-F_{qs}}\right\} = \ln\frac{z}{z_{0u}} - 3\ln\frac{1+\left(k_{\{T,q\}}C_{\{\Theta,Q\}}\right)^{-1}(z/L_*)^{1/3}}{1+\left(k_{\{T,q\}}C_{\{\Theta,Q\}}\right)^{-1}(z_{0u}/L_*)^{1/3}} \ . \tag{7}$$

This solution in combination with newly introduced concept of the buoyancy dependence of the effective roughness length yield formulations summarised in Table 1, with empirical constants deduced from field and LES data.

Table 1. Recommended formulas and empirical constants

Constant	Empirical value	In formula	Eq. number, comments
C_{U0} C_{U1} C_{U2}	6 0.3 -2.56	$\frac{U_*}{W_*} = C_{U1}\left(\ln\frac{h/z_{0u}}{(\ln(h/z_{0u})-C_{U0})^3}+C_{U2}\right)^{-1}$	(8) smooth surface
C_{U3} C_{00}	0.56 0.3	$\frac{U_*}{W_*} = C_{U3}\left[\frac{z_{0u}}{h}+C_{00}\left(\frac{z_{0u}}{h}\right)^{8/7}\right]^{1/6}$	(9) rough surface
C_1 C_2	0.25 -2.5	$\frac{F_{\theta s}}{W_*\Delta\theta} = \frac{F_{qs}}{W_*\Delta q} = \frac{W_*^2}{(\beta\Delta\theta+0.61g\Delta q)h}$ $= C_1\left(\ln\frac{h/z_{0u}}{(\ln(h/z_{0u})-C_{U0})^3}+C_{U2}\right)^{-1}\left(\ln\frac{h/z_{0u}}{(\ln(h/z_{0u})-C_{U0})^3}+C_2\right)^{-1}$	(10) smooth surface
C_3	0.8	$\frac{F_{\theta s}}{W_*\Delta\theta} = \frac{F_{qs}}{W_*\Delta q} = \frac{W_*^2}{(\beta\Delta\theta+0.61g\Delta q)h} = C_3\left[\frac{z_{0u}}{h}+C_{00}\left(\frac{z_{0u}}{h}\right)^{8/7}\right]^{1/3}$	(11) rough surface

2. VALIDATION OF MODEL AND DETERMINATION OF DIMENSIONLESS CONSTANTS

The proposed model predicts that essential features of large-scale eddies depend on the surface roughness. LES data visualised in Figure 1 confirm this prediction and clarify physical mechanisms. Indeed, convection regimes over the smooth and the rough surfaces are clearly distinguished. Over the smooth surface (Figure 1a,c), more energetic eddies and stronger heated plumes develop closer to the surface. However, these eddies are smaller, so that the horizontal extension of IBL flows and therefore U_* are also smaller. Over the rough surface (Figure 1b,d), the number of eddies decreases, but they become taller and more intensive, the plumes become hotter, the velocity maxima are observed at higher distances from the surface, and U_* is larger.

Figures 2-3 show our theoretical curves (solid lines) representing the resistance law, Eqs. (8)-(9) and the heat/mass/buoyancy transfer laws, Eqs. (10)-(11), together with data from field measurements and LES. It is seen that the LES and the field data correlate very well, demonstrating that LES realistically reproduces the CBL turbulence. Both LES and field data strongly support the proposed theory. The matching points between the low-roughness and the high-roughness regimes are h/z_{0u}=$3.62\cdot10^5$ for the minimum friction velocity U_*, and h/z_{0u}=$1.8\cdot10^5$ for the turbulent fluxes of potential temperature and specific humidity $F_{\theta s}$ and F_{qs} .

3. CONCLUSIONS

Following prior works of Businger (1973) Schumann (1988), Sykes et al. (1993) and Zilitinkevich et al. (1998), it is demonstrated that the Monin-Obukhov similarity theory for the atmospheric surface-layer turbulence, as well as other local theories and turbulence closures, become insufficiently advanced in the sheer-free convection. In this regime, basic features of the surface layer are strongly affected by large-scale semi-organised convective eddies characterised by the length scale h (CBL depth) and the velocity scale $W_* = (F_{bs} h)^{1/3}$, both overlooked in the classical theories.

Although the key non-local mechanism of enhancing the turbulence, namely, strong shears in the near-surface convergence flow patterns driven by large eddies, has already been recognised, no one of prior models were applicable to sufficiently wide range of roughness lengths and CBL depths.

The proposed advanced model accounts for the newly recognised dependence of the effective roughness length on the convective instability (besides its well-know dependence on the surface geometry) and covers the whole range of convection regimes over natural surfaces from aerodynamically smooth to very rough ($3 \cdot 10^3 < h / z_{0u} < 10^9$).

This model fits all available experimental and LES data. Equations (8)-(11) provide realistic and simple calculation scheme for the surface fluxes in free convection.

Table 2: Summary of LES and field data, and corresponding symbols in Figures 2-3.

LES data	Red symbols	Reference
Present LES	●	Present paper
Schmidt and Schumann LES	■	Schmidt and Schumann (1989)
Sykes et al. LES	◆	Sykes et al. (1993)
Sorbjan LES	►	Sorbjan (1996)
Otte and Wyngaard LES	◄	Otte and Wyngaard (2001)
Noh et al. LES	★	Noh et al. (2003)
Field data	**Blue symbols**	**Reference**
SCOPE, September 1993, San Clemente Island, California	■	Fairall et al. (1996a, b, c), Grachev and Fairall (1997), Grachev et al. (1997, 1998)
TOGA COARE, 1992-1993, Western Pacific Ocean	●	
BOREX, July 1995, Borris site, Denmark	▼	Mikkelsen et al. (1996), Grachev et al. (1998)
PAFEX-1, January-February 1998, Phoenix, Arizona	◆	Grachev et al. (1999)
ATHENS, 1994-1999, Athens city centre and Attiki peninsula, Greece	►	Batcharova and Gryning (1998), Akylas et al. (2003)

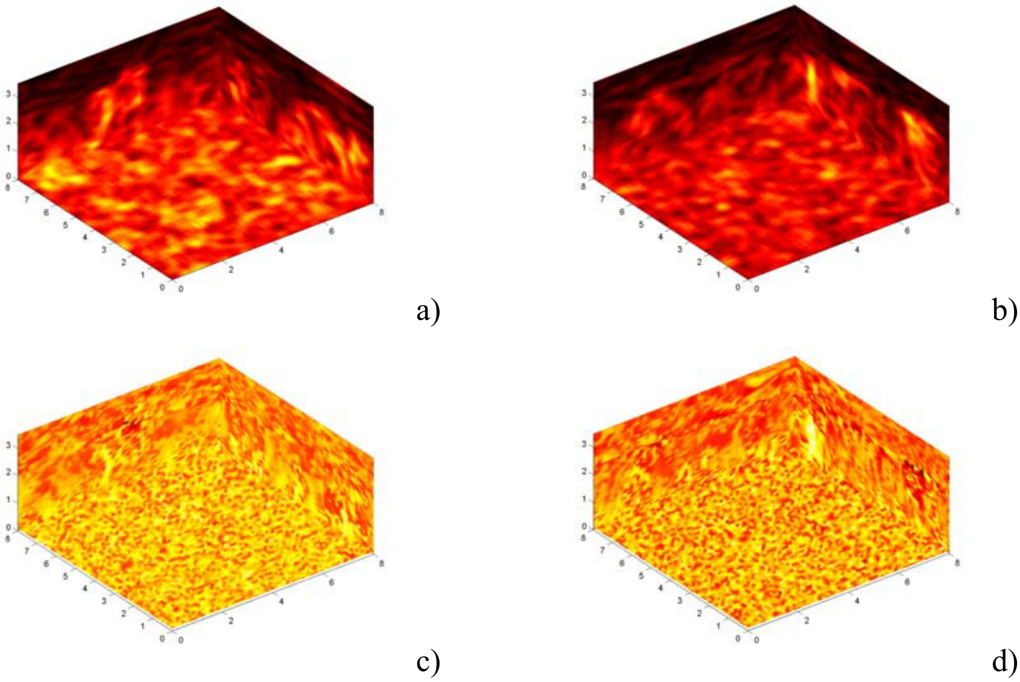

Figure 1. Turbulent kinetic energy (a,b) and potential temperature flux (c,d) in the CBLs over very smooth (a,c) and very rough (b,d) surface. Brighter areas correspond to more intensive transports of momentum and potential temperature. The domain sizes are given in kilometres.

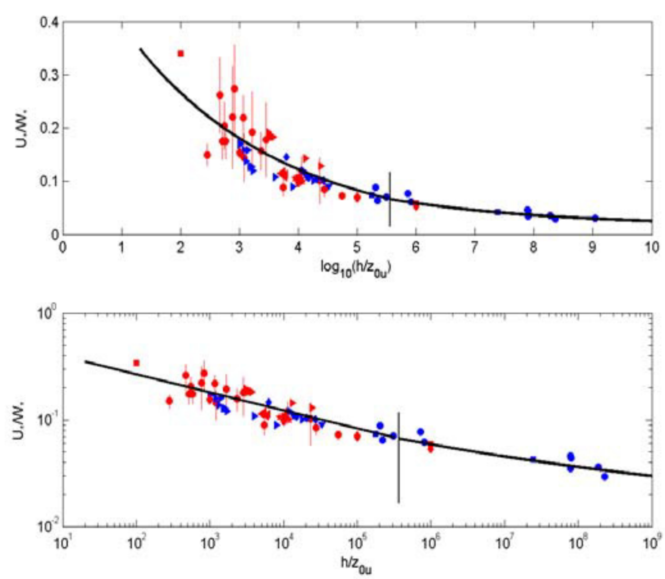

Figure 2. The resistance coefficient U_*/W_* versus h/z_{0u}. The solid curve shows our model: Eq. (8) for low-roughness surfaces $h/z_{0u} \geq 3.62 \cdot 10^5$, Eq. (9) for high-roughness surfaces $h/z_{0u} \leq 3.62 \cdot 10^5$. Vertical line shows the matching point $h/z_{0u} = 3.62 \cdot 10^5$ between these regimes. Symbols are given in Table 2.

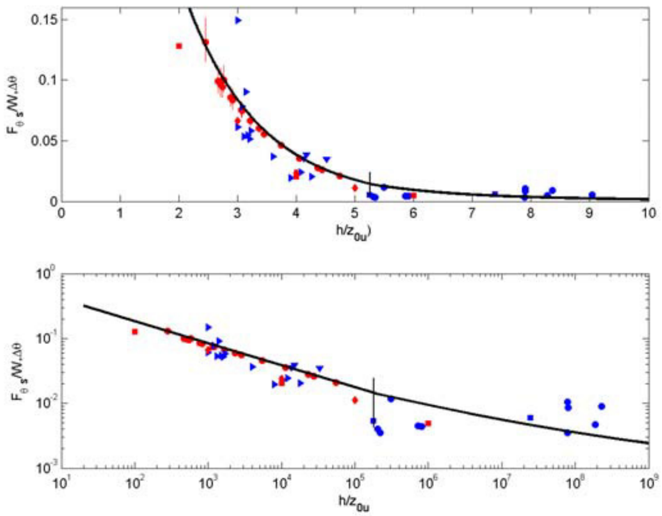

Figure 3. The heat transfer coefficient $C_H \equiv F_{\theta s} / W_* \Delta\theta$ versus h / z_{0u}. Solid curve shows our model: Eq. (10) for low-roughness surfaces $h / z_{0u} \geq 1.8 \cdot 10^5$, Eq. (11) for high-roughness surfaces $h / z_{0u} \leq 1.8 \cdot 10^5$. Vertical line shows the matching point $h / z_{0u} = 1.8 \cdot 10^5$. Symbols are given in Table 2.

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