

# Prediction of the Transformer Inrush Current Forces

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## Keywords

*Electromagnetic forces*  
*Finite element method*  
*Inrush current*  
*Transformer*

## Ključne riječi

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*Metoda konačnih elemenata*  
*Struja uklapanja*  
*Transformator*

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Preliminary note

When a power transformer is switched to power supply there is a transient inrush of current in each of the transformer phases. Its peak value mainly depends on the resistance of the primary winding, the point on the voltage wave at the instant of switching the transformer on, and the residual magnetic flux of the transformer. The inrush current is predicted by the time stepping 2D finite-element method (FEM) coupled with the external electrical circuit. During the switching on process the electromagnetic forces that appear as a result of inrush currents and their leakage magnetic fields are calculated. An experimental transformer is built and the measured results are given.

## Proračun sila uslijed struje uklapanja transformatora

Prethodno priopćenje

Uključivanjem transformatora na mrežu napajanja pojavljuje se u svakoj od faza tranzijentna struja uklapanja. Njezina vršna vrijednost ovisi o otporu primarnog namota, trenutnoj vrijednosti napona u trenutku uključivanja i remanentnom magnetskom tijeku. Struja uklapanja računata je primjenom 2D metode konačnih elemenata (MKE) spregnute s vanjskim električnim krugom, u vremenskoj domeni. Elektromagnetske sile na namote za vrijeme procesa uključivanja pojavljuju se uslijed tranzijentnih struja i njihovih rasipnih polja. Mjerenja struje uklapanja provedena su na eksperimentalnom transformatoru.

## 1. Introduction

Inrush currents are large transient currents generated when a transformer is switched to the power supply. The magnetizing inrush current has a close relationship with nonlinearity and hysteresis of the iron core. When a transformer is energized from a voltage source the peak magnetizing current may reach a very high value and cause a momentary dip in the voltage resulting in an unwanted tripping of the differential protective relay. The phenomenon of the inrush of the magnetizing current is due to the temporary overfluxing of the transformer core at the instant of energization. This temporary overfluxing effect is governed by a number of factors [1]; however, investigation in this paper is focused on:

1. the point-on-voltage wave at the instant of energization and
2. the magnitude and polarity of the remanent flux in the transformer core at the instant of energization;

Therefore the purpose of this paper is to determine the magnetizing inrush current taking into consideration the effect of different operating conditions such as residual flux and switching-on angle. The switch-on angles of the transformer primary voltage are taken into a random

process and could be determined by noticing the voltage waveforms on the digital oscilloscope. The magnitude of the magnetizing inrush current is in the range of the short circuit current and may result in severe dynamical stress in the transformer windings [2-5]. Therefore, accurate evaluation of such forces is essential during the design and manufacturing phase. In this work, carried out in 2D FEM software [6], components of forces on primary windings are calculated.

## 2. Basic equations

### 2.1. Circuit equations

The voltage equation of the primary winding can be written as:

$$v_1 = V_{m1} \sin(\omega t + \vartheta) = r_1 i_1 + N_1 \frac{d\Phi}{dt}, \quad (1)$$

where:  $V_{m1}$  is the peak of phase voltage;  $\omega$ , supply frequency;  $N_1$ , primary turns;  $i_1$  is the primary winding current;  $r_1$  is the primary winding resistance;  $\Phi$  is the total flux consisting steady state and residual fluxes and  $\vartheta$  is the voltage initial phase.

**Symbols/Oznake**

$v_1$	- supply voltage, V napon napajanja
$v_{m1}$	- supply voltage magnitude, V - vršna vrijednost napona napajanja
$N_1$	- primary winding turns number - broj zavoja primarnog namota
$i_1$	- inrush current, A - struja uključivanja
$r_1$	- primary winding resistance, $\Omega$ - ohmski otpor primarnog namota
$l_1$	- primary winding inductance, H - induktivitet primarnog namota
$t_0$	- opening time, s - trenutak isključivanja
$t_c$	- closing time, s - trenutak uključivanja
$e_x$	- DC decaying voltage, V - istosmjerni opadajući napon
$\bar{J}$	- current density, A/m <sup>2</sup> - gustoća struje
$\bar{B}$	- current density, T - gustoća struje

$\bar{f}$	- force density, N/m <sup>3</sup> - sila po jedinici volumena
$\Psi$	- flux linkage, Wb - ulančeni magnetski tijek
$\Psi_{m1}$	- flux linkage magnitude, Wb - vršna vrijednost magnetskog tijeka
$\Psi_r$	- residual flux linkage, Wb - residualni ulančeni tijek
$\vartheta$	- switching on angle, rad - kut uključjenja
$\omega$	- angular frequency, rad/s - električna kružna frekvencija

**Indices/Indeksi**

l	- primary winding - primarni namot
r	- residual flux linkage - residualni ulančeni tijek

If we let  $v_1 = V_{m1} \sin(\omega t + \vartheta)$  be the applied voltage before the opening time  $t_0$ , so that the linked magnetic flux after the opening remains at the value

$\Psi_{r0} = -\frac{V_{m1}}{\omega} \cos(\omega t_0 + \vartheta)$  up to the closing time  $t_c$  then this value is called residual flux,  $\Psi_r = \Psi_{r0}$ .

The linked magnetic flux at closing time  $t = t_c$  is:

$$\Psi_{tc} = -\frac{V_{m1}}{\omega} \cos(\omega t + \vartheta) + \frac{V_{m1}}{\omega} \cos(\omega t_c + \vartheta) - \frac{V_{m1}}{\omega} \cos(\omega t_0 + \vartheta) \quad (2)$$

and is equal to  $\Psi_r$ .

Supposing that the transformer operates within the non linear region of the magnetization curve (above the knee of the curve), the residual value of flux linkage for  $t \geq t_c$  is expressed as [2]:

$$\Psi_r(t) = \Psi_{tc} e^{-(r/l_1)t} = -\Psi_{m1} \cos(\omega t_0 + \vartheta) e^{-(r/l_1)t}, \quad (3)$$

where  $l_1$  is the inductance of the primary winding.

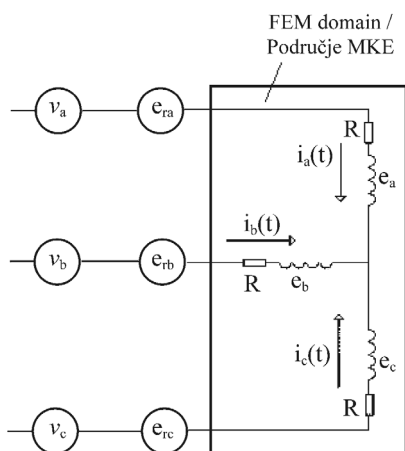
For simplicity if we chose  $t_c=0$  and rewriting (2), the flux linkage for  $t \geq t_c$  is

$$\Psi(t) = -\Psi_{m1} \cos(\omega t + \vartheta) + \Psi_{m1} \cos(\vartheta) e^{-(r/l_1)t} + \Psi_r(t). \quad (4)$$

The derivative of the (3) is

$$\frac{d\Psi_r}{dt} = \frac{r_1}{l_1} \Psi_{m1} \cos(\omega t_0 + \vartheta) e^{-(r/l_1)t}. \quad (5)$$

The term (5) represents the unidirectional decaying voltage  $e_r(t)$  which is added to the external electrical circuit. The FEM domain coupled with the external supply voltages  $v$  and DC decaying voltages  $e_r$ , is shown in Figure 1.



**Figure 1.** FEM domain coupled to external circuit

**Slika 1.** FEM područje spregnuto sa vanjskim krugom

The residual value of flux linkage can increase the peak value of the flux up to more than twice the rated peak value in the most critical case when the transformer is energizing and voltage is at  $\vartheta=0$  and  $\Psi_r$  has maximum amplitude.

Different residual flux in the transformer iron core is achieved by using proper values of DC excitation after initially demagnetizing the core.

### 2.2. FEM modelling

Calculation of the transformer inrush current is based on the transient 2D FEM analysis of the magnetic field coupled with the external electrical circuit. The magnetic field equations and the circuit equations are solved simultaneously. The analysis was done with the time-stepping method. The electromagnetic force on a small volume  $dV$  of primary winding, having a current density  $\vec{J}$  and placed in a point where the flux density is  $\vec{B}$  is expressed by:

$$\vec{f} = \vec{J} \times \vec{B}, \quad \text{N/m}^3. \tag{6}$$

Force per unit length is:

$$\vec{F} = \int_s \vec{f} d\vec{S}, \quad \text{N/m}. \tag{7}$$

## 3. Computational results

### 3.1. Inrush current

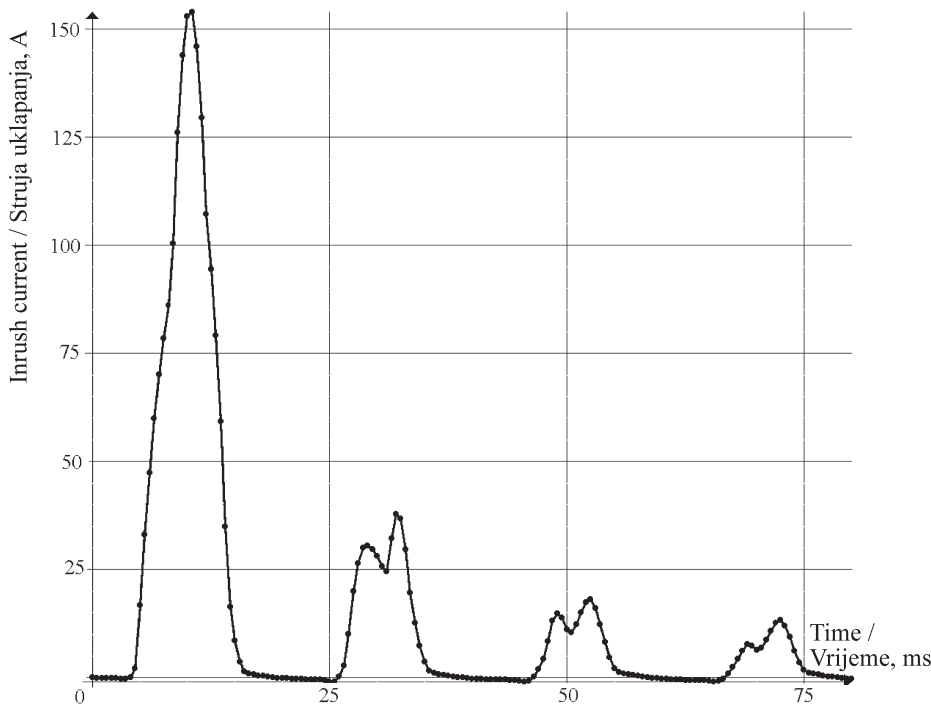
The inrush current is calculated under different values of residual flux and switching-on angle. The simulated waveform for the inrush current at switching-on angle ( $\vartheta=50^\circ$ ) and per unit residual fluxes  $\Psi_{ar} = -0.5$ ,  $\Psi_{br} = 0.5$  and  $\Psi_{cr} = 0$  is shown in Figure 3.

### 3.2. Electromagnetic force

The path (centerline) of the primary winding along which are computed forces is shown in Figure 4.

The vertical and horizontal components of the force density are shown in Figure 5 and Figure 6 respectively.

Strong vertical forces caused by fringing of the flux at either end of the winding are very clear. These forces cancel out in a simetric case. Figure 7 shows the time variation of the force of primary winding (left side of phase A). The force is obtained by multiplying (7) by axial winding length for each time step.



**Figure 3.** Waveform of the inrush current for  $\vartheta = 50^\circ$ ,  $\Psi_{ra} = -0.5$ ,  $\Psi_{rb} = 0.5$  and  $\Psi_{rc} = 0$   
**Slika 3.** Vremenski dijagram struje ukljućivanja za,  $\vartheta = 50^\circ$ ,  $\Psi_{ra} = -0.5$ ,  $\Psi_{rb} = 0.5$  i  $\Psi_{rc} = 0$

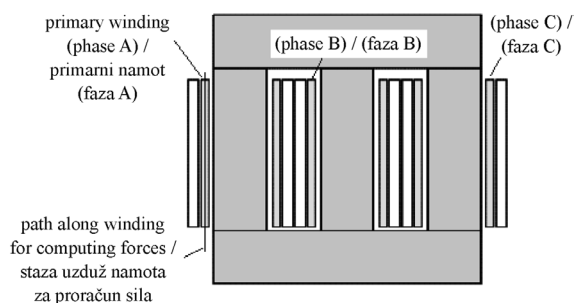


Figure 4. Path along the winding for computation of forces

Slika 4. Staza uzduž namota za proračun sile

## 4. Experimental verification

A 5-kVA power transformer is used for experimental verification of the computed inrush current. The transformer data are given in Table 1. In the experiment, the rms value of the supply voltage is 231 V. Figure 8 shows the three first cycles of the inrush current obtained in an inrush test. Before the experimental switch-on, the transformers were demagnetized using a variable ac source to eliminate the residual flux in the core.

Table 2. shows the peaks of inrush current obtained with simulation and experiment for the three first cycles.

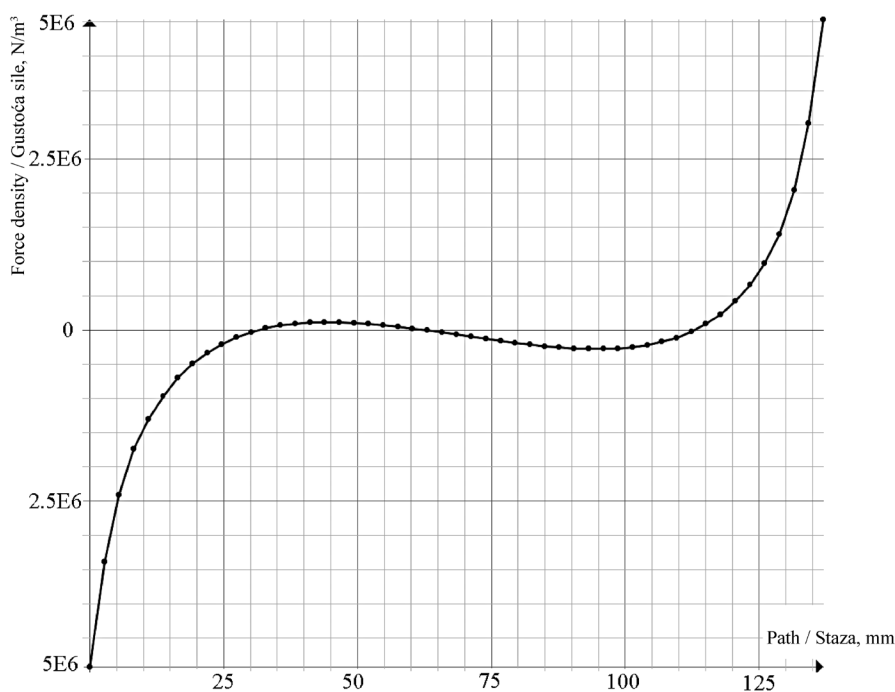


Figure 5. Vertical component of the force density curve along a path of primary winding

Slika 5. Vertikalna komponenta sile uzduž staze kroz primarni namot

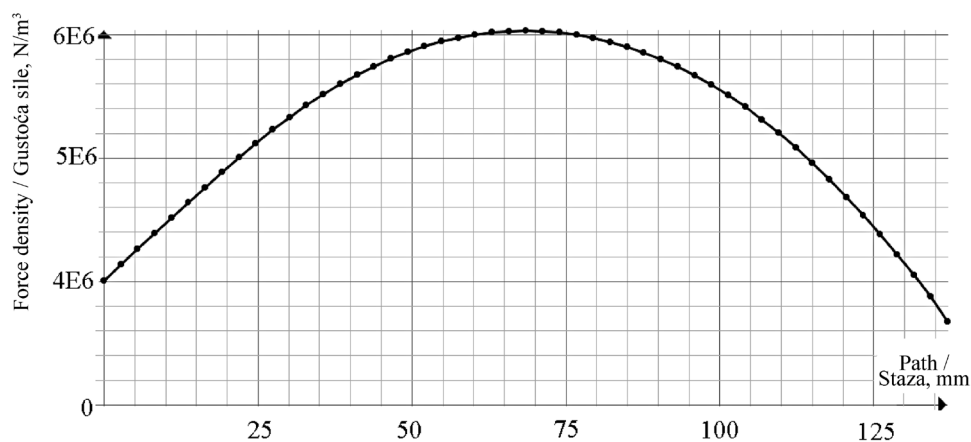
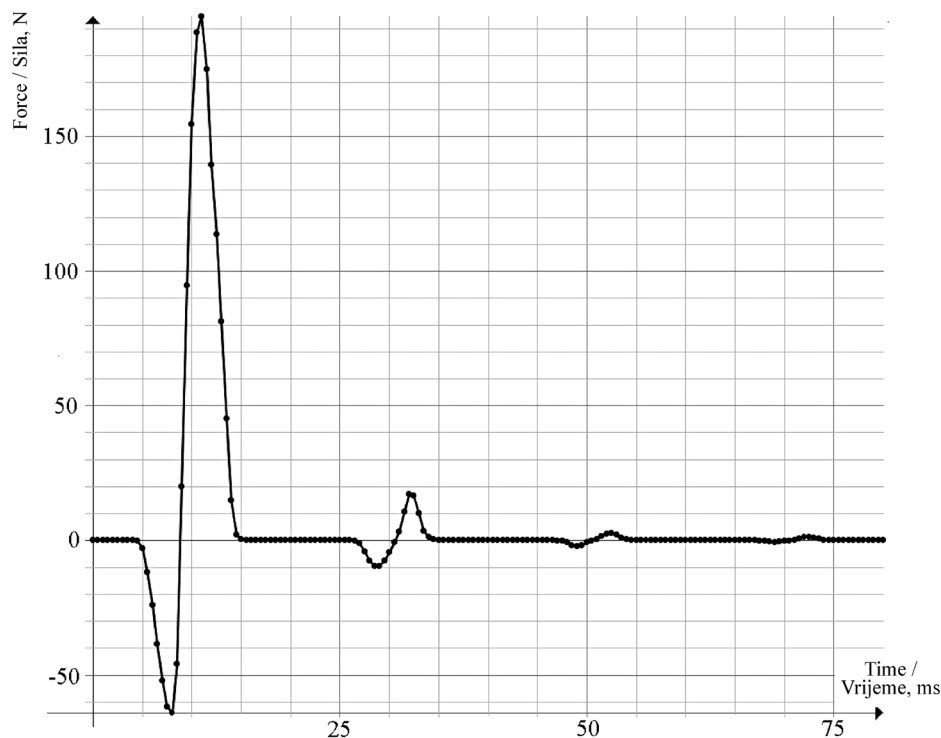
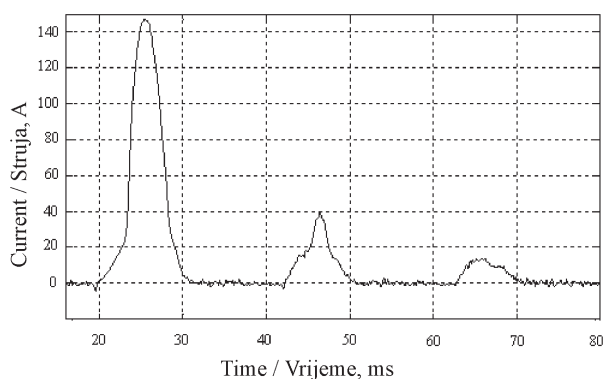


Figure 6. Horizontal component of the force density curve along a path of primary winding

Slika 6. Horizontalna komponenta sile uzduž staze kroz primarni namot



**Figure 7.** Horizontal component of the force on the left side of the primary winding (phase A, Figure 4.)  
**Slika 7.** Horizontalna komponenta sile na lijevu stranu primarnog namota (faza A, slika 4.)



**Figure 8.** Waveform of the inrush current for  $\vartheta = 50^\circ$  and  $\Psi_{ra} = -0.5$ ,  $\Psi_{rb} = 0.5$  and  $\Psi_{rc} = 0$  in the experiment

**Slika 8.** Vremenski dijagram struje uklapanja za  $\vartheta = 50^\circ$  i  $\Psi_{ra} = -0.5$ ,  $\Psi_{rb} = 0.5$  i  $\Psi_{rc} = 0$  u pokusu

**Table 1.** Transformer data

**Tablica 1.** Podaci transformatora

Power/Snaga	5kVA
Voltage/Napon	400/231 V
Current/Struja	7.2/12.5 A
Connection/Spoj	Y/y
No-load test/Prazan hod Current/Struja	0.077pu
Short-circuit test/Pokus kratkog spoja Voltage/Napon	0.026pu

**Table 2.** Inrush current peaks

**Tablica 2.** Amplitude struje uklapanja

Method / Metoda	Time (ms)	10	30	50
Calculation / Izračun	Current (A)	154	32	15
	Measurement / Dimenzija	147	39	14
Error / Greška	%	-4.8	+17.9	-7.1

## 5. Conclusion

This paper presents a prediction of the transformer magnetizing inrush current forces taking into consideration the residual flux of the core and the instant of windings energization. The simulation result for the inrush current waveform are compared with the experimental one. The relevance between measured and computed values renders this numerical calculation by FEM appropriate for transformer winding forces evaluation.

**REFERENCES**

- [1] LING, P.C.Y.; BASAK, A.: *Investigation of magnetizing inrush current in a single-phase transformer*, IEEE Trans. Magn., vol. 24, no. 6, pp. 3217–3222, Nov. 1988.
- [2] FAITZ, J. et al.: *Three and Two Dimensional Finite Element Computation of Inrush Current and Short Circuit Electromagnetic Forces on Windings of a Three Phase Core Type Power Transformer*, IEEE Transaction on Magnetics, Vol.44, No. 5, May 2008, pp.590-597.
- [3] ADLY, A.: *Computation of Inrush Current Forces on Transformer Windings*, IEEE Transaction on Magnetics, Vol.37, No. 4, July 2001, pp 2855-2857.
- [4] LING, P.; BASAK, A.: *Investigation of Magnetizing Inrush Inrush Current in a Single-phase Transformer*, IEEE Transaction on Magnetics, Vol.24, No. 6, November 1988, pp 3217-3222.
- [5] STEURER, M; FROHLICH, K.: *The impact of inrush currents on the mechanical stress of high voltage power transformer coils*, IEEE PWRD 17 (2002) (1), pp. 155–160.
- [6] FLUX V.10.3, *CAD package for electromagnetic and thermal analysis using finite elements*, CEDRAT, 2010.