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ENUMERATION OF SYMMETRIC (69,17,4) DESIGNS ADMITTING Z_6 AS AN AUTOMORPHISM GROUP

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ABSTRACT. All symmetric (69,17,4) designs admitting the cyclic group of order 6 as an automorphism group are classified and their full automorphism groups are determined.

1. INTRODUCTION AND PRELIMINARIES

A symmetric (v, k, λ) design is a finite incidence structure $(\mathcal{P}, \mathcal{B}, I)$, where $\mathcal{P}$ and $\mathcal{B}$ are disjoint sets and $I \subseteq \mathcal{P} \times \mathcal{B}$, with the following properties:

1. $|\mathcal{P}| = |\mathcal{B}| = v$,
2. every element of $\mathcal{B}$ is incident with exactly k elements of $\mathcal{P}$,
3. every pair of elements of $\mathcal{P}$ is incident with exactly λ elements of $\mathcal{B}$.

According to [4], there are 4 symmetric (69,17,4) designs known. All those designs admit an automorphism group of order 13.

The aim of this article is to prove the following

THEOREM 1.1. *There are 59 mutually nonisomorphic symmetric* $(69, 17, 4)$ *designs admitting the cyclic group of order 6 as an automorphism group. All of them are self-dual. Exactly 3 of those designs have full automorphism groups of order 6 isomorphic to group* Z_6 *and 54 designs have full automorphism groups of order 12 isomorphic to group* Z_{12}. *Furtermore, 2 designs have full automorphism groups of order 156; one of them is isomorphic to group* $F_{39} \times Z_4$ *and the other is isomorphic to group* $F_{39} : Z_4$.

Let $\mathcal{D} = (\mathcal{P}, \mathcal{B}, I)$ be a symmetric (v, k, λ) design and $G \leq Aut\mathcal{D}$. Group G has the same number of point and block orbits. Let us denote the number of G–orbits by t, point orbits by $\mathcal{P}_1, \ldots, \mathcal{P}_t$, block orbits by $\mathcal{B}_1, \ldots, \mathcal{B}_t$, and

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put $|\mathcal{P}_r| = \omega_r$, $|\mathcal{B}_i| = \Omega_i$. Further, denote by γ_{ir} the number of points of $\mathcal{P}_r$ which are incident with the representative of the block orbit $\mathcal{B}_i$. For these numbers the following equalities hold:

$$\sum_{r=1}^{t} \gamma_{ir} = k, \tag{1}$$

$$\sum_{r=1}^{t} \frac{\Omega_j}{\omega_r} \gamma_{ir}\gamma_{jr} = \lambda\Omega_j + \delta_{ij} \cdot (k - \lambda). \tag{2}$$

DEFINITION 1.2. *The $(t \times t)$-matrix (γ_{ir}) with entries satisfying properties* (1) *and* (2) *is called the orbit structure for parameters (v, k, λ) and orbit distribution $(\omega_1, \dots, \omega_t)$, $(\Omega_1, \dots, \Omega_t)$.*

2. PROOF OF THE THEOREM

From now on we shall denote by G an abelian group isomorphic to a cyclic group of order 6.

The first step in the construction of all symmetric designs admitting G as an automorphism group is to determine all possible orbit distributions and to find all possible orbit structures related to them. Action of G is semistandard, so it is sufficient to determine point orbit distributions $(\omega_1, ..., \omega_t)$.

LEMMA 2.1. *Symmetric* (69, 17, 4) *design admitting G as an automorphism group has one of the following orbit distributions:*

1. *(1,2,2,2,2,3,3,3,3,6,6,6,6,6,6,6,6),*
2. *(1,2,3,3,3,3,6,6,6,6,6,6,6,6,6),*
3. *(3,3,3,6,6,6,6,6,6,6,6,6,6).*

PROOF. The complete orbit structures satisfying (1) and (2) can be obtained only for orbit distributions from the statement of the lemma. ◇

Using the computer program by V. Ćepulić we obtained 7 orbit structures which correspond to the first type from lemma 1, 467 orbit structures corresponding to the second type and 257 orbit structures corresponding to the third type.

Next step in construction is to "lift" obtained orbital structures for the group $G \cong \langle \rho \mid \rho^6 = 1 \rangle$ to orbital structures for the cyclic group $\langle \rho^3 \rangle$ with the assumption that they admit ρ^2 as an automorphism. That can be done only for seven structures related to the first orbit distribution from lemma 1. So, by indexing structures corresponding to the second and third type cannot be obtained symmetric (69,17,4) designs admitting cyclic group of order 6 as an automorphism group.

The orbit structures related to the first type are the following.

OS1	1	2	2	2	2	3	3	3	3	6	6	6	6	6	6	6	6
1	1	2	2	0	0	0	0	0	0	6	6	0	0	0	0	0	0
2	1	0	0	1	0	3	0	0	0	3	0	3	3	3	0	0	0
2	1	0	0	0	1	3	0	0	0	0	3	0	0	0	3	3	3
2	0	1	0	2	2	0	0	0	0	3	0	3	0	0	3	3	0
2	0	0	1	2	2	0	0	0	0	0	3	0	3	3	0	0	3
3	0	2	2	0	0	1	0	0	0	0	0	2	2	2	2	2	2
3	0	0	0	0	0	0	1	0	0	2	2	4	2	0	2	0	4
3	0	0	0	0	0	0	0	1	0	2	2	2	0	4	0	4	2
3	0	0	0	0	0	0	0	0	1	2	2	0	4	2	4	2	0
6	1	1	0	1	0	0	1	1	1	0	2	2	2	1	1	2	1
6	1	0	1	0	1	0	1	1	1	2	0	1	1	2	2	1	2
6	0	1	0	1	0	1	2	1	0	2	1	0	1	2	2	1	2
6	0	1	0	0	1	1	1	0	2	2	1	1	2	1	0	2	2
6	0	1	0	0	1	1	0	2	1	1	2	2	1	2	2	0	1
6	0	0	1	1	0	1	1	0	2	1	2	2	0	2	2	1	1
6	0	0	1	1	0	1	0	2	1	2	1	1	2	0	1	2	2
6	0	0	1	0	1	1	2	1	0	1	2	2	2	1	1	2	0

OS2	1	2	2	2	2	3	3	3	3	6	6	6	6	6	6	6	6
1	1	2	2	0	0	0	0	0	0	6	6	0	0	0	0	0	0
2	1	0	0	1	0	3	0	0	0	3	0	3	3	3	0	0	0
2	1	0	0	0	1	3	0	0	0	0	3	0	0	0	3	3	3
2	0	1	0	2	2	0	0	0	0	3	0	3	0	0	3	3	0
2	0	0	1	2	2	0	0	0	0	0	3	0	3	3	0	0	3
3	0	2	2	0	0	1	0	0	0	0	0	2	2	2	2	2	2
3	0	0	0	0	0	0	1	0	0	2	2	4	2	0	2	0	4
3	0	0	0	0	0	0	0	1	0	2	2	2	0	4	0	4	2
3	0	0	0	0	0	0	0	0	1	2	2	0	4	2	4	2	0
6	1	1	0	1	0	0	1	1	1	0	2	2	1	2	2	1	1
6	1	0	1	0	1	0	1	1	1	2	0	1	2	1	1	2	2
6	0	1	0	1	0	1	2	1	0	2	1	0	2	1	1	2	2
6	0	1	0	0	1	1	1	0	2	1	2	2	2	1	0	2	1
6	0	1	0	0	1	1	0	2	1	2	1	1	1	2	2	0	2
6	0	0	1	1	0	1	1	0	2	2	1	1	0	2	2	1	2
6	0	0	1	1	0	1	0	2	1	1	2	2	2	0	1	2	1
6	0	0	1	0	1	1	2	1	0	1	2	2	1	2	2	1	0

OS3	1	2	2	2	2	3	3	3	3	6	6	6	6	6	6	6	6
1	1	2	2	0	0	0	0	0	0	6	6	0	0	0	0	0	0
2	1	0	0	1	0	3	0	0	0	3	0	3	3	3	0	0	0
2	1	0	0	0	1	0	3	0	0	3	0	3	0	0	3	3	0
2	0	1	0	2	2	0	0	0	0	3	0	0	3	0	3	0	3
2	0	0	1	2	2	0	0	0	0	0	3	3	0	3	0	3	0
3	0	2	0	0	0	1	0	0	0	0	2	4	2	0	2	2	2
3	0	0	2	0	0	0	1	0	0	2	0	2	2	2	0	2	4
3	0	0	0	0	0	0	0	1	0	2	2	2	0	4	4	0	2
3	0	0	0	0	0	0	0	0	1	2	2	0	4	2	2	4	0
6	1	1	1	1	0	0	1	1	1	0	1	1	2	2	2	1	1
6	1	0	0	0	1	1	0	1	1	1	2	1	1	1	1	2	3
6	0	1	1	0	1	2	1	1	0	1	1	0	1	2	2	2	1
6	0	1	0	1	0	1	1	0	2	2	1	1	0	2	1	2	2
6	0	1	0	0	1	0	1	2	1	2	1	2	2	2	0	1	1
6	0	0	1	1	0	1	0	2	1	2	1	2	1	0	2	2	1
6	0	0	1	0	1	1	1	0	2	1	2	2	2	1	2	0	1
6	0	0	0	1	0	1	2	1	0	1	3	1	2	1	1	1	2

OS4	1	2	2	2	2	3	3	3	3	6	6	6	6	6	6	6	6
1	1	2	2	0	0	0	0	0	0	6	6	0	0	0	0	0	0
2	1	0	0	1	0	3	0	0	0	3	0	3	3	3	0	0	0
2	1	0	0	0	1	0	3	0	0	3	0	3	0	0	3	3	0
2	0	1	0	2	2	0	0	0	0	3	0	0	3	0	3	0	3
2	0	0	1	2	2	0	0	0	0	0	3	3	0	3	0	3	0
3	0	2	0	0	0	1	0	0	0	0	2	2	2	2	4	2	0
3	0	0	2	0	0	0	1	0	0	2	0	0	2	4	2	2	2
3	0	0	0	0	0	0	0	1	0	2	2	4	0	2	2	0	4
3	0	0	0	0	0	0	0	0	1	2	2	2	4	0	0	4	2
6	1	1	1	1	0	0	1	1	1	0	1	2	2	1	1	1	2
6	1	0	0	0	1	1	0	1	1	1	2	0	1	2	2	2	2
6	0	1	1	0	1	1	0	2	1	2	0	2	1	1	1	2	1
6	0	1	0	1	0	1	1	0	2	2	1	1	0	2	1	2	2
6	0	1	0	0	1	1	2	1	0	1	2	1	2	2	0	1	2
6	0	0	1	1	0	2	1	1	0	1	2	1	1	0	2	2	2
6	0	0	1	0	1	1	1	0	2	1	2	2	2	1	2	0	1
6	0	0	0	1	0	0	1	2	1	2	2	1	2	2	2	1	0

OS5	1	2	2	2	2	3	3	3	3	6	6	6	6	6	6	6	6
1	1	2	2	0	0	0	0	0	0	6	6	0	0	0	0	0	0
2	1	0	0	1	0	3	0	0	0	3	0	3	3	3	0	0	0
2	1	0	0	0	1	0	3	0	0	0	3	3	3	0	3	0	0
2	0	1	0	2	2	0	0	0	0	3	0	3	0	0	3	3	0
2	0	0	1	2	2	0	0	0	0	0	3	0	3	3	0	0	3
3	0	2	0	0	0	0	1	0	0	2	0	2	2	2	2	0	4
3	0	0	2	0	0	1	0	0	0	0	2	4	0	2	2	2	2
3	0	0	0	0	0	0	0	1	0	2	2	2	4	0	0	4	2
3	0	0	0	0	0	0	0	0	1	2	2	0	2	4	4	2	0
6	1	1	0	1	0	1	0	1	1	0	2	1	1	1	2	2	2
6	1	0	1	0	1	0	1	1	1	2	0	1	1	2	1	2	2
6	0	1	1	1	0	0	1	2	1	1	1	2	2	2	1	1	0
6	0	1	1	0	1	2	1	1	0	1	1	0	2	1	2	2	1
6	0	1	0	0	1	1	1	0	2	1	2	2	1	2	0	2	1
6	0	0	1	1	0	1	1	0	2	2	1	1	2	0	2	1	2
6	0	0	0	1	0	1	2	1	0	2	2	1	0	2	1	2	2
6	0	0	0	0	1	1	0	2	1	2	2	2	1	1	2	0	2

OS6	1	2	2	2	2	3	3	3	3	6	6	6	6	6	6	6	6
1	1	2	2	0	0	0	0	0	0	6	6	0	0	0	0	0	0
2	1	0	0	1	0	3	0	0	0	3	0	3	3	3	0	0	0
2	1	0	0	0	1	0	3	0	0	0	3	3	3	0	3	0	0
2	0	1	0	2	2	0	0	0	0	3	0	3	0	0	3	3	0
2	0	0	1	2	2	0	0	0	0	0	3	0	3	3	0	0	3
3	0	2	0	0	0	0	0	1	0	0	2	4	2	2	0	2	2
3	0	0	2	0	0	0	0	0	1	2	0	2	4	0	2	2	2
3	0	0	0	0	0	1	0	0	0	2	2	2	0	2	4	0	4
3	0	0	0	0	0	0	1	0	0	2	2	0	2	4	2	4	0
6	1	1	0	0	1	1	0	1	1	1	1	0	2	1	2	2	2
6	1	0	1	1	0	0	1	1	1	1	1	2	0	2	1	2	2
6	0	1	1	1	0	2	1	0	1	0	2	1	1	1	2	2	1
6	0	1	1	0	1	1	2	1	0	2	0	1	1	2	1	1	2
6	0	1	0	1	0	0	1	1	2	2	1	1	2	2	2	0	1
6	0	0	1	0	1	1	0	2	1	1	2	2	1	2	2	1	0
6	0	0	0	1	0	1	1	2	0	2	2	1	2	0	1	2	2
6	0	0	0	0	1	1	1	0	2	2	2	2	1	1	0	2	2

OS7	1	2	2	2	2	3	3	3	3	6	6	6	6	6	6	6	6
1	1	2	2	0	0	0	0	0	0	6	6	0	0	0	0	0	0
2	1	0	0	1	0	3	0	0	0	3	0	3	3	3	0	0	0
2	1	0	0	0	1	0	3	0	0	0	3	3	3	0	3	0	0
2	0	1	0	2	2	0	0	0	0	3	0	3	0	0	3	3	0
2	0	0	1	2	2	0	0	0	0	0	3	0	3	3	0	0	3
3	0	2	0	0	0	0	0	1	0	0	2	2	2	4	2	2	0
3	0	0	2	0	0	0	0	0	1	2	0	2	2	2	4	0	2
3	0	0	0	0	0	1	0	0	0	2	2	0	4	0	2	4	2
3	0	0	0	0	0	0	1	0	0	2	2	4	0	2	0	2	4
6	1	1	0	0	1	1	0	1	1	1	1	1	1	1	2	1	3
6	1	0	1	1	0	0	1	1	1	1	1	1	1	2	1	3	1
6	0	1	1	1	0	1	1	2	0	1	1	2	2	0	1	1	2
6	0	1	1	0	1	1	1	0	2	1	1	2	2	1	0	2	1
6	0	1	0	1	0	1	2	0	1	2	1	0	1	2	2	1	2
6	0	0	1	0	1	2	1	1	0	1	2	1	0	2	2	2	1
6	0	0	0	1	0	1	0	1	2	1	3	2	1	1	2	1	1
6	0	0	0	0	1	0	1	2	1	3	1	1	2	2	1	1	1

The numbers of obtained new orbital structures for the cyclic group $\langle \rho^3 \rangle$ are as follows: OS1→51, OS2→51, OS3→10, OS4→49, OS5→10, OS6→23 and OS7→7.

We shall proceed by indexing obtained structures, that is by finding all possible (69,17,4) designs that can be constructed from those structures. That can be done only for some of them and the numbers of such structures are the following: OS1→27, OS2→27, OS3→0, OS4→3, OS5→0, OS6 →0 and OS7→2. From each structure that allows indexing we get 8 mutually isomorphic designs. Designs obtained from different structures are mutually nonisomorphic. So, we have 59 nonisomorphic designs. Using the computer program by V. Ćepulić we have found out that all obtained designs are self-dual. They are numbered as follows: OS1→1-27, OS2→28-54, OS4→55-57, OS7→58-59, and presented below. First number in each row, namely l, is the length of line orbit. It is followed by the line that is representative of line orbit and others can be obtained by applying $l-1$ times given automorphism σ. For brevity, designs are not presented with help of automorphism of order 6 (except those with full automorphism group of order 6). Also, common lines for more designs are written only once. We also presented the full automorphism groups of constructed designs.

common lines for designs 1-53
1: 0 1 2 3 4 21 22 23 24 25 26 27 28 29 30 31 32
3: 1 2 3 4 9 33 36 39 42 45 48 51 54 57 60 63 66
3: 12 21 24 27 30 33 34 36 37 41 44 53 56 63 64 66 67
3: 15 21 24 27 30 35 38 45 46 48 49 57 58 60 61 65 68
3: 18 21 24 27 30 39 40 42 43 47 50 51 52 54 55 59 62
4: 0 5 9 10 11 21 22 23 33 34 35 39 40 41 45 46 47
4: 1 5 6 7 8 21 22 23 36 37 38 51 52 53 60 61 62

- -

design 1
12: 0 1 6 12 15 18 29 31 33 34 39 43 48 53 59 61 68
12: 1 5 9 12 14 16 24 26 28 40 47 49 53 54 60 64 68
12: 1 8 9 13 18 20 23 24 31 38 40 44 45 58 59 63 67
12: 1 7 9 15 17 19 21 29 31 35 37 42 47 49 55 56 63

design 2
12: 0 1 6 12 15 18 28 32 33 34 40 42 48 53 58 62 68
12: 1 5 9 12 14 16 24 26 28 41 45 49 52 54 62 65 67
12: 1 8 9 13 18 20 21 26 31 37 41 43 47 57 58 63 68
12: 1 7 9 15 17 19 21 28 32 35 36 43 46 50 54 56 64

design 3
12: 0 1 6 12 15 18 29 31 33 34 39 43 50 53 58 60 68
12: 1 5 9 12 14 16 24 26 28 40 46 48 53 54 62 64 68
12: 1 8 9 13 18 20 23 24 31 38 40 44 47 57 58 63 67
12: 1 7 9 15 17 19 22 27 32 33 38 43 47 49 54 56 64

design 4
12: 0 1 6 12 15 18 29 31 33 34 41 42 48 52 59 61 68
12: 1 5 9 12 14 16 24 26 28 39 47 49 52 56 60 64 68
12: 1 8 9 13 18 20 21 25 32 36 40 44 46 57 59 64 68
12: 1 7 9 15 17 19 21 29 31 35 37 44 47 49 54 55 63

design 5
12: 0 1 6 12 15 18 28 32 33 34 39 44 48 52 58 62 68
12: 1 5 9 12 14 16 24 26 28 40 45 49 51 56 62 65 67
12: 1 8 9 13 18 20 22 24 32 38 41 43 45 58 59 64 66
12: 1 7 9 15 17 19 21 28 32 35 36 42 46 50 55 56 64

design 6
12: 0 1 6 12 15 18 29 31 33 34 41 42 49 52 57 62 68
12: 1 5 9 12 14 16 24 26 28 39 45 50 52 56 61 64 68
12: 1 8 9 13 18 20 21 25 32 36 40 44 47 57 58 64 68

12: 1 7 9 15 17 19 23 28 30 34 36 43 47 49 54 56 65

design 7
12: 0 1 6 12 15 18 28 32 33 34 39 44 49 52 59 60 68
12: 1 5 9 12 14 16 24 26 28 40 46 50 51 56 60 65 67
12: 1 8 9 13 18 20 22 24 32 38 41 43 46 57 59 64 66
12: 1 7 9 15 17 19 23 27 31 34 38 44 46 50 54 55 63

design 8
12: 0 1 6 12 15 18 29 31 33 34 41 42 50 52 58 60 68
12: 1 5 9 12 14 16 24 26 28 39 46 48 52 56 62 64 68
12: 1 8 9 13 18 20 21 25 32 36 40 44 45 58 59 64 68
12: 1 7 9 15 17 19 22 27 32 33 38 42 47 49 55 56 64

design 9
12: 0 1 6 12 15 18 28 32 33 34 39 44 50 52 57 61 68
12: 1 5 9 12 14 16 24 26 28 40 47 48 51 56 61 65 67
12: 1 8 9 13 18 20 22 24 32 38 41 43 47 57 58 64 66
12: 1 7 9 15 17 19 22 29 30 33 37 43 46 50 54 56 65

design 10
12: 0 1 6 12 15 18 28 32 33 35 40 42 48 53 58 62 67
12: 1 5 9 12 14 16 24 25 29 39 46 50 53 55 60 65 67
12: 1 8 9 13 18 20 21 26 31 36 41 43 47 57 58 65 67
12: 1 7 9 15 17 19 21 28 32 34 38 43 46 50 54 56 63

design 11
12: 0 1 6 12 15 18 29 31 33 35 39 43 48 53 59 61 67
12: 1 5 9 12 14 16 24 25 29 41 45 50 51 55 61 64 68
12: 1 8 9 13 18 20 23 24 31 37 40 44 45 58 59 65 66
12: 1 7 9 15 17 19 21 29 31 34 36 42 47 49 55 56 65

design 12
12: 0 1 6 12 15 18 28 32 33 35 40 42 49 53 59 60 67
12: 1 5 9 12 14 16 24 25 29 39 47 48 53 55 61 65 67
12: 1 8 9 13 18 20 21 26 31 36 41 43 45 58 59 65 67
12: 1 7 9 15 17 19 23 27 31 33 37 42 46 50 55 56 65

design 13
12: 0 1 6 12 15 18 29 31 33 35 39 43 49 53 57 62 67
12: 1 5 9 12 14 16 24 25 29 41 46 48 51 55 62 64 68
12: 1 8 9 13 18 20 23 24 31 37 40 44 46 57 59 65 66

12: 1 7 9 15 17 19 23 28 30 33 38 44 47 49 54 55 64

design 14
12: 0 1 6 12 15 18 28 32 33 35 40 42 50 53 57 61 67
12: 1 5 9 12 14 16 24 25 29 39 45 49 53 55 62 65 67
12: 1 8 9 13 18 20 21 26 31 36 41 43 46 57 59 65 67
12: 1 7 9 15 17 19 22 29 30 35 36 44 46 50 54 55 64

design 15
12: 0 1 6 12 15 18 29 31 33 35 39 43 50 53 58 60 67
12: 1 5 9 12 14 16 24 25 29 41 47 49 51 55 60 64 68
12: 1 8 9 13 18 20 23 24 31 37 40 44 47 57 58 65 66
12: 1 7 9 15 17 19 22 27 32 35 37 43 47 49 54 56 63

design 16
12: 0 1 6 12 15 18 29 31 33 35 41 42 48 52 59 61 67
12: 1 5 9 12 14 16 24 25 29 40 45 50 53 54 61 64 68
12: 1 8 9 13 18 20 21 25 32 38 40 44 46 57 59 63 67
12: 1 7 9 15 17 19 21 29 31 34 36 44 47 49 54 55 65

design 17
12: 0 1 6 12 15 18 28 32 33 35 39 44 48 52 58 62 67
12: 1 5 9 12 14 16 24 25 29 41 46 50 52 54 60 65 67
12: 1 8 9 13 18 20 22 24 32 37 41 43 45 58 59 63 68
12: 1 7 9 15 17 19 21 28 32 34 38 42 46 50 55 56 63

design 18
12: 0 1 6 12 15 18 29 31 33 35 41 42 49 52 57 62 67
12: 1 5 9 12 14 16 24 25 29 40 46 48 53 54 62 64 68
12: 1 8 9 13 18 20 21 25 32 38 40 44 47 57 58 63 67
12: 1 7 9 15 17 19 23 28 30 33 38 43 47 49 54 56 64

design 19
12: 0 1 6 12 15 18 28 32 33 35 39 44 49 52 59 60 67
12: 1 5 9 12 14 16 24 25 29 41 47 48 52 54 61 65 67
12: 1 8 9 13 18 20 22 24 32 37 41 43 46 57 59 63 68
12: 1 7 9 15 17 19 23 27 31 33 37 44 46 50 54 55 65

design 20
12: 0 1 6 12 15 18 29 31 33 35 41 42 50 52 58 60 67
12: 1 5 9 12 14 16 24 25 29 40 47 49 53 54 60 64 68
12: 1 8 9 13 18 20 21 25 32 38 40 44 45 58 59 63 67

12: 1 7 9 15 17 19 22 27 32 35 37 42 47 49 55 56 63

design 21
12: 0 1 6 12 15 18 28 32 33 35 39 44 50 52 57 61 67
12: 1 5 9 12 14 16 24 25 29 41 45 49 52 54 62 65 67
12: 1 8 9 13 18 20 22 24 32 37 41 43 47 57 58 63 68
12: 1 7 9 15 17 19 22 29 30 35 36 43 46 50 54 56 64

design 22
12: 0 1 6 12 15 18 29 31 33 35 40 44 48 51 59 61 67
12: 1 5 9 12 14 16 24 25 29 39 45 50 52 56 61 64 68
12: 1 8 9 13 18 20 22 26 30 36 40 44 47 57 58 64 68
12: 1 7 9 15 17 19 21 29 31 34 36 43 47 49 54 56 65

design 23
12: 0 1 6 12 15 18 28 32 33 35 41 43 48 51 58 62 67
12: 1 5 9 12 14 16 24 25 29 40 46 50 51 56 60 65 67
12: 1 8 9 13 18 20 23 25 30 38 41 43 46 57 59 64 66
12: 1 7 9 15 17 19 21 28 32 34 38 44 46 50 54 55 63

design 24
12: 0 1 6 12 15 18 29 31 33 35 40 44 49 51 57 62 67
12: 1 5 9 12 14 16 24 25 29 39 46 48 52 56 62 64 68
12: 1 8 9 13 18 20 22 26 30 36 40 44 45 58 59 64 68
12: 1 7 9 15 17 19 23 28 30 33 38 42 47 49 55 56 64

design 25
12: 0 1 6 12 15 18 28 32 33 35 41 43 49 51 59 60 67
12: 1 5 9 12 14 16 24 25 29 40 47 48 51 56 61 65 67
12: 1 8 9 13 18 20 23 25 30 38 41 43 47 57 58 64 66
12: 1 7 9 15 17 19 23 27 31 33 37 43 46 50 54 56 65

design 26
12: 0 1 6 12 15 18 29 31 33 35 40 44 50 51 58 60 67
12: 1 5 9 12 14 16 24 25 29 39 47 49 52 56 60 64 68
12: 1 8 9 13 18 20 22 26 30 36 40 44 46 57 59 64 68
12: 1 7 9 15 17 19 22 27 32 35 37 44 47 49 54 55 63

design 27
12: 0 1 6 12 15 18 28 32 33 35 41 43 50 51 57 61 67
12: 1 5 9 12 14 16 24 25 29 40 45 49 51 56 62 65 67
12: 1 8 9 13 18 20 23 25 30 38 41 43 45 58 59 64 66

12: 1 7 9 15 17 19 22 29 30 35 36 42 46 50 55 56 64

design 28
12: 0 1 6 12 15 18 29 31 33 34 42 46 48 53 55 62 65
12: 1 5 9 12 14 16 24 26 31 39 43 47 53 57 61 64 68
12: 1 8 9 13 18 20 21 29 31 35 36 40 44 49 57 59 67
12: 1 7 9 15 17 19 21 26 28 37 41 47 49 54 55 65 66

design 29
12: 0 1 6 12 15 18 28 32 33 34 42 45 49 52 56 62 65
12: 1 5 9 12 14 16 24 26 31 41 43 46 51 57 62 65 67
12: 1 8 9 13 18 20 21 28 32 35 37 41 43 48 58 59 66
12: 1 7 9 15 17 19 23 24 28 38 39 46 50 55 56 64 66

design 30
12: 0 1 6 12 15 18 29 31 33 34 42 45 50 53 55 61 65
12: 1 5 9 12 14 16 24 26 31 39 43 46 53 59 60 64 68
12: 1 8 9 13 18 20 21 29 31 35 36 40 44 48 58 59 67
12: 1 7 9 15 17 19 22 24 29 38 39 47 49 55 56 63 67

design 31
12: 0 1 6 12 15 18 28 32 33 34 42 47 48 52 56 61 65
12: 1 5 9 12 14 16 24 26 31 41 43 45 51 59 61 65 67
12: 1 8 9 13 18 20 21 28 32 35 37 41 43 50 57 58 66
12: 1 7 9 15 17 19 21 25 29 36 40 46 50 54 56 65 67

design 32
12: 0 1 6 12 15 18 29 31 33 34 42 47 49 53 55 60 65
12: 1 5 9 12 14 16 24 26 31 39 43 45 53 58 62 64 68
12: 1 8 9 13 18 20 21 29 31 35 36 40 44 50 57 58 67
12: 1 7 9 15 17 19 23 25 27 36 40 47 49 54 56 64 68

design 33
12: 0 1 6 12 15 18 28 32 33 34 44 45 49 51 55 62 65
12: 1 5 9 12 14 16 24 26 31 40 42 46 53 57 62 65 67
12: 1 8 9 13 18 20 22 29 30 33 38 41 43 49 57 59 67
12: 1 7 9 15 17 19 23 24 28 38 41 46 50 54 55 64 66

design 34
12: 0 1 6 12 15 18 28 32 33 34 44 47 48 51 55 61 65
12: 1 5 9 12 14 16 24 26 31 40 42 45 53 59 61 65 67
12: 1 8 9 13 18 20 22 29 30 33 38 41 43 48 58 59 67

12: 1 7 9 15 17 19 21 25 29 36 39 46 50 55 56 65 67

design 35
12: 0 1 6 12 15 18 29 31 33 34 44 45 50 52 54 61 65
12: 1 5 9 12 14 16 24 26 31 41 42 46 52 59 60 64 68
12: 1 8 9 13 18 20 22 27 32 33 37 40 44 49 57 59 68
12: 1 7 9 15 17 19 22 24 29 38 41 47 49 54 55 63 67

design 36
12: 0 1 6 12 15 18 28 32 33 35 42 45 49 52 56 62 64
12: 1 5 9 12 14 16 24 25 32 39 44 47 52 58 60 65 67
12: 1 8 9 13 18 20 21 28 32 34 36 41 43 48 58 59 68
12: 1 7 9 15 17 19 23 24 28 37 39 46 50 55 56 63 68

design 37
12: 0 1 6 12 15 18 28 32 33 35 42 45 49 52 56 62 64
12: 1 5 9 12 14 16 24 25 32 39 44 47 52 58 60 65 67
12: 1 8 9 13 18 20 21 28 32 34 36 41 43 48 58 59 68
12: 1 7 9 15 17 19 23 24 28 37 39 46 50 55 56 63 68

design 38
12: 0 1 6 12 15 18 28 32 33 35 42 47 48 52 56 61 64
12: 1 5 9 12 14 16 24 25 32 39 44 46 52 57 62 65 67
12: 1 8 9 13 18 20 21 28 32 34 36 41 43 50 57 58 68
12: 1 7 9 15 17 19 21 25 29 38 40 46 50 54 56 64 66

design 39
12: 0 1 6 12 15 18 29 31 33 35 42 45 50 53 55 61 64
12: 1 5 9 12 14 16 24 25 32 40 44 47 51 57 61 64 68
12: 1 8 9 13 18 20 21 29 31 34 38 40 44 48 58 59 66
12: 1 7 9 15 17 19 22 24 29 37 39 47 49 55 56 65 66

design 40
12: 0 1 6 12 15 18 28 32 33 35 42 46 50 52 56 60 64
12: 1 5 9 12 14 16 24 25 32 39 44 45 52 59 61 65 67
12: 1 8 9 13 18 20 21 28 32 34 36 41 43 49 57 59 68
12: 1 7 9 15 17 19 22 26 27 36 41 46 50 54 55 65 67

design 41
12: 0 1 6 12 15 18 29 31 33 35 42 47 49 53 55 60 64
12: 1 5 9 12 14 16 24 25 32 40 44 46 51 59 60 64 68
12: 1 8 9 13 18 20 21 29 31 34 38 40 44 50 57 58 66

12: 1 7 9 15 17 19 23 25 27 38 40 47 49 54 56 63 67

design 42
12: 0 1 6 12 15 18 28 32 33 35 43 45 49 53 54 62 64
12: 1 5 9 12 14 16 24 25 32 40 42 47 53 58 60 65 67
12: 1 8 9 13 18 20 23 27 31 33 38 41 43 50 57 58 67
12: 1 7 9 15 17 19 23 24 28 37 40 46 50 54 56 63 68

design 43
12: 0 1 6 12 15 18 29 31 33 35 43 46 48 51 56 62 64
12: 1 5 9 12 14 16 24 25 32 41 42 45 52 58 62 64 68
12: 1 8 9 13 18 20 23 28 30 33 37 40 44 48 58 59 68
12: 1 7 9 15 17 19 21 26 28 36 39 47 49 55 56 64 68

design 44
12: 0 1 6 12 15 18 28 32 33 35 43 47 48 53 54 61 64
12: 1 5 9 12 14 16 24 25 32 40 42 46 53 57 62 65 67
12: 1 8 9 13 18 20 23 27 31 33 38 41 43 49 57 59 67
12: 1 7 9 15 17 19 21 25 29 38 41 46 50 54 55 64 66

design 45
12: 0 1 6 12 15 18 29 31 33 35 43 45 50 51 56 61 64
12: 1 5 9 12 14 16 24 25 32 41 42 47 52 57 61 64 68
12: 1 8 9 13 18 20 23 28 30 33 37 40 44 50 57 58 68
12: 1 7 9 15 17 19 22 24 29 37 40 47 49 54 56 65 66

design 46
12: 0 1 6 12 15 18 28 32 33 35 43 46 50 53 54 60 64
12: 1 5 9 12 14 16 24 25 32 40 42 45 53 59 61 65 67
12: 1 8 9 13 18 20 23 27 31 33 38 41 43 48 58 59 67
12: 1 7 9 15 17 19 22 26 27 36 39 46 50 55 56 65 67

design 47
12: 0 1 6 12 15 18 29 31 33 35 43 47 49 51 56 60 64
12: 1 5 9 12 14 16 24 25 32 41 42 46 52 59 60 64 68
12: 1 8 9 13 18 20 23 28 30 33 37 40 44 49 57 59 68
12: 1 7 9 15 17 19 23 25 27 38 41 47 49 54 55 63 67

design 48
12: 0 1 6 12 15 18 29 31 33 35 44 46 48 52 54 62 64
12: 1 5 9 12 14 16 24 25 32 39 43 45 53 58 62 64 68
12: 1 8 9 13 18 20 22 27 32 35 36 40 44 50 57 58 67

12: 1 7 9 15 17 19 21 26 28 36 40 47 49 54 56 64 68

design 49
12: 0 1 6 12 15 18 28 32 33 35 44 45 49 51 55 62 64
12: 1 5 9 12 14 16 24 25 32 41 43 47 51 58 60 65 67
12: 1 8 9 13 18 20 22 29 30 35 37 41 43 49 57 59 66
12: 1 7 9 15 17 19 23 24 28 37 41 46 50 54 55 63 68

design 50
12: 0 1 6 12 15 18 29 31 33 35 44 45 50 52 54 61 64
12: 1 5 9 12 14 16 24 25 32 39 43 47 53 57 61 64 68
12: 1 8 9 13 18 20 22 27 32 35 36 40 44 49 57 59 67
12: 1 7 9 15 17 19 22 24 29 37 41 47 49 54 55 65 66

design 51
12: 0 1 6 12 15 18 28 32 33 35 44 47 48 51 55 61 64
12: 1 5 9 12 14 16 24 25 32 41 43 46 51 57 62 65 67
12: 1 8 9 13 18 20 22 29 30 35 37 41 43 48 58 59 66
12: 1 7 9 15 17 19 21 25 29 38 39 46 50 55 56 64 66

design 52
12: 0 1 6 12 15 18 29 31 33 35 44 47 49 52 54 60 64
12: 1 5 9 12 14 16 24 25 32 39 43 46 53 59 60 64 68
12: 1 8 9 13 18 20 22 27 32 35 36 40 44 48 58 59 67
12: 1 7 9 15 17 19 23 25 27 38 39 47 49 55 56 63 67

design 53
12: 0 1 6 12 15 18 28 32 33 35 44 46 50 51 55 60 64
12: 1 5 9 12 14 16 24 25 32 41 43 45 51 59 61 65 67
12: 1 8 9 13 18 20 22 29 30 35 37 41 43 50 57 58 66
12: 1 7 9 15 17 19 22 26 27 36 40 46 50 54 56 65 67

automorphism σ for designs 1-53:
$\sigma = (1,4,2,3)(5,8,6,7)(9,10,11)(12,13,14)(15,16,17)(18,19,20)$
$(21,31,26,27,22,32,24,28,23,30,25,29)$
$(33,67,38,63,34,68,36,64,35,66,37,65)$
$(39,55,44,51,40,56,42,52,41,54,43,53)$
$(45,61,50,57,46,62,48,58,47,60,49,59)$
full automorphism group for designs 1-53: Z_{12}

design 54
1: 19 22 25 28 31 40 41 43 44 45 48 52 53 55 56 57 60
3: 0 1 2 3 4 21 22 23 24 25 26 27 28 29 30 31 32
3: 1 2 3 4 10 34 37 40 43 46 49 52 55 58 61 64 67
3: 1 2 3 4 11 35 38 41 44 47 50 53 56 59 62 65 68
3: 13 22 25 28 31 34 35 37 38 39 42 51 54 64 65 67 68
4: 1 5 6 7 8 21 22 23 36 37 38 51 52 53 60 61 62
4: 1 8 11 12 19 20 23 27 31 34 36 40 42 48 58 59 68
12: 0 5 9 10 11 21 22 23 33 34 35 39 40 41 45 46 47
12: 0 1 6 12 15 18 28 32 33 34 42 46 50 52 56 60 65
12: 0 2 5 13 16 19 27 32 37 38 40 45 50 51 56 58 66
12: 2 6 10 12 13 17 21 22 29 41 42 48 55 58 62 65 66

automorphism σ for design 54:
$\sigma = (0,9,14)(1,66,26,4,36,29,2,63,23,3,33,32)$
$(5,7,6,8)(10,16,17)(11,12,20)$
$(13,15,18)(21,42,35,30,51,68,24,39,38,27,54,65)$
$(22,45,55,31,60,43,25,48,52,28,57,40)$
$(34,49,62,67,58,50,37,46,59,64,61,47)$ $(41,53,44,56)$
full automorphism group for design 54: $F_{39} \times Z_4$

common lines for designs 55-57
1: 0 1 2 3 4 21 22 23 24 25 26 27 28 29 30 31 32
2: 0 5 9 10 11 21 22 23 33 34 35 39 40 41 45 46 47
2: 0 7 12 13 14 21 22 23 36 37 38 51 52 53 57 58 59
2: 1 5 6 7 8 21 22 23 42 43 44 54 55 56 63 64 65
2: 3 5 6 7 8 27 28 29 33 34 35 48 49 50 57 58 59
3: 1 2 9 27 30 33 36 39 42 45 48 51 52 54 55 57 60
3: 3 4 12 21 24 39 42 46 47 49 50 53 56 57 60 63 66
3: 15 21 24 27 30 34 35 37 38 46 49 51 54 64 65 67 68
3: 18 21 24 27 30 33 36 40 41 43 44 58 59 61 62 63 66

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design 55
6: 0 1 3 6 12 15 18 29 34 36 39 43 47 52 62 64 68
6: 0 7 9 15 18 22 29 31 41 48 49 51 56 60 61 65 66
6: 1 4 8 9 16 17 18 21 26 35 37 41 50 52 57 61 64
6: 1 5 9 12 19 20 23 24 28 37 47 48 54 59 61 66 68
6: 1 7 9 13 14 17 25 30 32 34 39 43 46 50 59 65 66
6: 3 5 9 10 14 16 24 28 32 36 40 51 56 57 62 64 65
6: 3 8 9 12 19 20 22 30 32 35 36 41 43 49 53 55 67
6: 5 12 16 17 18 23 25 27 31 35 42 43 46 48 51 53 62

design 56
6: 0 1 3 6 12 15 18 29 34 36 41 42 47 52 61 64 68
6: 0 7 9 15 18 22 29 31 40 48 49 51 56 60 62 65 66
6: 1 4 8 9 16 17 18 21 26 35 37 40 50 52 59 60 64
6: 1 5 9 12 19 20 21 25 29 38 45 49 55 59 61 66 67
6: 1 7 9 13 14 17 25 30 32 34 41 42 46 50 58 65 66
6: 3 5 9 10 14 16 24 28 32 36 39 51 56 59 61 64 65
6: 3 8 9 12 19 20 23 30 31 33 37 41 43 50 51 56 68
6: 5 12 16 17 18 23 25 27 31 35 42 44 46 48 51 53 61

design 57
6: 0 1 3 6 12 15 18 29 34 36 40 44 47 52 60 64 68
6: 0 7 9 15 18 22 29 31 39 48 49 51 56 61 62 65 66
6: 1 4 8 9 16 17 18 21 26 35 37 39 50 52 58 62 64
6: 1 5 9 12 19 20 22 26 27 36 46 50 56 59 61 67 68
6: 1 7 9 13 14 17 25 30 32 34 40 44 46 50 57 65 66
6: 3 5 9 10 14 16 24 28 32 36 41 51 56 58 60 64 65
6: 3 8 9 12 19 20 21 31 32 34 38 41 43 48 52 54 66
6: 5 12 16 17 18 23 25 27 31 35 43 44 46 48 51 53 60

automorphism σ for designs 55-57:
$\sigma = (1,2)(3,4)(5,6)(7,8)(9,10,11)$
$(12,13,14)(15,16,17)(18,19,20)(21,25,23,24,22,26)$
$(27,31,29,30,28,32)(33,37,35,36,34,38)$
$(39,43,41,42,40,44)(45,49,47,48,46,50)$
$(51,55,53,54,52,56)\ (57,61,59,60,58,62)(63,67,65,66,64,68)$
full automorphism group for designs 55-57: Z_6

design 58
1: 1 5 6 7 8 21 22 23 36 37 38 51 52 53 57 58 59
1: 2 5 6 7 8 24 25 26 33 34 35 54 55 56 60 61 62
1: 3 5 6 7 8 27 28 29 39 40 41 48 49 50 63 64 65
1: 4 5 6 7 8 30 31 32 42 43 44 45 46 47 66 67 68
13: 0 1 2 3 4 21 22 23 24 25 26 27 28 29 30 31 32
13: 0 5 9 10 11 21 22 23 33 34 35 39 40 41 45 46 47
13: 0 6 9 10 11 24 25 26 36 37 38 42 43 44 48 49 50
13: 0 7 12 13 14 27 28 29 33 34 35 42 43 44 51 52 53
13: 0 8 12 13 14 30 31 32 36 37 38 39 40 41 54 55 56

automorphism σ for design 58:
$\sigma = (0,9,20,10,12,19,18,15,16,11,14,17,13)$
$(1,58,23,59,38,22,21,51,52,57,37,53,36)$
$(2,61,26,62,35,25,24,54,55,60,34,56,33)$

(3, 39, 49, 40, 63, 48, 50, 27, 28, 41, 65, 29, 64)
(4, 42, 46, 43, 66, 45, 47, 30, 31, 44, 68, 32, 67)
full automorphism group for design 58: $F_{39} : Z_4$

design 59
1: 0 1 2 3 4 21 22 23 24 25 26 27 28 29 30 31 32
4: 0 5 9 10 11 21 22 23 33 34 35 39 40 41 45 46 47
4: 1 5 6 7 8 21 22 23 36 37 38 51 52 53 57 58 59
6: 1 2 15 27 30 33 36 39 42 45 46 48 49 51 54 57 60
6: 9 21 24 29 32 39 41 42 44 52 55 57 58 60 61 63 66
12: 0 1 7 9 17 18 26 30 34 41 49 53 54 58 63 67 68
12: 1 4 6 9 14 15 16 22 28 33 37 39 44 56 62 63 68
12: 1 5 9 12 14 19 21 26 30 43 45 50 52 56 60 64 65
12: 5 9 17 19 20 22 27 28 32 33 38 43 49 54 55 59 66

automorphism σ for design 59:
$\sigma = (1, 4, 2, 3)(5, 8, 6, 7)$
(9, 13, 11, 12, 10, 14)(15, 20, 17, 19, 16, 18)
(21, 30, 26, 29, 22, 31, 24, 27, 23, 32, 25, 28)
(33, 39, 38, 44, 34, 40, 36, 42, 35, 41, 37, 43)
(45, 55, 50, 51, 46, 56, 48, 52, 47, 54, 49, 53)
(57, 68, 62, 64, 58, 66, 60, 65, 59, 67, 61, 63)
full automorphism group for design 59: Z_{12}

REMARK Only designs 54 and 58 admit an automorphism group of order 13. Hence, all constructed designs except them are new.

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