Feasibility study of construction investment projects assessment with regard to risk and probability of NPV reaching

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The paper presents an analysis of NPV and investment risk at the stage of assessment of the strategy and the feasibility study. This analysis, for a specific project value profile, allows for specification of probability of occurrence of a given value of cash flows and NPV and for presentation of their fuzziness.

INTRODUCTION

Decision-making with regard to investment is associated with analysis of risk related to achievement of the basic technical and economic parameters, including the updated value of NPV (Net Present Value). Prior to the decision-making stage, experienced investors analyse the revenues and expenditures for the investment project planned; usually, they have the knowledge sufficient to determine the value of deviations of these planned

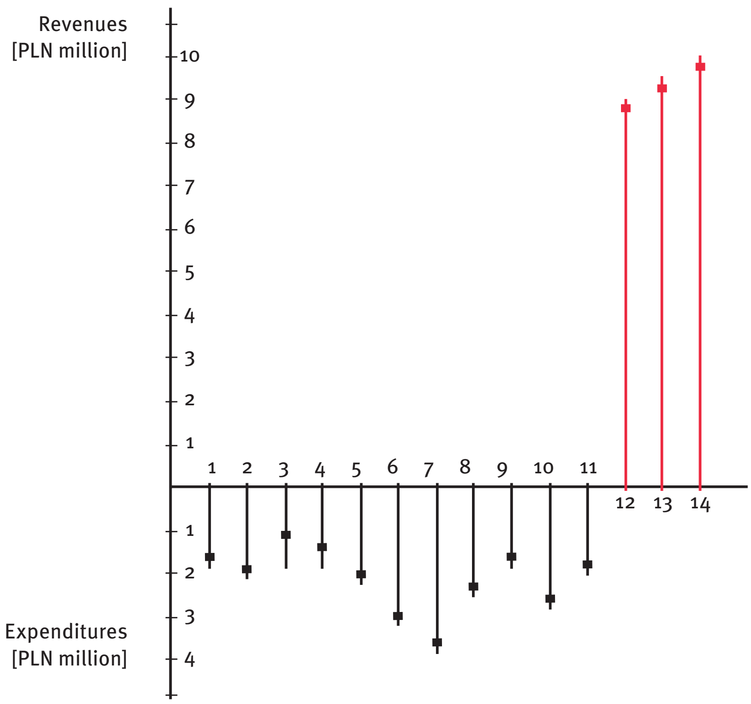

Figure 1: **Individual cash flows, without taking into account the risk.**

parameters from the really achieved ones. Such analysis may also be subject to opinion of construction experts, who may assess the value of such deviation in verbal form. The paper presented discusses the procedure of determination of probability of reaching the planned NPV value on the basis of linguistic assessments using fuzzy sets.

Assessment of the investment project – Assumptions

Any investment project, analysed in accordance with the NPV method, consists of the appropriate cash flows – revenues or expenditures. These flows finally result in the expected NPV value. If we fail to take into account the risks, individual flows may be presented as illustrated by fig. 1. In this case, we disregard the impact of the discount coefficient and we assume that these flows have already been discounted.
The precise flow values are provided in the second column of table 1. Expenditures (K_o) are represented by negative values, and revenues (Z_o) are represented by positive values. In general, expenditures and revenues will be marked as S_o – cash flows.

	i	Activity	S_o
Expenditures	1	Contractual proceedings	-1,8
	2	Project	-2,1
	3	Measurement works	-1,3
	4	Earthworks	-1,6
	5	Boarding	-2,2
	6	Concrete works	-3,2
	7	Steel structures	-3,8
	8	Water supply system	-2,5
	9	Ventilation	-1,8
	10	Power supply works	-2,8
	11	Sewage system	-2
Revenues	12	Revenues year 1	9
	13	Revenues year 2	9,5
	14	Revenues year 3	10
			NPV = 3,4

Table 1: The assumed cash flows for the project.

In fig. 2, variables have the following meanings:

- Z_{pes} – extreme pessimistic value of revenues, assuming probability = 0,
- Z_{opt} – extreme optimistic value of revenues, assuming probability = 0,
- ΔZ_{pes} – difference between the most probable value and the extreme pes simistic value of revenues,
- ΔZ_{opt} – difference between the extreme optimistic value and the most probable value of revenues,
- K_{pes} – extreme pessimistic value of expenditures, assuming probability = 0,
- K_{opt} – extreme optimistic value of revenues, assuming probability = 0,
- ΔK_{pes} – difference between the most probable value and the extreme pessi mistic value of expenditures,
- ΔK_{opt} – difference between the extreme optimistic value and the most prob able value of expenditures,
- p_{oz} – the probability of revenue being equal to Z_o,
- p_{ot} – the probability of expenditure being equal to K_o.

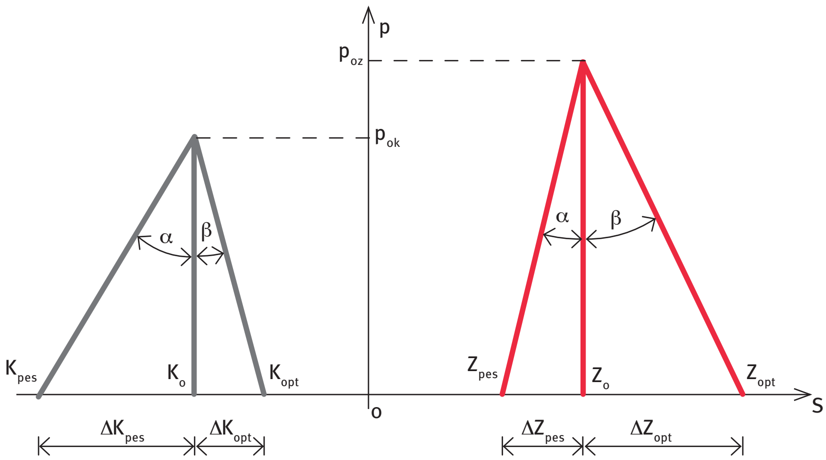

Figure 2: **Presentation of the flow distribution in triangular form.**

Although the flows calculated within a given investment project have been presented as the most probable ones, their probability will not be equal to 1. In other words – we are not 100% sure that a given flow will assume the value of S_o, and not a greater or smaller one. This will be influenced by the differences between the expected (most probable) values of cash flows and the real values of these flows that emerge during the project implementation.

Therefore, the flow can be reasonably presented as a random variable through its probability distribution, specifying the maximum probability, corresponding to flow S_o and cash flow values, which are characterised by lower probability. We assume that the probability of emergence of value S_o is the highest. The vertical axis of this chart is probability, while the horizontal axis consists of flow values.
Application of this type of distribution can be explained by the fact that at the stage of the feasibility study, we use approximate data, and application of more accurate distributions at this stage is not reasonable. The extreme values in the triangular distribution presented are, in fact, characterised by such a low probability hat at the stage of feasibility study, their probability has been assumed to be equal to zero.

The arms of the triangular distribution show how rapidly a revenue or an expenditure may change. The character of this change will be determined by providing tangents of inclination angles α and β. Angle α will determine the "risk of increase of expenditure" or the "risk of decrease of revenue", while angle β will stand for the "risk of decrease of expenditure" or the "risk of increase of revenue". In other words, α – a negative risk (associated with a decrease of NPV value), β – a positive risk (associated with an increase of the NPV value). We define risk as the ability, or susceptibility, to changes for individual cash flows. A high risk of an expenditure change means that the most probable value may undergo significant changes upon slight changes of external conditions. On the other hand, a low risk of changes will be typical for more certain activities, less susceptible to changes. Moreover, a high risk of changes determines the lower probability of emergence of expenditure K_o, which means we have to take into account substantial changes during the investment project implementation in case of a flow characterised by a high α or β. Value S_o will be obtained from the preliminary assessment of the investment. The probability p_o of the flow assuming value S_o is

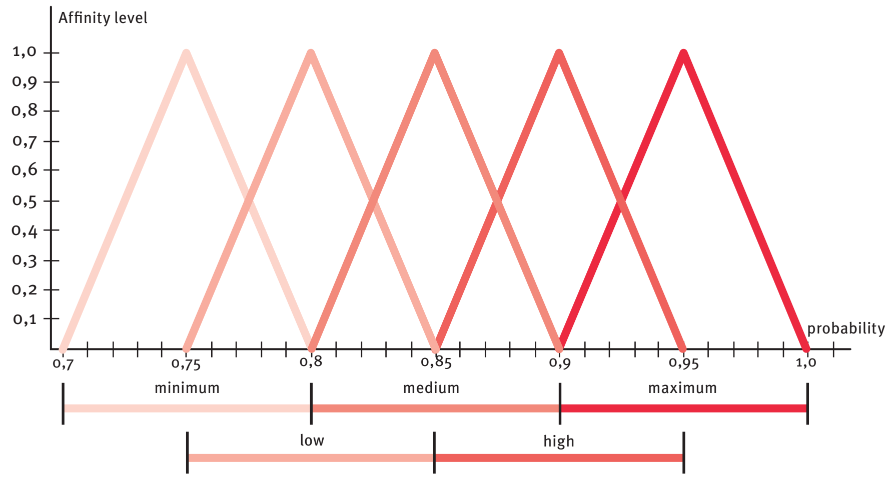

Figure 3: Presentation of probability as a linguistic variable.

	I	Activity	S_o	p_o(Eksp1)	p_o(Eksp2)	p_o(Eksp3)
Expenditures	1	Contractual proceedings	-1,8	medium	medium	low
	2	Project	-2,1	medium	low	minimum
	3	Measurement works	-1,3	maximum	high	high
	4	Earthworks	-1,6	medium	high	medium
	5	Boarding	-2,2	medium	medium	low
	6	Concrete works	-3,2	low	low	low
	7	Steel structures	-3,8	medium	medium	low
	8	Water supply system	-2,5	medium	high	high
	9	Ventilation	-1,8	high	medium	medium
	10	Power supply works	-2,8	medium	high	high
	11	Sewage system	-2	high	high	medium
Revenues	12	Revenues year 1	9	medium	high	medium
	13	Revenues year 2	9,5	medium	medium	low
	14	Revenues year 3	10	medium	low	medium

Table 2: A breakdown of expert opinions on po in linguistic form

	i	Activity	S_o	x	p_o	Δ_{pes}	Δ_{opt}	S_{pes}	S_{opt}	$\Delta_{pes}/\lvert S_o\rvert$	$\Delta_{opt}/\lvert S_o\rvert$	$S_{pes}/\lvert S_o\rvert$	$S_{opt}/\lvert S_o\rvert$
Expenditures	1	Contractual proceedings	-1,8	0,4	0,83	1,71	0,69	-3,51	-1,11	0,95	0,38	-1,95	-0,62
	2	Project	-2,1	0,6	0,80	1,56	0,94	-3,66	-1,16	0,74	0,45	-1,74	-0,55
	3	Measurement works	-1,3	0,5	0,92	1,45	0,73	-2,75	-0,57	1,12	0,56	-2,12	-0,44
	4	Earthworks	-1,6	0,5	0,87	1,54	0,77	-3,14	-0,83	0,96	0,48	-1,96	-0,52
	5	Boarding	-2,2	0,8	0,83	1,33	1,07	-3,53	-1,13	0,61	0,48	-1,61	-0,52
	6	Concrete works	-3,2	0,5	0,80	1,67	0,83	-4,87	-2,37	0,52	0,26	-1,52	-0,74
	7	Steel structures	-3,8	0,7	0,83	1,41	0,99	-5,21	-2,81	0,37	0,26	-1,37	-0,74
	8	Water supply system	-2,5	0,8	0,88	1,26	1,01	-3,76	-1,49	0,50	0,40	-1,50	-0,60
	9	Ventilation	-1,8	1	0,87	1,15	1,15	-2,95	-0,65	0,64	0,64	-1,64	-0,36
	10	Power supply works	-2,8	0,7	0,88	1,33	0,93	-4,13	-1,87	0,48	0,33	-1,48	-0,67
	11	Sewage system	-2	0,6	0,88	1,42	0,85	-3,42	-1,15	0,71	0,42	-1,71	-0,58
Revenues	12	Revenues year 1	9	0,7	0,87	1,36	0,95	7,64	9,95	0,15	0,11	0,85	1,11
	13	Revenues year 2	9,5	0,8	0,83	1,33	1,07	8,17	10,57	0,14	0,11	0,86	1,11
	14	Revenues year 3	10	0,9	0,83	1,26	1,14	8,74	11,14	0,13	0,11	0,87	1,11

Table 3: Characteristic values for each distribution

to be obtained from experts. To determine α or β, we also need the following ratio:

$$\frac{\Delta S_{+}}{\Delta S_{-}} = x \qquad (1)$$

$\Delta S+$ and $\Delta S-$ are deviations from values obtained on the basis of preliminary assessments.

Information concerning p_o can be provided by experts in linguistic form as one of the five expressions: minimum, low, medium, high, maximum. When we obtain such answer, it needs to be transformed into numerical values. For this purpose, we apply a presentation of linguistic space. In our case, it consists of fuzzy sets {minimum; low; medium; high; maximum}. The numerical values of linguistic variable in the case of p_o have been presented in fig. 3.
Presented below is information in linguistic form, concerning probability p_o, obtained from three experts, as an example is presented in table 2.
On the basis of the linguistic space assumed, numeric values have been determined for probability p_o as established by experts.

In this manner, each of the cash flows has been presented in form of two distributions – absolute and relative.
On the basis of cash flows, presented as a random variable, NPV has been determined:

$$NPV_{pes} = \sum_{i=1}^{n} S_{pes}$$
$$NPV_{0} = \sum_{i=1}^{n} S_{0} \qquad (2)$$
$$NPV_{opt} = \sum_{i=1}^{n} S_{opt}$$

Due to the fact that the value of NPV (2) has been presented as a random value, it is possible to calculate the value of p_o (3) of its emergence, as well as the values of α, β (4).

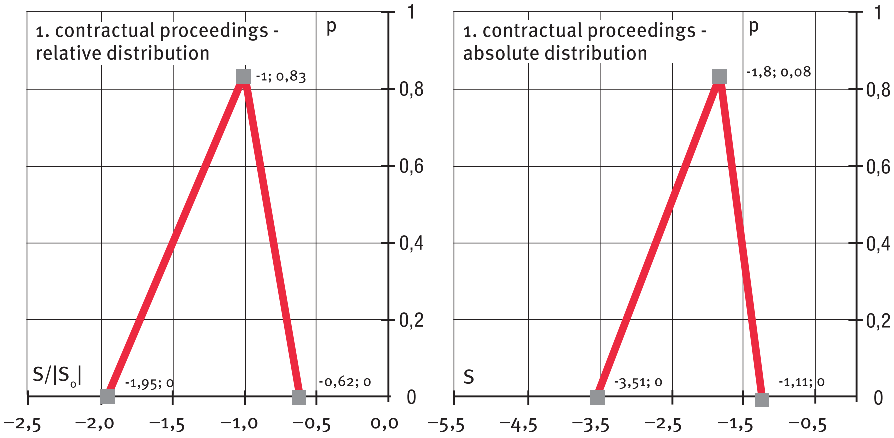

Figure 4: **Exemplary flows in form of a triangular probability distribution.**

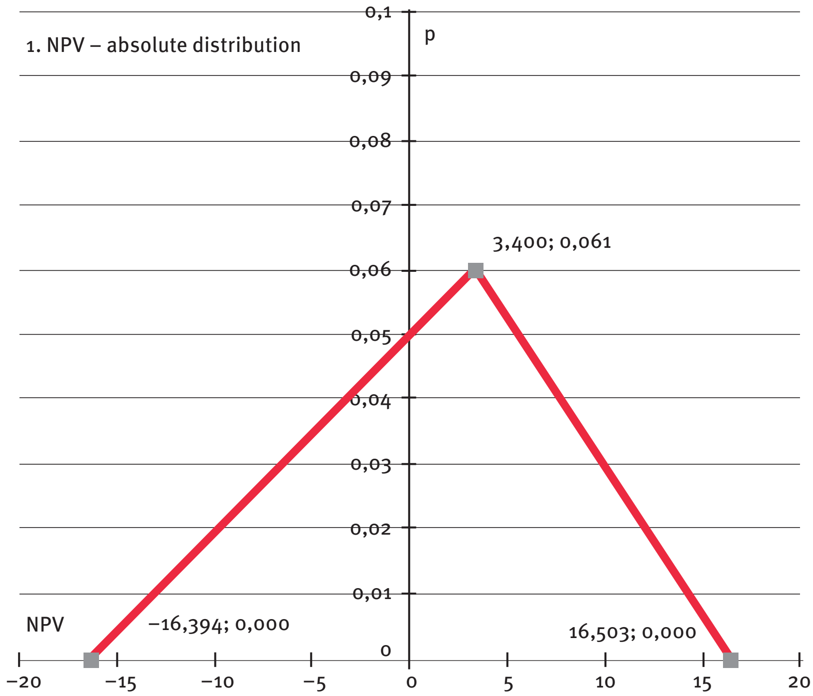

Figure 5: **Presentation of NPV value in form of a random variable distribution.**

$$P = \frac{1}{2} \cdot (NPV_{opt} - NPV_{pes}) \cdot p_0 = 1 \Rightarrow p_0 = \frac{2}{NPV_{opt} - NPV_{pes}} \qquad (3)$$

$$tg(\alpha) = \frac{\Delta S_{pes}}{p_0}$$
$$\alpha = arctg\left(\frac{\Delta S_{pes}}{p_0}\right)$$
$$tg(\beta) = \frac{\Delta S_{opt}}{p_0} \qquad (4)$$
$$\beta = arctg\left(\frac{\Delta S_{opt}}{p_0}\right)$$

The values of all distributions were determined not on the basis of the entire set of assessment of linguistic variables p_o, but only the values of affinity equal to 1, that is: minimum = 0.75; low = 0.80; medium = 0.85; high = 0.9; maximum = 0.95.
Therefore, the distribution values constitute acute distributions. In order to grasp the fully subjective and imprecise nature of expression of value p_o,

it is necessary to take into account all values of the linguistic variable. Thanks to the fact we have assumed a triangular function of affinity, it is now sufficient to conduct calculations for 3 values: L – extreme left of affinity level = 0; C – central of affinity level = 1; P – extreme right of affinity level = 0.

In our case, the linguistic value (L ; C ; P) will be as follows: minimum = (0,7 ; 0,75 ; 0,8); low = (0,75 ; 0,80 ; 0,85); medium = (0,85 ; 0,90 ; 095); high = (0,85 ; 0,9 ; 0,95); maximum = (0,9 ; 0,95 ; 1).

In this manner, instead of a single probability value p_o, we have obtained three values of p_o, and thus – three probability distributions for a single cash flow (extreme left – L, central – C, extreme right – P). On the basis of flow distributions, presented in this manner, the fuzzy distribution of NPV has been determined.

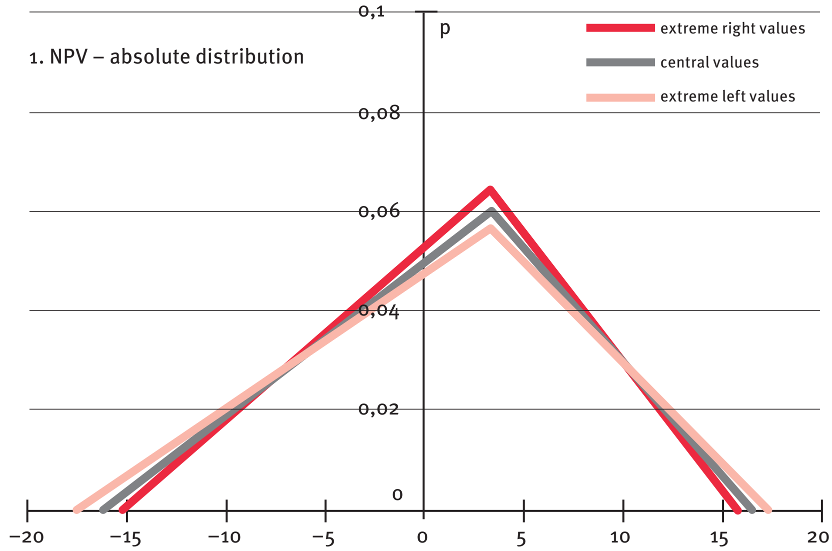

Figure 6: **Absolute distribution of NPV values.**

CONCLUSIONS

Such presentation of the distribution allows for an objective assessment of distribution of cash flows and the resulting NPV distribution. The levels of fuzziness of the parameters discussed allow for determination of subjectivity and imprecision of assumption of the basic values, which characterise the individual cash flow distributions and NPV values in linguistic form. The procedure discussed may be applied in practice at the stage of strategy assessment and the feasibility study. It allows for determination of probability of emergence of a given value of cash flows and NPV values and for presentation of their fuzziness.

REFERENCES

Morgan N.G., Henrion M. (1990) Uncertainty. A Guide to dealing with uncertainty in quantitative risk and policy analysis. Cambridge University Press, New York.

Piegat, A. (1999) Modelowanie i sterowanie rozmyte. Akademicka Oficyna Wydawnicza EXIT, Warsaw.

Minasowicz A. (2008) Analiza ryzyka w projektowaniu przedsi△wzi△cia budowanego, Oficyna Wydawnicza Politechniki Warszawskiej, Warszawa.