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Motion Control and Vibration Suppression of Flexible Lumped Systems via Sensorless LQR Control

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Original scientific paper

This work attempts to achieve motion control along with vibration suppression of flexible systems by developing a sensorless closed loop LQR controller. Vibration suppression is used as a performance index that has to be minimized so that motion control is achieved with zero residual vibration. An estimation algorithm is combined with the regular LQR to develop sensorless motion and vibration controller that is capable of positioning multi degrees of freedom flexible system point of interest to a pre-specified target position with zero residual vibration. The validity of the proposed controller is verified experimentally by controlling a sensorless dynamical system with finite degrees of freedom through measurements taken from its actuator.

Key words: Sensorless control, Vibration suppression, Reaction force observer, Optimal control, Action reaction state observer

Fleksibilni slijedni sustav s koncentriranim parametrima sa suzbijanjem vibracija korištenjem LQR-a bez senzora. U ovom radu opisan je fleksibilni slijedni sustav sa suzbijanjem vibracija upravljan LQR regulatorom u zatvorenom upravljačkom krugu bez senzora. Vibracije su korištene u težinskoj funkciji koja se minimizira s ciljem eliminiranja rezidualnih vibracija iz slijednog sustava. Kombinirajući algoritam estimacije s klasičnim LQR-om, razvijen je regulator za upravljanje gibanjem i vibracijama bez korištenja senzora, koji je sposoban pozicionirati određenu točku fleksibilnog sustava s više stupnjeva slobode u predefiniranu željenu točku bez preostalih vibracija. Validacija predloženog regulatora provedena je eksperimentalno upravljajući dinamičkim sustavom s konačnim brojem stupnjeva slobode uz korištenje mjerenja s aktuatora.

Ključne riječi: upravljanje bez senzora, suzbijanje vibracija, observer reakcijske sile, optimalno upravljanje, observer stanja akcije i reakcije

1 INTRODUCTION

Sensorless control techniques are of great importance because of the hardware sophistication that is added when sensors are utilized. Multiple sensors require multiple wirings along with their associated electronic setups. Moreover, using certain sensors increases the Mechatronics products' cost tremendously, especially if force/torque feedback is required. In addition, control of flexible systems require using sensors with certain specifications such as fatigue resistance to withstand the ever lasting fluctuations due to simplest manoeuvres. Furthermore, certain environmental conditions may cause sensor malfunctions that in turn implies obtaining unreliable results.

Much effort has been expended to suppress flexible system's residual vibrations during a motion control assignment. Among all the existing vibration suppression techniques, pre-filtering the control input is commonly utilized by passing the control signal through either a low pass or a notch filter to take away any energy at the system's resonance frequencies [1]. Indeed, such vibration suppression techniques succeed to minimize system's residual vibrations. However, fast responses cannot be achieved as the control input is trapped in the system low frequency range. D. Miu and S. Bhat pointed out that in order to achieve zero residual vibration, the flexible modes can be excited but the control input must be chosen such that all kinetic and potential energies trapped in the system's elastic elements are totally relieved at the end of the travel [2]. Furthermore, D. Miu and S. Bhat [3] demonstrated that the control input can be written as a linear combination of linearly independent basis functions to form an open loop control law that positions flexible system to the target position with zero residual vibrations [4]. It was commonly believed that minimum travel time can be accomplished by undergoing maximum acceleration followed by a maximum deceleration, constrained only by actuator saturation. However, in reality, flexibility of both plant and actuator causes the point of interest to vibrate, which must take time to settle. Surprisingly enough that the ultimate constraint in achieving faster time is not the availability of control voltage but rather the energy dissipation capability of the actuator that is presented by Copper [5]. Bellman [6] and Leitmann [7] demonstrated that undesired residual vibration can be eliminated by introducing additional switching time to the conventional bang-bang control input. A novel preshaping technique for eliminating residual vibration was presented by Singer and Seering [1], by convolving an arbitrary control input with a sequence of impulses that are chosen such that in the absence of control input, it would not cause residual vibration.

In this work, motion control of a multi-degrees-of-freedom flexible system is achieved along with residual vibration suppression by developing sensorless LQR controller. In other words, an estimation algorithm is combined with the regular LQR to develop a controller that is capable of achieving motion control and vibration suppression without taking any measurement from the flexible plant. However, actuator's variables and parameters are assumed to be available. Therefore, this paper attempts to keep the plant free from any attached sensors while performing a vibrationless motion control assignment. In order to demonstrate the visibility of the proposed algorithm, experiments are conducted on a flexible system with finite number of degrees of freedom. Then it can be extended to the more practical system with infinite modes.

This paper is organized as follows. In Section 2, the feedback like forces, namely the reaction forces are estimated through reaction force observer. Then the estimated reaction force along with the actuator velocity are used to identify parameters of a flexible dynamical system with three degrees of freedom. The action-reaction state observer is then utilized in Section 3 which allows estimating the flexible dynamical system states without measuring any of its outputs, the estimated reaction forces are rather injected onto the state observer structure to guarantee convergence of the estimated states to the actual ones. Section 4 includes the derivation of the flexible plant sensorless optimal motion and vibration control law that minimizes the residual vibration or the energy content performance index. In Section 5, experimental results are shown. Eventually, conclusions and final remarks are included in Section 6.

2 PROBLEM FORMULATION

Motion control and vibration suppression of flexible dynamical systems such as the one depicted in Fig.1 requires preknowledge of the dynamical system parameters, model and states in order to realize the optimal motion and vibration suppression control law. Nevertheless, the class of dynamical systems we consider has no accessible outputs. In other words, this work attempts to realize the optimal motion control and vibration suppression in the absence of dynamical system parameter and outputs. The only accessible measurement from the dynamical system is the actuator velocity or orientation, i.e., the actuator velocity $\dot{x}_m$ can only be measured. The dynamical system coordinates $x_1, x_2, \ldots, x_n$ are inaccessible. In addition, dynamical system parameters, namely, stiffness k and viscous damping coefficients b are unknowns.

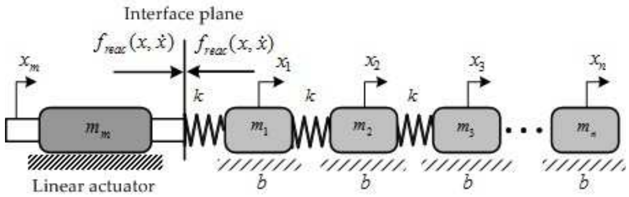


Fig. 1. Flexible system with inaccessible outputs

It is commonly agreed that state observer can be designed if the dynamical system is observable and if there exist some outputs that can be injected onto the observer structure in order to guarantee convergence of the estimated states to the actual ones. However, for the problem we consider, system outputs are not accessible. Therefore, a regular state observer cannot be utilized. The natural feedback concept was proposed in [8] and further utilized in [9]-[10] in order to estimate and observe dynamical system parameters and states respectively from measurements taken from its actuator by considering the reaction forces $f_{reac}(x, \dot{x})$ as feedback like forces from the dynamical system on the actuator as depicted in Fig.1. Necessary and sufficient conditions for observability of flexible dynamical systems with inaccessible outputs were shown in [9].

2.1 Reaction force estimation

The state space representation of the system we consider can be written as

$$\dot{\mathrm{x}} = \mathrm{A}\mathrm{x} + \mathrm{B}u \ , \ \mathrm{y} = \mathrm{C}\mathrm{x} \tag{1}$$

where x and y are system state and output vectors. A, B and C are system matrix, input and output distribution vectors with proper dimensions, respectively. Taking parameter deviations and disturbances into account, (1) can be rewritten as follows

$$\dot{\mathrm{x}} = (\mathrm{A}_o + \Delta\mathrm{A})\mathrm{x} + (\mathrm{B}_o + \Delta\mathrm{B})u + e\mathrm{d} \tag{2}$$

where, e is the distribution vector of disturbances d. $\Delta\mathrm{A}$ and $\Delta\mathrm{B}$ are the deviations between the nominal $(\mathrm{A}_o, \mathrm{B}_o)$ and actual ones. Rearranging (2)

$$\dot{\mathrm{x}} = \mathrm{A}_o\mathrm{x} + \mathrm{B}_o u + \underbrace{\Delta\mathrm{A}\mathrm{x} + \Delta\mathrm{B}u + e\mathrm{d}}_{d} \tag{3}$$

Applying (3) on the system illustrated in Fig.1, $\mathrm{d}(t)$ can be expressed as

$$\begin{aligned} d(t) &= -\Delta m_m \ddot{x}_m + \Delta k_f i_m - b(\dot{x}_m - \dot{x}_1) - k(x_m - x_1) \\ &= -\Delta m_m \ddot{x}_m + \Delta k_f i_m - f_{reac}(x, \dot{x}) \end{aligned} \quad (4)$$

where actual disturbance in (4) is $m_m \ddot{x}_m - k_f i_m$, Δk_f and Δm_m are the deviations of actuator's force constant and rod mass from their actual values. i_m is actuator's current. Disturbance d can be estimated through actuator's current and velocity through the following low-pass filter as follows [10]- [11].

$$\hat{d}(t) = \frac{g_{dist}}{s + g_{dist}}(m_{mn}\ddot{x}_m + k_{fn} i_m) \quad (5)$$

$$\hat{d}(t) = g_{dist} m_{mn} \dot{x}_m - \frac{g_{dist}}{s + g_{dist}}(g_{dist} m_{mn} \dot{x}_m + k_{fn} i_m)$$

where, g_{dist} is the corner frequency of the low-pass filter included in (5). The estimation error therefore is $\widetilde{d}(t) = d(t) - \widehat{d}(t)$. Consequently, the equation that governs the estimation error is

$$\widetilde{d}(t) = d_o e^{-g_{dist} t} + \int_0^t e^{-g_{dist}(t-\tau)} \Gamma(\tau) d\tau \quad (6)$$

$$\Gamma(t) \triangleq (s+g)(\Delta k_f i_m - \Delta m_m \ddot{x}_m) - g(m_{mn}\ddot{x}_m - k_{fn} i_m)$$

Therefore, (4) can be rewritten using the estimated disturbance instead of the actual one as

$$\widehat{d}(t) = -\Delta m_m \ddot{x}_m + \Delta k_f i_m - f_{reac}(x, \dot{x}) \quad (7)$$

Decoupling reaction force out of the disturbance signal $\widehat{d}(t)$ requires estimating both actuator's force-ripple $\Delta k_f i_m$ and varied-self mass force $\Delta m_m \ddot{x}_m$ which can be performed through an off-line experiment when the dynamical system is not attached to the actuator as both Δk_f and Δm_m are inherent properties of the actuator. Therefore, $f_{reac}(x, \dot{x}) = 0$ then (7) can be rewritten as follows

$$\widehat{d}(t) = -\Delta m_m \ddot{x}_m + \Delta k_f i_m \quad (8)$$

which can be considered as an over-determined system when $\widehat{d}(t)$, $\ddot{x}_m$ and i_m are considered as vectors of actuator acceleration and current data points. By constructing the following matrix $\mathrm{F} \triangleq [\underline{i}_m \underline{\ddot{x}}_m]$ [12], actuator parameter deviations can be estimated as follows

$$\begin{bmatrix} \widehat{\Delta k_f} \\ \widehat{-\Delta m_m} \end{bmatrix} = [\mathrm{F}^t \mathrm{F}]^{-1} \mathrm{F}^t \underline{\hat{d}} = \mathrm{F}^\dagger \underline{\hat{d}} \quad (9)$$

where $(\mathrm{F}^\dagger)$ is the pseudo inverse of F and $(\widehat{\Delta k_f})$ and $(\widehat{\Delta m_m})$ are the estimated deviations between actual and nominal actuator force constant and actuator inertia. Consequently, the reaction force can be estimated through the following low-pass filter

$$\widehat{f_{reac}}(x, \dot{x}) = P(s)\big(g_{reac}\widehat{\Delta m_m}\dot{x}_m + i_m\widehat{\Delta k_f} + \widehat{d}\big) - g_{reac}\widehat{\Delta m_m}\dot{x}_m \quad (10)$$

$$P(s) = \frac{g_{reac}}{s + g_{reac}}$$

where g_{reac} is the positive reaction force observer gain.

2.2 Parameter identification

System parameters can be identified from the reaction force signal if actuator position is measured along with a measurement from the dynamical system x_1. However, this work attempts to keep dynamical system free from any measurement while keeping the actuator side as a single platform for measurement and estimation. For the system depicted in Fig.1 there exist one rigid mode and $(n-1)$ flexible modes [12]. If any of the flexible modes of the dynamical system not including the actuator is not excited, the reaction force can be expressed as

$$\widehat{f_{reac}}(x, \dot{x}) = b(\dot{x}_m - \dot{\widehat{x}}) + k(x_m - \widehat{x}) \quad (11)$$

where $\widehat{x}$ is the position of the flexible system when none of its flexible modes is excited which can be determined through an off-line experiment at which the control input is filtered or Fourier synthesized such that its energy content is zero at the dynamical system resonances. Therefore, $\widehat{x}$ can be obtained by double integrating the estimated reaction force assuming that the total mass is known a priori. This requires the control input to be filtered just during the parameters identification procedure. Hereafter, the control input can excite any of the systems flexible modes. In other words, the control input is just filtered during the parameter identification off-line experiment.

By defining $\underline{\eta} \triangleq (\dot{x}_m - \dot{\widehat{x}})$ and $\underline{\zeta} \triangleq (x_m - \widehat{x})$, a matrix representation of (11) can be realized as follow

$$\underline{\widehat{f}}_{reac}(x, \dot{x}) = \begin{bmatrix} \underline{\eta} & \underline{\zeta} \end{bmatrix} \begin{bmatrix} b \\ k \end{bmatrix} \quad (12)$$

$\underline{\widehat{f}}_{reac}(x, \dot{x})$ is a vector of reaction force data points obtained through a rigid motion maneuver of the flexible dynamical plant, then defining matrix G as

$$\mathrm{G} \triangleq \begin{bmatrix} \underline{\eta} & \underline{\zeta} \end{bmatrix} \quad (13)$$

Equation (12) represents an over-determined system where the number of equations are more than the number of unknowns, joint stiffness and damping coefficients therefore can be estimated through the following expression

$$\begin{bmatrix} \widehat{b} \\ \widehat{k} \end{bmatrix} = [\mathrm{G}^t \mathrm{G}]^{-1} \mathrm{G}^t \underline{\widehat{f}}_{reac}(x, \dot{x}) = \mathrm{G}^\dagger \underline{\widehat{f}}_{reac}(x, \dot{x}) \quad (14)$$

where $\mathrm{G}^{\dagger}$ is the pseudo inverse of G. It is worth noting that the previous parameter identification procedure can be considered as an off-line experiment which has to be carried out on the flexible plant low-frequency range. In the next section, control and state estimation will be carried out along the entire frequency range of the plant.

3 ACTION-REACTION STATE ESTIMATION

In order to estimate dynamical system states from the actuator measurement, we utilize the action-reaction state observer [9] which can be written as follows

$$\dot{\hat{\mathrm{x}}} = \mathrm{A}\hat{\mathrm{x}} + \mathrm{B}u + \mathrm{M}\big(\widehat{f_{reac}}(x, \dot{x}) - f_{reac}(\hat{x}, \dot{\hat{x}})\big) \quad (15)$$

where $\widehat{f_{reac}}(x, \dot{x})$ is the estimated reaction force obtained through the reaction force observer (10), $f_{reac}(\hat{x}, \dot{\hat{x}})$ is the reaction force based on the estimated states $\hat{\mathrm{x}}$. M is the observer gain vector. The state matrix A includes the identified dynamical system parameters obtained through the off-line experiment outlined in the previous section. It is worth noting that the difference between the action-reaction state observer and any relevant existing state observer, is its ability to estimate dynamical system states in the absence of its outputs. $\widehat{f_{reac}}(x, \dot{x})$ is estimated through the reaction force observer while $f_{reac}(\hat{x}, \dot{\hat{x}})$ is computed through the estimated states and the model that is known a priori. The estimation error can be written as

$$e = x - \hat{x} \quad (16)$$

Therefore, the error dynamics can be shown to be

$$\dot{e} = (\mathrm{I} - c\mathrm{ML})^{-1}(A + k\mathrm{ML})e = \mathcal{A}e \quad (17)$$

$$\mathrm{L} = [1\ 0\ \cdots\ 0]$$

where I is the identity matrix with proper dimensions. Therefore, estimation error (e) will converge to zero if all eigenvalues of ($\mathcal{A} = (\mathrm{I} - c\mathrm{ML})^{-1}(A + k\mathrm{ML})$) lie on the left-half plane. Selection of the observer gain (M) is a regular pole placement problem. It can be shown now that the state observer (15) does not necessitate taking any measurement from the plant side, plant states (not including the actuator) are not measured at all. However, the incident reaction force is conceptually considered as a natural feedback from the sensorless plant. Thus, used to design the state observer (15) that only requires two measurements from the actuator to estimate disturbance force and reaction force through (5) and (10), respectively.

4 RESIDUAL VIBRATION SUPPRESSION

In order to perform vibrationless motion control, the following performance index is used

$$\mathrm{J}(\mathrm{x}(t), u(t), t) = \Upsilon + \frac{1}{2}\int_{T_0}^{T_f} (\mathrm{x}^t\mathrm{Q}\mathrm{x}(t) + u(t)^t\mathrm{R}u(t))dt \quad (18)$$

$$\Upsilon \triangleq \mathrm{x}^{\mathrm{t}}(t_f)\mathrm{H}\mathrm{x}(t_f)$$

where, R is a symmetric positive definite matrix, i.e., $\mathrm{R}^t = \mathrm{R}$, $\mathrm{R} > 0$, while Q is at least symmetric semi-definite matrix, i.e., $\mathrm{Q}^t = \mathrm{Q}$, $\mathrm{Q} \geq 0$. The previous performance index can be rewritten using the obtained estimated states through the action reaction state observer (15). Therefore, (18) can be written as

$$\mathrm{J}(\hat{\mathrm{x}}(t), u(t), t) = \Upsilon + \frac{1}{2}\int_{T_0}^{T_f} (\hat{\mathrm{x}}^t\mathrm{Q}\hat{\mathrm{x}}(t) + u(t)^t\mathrm{R}u(t))dt \quad (19)$$

$$\Upsilon \triangleq \hat{\mathrm{x}}^{\mathrm{t}}(t_f)\mathrm{H}\hat{\mathrm{x}}(t_f)$$

consequently the Hamiltonian can be written as follows

$$\mathscr{H}(\hat{\mathrm{x}}(t), u(t), \hat{\mathrm{p}}(t), t) \triangleq \Gamma + \hat{\mathrm{p}}^t(t)[\mathrm{A}\hat{\mathrm{x}} + \mathrm{B}u(t)] \quad (20)$$

$$\Gamma \triangleq g(\hat{\mathrm{x}}(t), u(t), t) = \frac{1}{2}(\hat{\mathrm{x}}^{\mathrm{T}}\mathrm{Q}\hat{\mathrm{x}} + u^t(t)\mathrm{R}u(t))$$

$\hat{\mathrm{x}}(t)$ is a vector of the estimated states through (15) while $\hat{\mathrm{p}}(t)$ is the corresponding vector of system co-states. Differentiating the Hamiltonian with respect to states, co-states and control, the necessary conditions for the plant sensorless optimal control can be represented as follows

$$\dot{\hat{\mathrm{x}}}^*(t) = \frac{\partial\mathscr{H}(\hat{\mathrm{x}}(t), u(t), \hat{\mathrm{p}}(t), t)}{\partial\hat{\mathrm{p}}} \quad (21)$$

$$\dot{\hat{\mathrm{p}}}^*(t) = -\frac{\partial\mathscr{H}(\hat{\mathrm{x}}(t), u(t), \hat{\mathrm{p}}(t), t)}{\partial\hat{\mathrm{x}}} \quad (22)$$

$$\frac{\partial\mathscr{H}(\hat{\mathrm{x}}(t), u(t), \hat{\mathrm{p}}(t), t)}{\partial\hat{\mathrm{u}}} = 0 \quad (23)$$

the following matrix differential equation can be obtained using the previous necessary conditions

$$\begin{bmatrix} \dot{\hat{\mathrm{x}}}^*(t) \\ \dot{\hat{\mathrm{p}}}^*(t) \end{bmatrix} = \begin{bmatrix} \mathrm{A} & -\mathrm{B}\mathrm{R}^{-1}\mathrm{B}^{\mathrm{t}} \\ -\mathrm{Q} & -\mathrm{A} \end{bmatrix} \begin{bmatrix} \hat{\mathrm{x}}^*(t) \\ \hat{\mathrm{p}}^*(t) \end{bmatrix} \quad (24)$$

solving the previous matrix differential equation for estimated states and co-states we obtain

$$\begin{bmatrix} \hat{\mathrm{x}}^*(t_f) \\ \hat{\mathrm{p}}^*(t_f) \end{bmatrix} = \begin{bmatrix} \Psi_{11} & \Psi_{12} \\ \Psi_{21} & \Psi_{22} \end{bmatrix} \begin{bmatrix} \hat{\mathrm{x}}^*(t) \\ \hat{\mathrm{p}}^*(t) \end{bmatrix} \quad (25)$$

where Ψ is the state transition matrix. Using the following boundary condition

$$\hat{\mathrm{p}}(t_f) \triangleq \mathrm{H}\hat{\mathrm{x}}(t_f) \quad (26)$$

combining (25) and (26), we obtain

$$\hat{\mathrm{p}}(t) = (\mathrm{H}\Psi_{12} - \Psi_{22})^{-1}(\Psi_{21} - \mathrm{H}\Psi_{11})\hat{\mathrm{x}}(t) \quad (27)$$

taking partial derivative of Hamiltonian with respect to the control input

$$u^*(t) = -\mathrm{R}^{-1}\mathrm{B}^t\hat{\mathrm{p}}(t) \quad (28)$$

using (27) in (28) we obtain

$$u^*(t) = -\mathrm{K}\widehat{\mathrm{x}}(t) \tag{29}$$

where $\mathrm{K} = \mathrm{R}^{-1}\mathrm{B}^{\mathrm{T}}(\mathrm{H}\Psi_{12} - \Psi_{22})^{-1}(\Psi_{21} - \mathrm{H}\Psi_{11})$. Implementation of the previous control law is illustrated in Fig.2 were the estimated states are used as input to (29) for the dynamical system with four degrees of freedom as depicted in Fig.1. Fig.2-a illustrates the phase portrait of the actuator while the end effector (Third mass) phase portrait is depicted in Fig.2-b. The previous plant sensorless control law guarantees convergence of system states to the origin with minimum oscillation. However, to perform a plant sensorless set point tracking motion control assignment along with vibration suppression, the origin of the system can be shifted to the desired reference position and the control law (29) can be modified as follows

$$u^*(t) = -\mathrm{R}^{-1}\mathrm{B}^{\mathrm{t}}\mathrm{K}(\widehat{\mathrm{x}}(t) - \mathrm{r}(t)) \tag{30}$$

Fig.3 illustrates the set point tracking simulation result using the optimal control law (28) for the same dynamical system. Similarly, actuator phase portrait is depicted in Fig.3-a while the third mass phase portrait is depicted in Fig.3-b.

Fig.4 illustrates a comparison between the PID controller with the controller gains included in Table.1 and the proposed sensorless linear quadratic regulator controller (28). The illustrated results are obtained during the control of the actuator as shown in Fig.4-a. Fig.4b-c-d illustrate the response of the other non-collocated masses for both the PID controller and the proposed sensorless LQR controller. The simulation parameters used during this control comparison are included in Table.1. Figure 4 indicates the effectiveness of the proposed controller in the sense of minimizing the residual vibration along the flexible dynamical system.

Table 1. Simulation parameters

Actuator force constant	k_{fn}	6.43	N/A
Nominal mass	m_{mn}	0.059	kg
Lumped masses	$m_{1,2,3}$	0.019	kg
Force observer gain	g_{reac}	100	Hz
Disturbance observer gain	g_{dist}	100	Hz
Low-pass filter gain	g_f	100	Hz
Proportional Gain	k_p	50	-
Integral Gain	k_i	5	-
Derivative Gain	k_d	20	-
Sampling time	T_s	1	msec

5 EXPERIMENTAL RESULTS

In order to verify the validity of the proposed sensorless motion and vibration suppression control law, experiments are conducted on a lumped flexible system with four degrees of freedom. As shown in Fig.5, the experimental setup consists of a linear motor connected to a flexible dynamical system with three degrees of freedom through an elastic element. Experimental parameters are included in Table.2. Linear encoders are attached to each lumped mass in order to compare the actual position of each lumped mass with the estimates obtained through the action reaction state observer. Velocities are determined through the following low-pass filter throughout all the experiments

$$\dot{\mathrm{x}} = \frac{sg_f}{s+g_f}\mathrm{x} \tag{31}$$

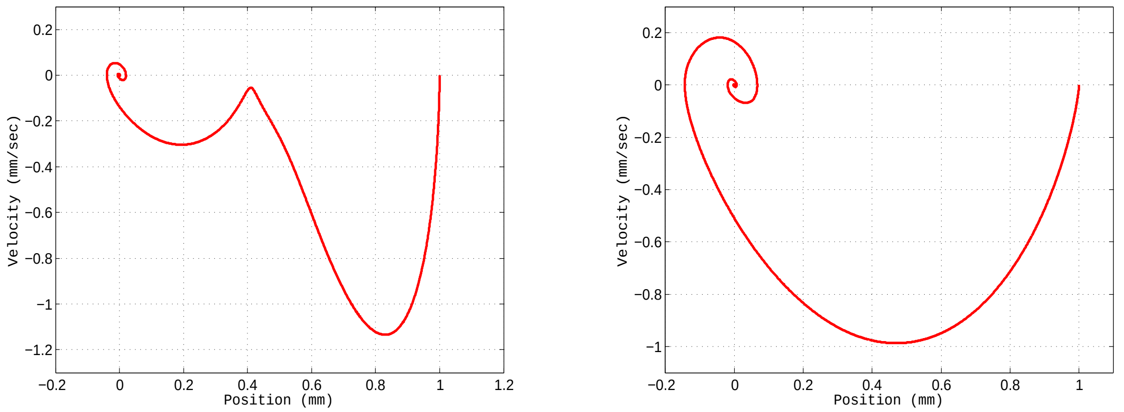
where g_f is the corner frequency of the low-pass filter. Experimentally, velocity of the actuator has to be measured or determined through (31) along with the knowledge of the reference input. These two variables are then used to estimate the disturbance force through (6).

An off-line experiment is performed in order to determine the actuator parameter deviations, in order to compute the actuator self-varied mass and force ripple. This experiment can be performed when the actuator is free from any attached load since force ripple and self-varied mass are inherent properties for the actuator. Fig.6a-b illustrates the difference between the estimated disturbance and the reconstructed disturbance using the identified actuator parameter deviation. Fig.6 indicates that the off-line experiment and utilization of (9) allows correct identification of the actuator parameter deviations from their actual values which in turn allows online determination of the actuator force ripple and self-varied mass that can be used to decouple the reaction force out of the disturbance force through (7). In order to avoid direct differentiation, reaction force is obtained through (10).

The estimated reaction force obtained through the reaction force observer (10) is used to identify the plant stiffness and damping coefficients. This requires another off-

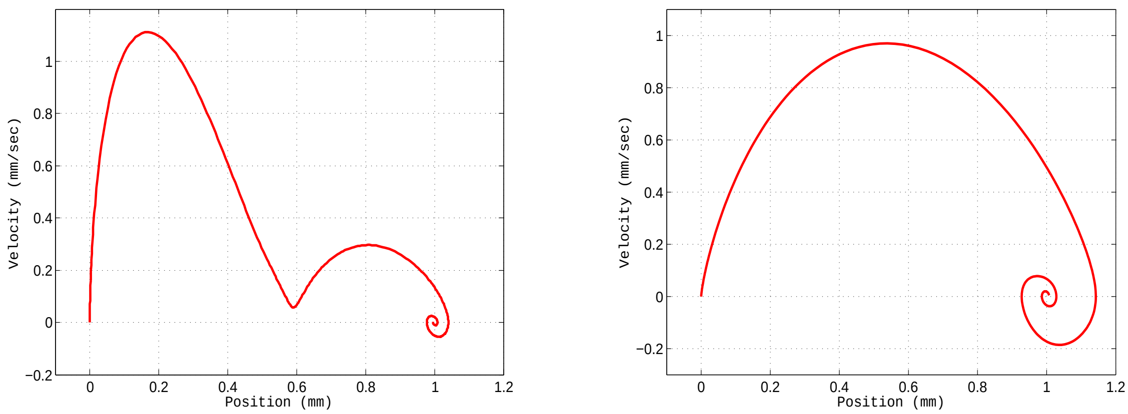
Table 2. Experimental parameters

Actuator force constant	k_{fn}	4.3	N/A
Nominal mass	m_{mn}	0.222	kg
Lumped masses	$m_{1,2,3}$	0.15	kg
Force observer gain	g_{reac}	50	Hz
Disturbance observer gain	g_{dist}	50	Hz
Low-pass filter gain	g_f	20	Hz
Sampling time	T_s	1	msec



(a) actuator phase portrait (b) third mass phase portrait

Fig. 2. Regulation control law simulation results

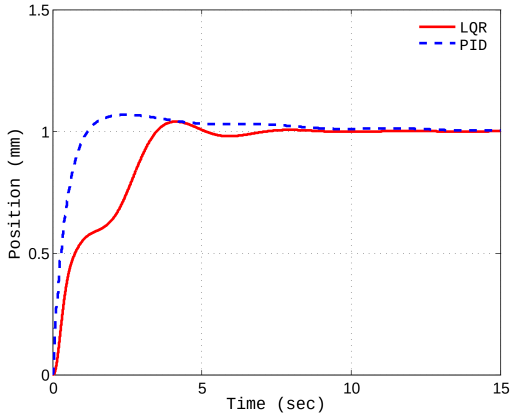


(a) actuator phase portrait (b) third mass phase portrait

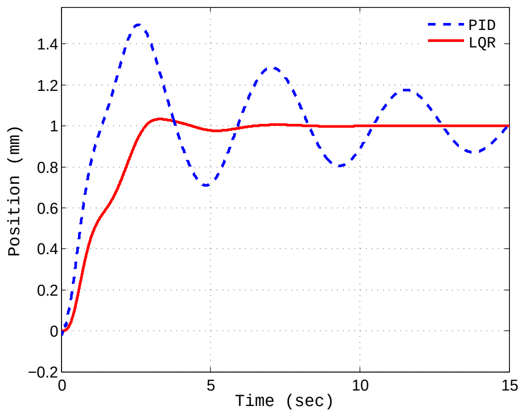
Fig. 3. Set point tracking control results

line (14) experiment at which the vectors $\underline{\eta}$ and $\underline{\zeta}$ are determined when any of the flexible modes of the plant is not excited. The estimated reaction force is plotted against the reconstructed reaction force using the identified parameters as depicted in Fig.6-c-d. This indicates that the flexible plant parameters can be identified correctly through the proposed identification process and therefore can be used to construct the system matrix A that will be further used in the action reaction state observer structure.

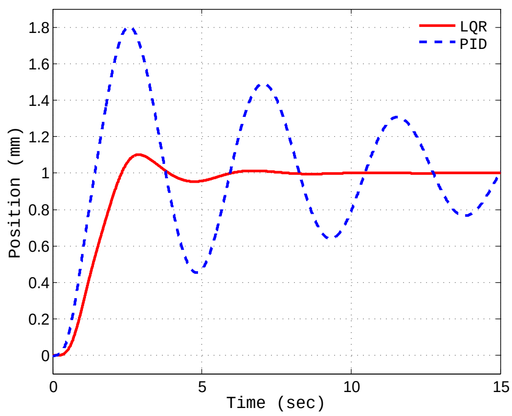
The action reaction state observer (15) is utilized in order to estimate the dynamical system plant without measuring any of its states. Actuator velocity is only measured and used along with the input in order to estimate the incident reaction force from the sensorless flexible plant onto the actuator. These reaction forces are conceptually considered as natural feedbacks from the plant. Fig.7 shows the experimental result of the state estimation experiment. x_1 and x_2 represent position of the first and second lumped masses of the system, $\dot{x}_1$ and $\dot{x}_2$ are their corresponding velocities, respectively. Experimentally, the actuator was exciting the system with arbitrary motions in order to examine the performance of the state observer. Meanwhile, position encoders that are attached to each lumped mass were used to compare the actual positions with the estimated ones through (15). Fig.7 indicates that the estimated states are accurately tracking the actual ones. Throughout the state estimation experiment, the difference between the actual positions and the estimated ones was at most 1.2% of the step input. The convergence time of the estimated states to the actual ones is as well satisfactory. Therefore, the previous experimental result indicates that the action-



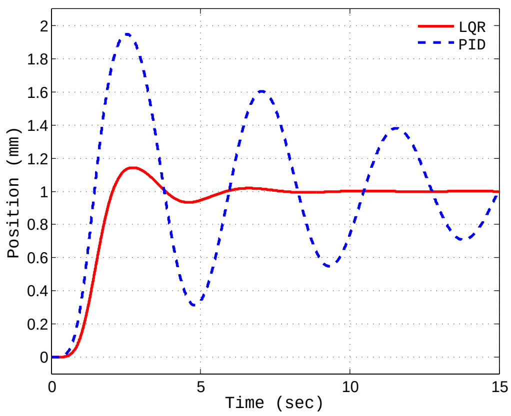
(a) Actuator position



(b) 1^{st} non-collocated mass position



(c) 2^{nd} non-collocated mass position



(d) 3^{rd} non-collocated mass position

Fig. 4. Sensorless LQR versus PID control simulation results

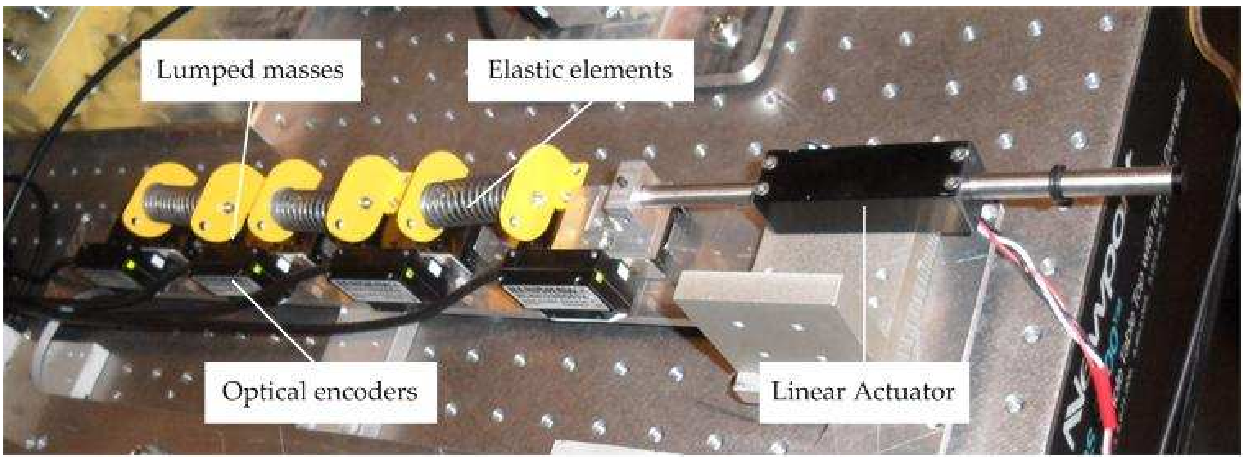
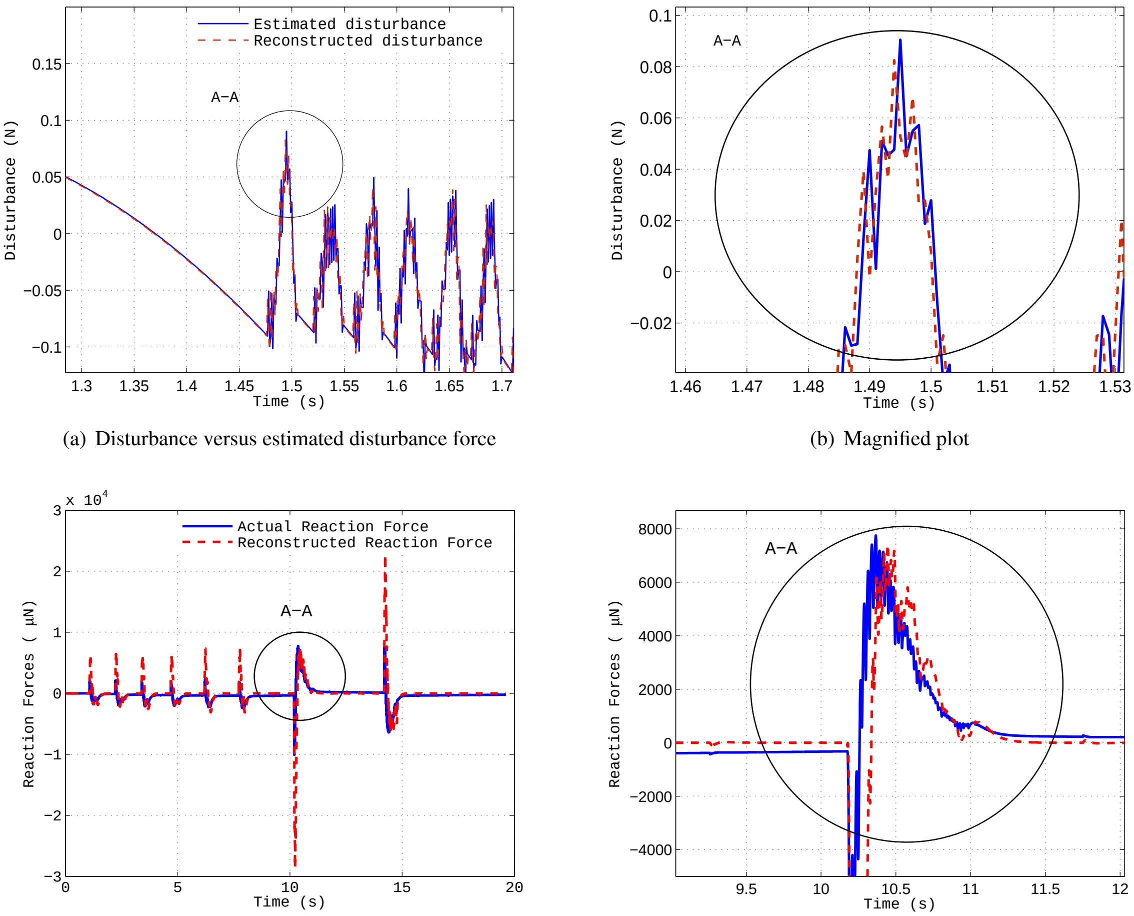


Fig. 5. Experimental setup with 4 DOF



(a) Disturbance versus estimated disturbance force

(b) Magnified plot

(c) reaction force versus estimated disturbance force

(d) Magnified plot

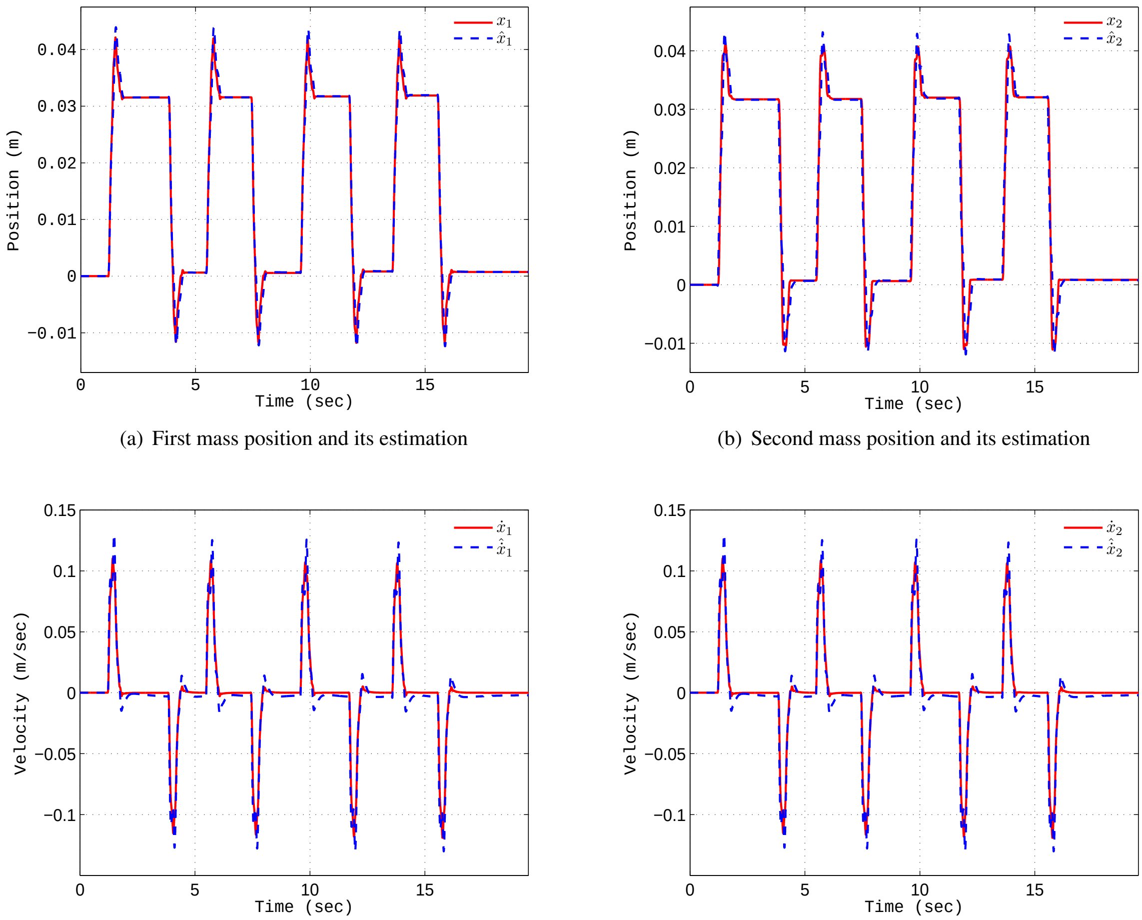
Fig. 6. Disturbance and reaction forces versus estimated disturbance and reaction forces

reaction state observer is satisfactory estimating the dynamical plant states from measurement taken from the actuator rather than plant outputs. Fig.8 further illustrates the effectiveness of the state observer when time varying arbitrary trajectories of the lumped masses are required to be observed.

The previous state estimation experimental results indicate that the action reaction state observer can be used in the realization of the plant sensorless optimal control law. The optimal regulation control law (29) requires estimation of all the flexible plant states. In this experiment, the estimated states are used in the optimal control law (29). The optimal motion control and vibration suppression experimental results are shown in Fig.9 where the flexible dynamical system is optimally regulated to the $0.015\ m$ from the origin. Fig.9a-b illustrates the behavior of the second and third non-collocated masses along with their estimates, respectively. The previous experiment indicates the validity of the proposed control technique which can be utilized whenever measurement can not be made due to several reasons such as micro-systems and micromanipulation operation in addition to control of dynamical system with inaccessible outputs. The entire identification, estimation and control technique depends on measurements taken from the actuator side, whereas the flexible plant is kept free from any attached sensor.

6 CONCLUSION

Optimal motion control and vibration suppression for multi degree of freedom flexible systems can be achieved from measurement taken from their actuator rather than having multiple sensors attached to their structure. The reaction force are conceptually considered as feedback like forces which can be used as replacement for system measurements which might be unavailable or can not be measured. Reaction force at the point of interface between



(a) First mass position and its estimation

(b) Second mass position and its estimation

(c) Second mass position and its estimation

(d) Second mass position and its estimation

Fig. 7. State estimation experimental results

the flexible dynamical system and its single input is estimated using a reaction force observer. The estimated reaction forces is then injected onto the state observer rather than the flexible system outputs in order to guarantee convergence of the estimated states to the actual ones. The estimated states are used instead of the actual ones in the realization of the optimal motion and vibration suppression control law.

The experimental results showed the validity of the proposed sensorless motion and vibration control algorithm where position control of a flexible system with four degrees of freedom is achieved from measurement taken from its actuator.

In order to ensure efficiency of the proposed plant sensorless motion and vibration suppression control, the disturbance observer gain, reaction force observer gain and the action reaction state observer gain vector have to be selected such that their induced phase lags do not cause instability by changing the system phase and gain margins.

The proposed controller can be utilized whenever measurement can not be taken from the flexible dynamical system or for a class of dynamical system with inaccessible outputs. Microsystems and micromanipulation operations are applications at which measurement can hardly be made. Therefore, the author of this manuscript believe that the proposed controller can assist automating Microsystems and micromanipulation operations.

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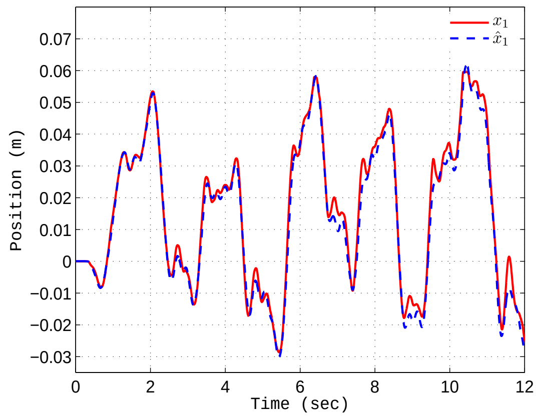
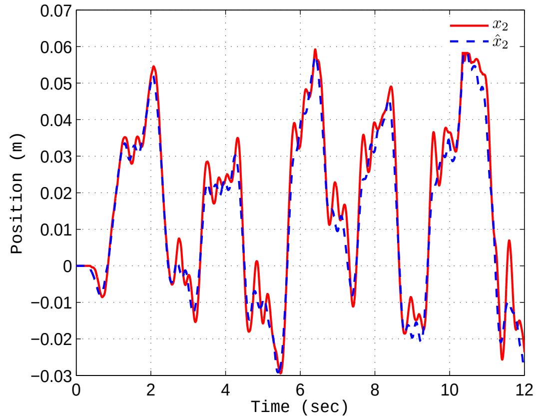
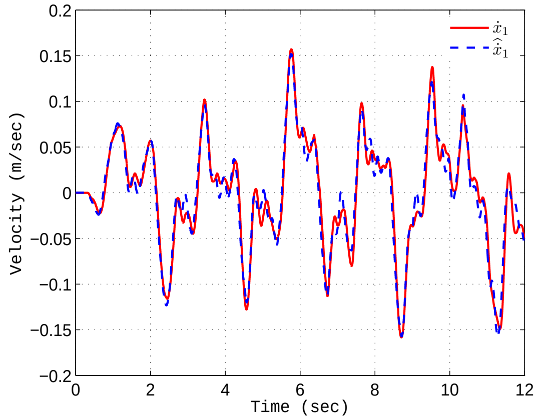


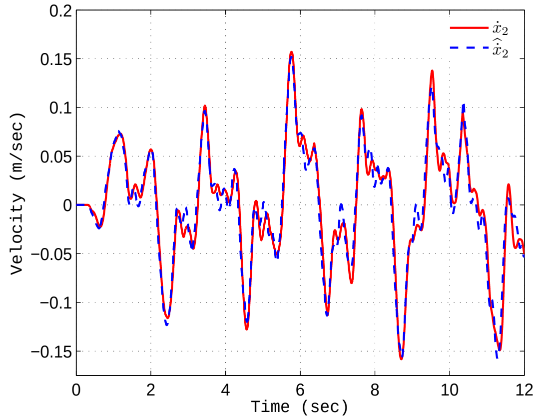
(a) First mass position and its estimation



(b) Second mass position and its estimation

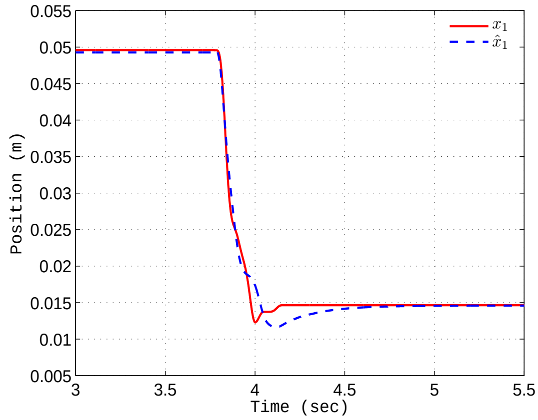


(c) Second mass position and its estimation

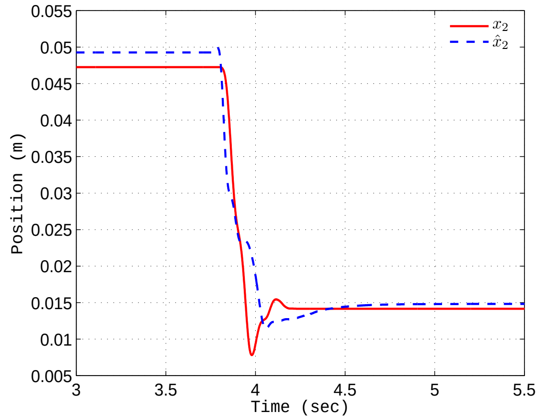


(d) Second mass position and its estimation

Fig. 8. State estimation experimental results



(a) First mass position versus its estimate



(b) Second mass position versus its estimate

Fig. 9. Optimal regulation experimental results when estimated states are used in the optimal control law

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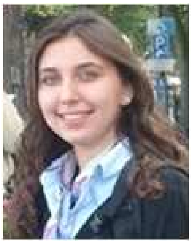

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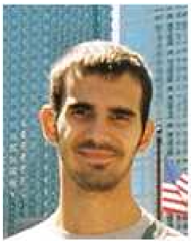

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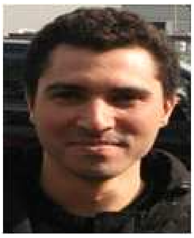

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