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On the Design of Discrete Time Repetitive Controllers in Closed Loop Configuration

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Original scientific paper

This paper deals with a discrete time repetitive control synthesis for non minimum phase plants. Two parts can be distinguished. The main design features of the repetitive controllers are discussed in the first part. More precisely one shows that one can realize two objectives; tracking with zero error and tracking with nonzero error. In the second part, a suitable plant model identification procedure for the repetitive control is proposed. An adequate input-output identification filter is designed such that the difference between the nominal and the actual repetitive control convergence conditions is minimized. Some illustrative examples are given to highlight the main features of the proposed approach.

Key words: Repetitive control, Non minimum phase systems, Tracking, Control relevant identification

Projektiranje vremenski diskretnih repetitivnih regulatora u konfiguraciji zatvorene petlje. Ovaj rad obrađuje sintezu vremenski diskretnih repetitivnih regulatora za neminimalno fazne sustave. Razlikuju se dva dijela. U prvom je dijelu razmatrano projektiranje glavnih obilježja repetitivnih regulatora. Točnije se pokazuje da se mogu ostvariti dva cilja; slijeđenje s pogreškom nula i slijeđenje s pogreškom različitom od nule. U drugom je dijelu predložen odgovarajući postupak identifikacije modela procesa za repetitivno upravljanje. Projektiran je adekvatan ulazno-izlazni identifikacijski filtar tako da je razlika između nominalnih i stvarnih uvjeta konvergencije repetitivnog upravljanja svedena na minimum. Dano je nekoliko ilustrativnih primjera, koji ističu glavna obilježja predloženog postupka.

Ključne riječi: repetitivno upravljanje, neminimalno fazni sustavi, slijeđenje, upravljački relevantna identifikacija

1 INTRODUCTION

Industrial processes make often repetitive or periodic tasks. Typical examples are industrial robots, which most of their tasks are of this kind; e.g. pick and place, painting, etc. Other examples are control of numerical control machines, hard-disc drive or many mechanical systems having revolving mechanisms inside. Repetitive control is an iterative approach that improves the transient response performance of such processes (Fig. 1).

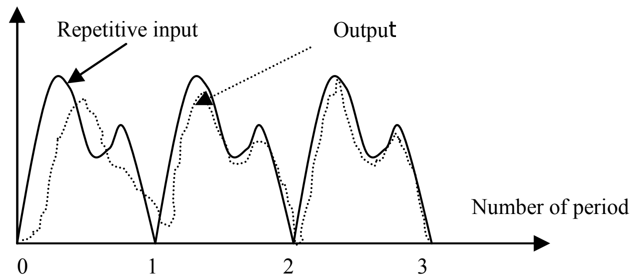

Fig. 1. Example of periodic output.

The repetitive control known also under "learning control" is a control law introduced in the early eighties to treat the systems which realize repetitive or periodic tasks. Most of the publications made around this control law, take into account an open loop structure, see [1, 2] and references therein.

The originality of our study is the treatment of the repetitive control in a closed loop configuration. A rather complete study is made in this paper for the synthesis of the repetitive controllers. We are especially interested in the case where the discrete-time system to be controlled possesses unstable zeros. Some the results presented here can be found in [3].

The concept of repetitive control systems was first introduced by Arimoto [4]. The idea was later developed, for continuous time systems, by several researchers (see [5] and references therein). The proposed control algorithms use past open loop tracking error signals to update actual input signal as shown in Fig. 2, where i refers to the number of the period which is different from the sampling instant k. One suppose that the reference signal is the same at each period i.e. that $y_d^i(k) = y_d^j(k)$ for any *i,j*, such that the index in the superscript can be omitted. At each instant k the control signal $u^i(k)$ and the output signal $y^i(k)$ are memorized. The repetitive control algorithm evaluates the error $e^i(k)=y_d(k)-y^i(k)$ and calculates the control signal $u^{i+1}(k)$ that will be used at the next period.

In [6] a discrete time repetitive control law based on classical closed loop systems is proposed. In this case, the controller output of the previous period is used to modify the present control signal. The main limitation of these algorithms is that they cannot be applied to non minimum phase processes [7, 8].

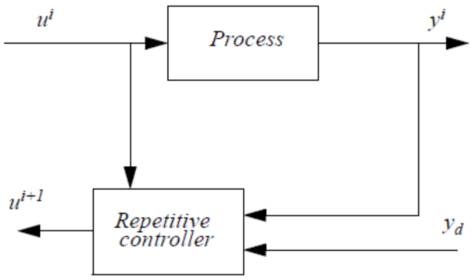

Fig. 2. Open loop repetitive control system.

In [3, 9, 10, and 11], it was shown that the asymptotic repetitive control algorithms inverts the process and hence the tracking error is always equal to zero. To overcome the process inversion, a promising approach has been developed in [3, 5, 11 and 12]. Indeed, the repetitive control objective is formulated as an optimization problem leading to a control signal that does not invert the process.

Furthermore, at the beginning of nineties, there was a particular interest in the relationship between control and identification involved in the design of a control system [13, 14, 15 and 16]. The concept of "control relevant identification" allows the identification criterion to be compatible with the control performance objective [17].

In this work, the main design features of the discrete time repetitive control in the case of non minimum phase plant (generally due to the discretization), are emphasized. More specifically, it is shown first that the difference between the desired trajectory and the output can be made arbitrarily small for non minimum phase plants. Second, a design taking into account both the control objectives and the model identification is presented and an adequate input-output identification filter is designed to minimize the difference between the nominal and the actual repetitive convergence conditions [18].

The paper is organized as follow. In section 2, the problem that we address is formulated. The repetitive control algorithm for non minimum phase plants is discussed in section 3. Section 4 deals with plant model identification.

2 PROBLEM FORMULATION

Consider the linear discrete time single input single output system described by the following transfer function

$$G(z^{-1}) = \frac{z^{-d}B(z^{-1})}{A(z^{-1})} \quad (1)$$

with

$$B(z^{-1}) = b_0 + b_1 z^{-1} + \cdots + b_m z^{-m}$$
$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_n z^{-n}$$

where z is the Z-transform complex variable and d is the number of delay steps. The numerator $B(z^{-1})$ can be factorized as: $B(z^{-1}) = B^+(z^{-1})B^-(z^{-1})$. Where $B^+(z^{-1})$, of order m^+, and $B^-(z^{-1})$, of order m^-, are respectively the stable and unstable parts of $B(z^{-1})$. In the sequel the operator z^{-1} will be omitted for the aim of simplification.

Consider the closed loop configuration of Fig. 3, where $y_d(k)$ is the reference signal and G_c is an a priori known controller that is designed to stabilize the system and to make the output $y(k)$ as closer as possible to the desired trajectory $y_d(k)$. It is clear, that the reference tracking will not be satisfactory due to two main reasons which are unavoidable in practice: disturbances and modeling uncertainties. Furthermore, when the desired trajectory is repetitive or periodic, the control system will perform the same errors, because the control does not take into account the errors made in the previous periods. It will be interesting to use all the information, obtained in the previous periods, in the actual control system to improve the reference tracking.

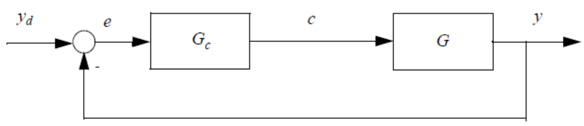

Fig. 3. Closed loop system.

Among those informations, we will particularly use the previous tracking errors and the control signal in closed loop configuration as shown in Fig. 4, where i refers to the number of the period, $y^i(k)$, $c^i(k)$, and $e^i(k)$ are respectively the output, the control and the tracking error signals at the i *th* period, α^i is an anticipation signal that is obtained by filtering respectively, $e^i(k)$ and $c^i(k)$ with G_e and G_u and it will be applied at the next period: $i+1$.

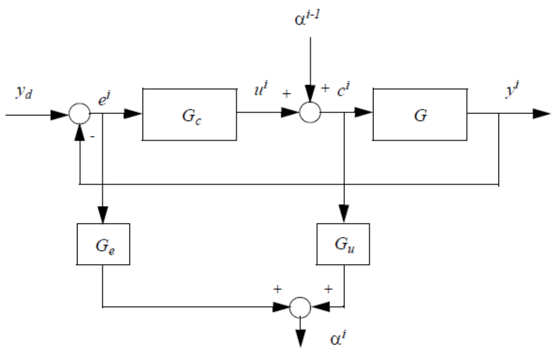

Fig. 4. Closed loop repetitive control system.

The design objective consists in the synthesis of the two filters G_e and G_u such that the asymptotic tracking error $(i \to \infty)$ goes to zero.

In the sequel, the sampling instant *k* will be omitted for simplification

3 REPETITIVE CONTROL

From Fig. 4, one can see that the repetitive control law is given by

$$c^i = G_c(y_d - y^i) + G_u c^{i-1} + G_e(y_d - y^{i-1}) \qquad (2)$$

or by

$$c^i = \frac{G_u - G_e G}{1 + GG_c} c^{i-1} + \frac{G_e + G_c}{1 + GG_c} y_d. \qquad (3)$$

Let $D = \dfrac{G_u - G_e G}{1 + GG_c}$ and $F = \dfrac{G_e + G_c}{1 + GG_c}$. (2) becomes

$$c^i = Dc^{i-1} + Fy_d. \qquad (4)$$

By developing the recurrence, one obtain

$$c^i = F(1 + D + \cdots + D^{i-1})y_d + D^i c^0. \qquad (5)$$

The control signal converges after an infinite number of periods to

$$c^\infty = \lim_{i\to\infty} c^i = \frac{F}{1-D} y_d \qquad (6)$$

if and only if

$$\|D\|_\infty < 1 \text{ i.e. } \left\| \frac{G_u - G_e G}{1 + GG_c} \right\|_\infty < 1 \qquad (7)$$

where the norm $\|.\|_\infty$ represents the maximum of $\|.\|_2$ norm on all frequency range.

The latter inequality is called the repetitive control convergence condition. In this case, the asymptotic control and output tracking error signal become

$$c^\infty = \left[(G_e + G_c)G + 1 - G_u\right]^{-1} (G_e + G_c) y_d \qquad (8)$$

and

$$\begin{aligned} e^\infty &= \lim_{i\to\infty} (y_d - y^i) \\ &= \left[1 - G\left[(G_e + G_c)G + 1 - G_u\right]^{-1}(G_e + G_c)\right] y_d. \end{aligned} \qquad (9)$$

Two cases can be distinguished depending on the choice of the filter G_u.

3.1 Perfect Tracking

If the control filter G_u is unity $(G_u = 1)$, from (8) it is clear that the control signal c^∞ becomes

$$c^\infty = \frac{1}{G} y_d \qquad (10)$$

and then

$$e^\infty = \lim_{i\to\infty} (y_d - y^i) = 0. \qquad (11)$$

It follows from (10) that the control signal after an infinite number of periods inverts the process dynamic which seems to be impossible when the plant to be controlled exhibits unstable zeros.

However, since $y_d(k)$ is an a priori known signal, it is possible to generate the off-line control signal even if the plant contains such zeros (see [3, 19] for more details).

In [5] it is shown that to satisfy the repetitive control convergence condition, the repetitive controller $G_e(z^{-1})$ will contain the inverse of the process. The question is then, what can we do when the process contains unstable zeros? For this, one distinguishes three types of repetitive controllers:

3.1.1 Complete Reverser Algorithm

In this case, the repetitive controller $G_e(z^{-1})$ is given, as in [9], by

$$G_e(z^{-1}) = k_e \frac{z^d A(z^{-1}) B^-(z)}{b \cdot B^+(z^{-1})} \qquad (12)$$

where $b \geq \max_{\omega\in[0,\pi]} |B^-(e^{-j\omega})|^2$.

The term k_e is called the repetitive control gain and $B^-(z)$ is obtained by replacing every z^{-1} in $B^-(z^{-1})$ by z. The terms z^d and $B^-(z)$ allow to realize a maximum advance equal to the number of unstable zeros plus the delay. The controller, in this case, uses the future input data to compute the output for the following period. This controller compensates also the poles and the stable zeros.

The repetitive control convergence condition for the closed loop configuration, as shown in Fig. 4, is then given by the following theorem [10]:

Theorem 1 *Consider the system* (1) *in closed loop with the repetitive controller* (12)*. Then, the repetitive control system is stable if the controller gain k_e satisfies:*

$$\delta < k_e < \beta \tag{13}$$

where

$$\delta = \max_{\omega\in[0,\pi]} \frac{b}{|B^-(e^{-j\omega})|^2}\left(1-\sqrt{1+2M\cos\varphi+M^2}\right) \tag{14}$$

and

$$\beta = \max_{\omega\in[0,\pi]} \frac{b}{|B^-(e^{-j\omega})|^2}\left(1+\sqrt{1+2M\cos\varphi+M^2}\right). \tag{15}$$

M and φ are the magnitude and the phase of $GG_c(e^{-j\omega})$, i.e:

$$GG_c(e^{-j\omega}) = M(\omega)e^{j\varphi(\omega)}. \tag{16}$$

Proof: The proof is given in Appendix A. ■

3.1.2 Partial Reverser Algorithm

In this case, the repetitive controller is derived from [20], and is given by

$$G_e(z^{-1}) = k_e \frac{z^{d+m^-}A(z^{-1})}{b\cdot B^+(z^{-1})} \tag{17}$$

where $b = B^-$ (1) Note that, as in the previous case, one realizes an advance of d+m^- in order to compensate the delay and the unstable zeros. Then, we have the following result:

Theorem 2 *Consider the system* (1) *in closed loop with the repetitive controller* (17)*. Then the repetitive control system is stable if*

$$|b_0| + \cdots + |b_{m-1}| < \frac{|b|\cdot MM}{2} \tag{18}$$

where MM is the modulus margin defined by $MM = \frac{1}{\|S\|_\infty}$ with S the sensitivity function of the closed loop system defined by $S = \frac{1}{1+GG_c}$.

Proof: See Appendix B for the proof. ■

3.1.3 Simple Anticipative Algorithm

In the two previous cases, one introduces, in the repetitive controller $G_e(z^{-1})$, an advance equal to the number of the delays d plus the number of unstable zeros m^-. One introduces also the poles and the stable zeros in order to compensate them. The last cancellation can be avoided, because it is not necessary to incorporate complicated expressions in the repetitive controller when it is enough to compensate only the delay and the unstable zeros [3].

Finally, one introduces a z^{-1} rational fraction $h(z^{-1})$, as simple as possible, in order to respect the repetitive control convergence condition. The repetitive controller is then

$$G_e(z^{-1}) = z^{d+m^-}h(z^{-1}). \tag{19}$$

3.1.4 Comparison

Two remarks concerning these repetitive controllers can be made. First, the three above repetitive controllers give quite the same asymptotic error. So, there is no difference between them from performances point of view. Second, the third controller is simpler than the others. In fact, it is not necessary to know exactly the process for designing the controller but it is sufficient to know the delay and the number of unstable zeros.

To illustrate the features of these algorithms, let us take an example. The process to be controlled is given by

$$G(z^{-1}) = \frac{z^{-1}(0.05+0.09z^{-1})}{1-0.3z^{-1}}.$$

It is a first order transfer function with an unstable zero ($z = -1.8$). The controller $G_c(z^{-1})$ is set to 1 in order to assume the stability of the loop.

Using the previous study, the filter $G_e(z^{-1})$ that satisfies the convergence condition, can be:

1- $G_e(z^{-1}) = \frac{k_e}{0.0196}\cdot z\left(1-0.3z^{-1}\right)(0.05+0.09z)$ from 3.1.1

2- $G_e(z^{-1}) = \frac{k_e}{0.014}\cdot z^2\left(1-0.3z^{-1}\right)$ from 3.1.2

3 - $G_e(z^{-1}) = 5\cdot z^2$ from 3.1.3

One can see that the third controller is simpler than the others. So, we use this controller in the repetitive control configuration. The reference input is shown in Fig. 5. The evolution of the error energy $\left\|e^i\right\|_2$ is shown in Fig. 6. One can see that it tends to zero and hence perfect tracking is ensured. Figure 7 shows the control signal after 30 periods. In spite that the control signal is finite, there are large oscillations near the discontinuities that can damage the actuator in real applications. This is the price to pay in order to get perfect tracking.

In the following section, a non perfect tracking algorithm is proposed to overcome this difficulty.

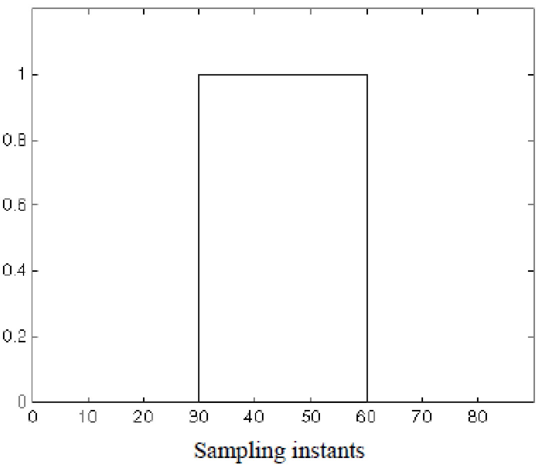

Fig. 5. Reference input signal.

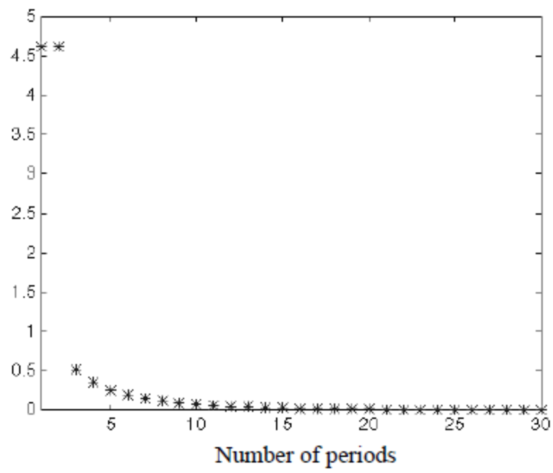

Fig. 6. Error Energy behaviour (Perfect tracking).

3.2 Non Perfect Tracking

As it appears from (9) when $G_u \neq 1$, the error after an infinite number of periods is not equal to zero. The task here is then to choose the filters G_e and G_u such that a norm of the final error is minimized. Note that the original convergence condition; $\left\| \dfrac{G_u - G_e G}{1 + GG_c} \right\|_\infty < 1$ is much less restrictive than $\left\| \dfrac{1 - G_e G}{1 + GG_c} \right\|_\infty < 1$ because we have the freedom to choose G_u.

Following the design approach proposed in [5], the repetitive control algorithm can be cast as the following minimization problem:

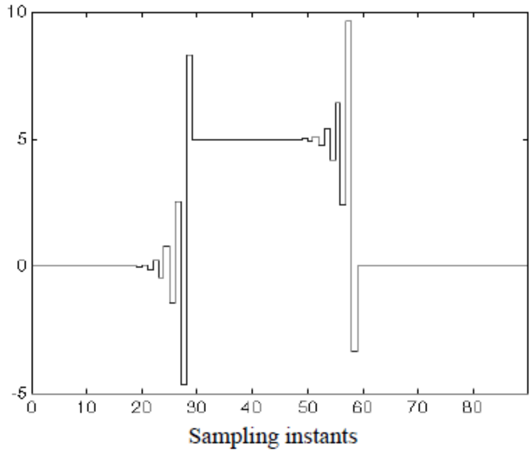

Fig. 7. Control signal behaviour at the 30th period (Perfect tracking).

Problem P1

Given the desired trajectory y_d, the plant and the controller transfer functions G and G_c, we have to find the filters G_e^* and G_u^* to minimize the total energy of the error signal $e^\infty(k)$, i.e:

$$\min_{G_e, G_u} \left(\sum_{k=0}^{N-1} [e^\infty(k)]^2 \right)^{1/2} = \min_{G_e, G_u} \|e^\infty(k)\|_2 \qquad (20)$$

where *N* is the number of time samples in one period. Equation (20) is equivalent to

$$\min_{G_e, G_u} \left\| \left[1 - G\left[(G_e + G_c)G + 1 - G_u\right]^{-1}(G_e + G_c)\right] y_d \right\|_2 \qquad (21)$$

with the convergence constraint :

$$\left\| \frac{G_u - G_e G}{1 + GG_c} \right\|_\infty < 1.$$

The solution of P1 will give the repetitive control algorithm that produces the smallest final error energy. To state the solution of P1, let us introduce the following problem:

Problem P2

Given the desired trajectory y_d and the plant transfer function *G*, find the filter H^* to solve

$$\min_H \|(1 - GH)y_d\|_2 . \qquad (22)$$

The following theorem relates the solution of problems P1 and P2.

Theorem 3 *Let H^* be the solution of P2 and G_e^* be defined by the factorization [12]*

$$H^* = T^* \cdot (G_e^* + G_c) \quad (23)$$

where T^* is an invertible filter which is designed such that $\left\| 1 - \frac{(T^*)^{-1}}{1+GG_c} \right\|_\infty < 1.$
Let

$$G_u^* = 1 - (T^*)^{-1} + G(G_e^* + G_c) \quad (24)$$

then, G_e^* and G_u^* are solutions of P1.

Proof: The proof is given in Appendix C. ■

There are three remarks that can be made. First, to solve P1, we simply solve P2 which is equivalent to find an approximate inverse of the plant transfer function. Second, the factorization given in the above theorem is not unique and hence several solutions of P1 may exist. Finally, we have formulated this problem for a fixed reference signal y_d. If we want to solve the problem for any references, then we have to solve the following minimization problem:

Problem P1'

Given the plant and the controller transfer functions G and G_c, find the filters G_e^* and G_u^* to minimize the ratio of the final error signal energy to any non zero reference signal energy, i.e [21]:

$$\min_{G_e,G_u} \left\{ \sup_{y_d(k)\neq 0} \left(\frac{\sum_{k=0}^{N-1} [e^\infty(k)]^2}{\sum_{k=0}^{N-1} [y_d(k)]^2} \right)^{1/2} \right\} = \min_{\substack{G_e,G_u \\ y_d(k)\neq 0}} \frac{\|e^\infty(k)\|_\infty}{\|y_d(k)\|_\infty} \quad (25)$$

which is equivalent to

$$\min_{G_e,G_u} \left\| 1 - G\left[(G_e+G_c)G + 1 - G_u\right]^{-1}(G_e+G_c) \right\|_\infty \quad (26)$$

with the constraint

$$\left\| \frac{G_u - G_e G}{1+GG_c} \right\|_\infty < 1.$$

As in the previous case, one can show that solving P1' is equivalent to solve the following problem:

Problem P2'

Given the plant transfer function G, find the filter H^* to solve

$$\min_H \|1 - GH\|_\infty . \quad (27)$$

3.3 Proposed Solution and Convergence Analysis

We have seen that in order to solve P1', it is sufficient to solve P2' which is equivalent to find an approximate inverse of the plant transfer function. We propose to choose one of the two following forms for H^*:

The first one is given by Tomizuka *et al.* [9]:

$$H_1(z^{-1}) = k_e \frac{z^d A(z^{-1}) B^-(z)}{b \cdot B^+(z^{-1})} \quad (28)$$

with: $b \geq \max_{\omega\in[0,\pi]} |B^-(\mathrm{e}^{-j\omega})|^2$.

The second is given by Landau [20]:

$$H_2(z^{-1}) = \frac{z^{d+m^-} A(z^{-1})}{B^-(1) B^+(z^{-1})}. \quad (29)$$

We suggest to use the approximation that gives the smallest H_∞ norm expressed by (27). Moreover, we have previously shown that any solution must satisfy the convergence condition

$$\left\| \frac{G_u - G_e G}{1+GG_c} \right\|_\infty < 1. \quad (30)$$

Taking into account (23) and (24), (30) becomes

$$\left\| 1 - \frac{(T^*)^{-1}}{1+GG_c} \right\|_\infty < 1. \quad (31)$$

Let (M,φ) and (Γ,η) be respectively the gain and the phase of $GG_c(e^{-j\omega})$ and $(T^*(e^{-j\omega}))^{-1}$, this leads to

$$\left[\Gamma - 2\cos\eta - 2M\cos(\varphi - \eta)\right]_{\omega\in[0,\pi]} < 0. \quad (32)$$

One can distinguish two cases:

Case 1

If the filter T^* is chosen to be constant, then (32) becomes

$$0 < \Gamma < \min_{\omega\in[0,\pi]} \left[2(1 + M\cos\varphi)\right]. \quad (33)$$

In order to satisfy this inequality, the term $(1 + M\cos\varphi)$ must be positive for every $\omega \in [0,\pi]$. Hence, the phase φ has to satisfy the following condition

$$-a\cos\left(-\frac{1}{M}\right) \leq \varphi \leq a\cos\left(-\frac{1}{M}\right). \quad (34)$$

Case 2

If T^* is a dynamic filter, η must verify the following inequality

$$-\frac{\pi}{2} + \chi < \eta < \frac{\pi}{2} + \chi \quad (35)$$

where

$$\chi = a\tan\left(\frac{R_2}{R_1}\right)$$

with

$$\begin{array}{rcl} R_1 & = & 1 + M\cos\varphi \\ R_2 & = & M\sin\varphi. \end{array}$$

The latter inequality defines the space containing the phase η. When η is chosen, it is sufficient to determine Γ such that the following inequality is satisfied:

$$0 < \Gamma < 2\sqrt{R_1^2 + R_2^2}\cos(\eta - \chi). \qquad (36)$$

To illustrate the features of the proposed repetitive algorithm with regard to the previous one (3.1), let us take the same example.

Simulation example

Figure 8 shows the behaviour of $2(1+M\cos\varphi)$ derived from (33). One can see that we must take $\Gamma < 1.8044$. In our simulations we have chosen $\Gamma = 1$ which correspond to the filter $T^* = 1$. Then, from (29), H^* is given by

$$H^*(z^{-1}) = \frac{z^2(1 - 0.3z^{-1})}{0.14}$$

hence

$$\begin{array}{rcl} G_e^* & = & -1 - 2.14z - 7.14z^2 \\ & & G_u^* = 0.64 + 0.36z \end{array}$$

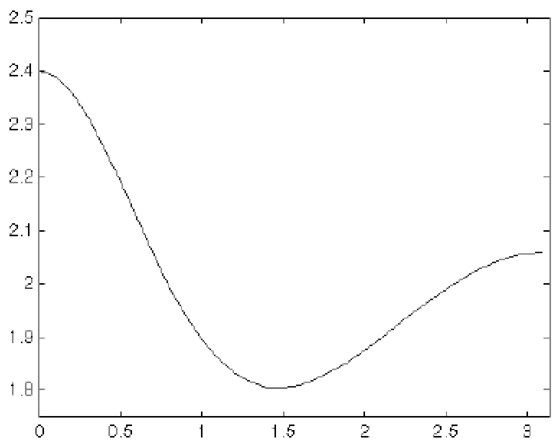

Fig. 8. Behaviour of $2(1 + M\cos\varphi)$.

Figure 9 shows the behaviour of the error energy versus the number of periods. Notice that in this case, the error energy does not tend to zero. Figure 10 shows the control signal behaviour after 30 periods. One can see that the control signal does not show any oscillations near the discontinuities as in the previous case. This is mainly due to the fact that the repetitive algorithm does not invert the process.

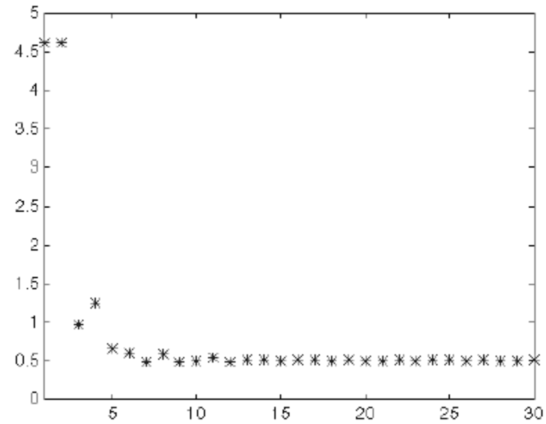

Fig. 9. Error Energy behaviour (Non perfect tracking).

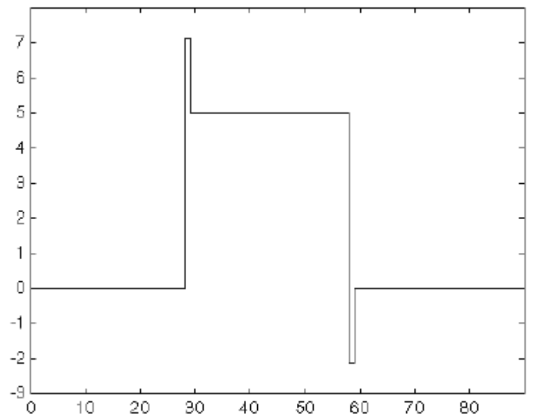

Fig. 10. Control signal behaviour at the 30th period (Non perfect tracking).

Application Example

This section concludes with an application to magnetic bearings [3, 8] control. A magnetic bearing is a device made of two main parts: an inertial wheel (rotor) and a stator (Fig. 11).

The guiding forces between the fixed part and the moving part are magnetic: the vertical sustentation is ensured by the passive magnetic bearing and the positioning in the horizontal plan is mainly due to two active magnetic bearings. When the rotor turns at high speed, there is an unbalanced movement of the inertial wheel induced by the non concordance between the geometric and inertial centers. This negative effect produces a repetitive disturbance which has to be rejected.

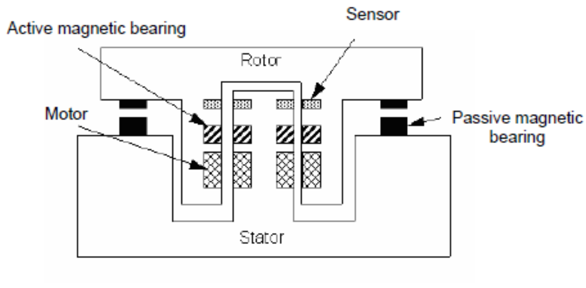

Fig. 11. Scheme of the inertial wheel.

The results obtained for this application are given on Fig. 12. They show that the repetitive algorithm is able to improve the centering of the inertial wheel. Figure 13 gives the evolution of the peak-to-peak error during the ten periods of trial where the repetitive algorithm has been applied.

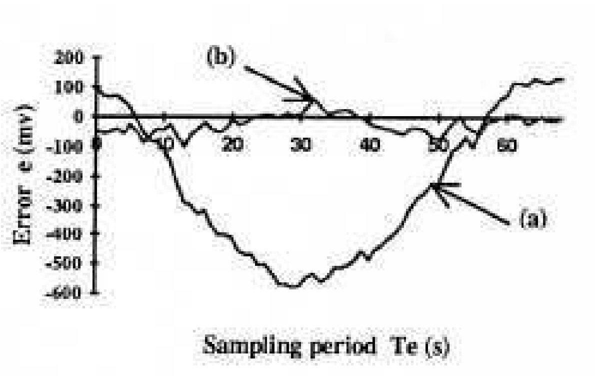

Fig. 12. Behavior of the error positioning (1 turn): (a) without repetitive algorithm; (b) with repetitive algorithm after the 10th turn.

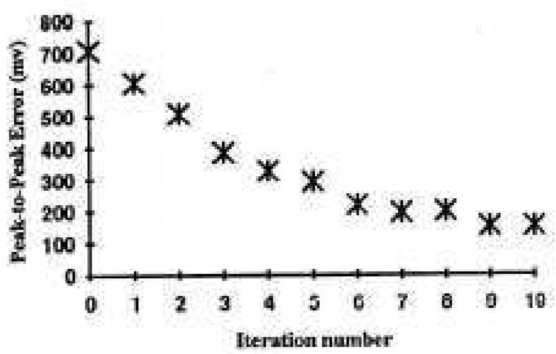

Fig. 13. Evolution of the peak-to-peak error on ten periods.

4 REPETITIVE CONTROL VIA PARAMETER ESTIMATION [18]

In the previous section we were interested in a qualitative evaluation of the behaviour of a linear repetitive control scheme. Due to the fact that we wanted to get an idea of the best possible performances, we assumed that the system to be controlled is known. In this section we partially relax this assumption. We now consider the repetitive control problem when the plant has a known structure with unknown parameters. Our approach to this problem will be based on parameter estimation technique that takes into account the control objective for finding the nominal model which is necessary for the design of the repetitive controllers [22]. Before giving this approach, let us review the prediction error identification method that will be used.

4.1 Process Identification Based on Prediction Error Identification Method

The plant model can be obtained using prediction error identification method [23] from the following model set:

$$y_m(k) = G(z^{-1},\theta)\cdot u(k) + H_n(z^{-1})\cdot v(k) \qquad (37)$$

where k denotes the sampling instant, θ denotes the parameter vector, $G(z^{-1},\theta)$ is the nominal transfer function, $H_n(z^{-1})$ represents the noise model which is assumed to be known and $v(k)$ is a white noise sequence. The best estimate of the output $y(k)$ using the measured data set $\{u(0), y(0), \ldots, u(k\text{-}1), y(k\text{-}1)\}$ is given by

$$\hat{y}(k/k-1) = H_n^{-1}(z^{-1})G(z^{-1},\theta)u(k) + \left[1 - H_n^{-1}(z^{-1})\right] y(k). \qquad (38)$$

The corresponding filtered prediction error is then given by

$$\begin{aligned}\varepsilon_f(k) &= D(z^{-1})(y(k) - \hat{y}(k/k-1)) \\ &= D(z^{-1})H_n^{-1}(z^{-1})\left[\left(G(z^{-1}) - G(z^{-1},\theta)\right)u(k)\right] + \nu(k)\end{aligned} \qquad (39)$$

where $D(z^{-1})$ is the identification filter.

The parameter vector is determined from N input/output data such that the following norm function is minimized:

$$V_N = \frac{1}{N}\sum_{k=0}^{N}\varepsilon_f^2(k). \qquad (40)$$

When $N \to \infty$, the parameter vector is given by

$$\theta^* = \underset{\theta}{\operatorname{argmin}}\left\{\frac{1}{2\pi}\int_{-\pi}^{+\pi}\left|G\left(e^{-j\omega}\right) - G\left(e^{-j\omega},\theta\right)\right|^2 \times \left|u\left(e^{-j\omega}\right)\right|^2 \left|\frac{D\left(e^{-j\omega}\right)}{H_n\left(e^{-j\omega}\right)}\right|^2 d\omega\right\}. \qquad (41)$$

4.2 Plant Model Identification for Repetitive Control

In this section, we will derive a repetitive control plant model identification procedure. More precisely, an adequate input-output identification filter is designed such that the difference between the nominal and the actual repetitive control convergence conditions is minimized.

It can easily be shown that the following inequality holds:

$$\begin{aligned}\left\|\frac{G_u(z^{-1})-G_e(z^{-1})G(z^{-1})}{1+G(z^{-1})G_c(z^{-1})}\right\|_\infty < \left\|\frac{G_u(z^{-1})-G_e(z^{-1})G(z^{-1},\theta)}{1+G(z^{-1},\theta)G_c(z^{-1})}\right\|_\infty \\ + \left\|\frac{G_u(z^{-1})-G_e(z^{-1})G(z^{-1})}{1+G(z^{-1})G_c(z^{-1})} - \frac{G_u(z^{-1})-G_e(z^{-1})G(z^{-1},\theta)}{1+G(z^{-1},\theta)G_c(z^{-1})}\right\|_\infty\end{aligned} \quad (42)$$

The transfer functions $G(z^{-1})$ and $G(z^{-1},\theta)$ are respectively the actual and the nominal plant transfer functions.

Notice that the left hand side as well as the first term of the right hand side of inequality (42) have to be less than one to ensure the convergence of the repetitive control algorithm when it is applied on both the actual system $G(z^{-1})$ and the plant model $G(z^{-1},\theta)$, i.e.:

$$\left\|\frac{G_u(z^{-1})-G_e(z^{-1})G(z^{-1})}{1+G(z^{-1})G_c(z^{-1})}\right\|_\infty < 1 \quad (43)$$

and

$$\left\|\frac{G_u(z^{-1})-G_e(z^{-1})G(z^{-1},\theta)}{1+G(z^{-1},\theta)G_c(z^{-1})}\right\|_\infty < 1. \quad (44)$$

It is clear that inequality (44) does not imply inequality (43). In order to satisfy (43), one should satisfy (44) and at the same time

$$J_{rp} = \left\|\frac{G_u(z^{-1})-G_e(z^{-1})G(z^{-1})}{1+G(z^{-1})G_c(z^{-1})} - \frac{G_u(z^{-1})-G_e(z^{-1})G(z^{-1},\theta)}{1+G(z^{-1},\theta)G_c(z^{-1})}\right\|_\infty \quad (45)$$

must be kept small. Since inequality (43) is satisfied in the repetitive control design step with respect to the filters $G_e(z^{-1})$ and $G_u(z^{-1})$, one has to minimize J_{rp} in the identification step with respect to the plant model $G(z^{-1},\theta)$. Notice that, the identification step involves minimization of H_∞ norm. Unfortunately, methods for direct optimization of the identification criterion in an H_∞ sense are not presently available [15]. To overcome this problem, a common design strategy is to minimize its H_2 norm, i.e:

$$J'_{rp} = \left\|\frac{G_u(z^{-1})-G_e(z^{-1})G(z^{-1})}{1+G(z^{-1})G_c(z^{-1})} - \frac{G_u(z^{-1})-G_e(z^{-1})G(z^{-1},\theta)}{1+G(z^{-1},\theta)G_c(z^{-1})}\right\|_2 . \quad (46)$$

The reason of this replacement is that H_2 approximation will generally yield to a reasonable nominal plant model in H_∞ sense. Such an optimization can be handled using a prediction error method together with an appropriate input-output identification as shown in the following lemma.

Lemma 1 *Assume that the plant $G(z^{-1})$ is used in a repetitive control configuration with filters $G_e(z^{-1})$ and $G_u(z^{-1})$and that the noise model $H_n(z^{-1})$ is known. Then, the limiting parameter vector θ^* minimizes J'_{rp} provided that the filter of identification is chosen as*

$$D^*(z^{-1}) = \frac{H_n(z^{-1})}{L(z^{-1})} \cdot F(z^{-1}) \cdot S_\theta(z^{-1}) \quad (47)$$

with

$$\begin{aligned}|F(z^{-1})| &= |G_u(z^{-1})G_c(z^{-1}) + G_e(z^{-1})| \\ |L(z^{-1})| &= |G_c(z^{-1})y_d(z^{-1}) + \alpha^{i-1}(z^{-1})| \\ S_\theta(z^{-1}) &= \frac{1}{1+G(z^{-1},\theta)G_c(z^{-1})}.\end{aligned}$$

Proof: The proof is given in Appendix D. ■

There are two remarks that should be pointed out. First, the definition of $D^*(z^{-1})$ involves the nominal sensitivity function $S_\theta(z^{-1})$. This property is consistent with the fact that the best model for control design requires a good knowledge of the frequency band of the control system. Second, the identification filter $D^*(z^{-1})$depends on both the estimated plant model and the repetitive controllers which are initially unknown. The implementation of such filters can only be achieved using an iterative approach. The iterative approach should alternate between identification and control design steps as shown in Fig. 14. More precisely, assuming that an appropriate plant model is available, the nominal repetitive control objective is optimized over the class of admissible controllers to obtain $G_e(z^{-1})$ *and* $G_u(z^{-1})$. Then, using these controllers, the identification objective J'_{rp} is minimized in the identification experiment with respect to the parameter vector θ leading to a new plant model $G(z^{-1},\theta)$. The entire procedure is repeated until a satisfactory performance level for the real plant is achieved.

To illustrate the behaviour of this procedure, one takes the same example as presented in (3.1) but with a parametric variation of the model at the 9th period. Figure 15 shows the error energy behaviour with this parametric variation. One can see that before the 9th period, the behaviour is the same as shown in Fig. 5 and when the parametric variation of the model is done, the error energy grows. After that it decreases until it becomes zero.

This simple example shows the effectiveness to introduce in the repetitive control structure an identification procedure that takes into account the control objective. Another simulation example concerning this procedure can be found in [18]. It shows the application of this method to a flexible transmission system. The obtained results are very satisfactory.

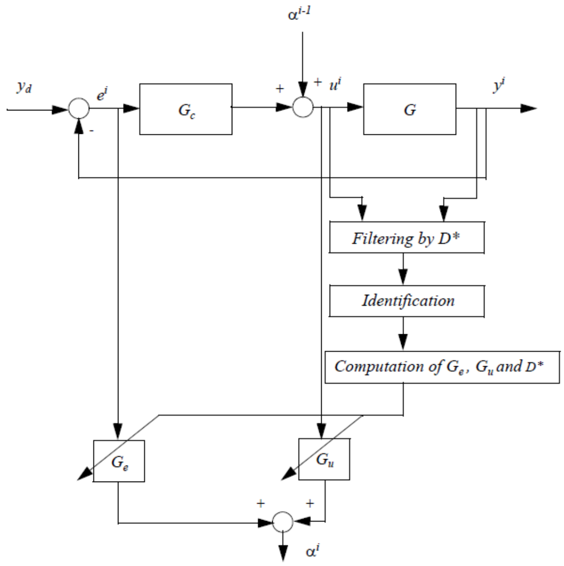

Fig. 14. Repetitive control algorithm with system identification.

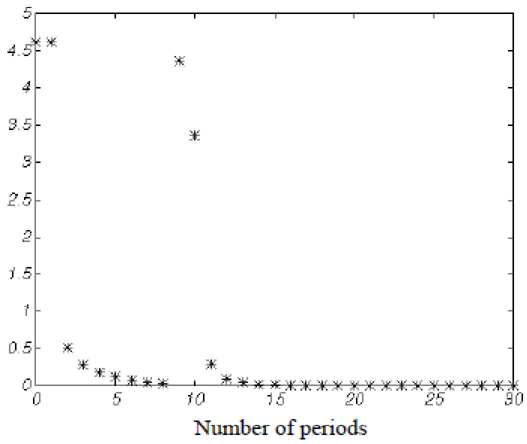

Fig. 15. Error energy behaviour with a parametric variation of the model.

5 CONCLUSION

In this paper we have considered the problem of a closed loop repetitive control scheme. First, our interest was to design the discrete time repetitive controllers that permit us to realize two objectives: perfect and non perfect tracking. In the first case the control signal after an infinite number of periods is more perturbed than in the second one. Moreover, we have considered the problem of repetitive control when the process model is unknown. Our approach was based on a parameter estimation technique that takes into account the control objective for finding the nominal model. This work permits us to obtain a robust plant model for increasing the performances of the repetitive control algorithm.

APPENDIX A PROOF OF THEOREM 1

Proof: The repetitive algorithm converges if and only if $\left\|\frac{1-G_eG}{1+GG_c}\right\|_\infty < 1$. In the frequency domain, one obtains for every $\omega \in [0,\pi]$ the following condition:

$$\left|\frac{1-G_e(e^{-j\omega})G(e^{-j\omega})}{1+GG_c(e^{-j\omega})}\right|_{\omega\in[0,\pi]} < 1.$$

Replacing G, G_e and GG_c by their respective expressions given by (1), (12) and (16), one has

$$\left|\frac{1-\frac{k_e}{b}\left|B^-(e^{-j\omega})\right|^2}{1+M(\omega)e^{j\varphi(\omega)}}\right|_{\omega\in[0,\pi]} < 1.$$

This implies that

$$\left|1-\frac{k_e}{b}\left|B^-(e^{-j\omega})\right|^2\right|_{\omega\in[0,\pi]} < \left|1+M(\omega)e^{j\varphi(\omega)}\right|_{\omega\in[0,\pi]}$$

then

$$\left|1-\frac{k_e}{b}\left|B^-(e^{-j\omega})\right|^2\right|_{\omega\in[0,\pi]} < \left(\sqrt{1+2M\cos\varphi+M^2}\right)_{\omega\in[0,\pi]}.$$

Hence, for every $\omega \in [0,\pi]$ we have $-\sqrt{1+2M\cos\varphi+M^2} < 1-\frac{k_e}{b}\left|B^-(e^{-j\omega})\right|^2 < \sqrt{1+2M\cos\varphi+M^2}$ that permits to obtain the condition.

■

APPENDIX B PROOF OF THEOREM 2

Proof: For $G_u = 1$ and from inequality (7), one has

$$\|1-G_eG\|_\infty < \|1+GG_c\|_\infty = \frac{1}{\|S\|_\infty} = MM.$$

Substituting G_e by (17) one obtains:

$$\left\|1-\frac{z^{m^-}B^-(z^{-1})}{b}\right\|_\infty < MM$$

which is equivalent to

$$\left\|b_0\left(1-z^{m^-}\right)+\cdots+b_{m^--1}(1-z)\right\|_\infty < |b|\cdot MM.$$

Moreover, one has

$$\left\|b_0\left(1-z^{m^-}\right)+\cdots+b_{m^--1}(1-z)\right\|_\infty$$
$$<\left\|b_0\left(1-z^{m^-}\right)\right\|_\infty+\cdots+\|b_{m^--1}(1-z)\|_\infty \text{ and}$$

$$\left\|\left(1-z^{m^-}\right)\right\|_\infty=\cdots=\|(1-z)\|_\infty=2$$

hence, one has $\left\|b_0\left(1-z^{m^-}\right)+\cdots+b_{m^--1}(1-z)\right\|_\infty < 2\left(|b_0|+\cdots+|b_{m^--1}|\right)$. The convergence condition is then satisfied if

$$|b_0|+\cdots+|b_{m^--1}|<\frac{|b|\cdot MM}{2}.$$

■

APPENDIX C PROOF OF THEOREM 3

Proof: If G_u^* and G_e^* are defined as $G_u^*=1-(T^*)^{-1}+G(G_e^*+G_c)$ and $G_e^*=(T^*)^{-1}H^*-G_c$,
where H^* is the solution of problem P2, then we have

$$\min_{G_e,G_u}\left\|\left[1-G\left[(G_e^*+G_c)G+1-G_u^*\right]^{-1}(G_e^*+G_c)\right]y_d(k)\right\|_2$$
$$=\|(1-GH^*)\,y_d(k)\|_2$$
$$=\min_H\|(1-GH)\,y_d(k)\|_2\,.$$

Note that G_u^* and $G_e^*$are candidate solutions for problem P1 as far as they verify the constraint

$$\left\|\frac{G_u^*-G_e^*G}{1+GG_c}\right\|_\infty=\left\|1-\frac{(T^*)^{-1}}{1+GG_c}\right\|_\infty<1.$$

So, all solutions of P2 are candidate solutions for P1.
To show that G_u^* and G_e^* are unique solution of P1, we are going to show that it does not exist any solution of P1 that does not lead to a solution of P2.
To do this, let $\overline{G_u}\neq G_u^*$ and $\overline{G_e}\neq G_e^*$ the solutions of problem P1, but we assume that they are not solutions of problem P2, i.e. $\overline{G_u}$and $\overline{G_e}$ do not verify the relation defined by
$\overline{G_u}=1-(T^*)^{-1}+G(\overline{G_e}+G_c)$ with $H^*=T^*\cdot(\overline{G_e}+G_c)$.
One defines $\overline{T}=\left((\overline{G_e}+G_c)G+1-\overline{G_u}\right)^{-1}$. Note that $\overline{T}$ exists and is invertible because $\overline{G_u}$ and $\overline{G_e}$ are solutions of Problem P1. Moreover $\overline{G_u}$ and $\overline{G_e}$ satisfy the constraint $\left\|\frac{\overline{G_u}-\overline{G_e}G}{1+GG_c}\right\|_\infty<1.$
One can then write:

$$\min_{G_e,G_u}\left\|\left[1-G\left[(G_e+G_c)G+1-G_u\right]^{-1}(G_e+G_c)\right]y_d(k)\right\|_2$$
$$=\min_{G_e,T}\|[1-GT(G_e+G_c)]\,y_d(k)\|_2$$
$$=\left\|\left[1-G\left[(\overline{G_e}+G_c)G+1-\overline{G_u}\right]^{-1}(\overline{G_e}+G_c)\right]y_d(k)\right\|_2$$
$$=\left\|\left[1-G\overline{T}(\overline{G_e}+G_c)\right]y_d(k)\right\|_2\,.$$

Further, let $\overline{H}=\overline{T}\cdot(\overline{G_e}+G_c)$, then one has

$$\min_{G_e,T,T^{-1}}\|[1-GT(G_e+G_c)]y_d(k)\|_2=\min_H\|(1-GH)y_d(k)\|_2$$
$$=\left\|\left[1-G\overline{T}(\overline{G_e}+G_c)\right]y_d(k)\right\|_2=\left\|(1-G\overline{H})y_d(k)\right\|_2\,.$$

This means that $\overline{H}$ is solution of P2 and hence

$$\left\|(1-G\overline{H})y_d(k)\right\|_2\leq\|(1-GH^*)y_d(k)\|_2$$

which is contradictory because H^* is the solution of P2. Then all solutions (G_u,G_e) for Problem P1 will give the solution for Problem P2.

■

APPENDIX D PROOF OF LEMMA

Proof: One has

$$J'_{rp}=\left\|\frac{G_u(z)-G_e(z)G(z)}{1+G(z)G_c(z)}-\frac{G_u(z)-G_e(z)G(z\theta)}{1+G(z\theta)G_c(z)}\right\|_2$$
$$=\left\|\frac{(G_u(z)G_c(z)+G_e(z))(G(z)-G(z\theta))}{(1+G(z)G_c(z))(1+G(z\theta)G_c(z))}\right\|_2.$$

Using Parseval's theorem, it yields

$$J'_{rp}=\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{|G_u(e^{j\omega})G_c(e^{j\omega})+G_e(e^{j\omega})|^2|G(e^{j\omega})-G(e^{j\omega},\theta)|^2}{|(1+G(e^{j\omega})G_c(e^{j\omega}))(1+G(e^{j\omega},\theta)G_c(e^{j\omega}))|^2}d\omega.$$

Taking into account that

$$[1+G(z)\,G_c(z)]\,u^i(z)=G_c(z)\,y_d(z)+\alpha^{i-1}(z)$$

it yields

$$J'_{rp}=\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{\left|F\left(e^{j\omega}\right)u^i\left(e^{j\omega}\right)\right|^2\left|S_\theta\left(e^{j\omega}\right)\right|^2}{|L(e^{j\omega})|^2}\times$$
$$\left|G\left(e^{j\omega}\right)-G\left(e^{j\omega},\theta\right)\right|^2d\omega\cdot$$

Moreover, the prediction error identification criterion is given by

$$\theta^*=\underset{\theta}{\operatorname{argmin}}\left\{\frac{1}{2\pi}\int_{-\pi}^{+\pi}|G(e^{j\omega})-G(e^{j\omega},\theta)|^2\right.$$
$$\left.\times|u(e^{j\omega})|^2\left|\frac{D(e^{j\omega})}{H_n(e^{j\omega})}\right|^2d\omega\right\}.$$

The result readily follows by substituting *D*(z),* in this equality.

■

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