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THE INFLUENCE OF THE TEMPERATURE FACTOR ON DEFORMABILITY OF THE PLASTIC MEDIUM

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Using the solution of a closed problem of the theory of plasticity some analytic expressions were obtained for determination of the strain parameters of zone of deformation in view of the temperature factor.

Key words: *plastic deformation, analytic expressions, strain, zone of deformation*

Utjecaj temperaturnog čimbenika na deformabilnost plastičnog sredstva. Rabeći rješenje završnog problema teorije plastičnosti ustrojeni su analitički izrazi za određivanje parametra naprezanja u zoni deformacije ovisno od temperaturnog čimbenika.

Ključne riječi: *plastična deformacija, analitički izrazi, naprezanje, zona deformacije*

INTRODUCTION

The system of equations of the theory of plasticity for a flat problem is [1]:

$$\frac{\partial\sigma_x}{\partial x}+\frac{\partial\tau_{xy}}{\partial y}=0,\qquad \frac{\partial\tau_{yx}}{\partial x}+\frac{\partial\sigma_y}{\partial y}=0,$$

$$\left(\sigma_x-\sigma_y\right)^2+4\tau_{xy}^2=4\kappa^2,\qquad \frac{\sigma_x-\sigma_y}{2\tau_{xy}}=\frac{\xi_x-\xi_y}{\gamma_{xy}}=f,$$

$$\xi_x+\xi_y=0,\qquad \frac{\partial^2\xi_x}{\partial y^2}+\frac{\partial^2\xi_y}{\partial x^2}=\frac{\partial^2\gamma_{xy}}{\partial y\partial x},$$

$$\frac{\partial T}{\partial t}=a^2\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right). \tag{1}$$

Boundary conditions are set in stresses:

$$\tau_n=-k\cdot\sin\left(A\Phi-2\alpha\right),$$

or

$$\tau_n=\left(\frac{\sigma_x-\sigma_y}{2}\cdot\sin 2\alpha-\tau_{xy}\cdot\cos 2\alpha\right).$$

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Conditions set the trigonometric distribution of the contact stresses on en oblique area. Function $A\Phi$ is determined by the solution of problem, angle on a contact surface - by α-geometry and in the general case is a variable value.

It is necessary to note that the system (1) contain seven equations and eight unknown values. For the present instance it is necessary to have one more equation. Really, the boundary conditions will be identically satisfied, if assume

$$\tau_{xy}=k\cdot\sin A\Phi.$$

The last equation is equation of thermal conductivity [2]. The first 6 equations (1) can be reduced to equations of the second order in the form [3]:

$$\frac{\partial^2\tau_{xy}}{\partial x^2}-\frac{\partial^2\tau_{xy}}{\partial y^2}=\pm 2\frac{\partial^2}{\partial x\partial y}\kappa\cdot\sqrt{1-\left(\frac{\tau_{xy}}{\kappa}\right)^2},$$

$$\frac{\partial^2\xi_x}{\partial y^2}-\frac{\partial^2\xi_x}{\partial x^2}=\pm 2\frac{\partial^2}{\partial y\partial x}\cdot\frac{1}{f}\xi_x. \tag{2}$$

THEORETICAL DEVELOPMENS

The value Φ is used to establish connection between shearing and linear rates of deformations. One can see from the papers [4, 5] that solutions for stresses and rates of deformations have the same formal form and can be presented as:

$$\tau_{xy} = \kappa \cdot \sin A\Phi, \quad \xi_x = -\xi_y = \beta \cdot \cos B\Phi \tag{3}$$

where

$$k = k(x,y), \quad \beta = \beta(x,y).$$

Then

$$\kappa = H_\sigma \cdot \exp\theta', \quad \beta = H_\xi \cdot \exp\theta''.$$

Substituting (3) and the last relations into (2) we obtain

- for stresses:

$$\Big[H_\sigma \cdot \Big[\theta'_{xx} + \left(\theta'_x + A\Phi_y\right)^2 - \theta'_{yy} - \left(\theta'_y - A\Phi_x\right)^2\Big] +$$
$$+ (H_\sigma)_{xx} + 2(H_\sigma)_x \cdot \left(\theta'_x + A\Phi_y\right) - (H_\sigma)_{yy} -$$
$$- 2(H_\sigma)_y \cdot \left(\theta'_y - A\Phi_x\right)\Big] \cdot \sin A\Phi +$$
$$+ \Big[2H_\sigma \cdot \left(A\Phi_x - \theta'_y\right) \cdot \left(\theta'_x + A\Phi_y\right) -$$
$$- 2(H_\sigma)_y \cdot \left(A\Phi_y + \theta'_x\right) + 2(H_\sigma)_x \cdot \left(A\Phi_x - \theta'_y\right) +$$
$$+ H_\sigma \cdot \left(A\Phi_{xx} - A\Phi_{yy}\right)\Big] \cdot \cos A\Phi =$$
$$-2H_\sigma \cdot A\Phi_{xy} \cdot \sin A\Phi + \left[2(H_\sigma)_{xy} + 2H_\sigma \cdot \theta'_{xy}\right] \cdot \cos A\Phi, \tag{4}$$

- for rates of deformations:

$$\Big[H_\xi \cdot \Big[-\theta''_{xx} - \left(\theta''_{xx} + B\Phi_y\right)^2 + \theta''_{yy} + \left(\theta''_y - B\Phi_x\right)^2\Big] -$$
$$- (H_\xi)_{xx} - 2(H_\xi)_x \cdot \left(\theta''_x + B\Phi_y\right) + (H_\xi)_{yy} +$$
$$+ 2(H_\xi)_y \cdot \left(\theta''_y - B\Phi_x\right)\Big] \cdot \sin B\Phi +$$
$$+ \Big[2H_\xi \cdot \left(B\Phi_x - \theta''_y\right) \cdot \left(\theta''_x + B\Phi_y\right) -$$
$$- 2(H_\xi)_y \cdot \left(B\Phi_y + \theta''_x\right) + 2(H_\xi)_x \cdot \left(B\Phi_x - \theta''_y\right) +$$
$$+ H_\xi \cdot \left(B\Phi_{xx} - B\Phi_{yy}\right)\Big] \cdot \cos B\Phi =$$
$$2H_\xi \cdot B\Phi_{xy} \cdot \sin B\Phi + \left[2(H_\xi)_{xy} + 2H_\xi \cdot \theta''_{xy}\right] \cdot \cos B\Phi. \tag{5}$$

Then $\theta' = -A \cdot \theta$, $\theta'' = -B \cdot \theta$. If $A = B$ then $\theta' = \theta''$.

While analyzing (4), (5) we make sure that the last equations belong formally to the same type, and in this case appear the identical simplifying conditions for stresses as well as for rates of deformations.

For stresses

$$\theta'_x = -A\Phi_y, \quad \theta'_y = A\Phi_x,$$

in this case

$$\theta'_{xx} = -A\Phi_{yx}, \quad \theta'_{yy} = A\Phi_{xy}, \quad \theta'_{xy} = A\Phi_{yy} = -A\Phi_{xx}.$$

For rates of deformations

$$\theta''_x = -B\Phi_y, \quad \theta''_{yy} = B\Phi_x,$$

whence

$$\theta''_{xx} = -B\Phi_{yx}, \quad \theta''_{yy} = B\Phi_{xy}, \quad \theta''_{xy} = B\Phi_{yy} = -B\Phi_{xx}.$$

That allows, with above presented limitations imposed on functions to obtain solutions in stresses and rates of deformations. Functions $\theta, A\Phi, B\Phi$ are harmonically ones satisfying Laplace's equation. In general case take place equations for determination of the components of tensor of stresses and rates of deformations:

$$\tau_{xy} = C_\sigma \cdot \exp\theta' \cdot \sin A\Phi,$$
$$\sigma_x = C_\sigma \cdot \exp\theta' \cdot \cos A\Phi + f(x) + C,$$
$$\sigma_y = -C_\sigma \cdot \exp\theta' \cdot \cos A\Phi + f(x) + C,$$
$$\gamma_{xy} = C_\xi \cdot \exp\theta'' \cdot \sin B\Phi,$$
$$\xi_x = -\xi_y = C_\xi \cdot \exp\theta'' \cdot \cos B\Phi$$

with

$$\theta'_x = -A\Phi_y, \quad \theta'_y = A\Phi_x,$$
$$\theta''_x = -B\Phi_y, \quad \theta''_y = B\Phi_x.$$

Let us consider the deformational problem. Analysis shows that the index of exponent θ'' in equation (5) can be presented as a complex function depending on non-uniformity of deformed state (contact friction F) end temperature T. In the last case it is possible, if the temperature fields are described by the same dependences on coordinates, as the rates of deformations. After suitable transformations one can show that

$$\theta''_x = \theta''_F \cdot F_x + \theta''_T \cdot T_x, \quad \theta''_y = \theta''_F \cdot F_y + \theta''_T \cdot T_y,$$

or

$$\theta''_x = \theta''_F \cdot F_x + \theta''_T \cdot T_x = -B\Phi_y,$$
$$\theta''_y = \theta''_F \cdot F_y + \theta''_T \cdot T_y = B\Phi_x.$$

The second derivatives have appearance

$$\theta''_{xx} = \left(\theta''_F \cdot F_x + \theta''_T \cdot T_x\right)_x = -B\Phi_{yx},$$

$$\theta''_{yy} = \left(\theta''_F \cdot F_y + \theta''_T \cdot T_y\right)_y = B\Phi_{xy},$$

$$\theta''_{xy} = \left(\theta''_F \cdot F_x + \theta''_T \cdot T_x\right)_y = -B\Phi_{yy},$$

$$\theta''_{yx} = \left(\theta''_F \cdot F_y + \theta''_T \cdot T_y\right)_x = B\Phi_{xx}.$$

Equating the mix derivatives with θ, we shall obtain

$$\mathrm{d}\theta''_2 = \theta''_F \mathrm{d}F, \quad \mathrm{d}\theta''_3 = \theta''_T \mathrm{d}T, \quad \mathrm{d}\theta'' = \mathrm{d}\theta''_2 + \mathrm{d}\theta''_3.$$

Whence

$$\theta'' = \theta''_2 + \theta''_3, \quad \theta'' = -\left(B_2\theta + B_3\theta\right),$$

or

$$B = B_2 + B_3,$$

where B_2 and B_3 are the constant coefficients determining the influence of the contact friction and the temperature the strain state. Then

$$\gamma_{xy} = C_\xi \cdot \exp\left(-B_2\theta\right) \cdot \exp\left(-B_3 \cdot \theta\right) \cdot \sin B\Phi,$$

$$\xi_x = -\xi_y = C_\xi \cdot \exp\left(-B_2 \cdot \theta\right) \cdot \exp\left(-B_3 \cdot \theta\right) \cdot \cos B\Phi. \quad (6)$$

It is necessary to bring the expressions (6) to correspondence with solutions of temperature problem. As the functions θ and Φ are harmonic ones satisfying Cauchy-Riemann's relations, the proposed approaches can be used while solving the stationary equation of thermal conductivity too. Let us sleek at first the solution for the stationary problem in the form [1]:

$$T = C_T \cdot \exp\theta''' \cdot \left(\cos B_4\Phi + \sin B_4\Phi\right) \quad (7)$$

at

$$\theta'''_x = -B_4\Phi_y, \quad \theta'''_y = B_4\Phi_x.$$

Substituting (7) in equation of thermal conductivity and simplifying it we shall obtain

$$\left[\theta'''_{xx} + \left(\theta'''_x\right)^2 - \left(B_4\Phi_y\right)^2 + \theta'''_{yy} + \left(\theta'''_y\right)^2 - \left(B_4\Phi_x\right)^2\right] \cdot$$

$$\cdot \left(\cos B_4\Phi + \sin B_4\Phi\right) - \left(2B_4\Phi_x \cdot \theta_x + B_4\Phi_{xx} +\right.$$

$$\left. +2B_4\Phi_y \cdot \theta_y + B_4\Phi_{yy}\right) \cdot \left(\sin B_4\Phi - \cos B_4\Phi\right) = 0.$$

Solution will take place when $\theta'''_x = -B_4\Phi_y$, $\theta'''_y = B_4\Phi_x$, in the present case $\theta'''_{xx} = -B_4\Phi_{yx}$, $\theta'''_{yy} = B_4\Phi_{xy}$, $B_4\Phi_{xx} = -B_4\Phi_{yy} = \theta'''_{yx}$. Substituting the last relations in equation of thermal conductivity we obtain identity. Like in the paper [1] one can show that here takes place the solution for the non-stationary problem

$$T = C_T \cdot \exp\theta''' \cdot \left(\cos B_4\Phi + \sin B_4\Phi\right) +$$

$$+\sum_{k=1}^{\infty}\left[C'_1 \cdot \exp\left(-a^2 \cdot \lambda_k \cdot t\right) \cdot \cos\left(\sqrt{\frac{\lambda_k}{2}} \cdot (y - x) + C_{1n}\right)\right] +$$

$$+\sum_{k=1}^{\infty}\left[C''_2 \cdot \exp\left(-a^2 \cdot \lambda_k \cdot t\right) \cdot \sin\left(\sqrt{\frac{\lambda_k}{2}} \cdot (y - x) + C_{2m}\right)\right].$$

In general form we have

$$T = C_T \cdot \exp\theta''' \cdot \left(\cos B_4\Phi + \sin B_4\Phi\right) + T',$$

where T' is non-stationary part of the temperature field, and

$$T'' = T - T' = C_T \cdot \exp\theta''' \cdot \left(\cos B_4\Phi + \sin B_4\Phi\right),$$

where T'' is a stationary part of the temperature field.

The different types of differential equations of the system (1) are connected really though identical function dependences on coordinates. The same variable values are presents in the fields of stresses, rates of deformations and temperatures. In this case

$$\theta' = -A\theta, \quad \theta'' = -\left(B_2\theta + B_3\theta\right), \quad \theta''' = B_4\theta.$$

Thus, it is possible to express through the commune functions the intensities of tangential stresses and rates of shearing deformations, i.e.

$$T_i = k = C_\sigma \cdot \exp\left(-A\theta\right),$$

$$H_i = 2\beta = 2C_\xi \cdot \exp\left[-\left(B_2\theta + B_3\theta\right)\right].$$

Consequently, $T_i = \alpha \cdot \left(H_i\right)^m$, where $m = \dfrac{A}{B}$.

Besides that, it follows from solution (7) that correspondence takes place not only between the fields of stresses and rates of deformations, but between the fields of rates of deformations and temperatures. Really,

$$\exp(-\theta) = \left(\frac{H_i}{2C_\xi}\right)^{\frac{1}{B}} = \left(\frac{T''}{C_T \cdot \left(\cos B_4\Phi + \sin B_4\Phi\right)}\right)^{\frac{1}{B_4}}.$$

Substituting the value of exponent $\exp(-\theta)$ in expression for intensity of rates of shearing deformations H_i, we shall obtain

$$H_i = 2\beta = A_2 \cdot \exp(-B_2\theta) \cdot T^{m_2}, \tag{8}$$

where $m_2 = -\frac{B_3}{B_4}$, is the variable value determined by coordinates of zone, of deformation, is the constant parameter taking info account the non-uniformity of the strain state.

Or $H_i = H_i' \cdot (T'')^{m_2}$, where H_i' is intensity of rates of shearing deformations taking into account the non-uniformity of plastic deformation.

EXPERIMENT CONFIRMATION

The expression (8) shows that the non-uniformity of the plastic deformation is determined non only by the contact friction, but by temperature distribution too. In the paper [6] are presented the results of experimental investigations determining the influence of the strip temperature gradient on distribution of the section.

The obtained theoretical result is in correspondence with experimental data and it can be used for analysis of the strain state of metal the different initial temperature condition and techniques of metal forming.

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List of symbols

- σ - normal components of the stress tensor
- τ - tangential components of the stress tensor
- ξ - linear components of the strain rate tensor
- γ - shear components of the strain rate tensor
- τ_n - tangential contact stress on the arbitrary inclined area
- α - angle of inclination of the contact area
- k - strength of the shearing plastic deformation
- Φ - a harmonic function depending on coordinates of zone of deformation and being the argument of a trigonometric function
- A - a constant value characterizing the trigonometric function for the state of stress of plastic medium
- B - a constant value characterizing the trigonometric function for the state of strain of plastic medium
- θ' - a harmonic function, exponential index, characterizing distribution of the strength of shearing in zone of reduction
- θ'' - a harmonic function, exponential index, characterizing distribution of the rate intensity for shearing in zone of reduction
- θ''' - a harmonic function, exponential index, characterizing distribution of temperature in zone of reduction
- H_σ - function of coordinates determining the state of stress of plastic medium
- H_ξ - function of coordinates determining the state of strain of plastic medium
- C_σ - a constant value determining the state of stress of plastic medium
- C_ξ - a constant value characterizing the state of strain of plastic medium
- θ''_x - derivate of harmonic function θ'' with respect to the coordinate x
- T - temperature of k-th point
- T_i - intensity of tangential stress
- H_i - intensity of shearing rates
- C_T - a constant value characterizing the temperature field
- t - time
- a - coefficient of temperature conductivity
- λ - coefficient determining relation between function and its derivate