

APPLICATION OF FINITE ELEMENT ANALYSIS OF THIN STEEL PLATE WITH HOLES

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Original scientific paper

Stress and strain under the influence of external loads, in many elements of structures, cannot be obtained in closed form. Usually, the real loads and conditions of support of structures are such that the analysis cannot be applied exactly, but only approximately. On the other hand, the stress distribution is governed not only by load and exploitation conditions, but also by the geometric shape of the considered parts, which may be very complex. A special problem occurs in structural elements, which are weakened by some of the holes of various shapes. In the analysis and calculation, these structural systems are, typically, introduced as plates and frames, which is sufficiently accurate for the global behaviour of structural building systems. However, consideration of local influences and stress concentrations around the hole are not included in these calculations. Numerical methods are commonly used for such cases. The finite element method is successfully applied in solving such problems in the linear and nonlinear analysis, and it is applied in the presented paper. In this paper the metal plates with rectangular holes are examined. Numerical results are presented. It is shown that the position of the hole, relative to the direction of external stress, significantly influences distribution of stress around the hole. Also, there are clearly observed places with significant concentration of stress.

Key words: *finite elements method, plates, rectangular holes, stress concentration, stress distribution*

Primjena metode konačnih elemenata u analizi tankih čeličnih ploča s otvorima

Izvorni znanstveni članak

Raspodjela naprezanja i deformacija pod djelovanjem vanjskog opterećenja, kod mnogih elemenata konstrukcija, ne može se dobiti u zatvorenom obliku. Obično su realna opterećenja i uvjeti oslanjanja konstrukcija takvi da se u analizu ne mogu unijeti egzaktno, već samo približno. S druge strane, raspodjela naprezanja je uvjetovana ne samo opterećenjem i eksploatacijskim uvjetima, već i geometrijskim oblikom razmatranih elemenata, koji mogu biti vrlo složeni. Poseban problem se javlja kod konstrukcijskih elemenata koji su oslabljeni otvorima različitog oblika. U analizi i proračunu, ovi se konstrukcijski sustavi, obično, uvode kao ploče i okviri nosači, što je za globalno ponašanje konstrukcijskog sustava objekta dovoljno točno. Međutim, razmatranje lokalnih utjecaja, odnosno koncentracije naprezanja oko otvora, nije obuhvaćeno takvim proračunima. Za takve slučajeve obično se primjenjuju numeričke metode. Metoda konačnih elemenata se vrlo uspješno primjenjuje u rješavanju ovakvih problema u okviru linearne i nelinearne analize, pa je primijenjena i u ovom radu. U ovom radu razmatrane su metalne ploče s otvorima pravokutnog oblika. Prikazani su numerički rezultati. Pokazano je da položaj otvora u odnosu na pravac djelovanja vanjskih opterećenja značajno utječe na raspodjelu naprezanja oko otvora. Također, jasno se zapažaju mjesta s povećanom koncentracijom naprezanja.

Ključne riječi: *koncentracija naprezanja, metoda konačnih elemenata, ploče, pravokutni otvori, raspodjela naprezanja*

1

Introduction

Uvod

It is known that elements of structures, during the exploitation, are exposed to different types of loads. During certain time intervals load may be static or dynamic. The same, in certain parts of the construction, it can cause states such as dynamical, thermal and other stresses. Indicators of these states are stress, strain, vibration, temperature, etc. The most significant stress state is the one on the basis of which may be determined basic dimensions of parts of a structure. Besides, loads can cause such states in which elements of structures cannot properly perform their functions. Therefore, for evaluation of safety and reliability of components and structures as a whole, [1, 2, 3, 12], it is necessary to know the size and distribution of stress, which depends on the size and type of load. These are also influenced by the size and type of stress, as well as by the shape of that element, material, location in relation to the direction of external loads, etc.

Structural components that form part of a structure are, in most cases, of complex geometric shapes. Examinations have shown that, in places where cross section decreases, the stress increases. This phenomenon is called the stress concentration. Typical sources of stress concentration in the elements of design are the openings that may be of different shapes (square, rectangular, trapezoidal, triangular, elliptical, circular, semicircular, etc.).

The rectangular shaped hole is very distinctive, because of sharp transitions between sides. In order to mitigate the

effect of stress concentration, these cross section shapes are made with roundness. This paper describes how the size and distribution of stress is affected by the position of the rectangular opening in relation to the direction of external load.

The plate with a rectangular hole has been examined. The problem comes down to solving of partial differential equations that provide a link between the stress and strain and external loading. The procedure of analytical solution in this case is quite complex, since then the boundary conditions at the edges of the hole have to be considered.

Methods of approximate solving of these specific tasks can be seen, for example, in the works of N. F. Gurijev [10], M. P. Šermetjev [11] and S. Timoshenko [5], who applied one of the basic numerical methods – Finite Difference Method based on replacing of differential equations with corresponding difference equations, and others. Mushelišvili N. I. [7] showed that the solution of such problems could be obtained via analytic functions of complex variables and mapping functions, which are also a complex mathematical task.

Here is applied the finite element method (FEM), which has already been efficiently used in similar problems, [14, 15, 16, 18, 19, 20], where the authors have provided solutions for different forms of one or more openings.

2

FEM in linear elasticity

MKE u linearnoj elastičnosti

Most problems in the Theory of Elasticity [4, 5, 6, 7, 9],

are described with partial differential equations, with appropriate boundary conditions, or as variational methods. To find some analytic solutions of particular problems is a difficult and often insoluble task. Also, differential equations, which describe the state of stress and the state of strain, of some complex geometric shapes and complicated boundaries, very often cannot be solved analytically. In that case approximate solutions may be found by application of numerical methods, like in [8, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 26].

The main characteristic of numerical methods is that the fundamental equations of the Theory of Elasticity, including boundary conditions, can be solved numerically [13]. During this process, the obtained solutions are approximate, but they can always be as accurate as it is wanted.

The Finite Elements Method [13] is one of the most often applied methods in many fields, especially in technical considerations, as is shown in [18, 19, 20, 21, 22, 23, 24, 26]. Because of cumbersome calculations, the application of this method includes the use of computer with both great capacity and speed of calculating. Unlike analytic methods, this method is used to solve very complex problems, with real boundary conditions, complex geometric shapes and external loads, etc.

In this paper, for the modelling of the continuum in plane condition of stress and strain, where plates belong too, 2D finite elements are used [13]. The principle of minimum potential energy and the displacement method, as a form of the finite element method, are used [8, 13]. Special mathematical models have been established for simulation of real loads and boundary conditions in the parts of metal structures with geometrical discontinuity.

For the modelling of the continuum in plane condition of stress and strain, where plates belong too, 2D finite elements are used. In plane condition of strain the material is deformed in identical way in all planes parallel to one plane, for example, xy plane. Internal distribution of displacement in the finite element itself is defined by interpolation functions.

The finite element, which approximates the displacement field, must have nodes lying in the xy plane with displacements having components in the direction of x and y axis. Thus, the displacement field of points can be defined by a vector field, [8, 13, 18], in the following way:

$$\{s\} = \mathbf{s} = \begin{Bmatrix} u \\ v \end{Bmatrix} = [h] \mathbf{q}_K = [h] \{q\}_{(K)} \tag{1}$$

Interpolation of geometry can be written in the following form:

$$x = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} x \\ y \end{Bmatrix} = HX, \tag{2}$$

where H is interpolation matrix.

The equations (1) and (2), as a general case, can be written in component form in the following way:

$$x_i = \sum_{k=1}^N h_k X_i^k \quad i = 1,2 \tag{3}$$

$$u_i = \sum_{k=1}^N h_k U_i^k \quad i = 1,2. \tag{4}$$

The interpolation functions can be linear or higher order polynomials [8, 13, 18]. Fig. 1 shows a four node linear 2D finite element, as a general case, where the interpolation functions $h_k(r, s)$ may be expressed as:

$$\begin{aligned} h_1 &= \frac{1}{4}(1+r)(1+s) \\ h_2 &= \frac{1}{4}(1-r)(1+s) \\ h_3 &= \frac{1}{4}(1-r)(1-s) \\ h_4 &= \frac{1}{4}(1+r)(1-s) \end{aligned} \tag{5}$$

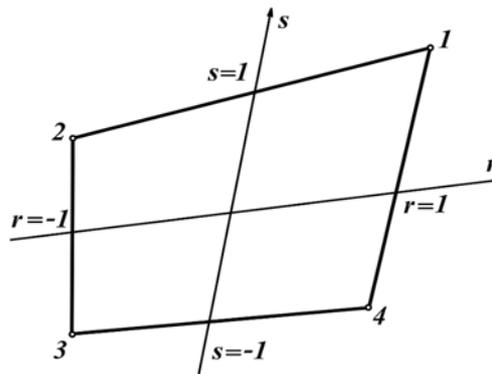


Figure 1 Two dimensional element
Slika 1. Dvodimenzionalni konačni element

The derivatives of interpolation functions per x and y coordinates are determined by using Jacobian or inverse Jacobian in the following manner:

$$\begin{aligned} h_{k,j} &= \frac{\partial h_k}{\partial r} \frac{\partial r}{\partial x_j} + \frac{\partial h_k}{\partial s} \frac{\partial s}{\partial x_j} = \\ &= J_{j1}^{-1} \frac{\partial h_k}{\partial r} + J_{j2}^{-1} \frac{\partial h_k}{\partial s}. \end{aligned} \tag{6}$$

In this case Jacobian J and inverse Jacobian J^{-1} are in the following form:

$$J = \begin{bmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \frac{\partial x}{\partial s} & \frac{\partial y}{\partial s} \end{bmatrix} \tag{7}$$

$$J^{-1} = \begin{bmatrix} \frac{\partial r}{\partial x} & \frac{\partial s}{\partial x} \\ \frac{\partial r}{\partial y} & \frac{\partial s}{\partial y} \end{bmatrix},$$

and the components (members) of the Jacobian:

$$J_{mn} = \sum_{k=1}^4 \frac{\partial h}{\partial r_m} X_n^k, \tag{8}$$

where: $r_1=r$ and $r_2=s$.

In tensor notation, Cauchy's kinematic equations can briefly be written:

$$\{\varepsilon\} = [d] \{s\} \text{ or } \varepsilon = d s, \quad (9)$$

where d is differential operator and s is displacement vector. Equilibrium conditions between the internal and external forces on the part of the contour where the contour conditions are given by surface forces are expressed in Cauchy's equations

$$[d_s]^T \{\sigma\} = \{p_n\} \text{ or } d_s^T \sigma = p_n, \quad (10)$$

where $[d_s]^T$ is transposed matrix of the matrix d_s whose elements are cosines of the angles which the normal line n makes with the axes x, y at the points of the contour area.

The general form of constitutive equations, that is, the relation between the components of stress and components of strain for elastic material which represents a generalization of the known *Hooke's law*, can be shown as

$$\{\sigma\} = [D] \{\varepsilon\} \text{ or } \sigma = D \varepsilon, \quad (11)$$

where D is matrix of elastic constants or constitutive matrix, in the form

$$D = \frac{E(1-\nu)}{(1+\nu) \cdot (1-2\nu)} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1-\nu & 0 \\ 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix}. \quad (12)$$

Now, the strain tensor can be written in the form:

$$\varepsilon = C \sigma, \quad (13)$$

where $C = D^{-1}$ is flexibility matrix of the material.

If we ignore temperature stresses, we obtain the basic equation of the finite element which gives the relation between the nodal displacements and nodal forces for the finite element:

$$\{F\} = [K^e] \{S\}. \quad (14)$$

The matrix $[K^e]$ is stiffness matrix of the finite element, which depends on the physical properties of materials, the coordinates of elements and interpolation functions.

Construction equation is obtained by joining e equations (14) into a group equation as form:

$$\{F\} = [K] \{S\} \quad (15)$$

where $\{F\}$ is matrix of the columns whose elements are components of the generalized nodal forces and $\{S\}$ is matrix of the columns whose elements are components of the nodal displacement vectors.

The applying of numerical methods includes the usage of computers and appropriate program packages. During this research to generate networks of finite elements and to obtain results of distribution of stress, the ANSYS program package has been used. It is a program which satisfies demands of modern computer technology achievements and enables to carry out the relatively complex and large procedure in a faster and simpler way.

Certain operations in this program package, just as well as in other program packages, are automated, what enables automatic generating of the finite elements network.

3 Numerical results Numerički rezultati

The subject of analysis in this study is the influence of hole position on distribution of stress at the isotropic plane field, i.e. in the plate.

The plates are weakened by rectangular holes and they are also uniaxially loaded by unit surface forces with intensity of $p = 1 \text{ N/m}^2$. The dimensions of plates in all examples are $2 \times 5 \times 0,1 \text{ m}$. The sides length of the hole are $a = 100 \text{ mm}$ and $b = 20 \text{ mm}$, and the radius of rounded angles of the hole is $r = 6 \text{ mm}$.

The hole is in the middle of the plate and its sides are inclined to the lines of external load. The plate is made of steel with modulus of elasticity $E = 2,1 \times 10^5 \text{ MPa}$ and Poisson's ratio $\nu = 0,33$. In the examples we have used 2D finite elements (as shown in Figure 1), but the computer program has the ability to split these into two triangular solid finite elements. Figures 2, 5, 8 and 11 show details of the mesh of plates (the mesh has been generated automatically by using the program package ANSYS).

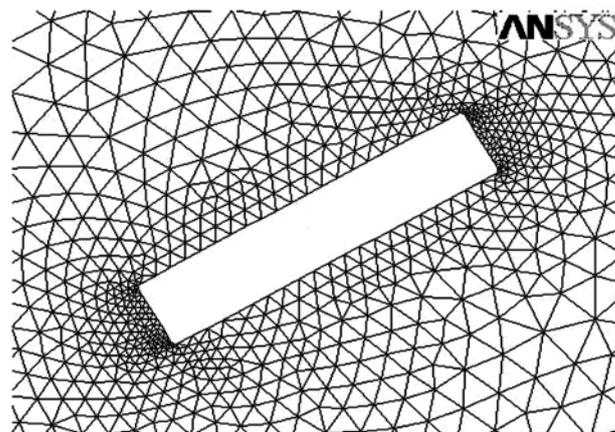


Figure 2 Finite element mesh (FEM model) for $\alpha = 60^\circ$
Slika 2. Mreža konačnih elemenata (MKE model) kada je $\alpha = 60^\circ$

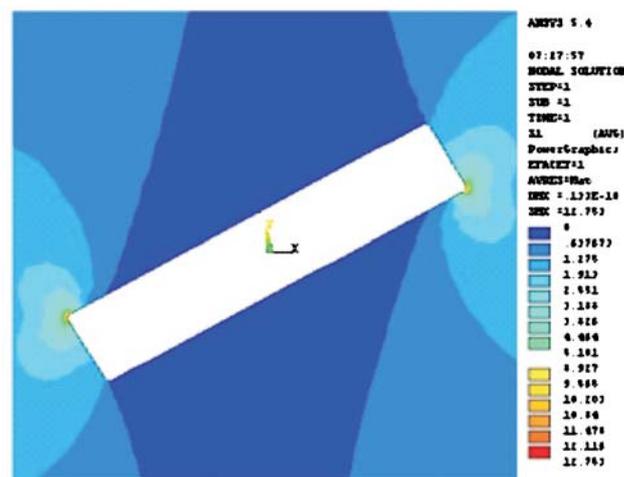


Figure 3 Distribution of maximum stress (σ_{\max}) for $\alpha = 60^\circ$
Slika 3. Raspodjela maksimalnog naprezanja (σ_{\max}) za $\alpha = 60^\circ$

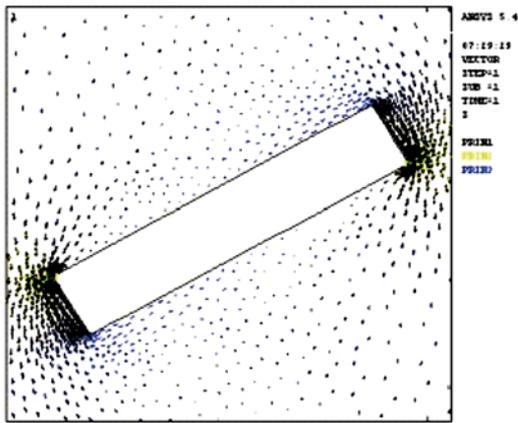


Figure 4 The trajectories of the stress for $\alpha = 60^\circ$
Slika 4. Trajektorije naprezanja za $\alpha = 60^\circ$

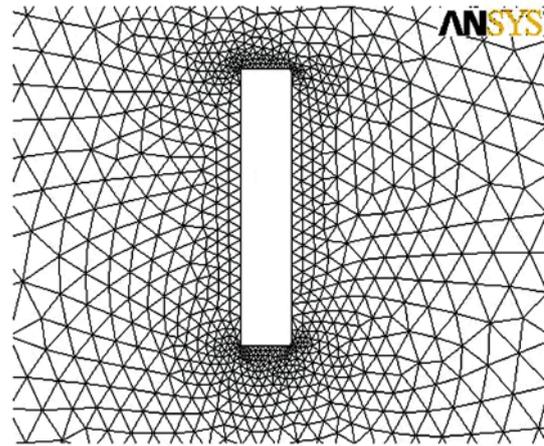


Figure 8 Finite element mesh (FEM model) for $\alpha = 0^\circ$
Slika 8. Mreža konačnih elemenata (MKE model) za $\alpha = 0^\circ$

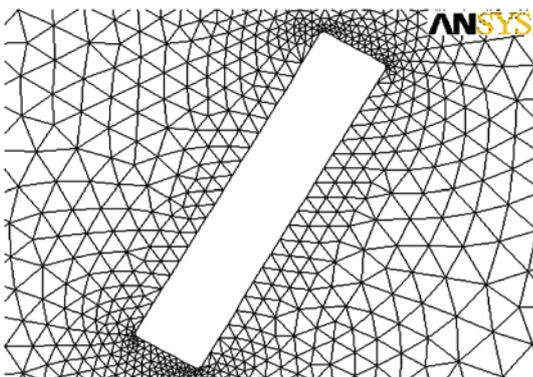


Figure 5 Finite element mesh (FEM model) for $\alpha = 30^\circ$
Slika 5. Mreža konačnih elemenata (MKE model) za $\alpha = 30^\circ$

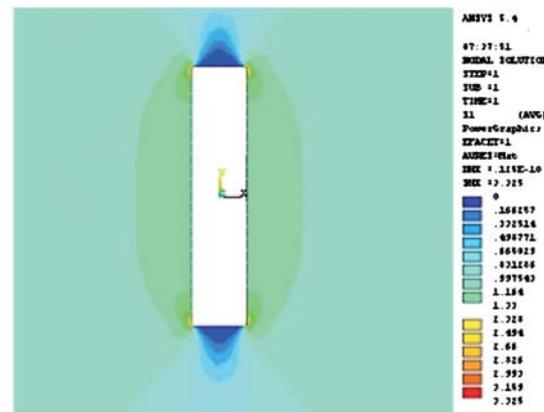


Figure 9 Distribution of maximum stress (σ_{max}) for $\alpha = 0^\circ$
Slika 9. Raspodjela maksimalnog naprezanja (σ_{max}) za $\alpha = 0^\circ$

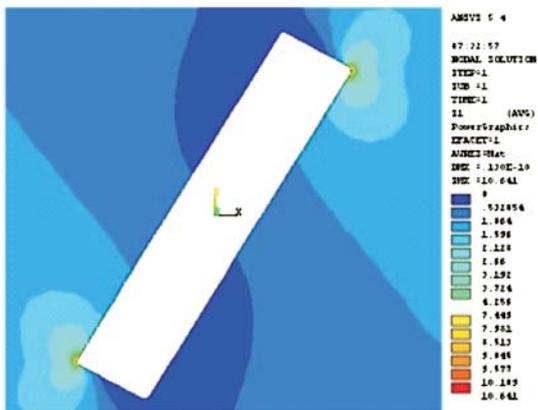


Figure 6 Distribution of maximum stress (σ_{max}) for $\alpha = 30^\circ$
Slika 6. Raspodjela maksimalnog naprezanja (σ_{max}) za $\alpha = 30^\circ$

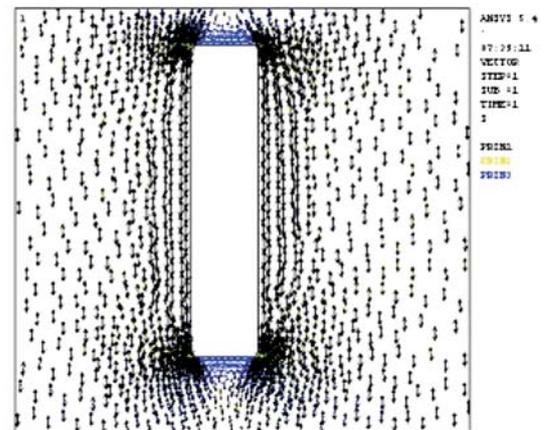


Figure 10 The trajectories of the stress for $\alpha = 0^\circ$
Slika 10. Trajektorije naprezanja za $\alpha = 0^\circ$

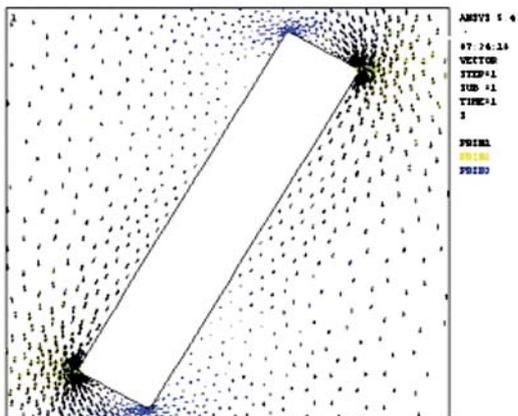


Figure 7 The trajectories of the stress for $\alpha = 30^\circ$
Slika 7. Trajektorije naprezanja za $\alpha = 30^\circ$

The trajectories of the main stress are shown in Figures 4, 7, 10 and 13.

Figures 3, 6, 9 and 12 show details of distribution of maximum stress (σ_{max}) close to the hole (the review of stress distribution on the whole plate would be unclear and confusing). According to this, the angle between striking lines of external load and the longer side of rectangular hole, shown in Fig. 3, is $\alpha = 60^\circ$, in Fig. 6, $\alpha = 30^\circ$, in Fig. 9, $\alpha = 0^\circ$, and in Fig. 12, $\alpha = 90^\circ$.

In order to compare the obtained results of distribution of stress, Tab. 1 shows the σ_{max} and σ_{nom} [2, 3, 4, 5] values and values of stress concentration factor α_k for the given values of angle α , which determines the position of the hole.

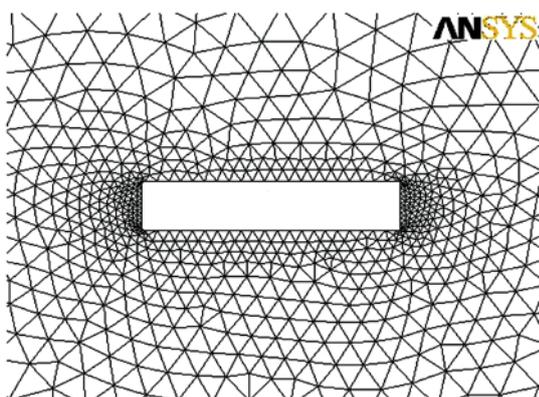


Figure 11 Finite element mesh (FEM model) for $\alpha = 90^\circ$
Slika 11. Mreža konačnih elemenata (MKE model) za $\alpha = 90^\circ$

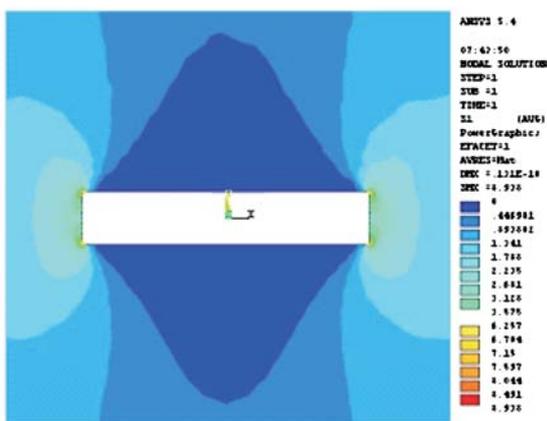


Figure 12 Distribution of maximum stress (σ_{max}) for $\alpha = 90^\circ$
Slika 12. Raspodjela maksimalnog naprezanja (σ_{max}) za $\alpha = 90^\circ$

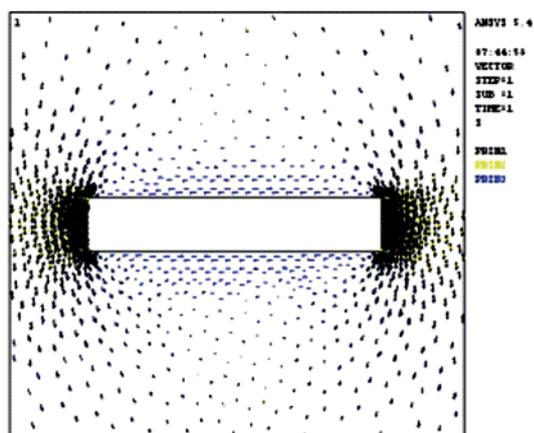


Figure 13 The trajectories of the stress for $\alpha = 90^\circ$
Slika 13. Trajektorije naprezanja za $\alpha = 90^\circ$

Table 1 Stress intensity σ_{max} and σ_{nom} and values of stress concentration factor α_k

Tabela 1. Intenziteti naprezanja σ_{max} i σ_{nom} i vrijednosti faktora koncentracije naprezanja α_k

	α°			
	60	30	0	90
$\sigma_{max} / N/m^2$	12,753	10,641	3,325	8,938
$\sigma_{nom} / N/m^2$	1,020	1,012	1,010	1,053
$\alpha_k = \sigma_{max} / \sigma_{nom}$	12,503	10,515	3,292	8,488

4 Conclusions Zaključak

The results obtained by the Finite Elements Method, using the ANSYS computer program, confirm the suitability and also justify the application of the same method for solving these or similar problems of design.

These researches have proved the influence of the hole position to the appearance of the state of stress, especially at the edge of the hole. According to the research results, it can be concluded that any change of the hole position leads to disorder in the stress state of stressed element and to stress concentration.

The hole position in the direction of striking lines of the external load in Fig. 9 is more suitable, regarding the stress concentration, than the other positions. In other words, it can be concluded that the source of the stress concentration, which lies in the direction of load lines, often creates a lower stress concentration than it was the case when the same source lay transversely to the direction of load line.

As a rule, the concentration of stress leads to the increase of real stress, in comparison with the nominal stress. According to the obtained results it can be concluded that a slight change of the cross sections' shape creates small concentration of stress, often irrelevant, but a large change of shape creates great concentration, which must be well controlled.

At the concentration of stress source, the real stress is the strongest, and at that point, according to the geometric proportions of elements, sometimes great differences between real and nominal stress can be found. This shows the values of stress concentration factors, given in Tab. 1.

Precisely, the significance of these results is in the fact that the original scientific data (which were the bases for stress concentration factors definition) are unknown in literature.

Based on researches the results show that the stress concentration decreases with the increase of the radius of curves at the corners of a rectangular hole.

The results obtained can be used in practice and scientific research because their credibility has already been confirmed.

Special problem arises when there are the initial damages, in the form of cracks on the edges of holes, like in [23], where authors showed the initial research results. Analysis of cracks propagation, and analysis of life cycle, will be the subject of future investigations.

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5

References

Literatura

- [1] Shigley, E. J.; Mischke, R. C. Mechanical Engineering Design, Mc Graw Hill, International Edition, 2001.
- [2] Durelli, A. J. Applied stress analysis, Prentice Hall, 1976.
- [3] Nikolić, V. Machine elements, theory, calculation, examples, University of Kragujevac, Faculty of Mechanical Engineering, Kragujevac, 2004.
- [4] Rašković, D. The Theory of Elasticity, Science book, Belgrade, 1985.
- [5] Timošenko, S.; Gudier, J. N. Theory of Elastic Stability, New York, Mc Graw Hill, 1962.
- [6] Hedrih (Stevanović), K. Chosen chapters of the Theory of Elasticity, Niš, 1976.
- [7] Mushelišvili, N. I. Nekotore osnovne zadači matematičeskoj teoriji uprugosti, Izdvo AN SSSR, 1964.
- [8] Nikolić, V. Mechanical analysis of gears, Monograph, Cipmes, University of Kragujevac, Faculty of Mechanical Engineering, Kragujevac, 1999.
- [9] Mahavikov, V. I. Pro nabljiženi konformni vidobraženja i ih zastosuvanja v teoriji pružnosti // Prikl. meh., 1957.
- [10] Gurjev, N. F. O naprjaženom sostojaniji plastinki oslabljenoj krivolinijejnim otverstijem // Inž. žurn. 1974.
- [11] Šermetjev, M. P. Plastinki s podkrepļjenim krajem, Izdvo Ljvovsk, 1960.
- [12] Savin, G. N. Raspređeljenje naprjaženij okolo otverstij, Izdvo "Naukova dumka", Kijev, 1968.
- [13] Zienkiewicz, O. C., The Finite Element Method, Third edition, McGraw-Hill, London, 1977.
- [14] Nikolić, V.; Ratković, N. The application of the MOIRE Method and the Finite Elements Method in analysis of tensile deformational state at the plane tensed element with an aperture and the notches // Technic, 7, 1994.
- [15] Pavlović, A.; Nikolić, V. In the planar isotropic field reduced with holes // Proceedings of the 4th Youth Symposium on experimental Solid Mechanics, Bologna, Castrocaro Terme, Italy, May 4-7, 2005, 159-160.
- [16] Radojković, M.; Nikolić, V.; Vasić, Z. The influence of rounded radius of apertures upon distribution of tension at the isotropic plane field // Proceedings of the first Serbian (26th YU) Congress on Theoretical and Applied Mechanics, p. 837-840, Kopaonik, Serbia, April 10-13, 2007.
- [17] Radojković, M.; Vasić, Z. Application of numerical methods on calculation of tension and deformity states of the plates // Proceedings of the IRMES '04, Kragujevac, 2004.
- [18] Nikolić, V.; Dolićanin, Č.; Radojković, M. Application of Numerical Methods in solving a Phenomenon of the Theory of thin Plates // Scientific Technical Review, 60, 1(2010), 61-66.
- [19] Dolićanin, Č.; Nikolić, V.; Radojković, M. Matematičeski Modeli and application Numerical Methods in solving a phenomenon of the Theory of thin Plates // Proceedings of the Mathematics and information technology, MIT 2009, Kopaonik, 08.2009., 27-31.
- [20] Dolićanin, Č.; Nikolić, V.; Radojković, M. Development of the dynamic and mathematical models for the analysis of discrete systems // Proceedings of the 2nd International Congress of Serbian Society of Mechanics (IConSSM 2009), Palić (Subotica), Serbia, 1-5.06.2009.
- [21] Maksimović, K.; Nikolić, V.; Maksimović, S. Efficient computation method in fatigue life estimation of damaged structural components // Facta Universitatis, Series Mechanics, Automatic control and Robotics, 4, 16(2004), 101-114.
- [22] Maksimović, K.; Nikolić, V.; Maksimović, S. Modeling of the surface cracks and fatigue life estimation // Proceedings of the ECF 16 (2006), Minisymposium integrity of dinamical systems, Alexandroupolis, July 3-7, 2006, 113-116.
- [23] Jovičić, G.; Živković, M.; Jovičić, N. Numerical Simulation of Crack Modeling using Extended Finite Element Method // Strojniški vestnik – Journal of Mechanical Engineering, 55, 9(2009), 549-555.
- [24] Maksimovic, S.; Posavljak, S.; Maksimovic, K.; Nikolic, V.; Djurkovic, V. Total fatigue life estimation of notched structural components using low cycle fatigue properties // Journal STRAIN, ISSN 0039-2103, (2010).
- [25] Hedrih (Stevanovic), K.; Nikolic-Stanojevic, V. A Model of Gear transmission: fractional order System dynamics, (Research Article) // Mathematical Problems in Engineering, Received: January 24, 2010; Revised: March 2, 2010; Accepted: May 6, 2010, (<http://www.hindawi.com/journals/mpe/aip.972873.html>), ISSN 1024-123x.
- [26] Stamenković, D.; Maksimović, K.; Nikolić-Stanojević, V.; Stupar, S.; Vasovic, I. Fatigue Life Estimation of Notched Structural Components // Journal of Mechanical Engineering (Strojnski vestnik) (2010). In Press: <http://www.sv-jme.eu/articles-in-press/>, Volume 13, ISSN 0354-9836, UDC 621.

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