ISSN 1330-3651
UDC/UDK 535.34/.36:676.017.55

KUBELKA-MUNK THEORY IN DESCRIBING OPTICAL PROPERTIES OF PAPER (I)

Vesna Džimbeg-Malčić, Željka Barbarić-Mikočević, Katarina Itrić

Subject review

One of the greatest simplifications of the transfer equation is where all incident and scattered light is assumed to be perfectly diffuse, only two opposite directions of light transport are considered, and the light intensity is assumed to vary along one axis only. Such two-flux approximations have been presented by many authors, starting with a paper by Schuster [1]. One of the most famous two-flux approximations is the approximation presented by Kubelka and Munk [2] in 1931 and further developed by Kubelka [3,4]. Kubelka and Munk gave a comprehensive formulation and a treatment with a clear aim towards practical methods of measurement. It was quickly adapted for use by the papermaking industry [5-8] and has now been in widespread use for decades in the measurement and prediction of colour, brightness and opacity of paper sheets. Most of the information reported here is taken from the references cited at the end of the article which should be consulted for a more in-depth study.

Keywords: *absorption, Kubelka-Munk, paper, scattering*

Kubelka-Munk teorija u opisivanju optičkih svojstava papira (I)

Pregledni članak

Jedno od najvećih pojednostavnjenja jednadžbe transfera pretpostavlja: upadno i raspršeno zračenje je savršeno difuzno, promatraju se samo dva suprotna smjera prijenosa svjetlosti, i intenzitet svjetlosti se mijenja samo duž jedne osi. Takvu aproksimaciju dva smjera prijenosa svjetlosti započeo je Schuster [1], a nastavili su Kubelka i Munk [2] 1931. a model je dalje usavršio Kubelka [3-4]. Kubelka i Munk su dali sažeti oblik i pristup s jasnim ciljem ka praktičnim metodama mjerenja. Ta metoda je bila brzo prihvaćena u papirnoj industriji [5-8] i danas ima široku primjenu u mjerenju i procjeni obojenja, svjetline i opaciteta pri proizvodnji papira. Većina informacija u ovom članku preuzeta je iz referenca čiji se popis nalazi na kraju članka i koje je potrebno konzultirati za detaljniju analizu.

Ključne riječi: *apsorpcija, Kubelka-Munk, papir, raspršenje*

1 Introduction
Uvod

This review article is a general summary of the widely used Kubelka-Munk model. Kubelka and Munk carried out a simplified analysis of the interaction of incoming light with a layer of material such as a layer of paint. The material is assumed to be uniform, isotropic, non-fluorescent, non-glossy and the sample has to be illuminated by diffuse, monochromatic light.

Though real paper never completely satisfies all of the assumptions, and sometimes significantly deviates from them, the K-M model has been widely used in paper industry for many years probably due to its explicit form, simple use and its acceptable prediction accuracy. It is used for multiple-scattering calculations in paper, paper coatings, printed paper, deinked paper (recycled paper and hand sheets), paint, plastic and textile. The theory has also found numerous applications in the paint and colorant industry.

2 Kubelka-Munk model
Model Kubelka-Munk

Kubelka and Munk proposed, as Schuster had (this work was not known to Kubelka and Munk), a system of differential equations based on the model of light propagation in dull colored layer that is parallel to a plane substrate.

The K-M model is based on several assumptions:

1. The medium (sample) is modeled as a plane layer of finite thickness, but infinite width and length (infinite sheet approximation), so there are no boundary effects.
2. A perfectly diffuse and homogeneous illumination incident on the surface.
3. The only interactions of light with the medium are scattering and absorption; polarization and spontaneous emission (fluorescence) are ignored.
4. The medium is considered isotropic and homogeneous, containing optical heterogeneities (small compared to the thickness of the layer) able to disperse light.
5. No external or internal surface reflections occur.
6. Parameters S and K are constant whatever the thickness of layer is.

Since the lateral extent of the medium is infinite, only the thickness direction is incorporated in the equations. The incident, reflected and transmitted intensities are all assumed to be perfectly diffuse, and are assigned either of two directions: upwards or downwards.

The reflectance of the medium is denoted R, and its transmittance T. The sample is placed in optical contact with an opaque substrate with a known reflectance R_g, as shown in Fig. 1, and is split into a series of layers of equal thickness dx. That layer receives a flux i traveling downward and flux j traveling upward, the reflectance being simply j/i (Fig. 1).

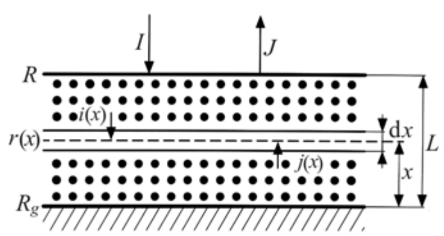

Figure 1 *Light path of the Kubelka-Munk model in conjunction with the depth coordinate x*
Slika 1. *Put svjetlosti u Kubelka-Munk modelu u ovisnosti o debljini sloja x*

Passing from one layer to the next, these fluxes are changed. Without scattering, the change in irradiance of the flux of light in the down direction would be dI=–Kdx according to the Beer-Lambert law. If x is expressed in mm, the constant K, is in units of 1/mm and is called an absorption coefficient. The larger the value of K, the greater is the probability that a photon will be absorbed.

Scattering also decreases the flux of light in the down direction. Kubelka and Munk [1] suggested that the scattering phenomenon, like absorption, is a first order phenomenon. S is the Kubelka-Munk scattering coefficient and has the same units (inverse length) as the Kubelka-Munk absorption coefficient K. The flux in down direction I has a positive value when the flux is going down. According to Kubelka-Munk sign conventions, J is the flux of light in the up direction. The value of J is positive when light is going up. The term $+SJdx$ describes that the up moving flux is scattered to add to the down moving flux. The first order differential equation is expanded to include two other terms:

$$\mathrm{d}I = -KI\mathrm{d}x - SI\mathrm{d}x + SJ\mathrm{d}x. \tag{1}$$

A second differential equation describes the rate of change in the up moving flux:

$$\mathrm{d}J = -KJ\mathrm{d}x - SJ\mathrm{d}x + SI\mathrm{d}x. \tag{2}$$

When passing through the layer, light that is absorbed is lost, and some part of the light that is scattered changes direction from upwards to downwards, or vice versa, and is exchanged between i and j. Two coefficients K and S are introduced to denote the amount of absorption and direction changing scattering, respectively. The scattering and absorption are both proportional to the intensities and to the thickness of the layer L.

In a bulk material of infinite lateral extent, the lateral dimensions are assumed to be much larger than the mean free paths, $1/K$ and $1/S$, for absorption and scattering in the material. This means that no light leaks out of the edges of the material due to lateral scattering, so that the light flux in the horizontal direction is ignored. (We will see later how lateral scattering has a significant impact on image tone reproduction by halftone processes.)

Equations (1) and (2) are two differential equations with two flux terms, I and J. If we consider the value of $I = I_t$ at the bottom of the material, then we can define a transmittance:

$$T = \frac{I_t}{I_0}. \tag{3}$$

Similarly, we can define a reflectance factor, R, in terms of the up flux at the surface of the paper:

$$R = \frac{J_r}{I_0}. \tag{4}$$

Kubelka and Munk solved these differential equations to obtain analytical expressions for R and T associated to a given wavelength in the case of an infinite (R_∞) or finite (R) painted layer, in terms of the absorption and scattering coefficients and the reflectance of the support (R_g):

$$R(\lambda) = \frac{\dfrac{R_g - R_\infty}{R_\infty} - R_\infty\left(R_g - \dfrac{1}{R_\infty}\right)\exp\left[SL\left(\dfrac{1}{R_\infty} - R_\infty\right)\right]}{(R_g - R_\infty) - \left(R_g - \dfrac{1}{R_\infty}\right)\exp\left[SL\left(\dfrac{1}{R_\infty} - R_\infty\right)\right]}. \tag{5}$$

In practice, R_∞ is the reflectance of a layer so thick to completely hide the substrate, i.e. the limiting reflectance that is not modified by any additional thickness of the same material [9]. Kubelka treated the same problem [3] as he had with Munk in 1931, but in his generalization the incident beam light comes to the colored layer under the angle $\Phi \neq 0$ [10].

There are other theories which describe in greater detail how light interacts with material, but they have not been so widely accepted as the simple K-M theory. It still has a special position not only in paper optics and in paper industry [11-16], but also within the colorant branch where it is used to calculate color pigment mixtures.

3 Useful Solutions to the Kubelka-Munk Differential Equations
Korisna rješenja Kubelka-Munk diferencijalnih jednadžbi

All the well known results of the K-M theory are listed in literature. Wyszecki and Styles [17] give solutions of differential Eqs. (1) and (2) in terms of R and T. The solutions are complicated transcendental functions, but in terms of only four parameters of the system:

$$R = f_1(S, K, L, R_g) \text{ and } T = f_2(S, K, L, R_g). \tag{6}$$

The particular form of the functions, f_1 and f_2, depend on the boundary conditions of the system. The general solutions for reflectance and transmittance are:

$$R = \frac{1 - R_g\left[a - b\coth(bSL)\right]}{a - R_g + b\coth(bSL)}, \tag{7}$$

$$T = \frac{b}{a\sinh(bSL) + b\cosh(bSL)}. \tag{8}$$

With definition for a and b:

$$a = \frac{S+K}{S},\ b = \sqrt{a^2 - 1}. \tag{9}$$

3.1 Some special cases of solutions of K-M differential equations
Neki posebni slučajevi rješenja K-M diferencijalnih jednadžbi

a) R_g=0 and S=0, and K>0.

This is a transparent medium that absorbs light but does not scatter it. It is the "Beer-Lambert" case. Derivation of the limit of the Eq. (10) as S approaches zero, converges to the Beer-Lambert law.

b) $R_g > 0$, $K > 0$, but $S = 0$.

This is a Beer-Lambert system placed against a Lambertian scattering reflector (ink on paper, photographic emulsion on paper).

c) $S > 0$ and $K > 0$, and L (thickness) $\to$ infinity.

This is the case of an infinite thickness of a scattering/absorbing material. It is a system that is sufficiently thick to reflect the same amount of light with a background of $R_g = 1$ or $R_g = 0$. This corresponds physically to a layer that is so thick that it is effectively opaque and therefore hides the substrate completely. The thickness required in order to approximate the complete hiding case is often quite small. Most commercial paints are formulated to have sufficient scattering and absorbing powers so that only a thin coating will be needed to completely hide the color of the substrate.

In such systems, the following form of the Kubelka-Munk equation applies, so-called Kubelka-Munk function:

$$\frac{K}{S} = \frac{(1-R_\infty)^2}{2R_\infty}. \tag{10}$$

The term R_∞ reminds us that this is the reflectance when the sample is infinitely thick (Fig. 2a). It is in fact R of equation (7) in the limit of $L \to \infty$.

Solving Eq. (10) for R_∞ leads to another useful expression, in which a and b are defined as in Eq. (9):

$$R_\infty = \lim_{x\to\infty} R_0 = a - b = 1 + \frac{K}{S} - \sqrt{\frac{K^2}{S^2} + 2\frac{K}{S}}. \tag{11}$$

Since the thickness of the medium is infinite and therefore irrelevant the only thing that matters are the relative proportions between absorption and scattering, and the reflectance may be expressed as a function of K/S only.

It is usual to use a black background with a reflectance value $R_g < 0{,}5$ %, i.e. $R_g = 0$. The reflectance of the paper measured in that way is the reflectance factor over black baking $R = R_0$:

$$S = \frac{1}{L\left(\frac{1}{R_\infty} - R_\infty\right)} \ln\left[\frac{(1-R_0R_\infty)R_\infty}{R_\infty - R_0}\right]. \tag{12}$$

$$K = \frac{S\cdot(1-R_\infty)^2}{2R_\infty}. \tag{13}$$

Definitions of the quantities used in Eqs. (12) and (13)[19]:

R – reflectance of the layer (sheet, coating, ply) which has behind it a surface with a reflectance of R_g

R_g – reflectance of the background behind the layer whose reflectance is being considered

R_0 – reflectance of the layer with ideal black background, $R_g = 0$

R_∞ – reflectivity = reflectance of the layer so thick that further increase in thickness does not change the reflectance.

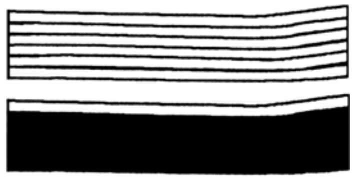

Figure 2 *a) the reflectance factor for an opaque pad of paper, reflectivity R_∞; b) the reflectance factor for a single sheet over a black background (R_g=0) R_0.*
Slika 2. *a) faktor reflektancije za neprozirni kup papira, reflektivnosti R_∞; b) faktor reflektancije za jedan papir preko crne podloge (R_g=0) R_0.*

Fig. 2 illustrates the sample arrangements used for the definition of quantities R_0 and R_∞.

An alternative method, especially when the amount of material is limited, is to measure reflectance over two different backgrounds, white and black [18].

If density is homogeneous, the corresponding analysis can be made where the thickness L is replaced by the grammage of the sheet. Van der Akker [5] showed that the K-M differential equations remain the same if the original scattering and absorption coefficients S and K (unit 1/m) are replaced with the specific scattering and absorption coefficients s and k (unit m^2/kg), and the thickness L (unit m) is replaced with grammage w (unit kg/m^2). The light scattering power of a paper sheet is sometimes given as dimensionless product $s\cdot w$.

The coefficients s and k can be determined for different wavelengths or with different functions such as the brightness function R_{457} or $YC/2^0$ function (ISO 9416). Determinations of k with R_{457} function are common within the pulp industry since k (R_{457}) is a measure of the content of chromophoric groups in the pulp. From Eq. (13) it is evident that R_∞ depends on the relationship between k and s. High values of the ratio s/k are required to attain the highest reflectivity values; s have to be large and/or k small, i.e. high light scattering and low absorption.

The values of k and s are typically [18]:
$k < 2$ m^2/kg for coated and uncoated fine papers made from bleached chemical pulps,
$3 < k < 6$ m^2/kg for mechanical pulps and around 14 m^2/kg for unbleached kraft pulp,
$s > 502$ m^2/kg for filled and coated fine papers,
$20 < s < 40$ m^2/kg for bleached and unbleached chemical pulps,
$40 < s < 702$ m^2/kg for mechanical pulps.

d) $R_g > 0$, $K = 0$ and $S > 0$.

This is a medium that scatters light, but does not absorb it (Lambertian scattering reflector). This is particular case for conservative scattering (zero absorption). For that system the definitions for a and b (9) become:

$$a = 1 \text{ and } b = 0 \tag{14}$$

and the reflectance is [20]:

$$R = \frac{R_g + SL(1-R_g)}{1 + SL(1-R_g)}. \tag{15}$$

4 Interactions of light with paper
Interakcije svjetlosti s papirom

Paper is a complex structure consisting largely of cellulose fibers, fillers and additives. The appearance of a typical sheet of paper is a function of its detailed structure, the presence and concentration of light absorbing groups (chromophores), the refractive indices of its components, its grammage and its surface reflective characteristics. When light strikes the paper, part of it is reflected (no resonant type of interaction) at fiber and pigments in the surface layer and inside the paper structure, part of it is absorbed (ray 2) (resonant type of interaction) and the remainder part passes into the air like transmitted light (no resonant type of interaction).

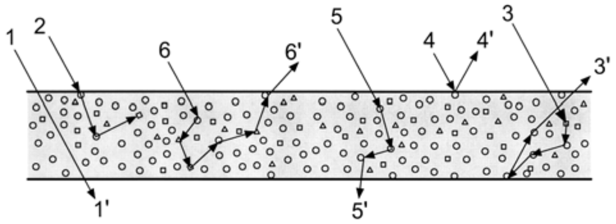

Figure 3 *Possible events for photons in a turbid medium (paper)*
Slika 3. *Mogući događaji za fotone u zamućenom sredstvu (papiru)*

4.1 Direct and diffuse transmission
Direktna i difuzna transmisija

In a more or less transparent medium, some portion of the incident photons might pass straight through and exit on the opposite side. Paper is a strongly scattering medium, so this is a rare event, and direct transmission (ray 11') can be neglected for any reasonably thick paper.

Paper is a turbid medium, but a paper sheet is also quite thin, and therefore translucent. The internal scattering of light might well result in the photon leaving the paper at the opposite side from where it entered. There is a transmission through a paper sheet, but that transmission is almost diffuse (ray 55').

4.2 Surface and bulk reflection
Površinska refleksija i refleksija uzrokovana raspršenjem

Light is reflected at the interface between materials with different refractive indices. The direction of specularly reflected light depends on the incidence angle and the orientation of the surface normal. Since the surface of a paper is often rough, the specular surface reflection (ray 44') does not usually occur.

Paper is a translucent material, and multiple light scattering in the bulk of the paper is really the main reason for most of its macroscopic optical properties, for example the fact that it is white. Light that enters the paper is typically scattered several times before it either exits the paper or is absorbed. This reflection of scattered photons is bulk reflection (ray 66'). A photon that enters the paper at one point does not necessarily exit at the same point, although it is more probable that it exits close to the point of incidence than far from it. This lateral spreading of light is quite different from the local surface reflection.

A common reflection classification is to distinguishe between *diffuse* and *specular reflection*. Diffuse reflection is a light reflection that has no strong directional properties and might occur also from a totally opaque, highly reflective but rough surface. Thus the term "specular reflection" could refer to either a part of the first surface reflection, or all of it, depending on the topography of the surface. Since the interaction of light with micro-rough surfaces is really a scattering phenomenon, the term *surface scattering* is often used in optics literature [12].

4.3 Internal surface reflection
Refleksija na drugoj granici sredstva (papira)

Any interface between two media with different refractive indices will give rise to surface reflection (ray 33'), and this also concerns light that hits the paper surface from within. Light that tries to exit from the paper might actually be reflected and enter the paper again. If this effect is strong, it will likely have a considerable impact on the halftone imaging properties of the medium [21]. For prints on rough paper surfaces, this effect is not so prominent, but for higher paper grades with a glossy and smooth coating layer it could be important to consider internal surface reflections.

4.4 Saunderson correction
Saundersonova korekcija

Kubelka and Munk first solved differential equations (1) and (2) for the special case of a colorant layer with opaque thickness R_∞ (5). Analyzing the layer of a lesser thickness the colorant appears translucent and the backing of reflectance R_g starts to shine through. Kubelka and Munk gave a general solution for the reflectance of such translucent layer, but they ignored any surface or internal reflections at the colorant boundaries caused by different refractive indices. Saunderson [22] presented a conventional adjustment (correction) of the estimate value to the experimental results taking account of the ignored internal reflectance R_i and transmittance T_i factors at the inner top boundary surface:

$$R_{\exp} = R_s + T_s T_i \frac{R_{KM}}{1 - R_i R_{KM}}, \quad (16)$$

where R_s and T_s are the specular reflectance and transmittance factors of the incident electromagnetic radiation at the top surface of the layer.

5 Additivity principle
Princip adicije

5.1 One layer paper
Jednoslojni papir

As it is mentioned before, the paper is a complex structure consisting of cellulose, fibers, fillers and additives. The K-M equations can be used in description of such paper in form of additivity principle:

$$S_{\text{paper}} = \sum_{i=1}^{n} S_i x_i = S_1 x_1 + S_2 x_2 + S_3 x_3 + \dots \tag{17}$$

$$K_{\text{paper}} = \sum_{i=1}^{n} K_i x_i = K_1 x_1 + K_2 x_2 + K_3 x_3 + \dots \tag{18}$$

Where the S_i and K_i coefficients indicate different scattering and absorption coefficients for different ingredients and x_i indicate the mass fraction of the particular component. First it is necessary to obtain values for S_1 and K_1 from Eqs. (12) and (13) which correspond to the fiber component [23]. For that purpose unfilled paper sheet (fiber sample) has been made under the same experimental conditions as examined paper. So, a second set of handsheets (fiber + filler sample) has to be prepared with a known fraction x_2 of the chosen filler. The values of the filler's scattering and absorption coefficient S_2 and K_2 can be calculated from [24]:

$$S_2 = \frac{1}{x_2}\left[S_{\text{paper}} - (1-x_2)S_1\right], \tag{19}$$

$$K_2 = \frac{1}{x_2}\left[K_{\text{paper}} - (1-x_2)K_1\right], \tag{20}$$

where

$$x_1 = 1 - x_2. \tag{21}$$

Addition of filler to the pulp does not always follow the additivity rule because of the complexity of interactions. The filler light scattering effect is reduced when the filler content is increased, so S_2 cannot be treated as a constant. Light scattering is dependent on the structure and the structure is changed when the filler is mixed with the pulp. Treatments such as calendering and wet pressing can also change the light scattering in the paper.

5.2 Two-layer paper
Dvoslojni papir

Kubelka published [3] a number of equations which are used for coated paper. For a system with layer 1 above layer 2 it is possible to analyze how the light is reflected between them. The result of the analysis was a number of new equations for transmittance and reflectance:

$$T_{12} = \frac{T_1 T_2}{1 - R_{01}R_{02}}, \tag{22}$$

$$R_{12} = R_{01} + \frac{T_1^2 R_{02}}{1 - R_{01}R_{02}}. \tag{23}$$

From the structure of the equations it is evident that transmittance is a symmetrical process, i.e. the transmittance is the same if the system with two layers is reversed ($T_{12}=T_{21}$), while the process of reflectance is not symmetrical ($R_{12}\neq R_{21}$). The following relationship is valid for a system of n two layer sheets, i.e. two layer sheets over $n-1$ two layer sheets [18]:

$$R_{n12} = R_{012} + \frac{T_{12}^2 R_{(n-1)12}}{1 - R_{012}R_{(n-1)12}}. \tag{24}$$

5.3 Multi-layer paper
Višeslojni papir

Taking limes $n\to\infty$ and rearrangement we obtain:

$$R_{\infty 12} = \frac{1 + R_{012}R_{021} - T_{12}^2}{2R_{021}} - \sqrt{\left(\frac{1 + R_{012}R_{021} - T_{12}^2}{2R_{021}}\right)^2 - \frac{R_{012}}{R_{021}}} \tag{25}$$

with:

$$T_{12} = \sqrt{\left(\frac{1}{R_{\infty 12}} - R_{021}\right)\left(R_{\infty 12} - R_{012}\right)}. \tag{26}$$

A three layer system 123 is now easy to obtain considering the system as a two layer 12 placed on top of layer 3 [11]. This simulation of three layers can be used to calculate reflectivity for coated paper. For that purpose it is necessary to know the absorption K and scattering S coefficients of the coating and of the substrate (paper) as well as their weight and grammage.

6 Interactions of light with a halftone printed paper
Interakcija svjetlosti s otiskom na papiru

The theory of Kubelka-Munk has found a wide acceptance for modeling the reflectance and transmittance behavior of scattering dull materials having the same surface characteristics over the whole area of examined substrate. Still, halftone prints cannot be regarded as infinitely wide colorant layers and the original concept fails to predict their measured color. K-M theory has to adapt for analyzing the light scattering inside paper printed with halftone. Halftone ink or toner prints consist of microscopically varying transmittances and cannot be regard as an infinitely wide layer. Interaction of light with a halftone printed paper is a very complex process and possible events for a photon are shown in Fig. 4.

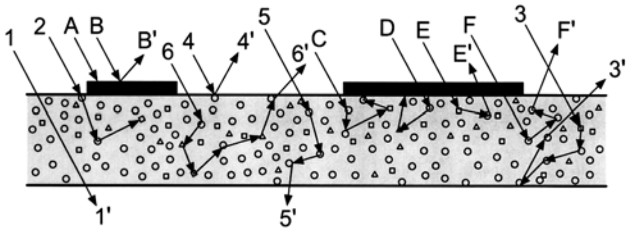

Figure 4 *Possible events for photons in a halftone print on the paper substrate. Arrows marked by numbers 1-6 denote possible interactions with paper as in Fig. 3., while arrows marked by letters A-F denote interactions caused by halftone print.*

Slika 4. *Moguće putanje fotona za otisak na papiru. Strelice označene brojevima od 1 do 6 odgovaraju mogućim interakcijama fotona s papirom kao na Sl. 3., dok strelice označene sa slovima od A do F odgovaraju interakcijama fotona s rasterskim elementom.*

There are no strongly specular surfaces in a normal print, but specular reflection might happen at the surface of the ink (arrow B). A is a photon that is absorbed by the ink layer before reaching the paper substrate. The cause of optical dot gain is the type of event illustrated by arrow C, namely a photon that enters the paper at a point not covered by ink, but is absorbed on its way out in the printed pattern and might cause decreasing of bulk reflection. If the ink has a low absorption, considerable amounts of light might pass through the ink film without being absorbed, and the events indicated by arrows D, E and F also become important [20]. This is for example the case in four color printing, where the colored inks are rather transparent for many wavelengths [25].

The first optical model of tone reproduction in the halftone process was the Murray-Davies equation [26] which was extended by Neugebauer [27]. These models describe a linear relationship between the reflectance of the halftone image and the fractional dot area of the image that is covered by halftone pattern, i.e. these models assume only the occurrence of a direct reflection on the surface which in reality is far from real (Fig. 4.). Therefore, lateral scattering cannot be ignored. The entrance point of incident light differs from its exit point which is well known as the Yule-Nielsen effect [28] (optical dot gain).

The work of Arney, Engeldrum and Zeng [29] has shown that the dot gain depends both on the scattering properties of the paper and on the dots themselves. They conclude that the model should include two empirical parameters instead of the one Yule-Nielsen parameter n.

P. Emmel and R. D. Hersch introduced unified model [30] and a new mathematical formulation based on matrices describing the light scattering and ink spreading in printing. This model generalized the K-M theory and unified it with the Neugebauer model. The Saunderson correction, the Clapper-Yule equation, the Murray-Davies relation, are particular cases of that unified model.

6.1 Lateral scattering
Lateralno (bočno) raspršenje

Two theories commonly applied to describe the optical behavior of paper are the Kubelka-Munk theory and the Linear System theory. Kubelka-Munk theory describes reflectance and transmittance properties of scattering materials in terms of the scattering and absorption coefficients, while the Linear System theory uses a point spread function $PSF(x,y)$. $PSF(x,y)$ is a probability density function that describes the probability of a photon returning to the surface of the paper at the location (x,y) away from the point of entry into the paper [31]. The Fourier transform of the point spread function, *FFT*(*PSF*) is called the modulation transfer function *MTF* of paper. The characteristic lateral scattering of the paper is often described by either the *PSF* function or *MTF* function. In literature, several approaches of the point spread function for paper are known. Most of them *MTF* determined empirically [32] or assuming a specific type of function [33-39]. Yule and Nielsen [33] suggested that the LSF characteristic of paper is Gaussian and the corresponding *MTF* is also a Gaussian function, while Wakeshima [34] reported that PSF is an exponential function which would make the *LSF* a Bessel function and the *MTF* a complex inverse power function.

Some of approaches are based on numerical simulations [20, 40], microscopic reflectance measurements [41], image processing, multi-flux theory [42], modeling [43-46] or radiative diffusion [31].

Because of the negligence of lateral light streams, the original concept of Kubelka-Munk is not adequate for modeling optical properties of halftone prints. In this respect, a recent improvement was made by Berg [47] who has introduced isotropic light scattering into a Kubelka-Munk oriented approach. He introduced a lateral analysis of scattering light and presented a Kubelka-Munk based approach, which, by numerical evaluation, predicts the point transmittance and reflectance profile of an ideal print. His approach remains limited to a two-dimensional model and ignores the influence of brightened paper or of internal reflections at the interfaces of the paper. Berg´s model was extended by Mourad in order to reduce the gap between the mathematical description of the paper's point spread function and the experimental results of simple reflectance measurements [48, 49]. This model analyzes the scattering light fluxes in a general three- dimensional space.

Arney and co-workers [50] discussed three experimental techniques for measuring light scattering and resolution characteristics of paper:

a) direct measurements of lateral light scatter (*MTF*) of paper by microdensitrometric scans of an illuminated knife edge
b) derivate *MTF* from the K-M equations and experimental measurements of paper reflectance
c) modeling of Yule-Nielsen effect of optical dot gain and fitting the model to experimental data.

The first of these techniques is the most difficult and the least precise one, while the third presents the line screen method (ideal one-dimensional halftone) which is much easier to perform and can be done with reasonably high precision. Reflectance measurements (the second of these techniques) are the easiest way of measurements and instrumentation and they can be done in many laboratories. However, the K-M theory appears not to provide as exact an estimate of lateral spread as can be achieved by the line screen technique. The possibility of poor correlation between the K-M theory and the line screen experiment is the assumption of homogeneity in light scatter inherent in the K-M model.

Oittinen [51] and Engeldrum and Pridham [52] suggested that K-M theory may provide a basis for an a priori derivation of the *PSF*, *LSF* and *MTF* of paper, assuming that light scatters homogeneously in all directions in the paper. Engeldrum derived *MTF* of the paper by taking the derivative of the K-M function for reflectance and then taking the Hankel transform:

$$MTF(\omega) = \frac{1}{-\ln(1-R_\infty{}^2)} \cdot \sum_{j=1}^{\infty} \frac{R_\infty{}^{2j}}{j} \cdot \left[1+\left(\frac{2\pi\omega}{2jbS}\right)^2\right]^{-\frac{3}{2}}. \quad (27)$$

This equation overestimates the influence of K in the *MTF* of paper. Arney [53] suggests an *MTF* model with K=0, because the majority of paper samples used to print halftone images have low absorption coefficients. He has to make empirical modification of that model by adding the empirical constant k_0:

$$k_\mathrm{p} = \left(\frac{5{,}4}{S}\right) \cdot \left(1-\mathrm{e}^{-LS}\right) + k_0. \quad (28)$$

Arney shows [53] that the K-M parameters *S* and *L* are not sufficient descriptors of paper *MTF*, even for paper with the same background (R_g= 0) and negligible values of *K*. This additional parameter may relate to the degree of homogeneity of the substrate. Engeldrum pointed out that any model derived from the K-M theory implies the assumption that the scattering coefficient is homogenous throughout the paper (coated papers are good example of inhomogeneous substrates). In addition, directional homogeneity is assumed so that *S* in the vertical direction is the same as *S* in the lateral direction, while the intrinsic directionality of paper suggests that vertical and lateral scattering distances are not the same.

6.2 PSF correction for fluorescence
PSF korekcija za fluorescenciju

The point spread function of brightened papers is affected by the included fluorescent additives. The supplied brighteners absorb certain energy of the invisible radiations (ultraviolet part of electromagnetic spectrum). A specific amount of that energy is then released by radiative relaxation (fluorescence spectrum) in the visible part of electromagnetic radiation. This technique compensates the yellowish of natural non-brightened papers. As the fluorescence emission happens in all space directions inside the paper, it acts like a diffuse partial light source or converter. Because of that the brightening effect amplifies the point spread function of the paper in the blue part of visible spectrum and must be taken into account when analyzing the *PSF*.

As printed color layers behave like spectral light filters, they can reduce the energy of fluorescence emission of the paper by absorbing a part of excitation spectrum of the fluorescent additives.

7 Conclusion
Zaključak

The Kubelka-Munk theory, though it remains the most used in practice, has some disadvantages, like imprecision in some cases (fluorescence problems for instance), initial assumptions that are too simplified. Real paper never completely satisfies all those assumptions, but researchers are interested in finding a model to upgrade that theory probably due to its explicit form, simple use and its acceptable prediction accuracy in many cases useful in paper, paint and colorant industry.

To improve the results obtained by the use of the K-M theory, numerical methods are common nowadays which simulate individual photon paths (Monte Carlo simulation model, GRACE, DORT).

8 References
Literatura

[1] Schuster, A. Radiation through a foggy atmosphere. // Astrophys. J. 21, 1(1905), 1–22.
[2] Kubelka, P.; Munk, F. Ein Beitrag zur Optik der Farbanstriche, Z. Tech. Phys. (Leipzig), 12(1931), 593–601.
[3] P. Kubelka, New contributions to the optics intensely light-scattering materials. Part I // J. Opt. Soc. Am., 38(1948), 448–457.
[4] Kubelka, P. New contributions to the optics intensely light-scattering materials. Part II: Nonhomogeneous layers. // J. Opt. Soc. Am., 44(1954), 330–335.
[5] Van den Akker, J. A. Scattering and absorption of light in paper and other diffusing media. // Tappi J., 32(1949), 498–501.
[6] Stenius, Å. S. The Application of the Kubelka-Munk Theory to the Diffuse Reflection of Light from paper, I, Svensk Papperstidning, 54(1951), 663–670.
[7] Stenius, Å. S. The Application of the Kubelka-Munk Theory to the Diffuse Reflection of Light from paper, II, Svensk Papperstidning, 54(1951), 701–709.
[8] Stenius, Å. S. The Application of the Kubelka-Munk Theory to the Diffuse Reflection of Light from paper, III, Svensk Papperstidning, 56(1953), 607–614.
[9] Vollz, H. G. Practical pigment testing with the aid of the Kubelka- Munk theory, Progr. Organic Coatings, 13(1985), 153–169.
[10] Philips-Invernizzi, B.; Dupont, D.; Caze, C. Bibliographical review for reflectance of diffusing media. // Society of Photo-Optical Instrumentation Engineers, 40, 6(2001), 1082–1092.
[11] Judd, D. B. Optical specification of light scattering materials. // Tech. Assoc. Papers. 21(1938), 474–481.
[12] Adrian, A. P. New procedure for evaluating the opacifying properties of pigments.// Tech. Assoc. Papers., 27(1944), 465–471.
[13] Hillend, W. J.; Opacity problems in printing papers: Kubelka-Munk theory gives good, quick answers.// Tappi, 49, 7 (1966), 41A–47A.
[14] Borch, J.; Scallan, A. M. A comparison between the optical properties of coated and loaded paper sheets.// Tappi, 58, 2(1975), 34–135.
[15] Robinson, J. V. A summary of reflectance equations for application of the Kubelka-Munk theory of optical properties of paper. // Tappi, 58, 10 (1975), 152–153.
[16] Judd, D.; Wyszecki, G. Color in Business, Science and Industry, J. Wiley & Sons, New York, 1975.
[17] Wyszecki, G.; Stiles, W. S. Color Science, 2nd ed., J. Wiley & Sons, New York, 2000.
[18] Pauler, N. Paper Optics, AB Lorentzen & Wettre, Sweden, 2001.
[19] TAPPI. In TAPPI Test Methods, Interrelation of reflectance, R_0; reflectivity, R_∞; TAPPI opacity, $c_{0.89}$; scattering, s; and absorption, k Standard practice T 1214 sp-98.// TAPPI Press (1998).
[20] Gustavson, S. Dot Gain in Colour Halftones, PhD Dissertation No. 492, Dept. of Electrical Engineering, Linkoping University, Sweden, 1997.
[21] Clapper, F. R.; Yule, J. A. C. The effect of multiple internal reflections on the densities of half-tone prints on paper.// Journal of the Optical Society of America, 43, 7(1953), 600.
[22] Saunderson, J. L. Calculation of the Color Pigmented Plastics. // J. Opt. Soc. Am., 32(1942), 727–736.
[23] Hubbe, M. A.; Pavlak, J. J.; Koukoulas, A. A. Paper's appearance: a review, BioResources, 3, 2(2008), 627–665.
[24] Middleton, S. R.; Desmeules, J.; Scallan, A. M. The Kubelka-Munk coefficients of fillers. // J. Pulp Paper Sci., 20, 8(1994), 231–235.
[25] Clapper, F. R.; Yule, J. A. C. Reproduction of Color with Halftone Images, TAGA Proceedings, 7 (1955).
[26] Murray, A. Monochrome Reproduction in Photoengraving, Technical Report of the J. Franklin Institute, 221 (1936).
[27] Neugebauer, H. E. J.: Die theoretischen Grundlagen des Mehrfarbenbuchdrucks. // Zeitschrift für wissenschaftliche Photographie, Photophysik und Photochemie, 364(1937), 73–89.
[28] Yule, J. A. C.; Nielsen, W. J. The penetration of light into paper and its effect on halftone reproduction, TAGA Proceedings, 3 (1951), 65–76.
[29] Arney, J. S.; Engeldrum, P. G. ; Zeng, H. An Expanded Murray-Davies Model of Tone Reproduction in Halftone Imaging. // Journal of Imaging Science and Technology, 39(1995), 502–508.
[30] Emmel, P.; Hersch, R. D. A Unified Model for Color Prediction of Halftoned Prints. // J. Imaging Sci. Technology, 44(2000), 351–359.
[31] Rogers, G. Optical dot gain in a halftone print. // J. Imaging Sci. Technol. 41(1997), 643–656.

[32] Ruckdeschel, F.; Hauser, O. Yule-Nielsen Effect in Printing: a Physical Analysis, Appl. Opt., 17(1978), 3376–3383.
[33] Yule, J. A. C. ; Howe, D. J. ; Altman, J. H. Tappi, (1967) 337-346.
[34] Wakeshima, H.; Kunishi, T. J. Opt. Soc. Amer. 58(1968), 272.
[35] Oittinen, P.; Saareima, H. Paperi Ja Puu Paper & Timper, 75 (1993).
[36] Dainty, J. C.; Shaw, R. Image Science, Academic Press, New York, 1974.
[37] Fischer, G.; Rodriguez-Giles, J.; Scheuter, K. R. Ein physikalisches Modell fur die Beschreibung von Lichtstreuprozessen, Die Farbe, 30(1982), 199–220.
[38] Rogers, G. Lateral Scattering Probabilities. // J. Imaging Sci. Technol., 42(1998), 336.
[39] Rogers, G. Effect of light scatter on halftone color. // J. Opt. Soc. Am. A, 15(1998), 1813–1821.
[40] Yang, L.; Gooran, S. ; Kruse, B. Simulation of Optical Dot Gain in Multichromatic Tone Production. // J. Imaging Sci. Technol., 45 (2001), 198–204.
[41] Shinichi, I. ; Norimichi, T., Yoichi, M. Analyzing CTF of Print by MTF of Paper. // Journal of Imaging Science and Technology, 42, 6(1998), 572–576.
[42] Joshi, J. J.; Vaidya, D. B.; Shah, H. S. Application of Multi-Flux Theory Based on Mie Scattering to the Problem of Modeling the Optical Characteristics of Colored Pigmented Paint Films, Color Research and Application, 26, 3(2001), 234–245.
[43] Arney, J. S.; Arney, C. D. Modeling the Yule-Nielsen Halftone Effect. // J. Imaging Sci. Technol., 40(1996), 233–238.
[44] Pray, E. Kubelka-Munk Theory and Yule-Nielsen Effect on Halftones. // J. Imaging Sci. Technol., 43(1999), 365–370.
[45] Arney, J. S. A Probability Description of the Yule-Nielsen Effect I. // J. Imaging Sci. Technol., 41 (1997), 633–642.
[46] Arney J. S.; Katsube, M. A Probability Description of the Yule-Nielsen Effect II: The Impact of Halftone Geometry.// J. Imaging Sci. Technol., 41(1997), 637–642.
[47] Berg, F. Isotrope Lichtstreuung in Papier – Neue Überlegungen zur Kubelka-Munk Theorie, PhD thesis, Technische Hochschule Darmstadt, 1997.
[48] Mourad, M. S. Color Predicting Model for Electrophotographic Prints on Common Office Paper, PhD Dissertation No. 40, Lausanne, 2002.
[49] Mourad, M. S.; Emmel, P.; Simon, K.; Hersch, R. D. Extending Kubelka-Munk's Theory with Lateral Light Scattering, IS&T´s NIP 17: International Conference on Digital Printing Technologies, 2002, 469–473.
[50] Arney, J. S.; Arney, C. D.; Katsube, M. An MTF Analysis of Papers. // J. Imaging Sci. Technol., 40(1996), 19–25.
[51] Oittinen, P. Advances in Printing Science and Technology, W. H. Banks, Ed, Pentech Press, London, 16(1982), 121–128.
[52] Engeldrum, P. G.; Pridham, B. Application of Turbid Medium Theory of Paper Spread Function Measurements, TAGA Proceedings, 1995, 339–352.
[53] Arney, J. S.; Chauvin, J.; Nauman, J.; Anderson, P. G. Kubelka-Munk Theory and MTF of Paper. // J. Imaging Sci. Technol., 47(2003), 339–345.

Authors' addresses
Adrese autora

Vesna Džimbeg-Malčić, doc. dr. sc.
Željka Barbarić-Mikočević, doc. dr. sc.
Katarina Itrić
Faculty of Graphic Arts
University of Zagreb
Getaldićeva 2
10000 Zagreb, Croatia
vesna.dzimbeg.malcic@grf.hr