Asymmetric correlations on the Croatian equity market

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Abstract

This paper compares the equity market in Croatia in bad (bear or turbulent) and good (bull, calm) market conditions. The two market regimes are formally identified under the Markov Regime Switching (MRS) framework. The analysis conducted suggests that correlations between equity prices are more than twice as high during bear than in bull markets. This result holds both for the shares included in the CROBEX and for the relationship among various European equity indices. In the context of international diversification the result suggests only a limited benefit that foreign investors can count on when diversifying their portfolios by expanding to developing European markets. In addition, by evaluating a portfolio optimization model that takes asymmetric correlations into account in an out-of-sample exercise, this paper also illustrates the losses that may occur if the asymmetry is ignored in practice.

Keywords: portfolio optimization, Markov Regime Switching, CAPM

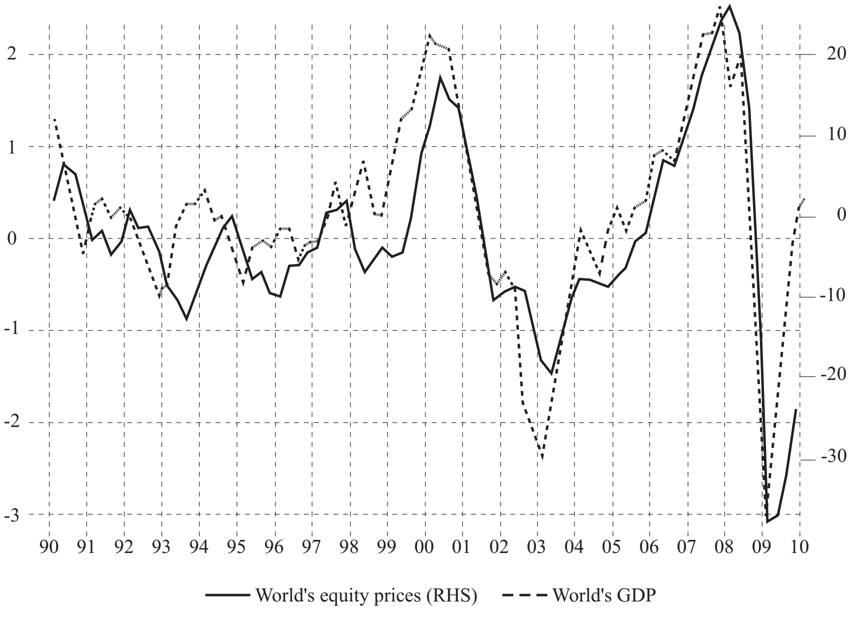
1 INTRODUCTION AND MOTIVATION

In financial economics research, special attention is paid to the phenomenon of *asymmetric correlations* among equity prices (Ramchand and Susmel, 1997; Longin and Solnik, 2001; Ang and Chen, 2002; Ang and Bekaert, 2004; Markose and Yang, 2008). In this context, asymmetry refers to correlation among returns varying much more during times of turbulence in the market than in periods of calm. In general, during calm periods in the capital markets the correlation among returns is low, which enables investors successfully to diversify their portfolios, depending on their attitude to risk. In contrast, turbulent times are characterised by a sudden rise in correlations, portfolio diversification under such conditions becoming difficult. Recent literature primarily studies the asymmetries in the developed markets and, in context of the new EU members, also some of the leading central and eastern European markets (Syriopoulos, 2004; Égert and Kočenda, 2007; Syllignakis and Kouretas, 2006). It is therefore difficult to find any information on the asymmetric nature of the domestic equity market. For that reason this paper formally identifies regimes in the Croatian equity market and also tries to identify their causes. Moreover it tests whether taking the regimes into account can help to optimize an equity portfolio on a daily basis in practice. In addition, the paper studies the asymmetric relations between the domestic market and the equity markets of the new and the old Europe, principally in order to derive a better insight into the level of integration of the domestic markets and developed European markets and also to analyse the potential of diversification of international portfolios by expanding onto developing markets.

What causes regime changes in capital markets? The prevalent opinion is that the main trigger of sudden changes in capital markets is the expected change in aggregate real activity. Assuming that the price of a share reflects the expected present value of all future cash flows relating to the share (i.e. dividends), markets react to changes in the real business cycle. For instance, if the market expects a drop in real activity in the near future, it also expects weaker business results of companies in the market and consequently lower profits in the form of smaller dividends. Thus, the ultimate consequence of the expected real drop in retail activity is the fall in the price of shares. As a result, stock exchange indices are often good leading indicators of real economic activity[1]. Figure 1 compares the world's output and world's equity indices in the last 20 years. The figure suggests that while developments in financial markets used to lead real developments up to several quarters ahead, in more recent years the two sectors have become more synchronised. At the global level this relationship is relatively stable. Still it is more difficult to establish at the country level, which is particularly the case for a small open economy where relations with foreign countries have a major impact on domestic developments.

Figure 1

World's equity prices and GDP



Note: The graph compares deviations from the long run equilibrium, i.e. the gaps for world's real GDP and equity market index for the period 1990-2010. Gaps are extracted using the Hodrick-Prescott filter and can be interpreted as percentage points deviations from the trend.

Correlation between national equity markets has a strong impact on the performance of an international equity portfolio. One of the basic reasons for holding an

[1] The most quoted remark in this context is due to Paul Samuelson: *Wall Street indexes predicted nine out of the last five recessions!* (Samuelson, 1966).

international portfolio is the potential for diversification. A precondition for a successful international diversification is that all the shocks hitting national markets are idiosyncratic. Since during turbulent periods the correlation between the markets suddenly increases, it is not easy to materialize the potential benefits of diversification. In order to be able to take advantage of correlation between the markets when optimizing an international portfolio it is crucial to identify the main channels of the international propagation of financial shocks. For example, a consequence of the increased economic integration at the global level is that global macro shocks may simultaneously affect financial markets via national channels between the financial sector and the real sector. Beside that, a real shock of a country may affect the fundamentals of another country and consequently its financial sector. Correlations between returns may also rise independently on the real developments, fundamentals, policy changes, etc. Correlation between equity markets may be surprisingly high, even for countries with different developments in their fundamentals and with no direct links between the markets. The kind of propagation is called contagion[2]. Finally, investors often prefer to monitor the aggregate market convinced it already includes all the relevant information and do not conduct their own, often costly and demanding analysis. International propagation of financial shocks is studied in details in Calvo and Reinhart (1996), Calvo and Mendoza (2000), and Forbes and Rigobon (2002).

At the micro level, while a portfolio is being optimised, a valid identification of the current market regime has a crucial impact on the performance of a portfolio. A reason for this is that expectations and variances of individual shares and correlations between them are important inputs when determining an optimal equity portfolio. Because the estimates of these moments may be different across the regimes, an investor will choose a non-optimal portfolio if it anticipates the wrong regime or even worse ignores the market regimes. To illustrate, the standard Markowitz Mean Variance (MV) theory of asset allocation can guarantee the optimal allocation only presuming that actual values of the moments are known (Markowitz, 1952). Of course, this is not a case in practice and one needs to operate with statistical estimates of the moments of interest. For that reason the MV methodology has often been criticized as one that delivers unusual asset allocation (Black and Litterman, 1992). The algorithm prefers shares with high estimates of expected returns and low estimates of their standard deviations. Yet these estimates are not stable in practice and often do not reflect the most recent dynamics of the share of interest. For that reason the MV model allocates the wealth according to wrong inputs and therefore often yields unexpected results. Although these instabilities have been known for some time, the first models really to take market regimes into account were developed only in the last few years.

[2] Mullainathan (1998) explains the increased correlation in capital markets from the psychological point of view. He argues that during bad periods investors often recall past crises imperfectly. As a consequence, correlation of memories and not the fundamentals triggers the rise in correlation.

Ang and Bekaert (2004) exploit asymmetric correlations within an international portfolio optimisation framework in the period from 1975-2000. Their analysis suggests that there exist two regimes in the world equity market. The first regime is a normal regime with *high returns and low volatility* and second regime is characterised by *low returns and high volatility.* In addition, they showed that asset returns are indeed more correlated under a high volatility regime. They also show that taking into consideration the existence of bull/bear (or calm/turbulent) market regimes may significantly improve international portfolio performance. Markose and Yang (2008) study regimes in the UK equity market and manage to identify bull and bear regimes in the FTSE 100 index. Similarly to Ang and Bekaert (2004), their out-of-sample portfolio optimisation exercise clearly illustrates the benefits of recognising market regimes. In both analyses the authors assume sudden and unexpected transitions between market states. In line with that the moments of the shares, the expected returns, standard deviations and correlations between shares, are also regime dependent. Transition between the states of the market is conveniently modelled under Markov Regime Switching framework (Hamilton, 1998; 1999; 2004), which implies the states of the market to be unobservable.

Building on the related literature, this paper studies the asymmetric correlation in the domestic equity market. In the first step it studies the relationship between the Croatian market and several European markets. Results of the Markov switching model suggest that all the markets under analysis, including the Croatian, operate under one of the two regimes. The first regime appears to be characterised with high expected returns and low volatility and is therefore identified as bull or calm regime. On the other hand, the second regime on average exhibits negative returns, followed by high volatility and is identified as a bear or turbulent regime. Furthermore, the increased synchronization of the market regimes in all the countries suggests an increased integration of the new and the old European countries. Croatia, the only non-member state of the EU, is not an exception in this context. In line with related literature on developed markets (Ramchand and Susmel, 1997; Longin and Solnik, 2001; Ang and Chen, 2002) this paper also finds that correlations between the markets under analysis are asymmetric. During the bad periods these correlations are more than twice as big as the good market periods. The result also holds for the links between the new and the old European countries. These results are important in the context of international diversification and suggest only a limited benefit that foreign investors can count on when diversifying their portfolios by expanding to developing European markets.

After the analysis of the link between domestic market and international environment, the paper studies the correlation among shares on domestic market. First, the two regimes under which the domestic market operates are identified by applying the Markov switching framework. Like developed markets, the two identified regimes of the CROBEX are characterized by negatively correlated expected returns and standard deviations. After that, an out-of-sample portfolio optimization exercise is conducted on the period 2007-2010, which suggests that taking the regimes into account can improve the performance of a standard MV model[3]. During the boom periods, MRS portfolios yield higher returns than a standard model. On the other hand, during the bad periods on the equity market, the standard model ignores the market regimes and therefore anticipates the crisis slowly. As a consequence, it holds a large share of risky assets during the bad periods and finally performs poorly in our out-of-sample exercise.

In the context of the methodology used, this paper builds on the existing framework with two important details. First, the model employed in this paper assumes that only the market index is explicitly regime-sensitive and individual shares are regime sensitive only via CAPM relations with the market index. These relations are not stable on the Croatian market and therefore this paper tests whether correction for this instability improves the performances of the standard Markov switching model. The results obtained in their paper suggest that taking this instability into account can significantly improve the model. The second technical detail deals with the reliability of a Markov switching model. By using a simple Monte Carlo analysis it was shown that it is possible to replicate reliably monthly returns of CROBEX by the Markov model which is not a case for a standard normal distribution.

The structure of the paper is as follows. The second section deals with the Markov Regime switching methodology. The third section studies the asymmetric correlation between the domestic market and its international environment. The fourth section develops a portfolio optimization model that takes asymmetric correlations into account and shows the result of an out-of-sample exercise. The fifth section concludes.

2 METHODOLOGY

The main results of the paper are based on an assumption that one can easily identify several regimes in equity returns. For example, the identification is performed by conditioning on various real or financial indicators. Furthermore, one can identify market regimes by assuming that the equity returns are generated from a different statistical distribution, depending on the current market regime. The latter strategy is adopted in this paper and is usually based on *Markov Regime Switching (MRS) methodology*.

2.1 MARKOV REGIME SWITCHING FRAMEWORK

Unexpected, sudden breaks in time series, say in returns series y_t are easily captured by a simple Markov switching model. Assuming that there are two states of the market/world, most often one consider bull and bear market regime, variable

[3] Mean Variance models for Croatian equity market are tested in Fruk and Huljak (2004), Jakšić (2007), Latković (2001), Latković and Boršić (2000), and Latković (2000). Although these studies deal with important issues for domestic equity market, implemented models ignore regimes on the market.

of interest is assumed to be drawn from one of the two Gaussian distributions, depending on the state of the market S_t:

$$y_t \mid (S_t = i) \sim N(\mu_i, \sigma_i^2), \text{ for } i = 1,2, t = 1,\ldots,T, \tag{1}$$

or equivalently, in *regression form*:

$$y_t = \begin{cases} \mu_1 + \sigma_1 \varepsilon_t & \text{if } S_t = 1 \\ \mu_2 + \sigma_2 \varepsilon_t & \text{if } S_t = 2, \end{cases}$$

where S_t denotes the current state of the market, ε_t denotes standard normal white noise and μ_i and σ_i parameters to be estimated. State variable S_t is unobservable and it is conveniently assumed that it obeys a discrete Markov process on the state space $S = \{\text{bull, bear}\} = \{1, 2\}$. In that case, the transition probabilities of the process are characterized by two properties:

$$P(S_t = j \mid S_{t-1} = i) = p_{ij,} \forall t (\text{time homogeneity}) \tag{2}$$

$$P(S_t = j \mid S_{t-1} = i, S_{t-2} = k, \ldots) = P(S_t = j \mid S_{t-1} = i) (\text{Markov property}) \tag{3}$$

and transition probabilities form a matrix to be estimated:

$$\begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix} = \begin{bmatrix} P(S_t = 1 \mid S_{t-1} = 1) & P(S_t = 2 \mid S_{t-1} = 1) \\ P(S_t = 1 \mid S_{t-1} = 2) & P(S_t = 2 \mid S_{t-1} = 2) \end{bmatrix} = \begin{bmatrix} P & 1-P \\ 1-Q & Q \end{bmatrix}.$$

In our context, the first property ensures that transition probabilities between bull and bear market periods are constant over time, while the second one implies that the market regime for the following period depends only on the current regime.

2.1.1 Parameter estimation

Before proceeding to the optimisation step we need to resolve the parameter estimation problem first. The set of parameters $\theta = (\mu_1, \mu_2, \sigma_1, \sigma_2, p_{11}, p_{22})$ to be estimated consists of expectations and standard deviations for two regimes $(\mu_1, \mu_2, \sigma_1, \sigma_2)$ and probabilities (p_{11} and p_{22}) for the market to stay in the current state. The regime process $\{S_t\}$ is unobservable and, therefore, the likelihood function (i.e. the joint density function for the market process $\{y_t\}$) needs to be evaluated in a series of steps.

The log-likelihood function is:

$$\ln L(\theta;\, y_1,\ldots, y_T) = \sum_{t=1}^{T} \ln f(y_t \mid I_{t-1}) = \sum_{t=1}^{T} \ln\Big(\sum_{s_t=1}^{2} f(y_t, s_t \mid I_{t-1})\Big)$$
$$= \sum_{t=1}^{T} \ln\Big(\sum_{s_t=1}^{2} f(y_t \mid s_t, I_{t-1}) f(s_t \mid I_{t-1})\Big)$$

where $I_{t-1} = \{y_1,\ldots, y_{t-1}\}$ denotes the investor's information set available at time t–1 and conditional normal densities $f(y_t \mid s_t, I_{t-1}) = f(y_t \mid s_t)$ are known from (1). In order to complete the likelihood function we need to calculate the $f(S_t \mid I_{t-1}) = P(S_t = s_t \mid y_1,\ldots,y_{t-1})$ iteratively using the *Hamilton's filter* (see Hamilton, 1989; 1994):

$$P(S_t = i \mid I_{t-1}) = P(S_t = i, \cup_{j\in\{1,2\}}(S_{t-1} = j) \mid I_{t-1}) \tag{4}$$
$$= \sum_{j=1}^{2} P(S_t = i, S_{t-1} = j \mid I_{t-1})$$
$$= \sum_{j=1}^{2} P(S_t = i \mid S_{t-1} = j, I_{t-1}) P(S_{t-1} = j \mid I_{t-1})$$
$$= \sum_{j=1}^{2} p_{ji} P(S_{t-1} = j \mid I_{t-1}).$$

$$P(S_t = i \mid I_t) = \frac{f_{S_t, y_t \mid I_{t-1}}(i, y_t \mid I_{t-1})}{f(y_t \mid I_{t-1})} \tag{5}$$
$$= \frac{f_{y_t \mid S_t, I_{t-1}}(y_t \mid i, I_{t-1}) P(S_{t-1} = i \mid I_{t-1})}{\sum_{j=1}^{2} f_{y_t \mid S_t, I_{t-1}}(y_t \mid j, I_{t-1}) P(S_{t-1} = j \mid I_{t-1})}.$$

In order to obtain the vector of estimates θ which is the most consistent with the observed data, we need to maximise ln $L(\theta; y_1,..., y_T)$ using numerical optimisation[4]. The *filtered probabilities* $P(S_t = i \mid I_t)$ can be estimated recursively from (5) given appropriate initial values (see Hamilton, 1994) and vector θ. These probabilities reflect *investor's inference* of the current world market regime given the observed world market return up to time *t*. In addition, one can also estimate ex-post inference, i.e. the *smoothed probabilities* $P(S_t = i \mid y_1,...,y_T)$ given full sample. The algorithm for calculating these ex-post probabilities can be found in Kim (1994), and Hamilton (1994). By construction, it is clear that filtered probability equals the smoothed probability for the last observation in our sample. The framework can be easily extended to more general Markov chains.

3 ASYMMETRIC CORRELATION ON EQUITY MARKETS IN EUROPE

This section studies the asymmetry on the Croatian equity markets and on several European equity markets of interest. New member states of the EU are represented by Czech Republic, Romania and Bulgaria while the old Europe is represented by Germany. For comparison purposes, the aggregate equity indices of a group of the Central and Eastern European Countries (CEEC), the EU market and overall world equity market are also considered. The analysis is conducted in two steps. First, for each equity index under analysis the two market regimes are identified by employing the Markov Regime Switching framework. After that, the correlation between markets is calculated for each of the identified regimes. All the calculation is conducted with the use of monthly data for the period 2001-2009. Data source is *Bloomberg*[5].

[4] Alternatively, parameters can be estimated via EM algorithm as shown in Hamilton (1990).
[5] For the purpose of the analysis the following equity indices are used: CROBEX (Croatia), PX (Czech Republic), BET (Romania), SOFIX (Bulgaria), DAX (Germany), CECE (CEEC), MXWD (World).

3.1 REGIMES ON EUROPEAN MARKETS

Table 1 reports expected nominal returns and standard deviations across identified *Markov* regimes for eight markets under analysis.

TABLE 1

Basic statistics for market regimes

	CRO	CZ	ROM	BU	CEEC	GER	EU	World
Expected return (%)								
Full sample	0.8	0.8	1.5	1.8	0.9	0.6	0.4	0.3
Bull regime (μ_1)	2.0	2.3	3.9	4.2	1.3	1.8	1.6	1.1
Bear regime (μ_1)	-1.0	-1.3	-8.8	-2.8	-2.7	-2.3	-0.4	-0.8
Standard deviation (%)								
Full sample	7.5	6.6	9.5	9.5	7.2	5.8	5.2	4.3
Bull regime (σ_1)	4.6	4.0	7.5	5.8	6.1	3.5	2.6	2.3
Bear regime (σ_1)	10.4	7.5	10.1	13.0	13.6	6.9	6.1	5.5
P	0.98	0.96	0.99	0.98	0.99	0.96	0.96	0.97
Q	0.97	0.97	0.91	0.97	0.89	0.90	0.97	0.95

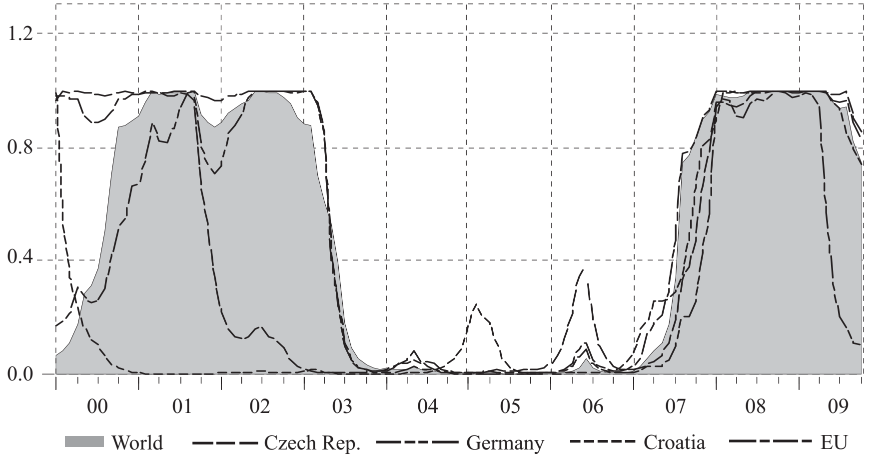
Note: CRO (Croatia), CZ (Czech Republic), ROM (Romania), BU (Bulgaria), GER (Germany).

The reported results suggest that the equity markets of interest operate under one of the two states of the world, i.e. regimes, characterized by negatively-related expected returns and standard deviations. The first regime appears to be characterised by high expected returns and low volatility and is therefore identified as bull or calm regime. On the other hand, the second regime on average exhibits negative returns, followed by high volatility and is identified as a bear or turbulent regime. The probability for the market to stay in the current state/regime (parameters P and Q) is close to unity for both regimes which is a property that suggests a high persistence of the estimated regimes in past years.

The equity returns in Croatia are well captured by a bull regime with an expected monthly return of 2% and standard deviation of 4.6% and a bear regime with monthly loss of 1% and standard deviation of 10.4%. On average, CEEC markets are to a great extent more volatile and report higher expected returns than developed markets. The Croatian equity market and the Czech market report statistics similar to those from developed markets. This is not the case with Romania and Bulgaria which report dramatic differences in this context.

Figure 2

Ex-post probabilities for various markets to be in the bear regime



By comparing market regimes across the markets of interest one can illustrate the integration dynamics of international equity markets. Figure 2 therefore shows the ex-post probability for different markets to be in a bear/turbulent regime during the last ten years. The first part of the period is largely characterized by still undeveloped financial markets of central and eastern Europe and also their weak integration into international markets as well as the poor general economic linkages with international environment. As a consequence, an increased (or decreased) volatility in transition countries is primarily a consequence of domestic financial and economic developments and to much less extent of some external activities. On the other hand, during the past several years, the links with developed markets have strengthened dramatically. A direct consequence of the increased integration is that the market regimes have become more synchronized.

Figure 3

Nominal monthly returns on CROBEX together with ex-post probabilities of being in a turbulent (bear) regime

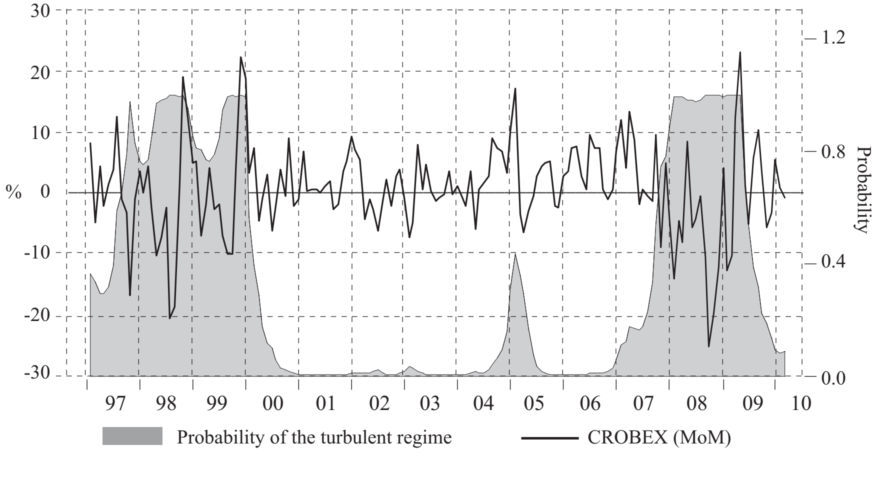


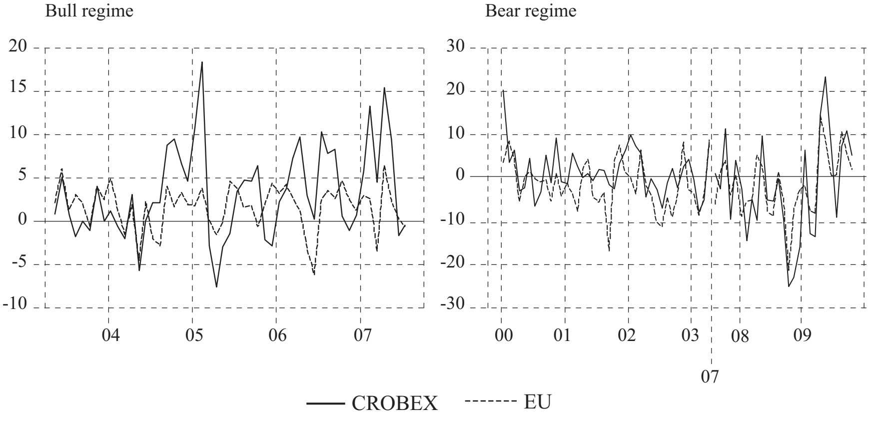
Figure 3 shows the ex-post probability for the Croatian equity market to operate under a turbulent regime. Markov switching methodology identified two turbulent regimes on that period. The estimated turbulent periods however have very different causes. The first crisis from 1997-1999 coincides with domestic recession triggered by the turbulences in the domestic banking sector. In contrast to that, the recent global recession caused by the problems of the international banking sector, has spilled over onto financial markets worldwide.

3.2 ASYMMETRIC CORRELATION IN EUROPEAN EQUITY MARKET

This section compares correlations of the nominal monthly returns among several European equity indices for two identified regimes – the bull and the bear market regime. Every market is considered to operate under a bull regime if ex post probability for the EU market to be in the bull state is larger than 0.5, i.e. $P(S_t^{eu} = bull \mid y_1^{eu},\ldots, y_T^{eu}) > 0.5$. We indicate a bear regime in a similar way. Due to a high co-movement between European equity markets, the state of the market in the EU really is a valid indicator for market conditions in other European countries under analysis (see figure 2).

FIGURE 4

Monthly returns on the CROBEX together with returns on the aggregate EU index for two identified regimes



In order to illustrate the asymmetric correlation among European markets figure 4 compares co-movement between returns of Croatian CROBEX and aggregate EU market for two identified regimes. The figure illustrates much tighter links between the two markets during the turbulent periods compared to the calm market periods.

Table 2 compares the correlations between European markets across the identified regimes. The estimated correlations in the bull market are shown under the diagonal and those in the bear periods above the diagonal. For all the pairs of indices the asymmetric correlation is established and correlations above the diagonal are much larger than those under the diagonal. The asymmetric nature of the links among the markets of interest is established for all the markets, both developed and transitional. The reported results therefore suggest the only limited benefit that foreign investors can count on when diversifying their portfolios by expanding to developing European markets.

TABLE 2
Asymmetric correlations

	BU	CEEC	CRO	CZ	ROM	GER	EU
BU	**1.00**	0.55	0.61	0.62	0.58	0.35	0.42
CEEC	0.30	**1.00**	0.62	0.89	0.64	0.76	0.83
CRO	0.04	0.27	**1.00**	0.58	0.59	0.57	0.63
CZ	0.18	0.86	0.29	**1.00**	0.72	0.71	0.77
ROM	0.18	0.43	0.34	0.45	**1.00**	0.36	0.47
GER	0.05	0.44	0.28	0.48	0.15	**1.00**	0.96
EU	0.07	0.57	0.38	0.66	0.30	0.92	**1.00**

Note: Turbulent (above the diagonal), calm (under).
BU (Bulgaria), CRO (Croatia), CZ (Czech Republic), ROM (Romania), GER (Germany).

4 ASYMMETRIC CORRELATION IN THE CROATIAN EQUITY MARKET

This section studies asymmetric correlations in the Croatian equity market. Moreover, a portfolio optimization model that incorporates the asymmetric nature of the market is introduced and finally, by using this model, we illustrate potential losses that may arise if an investor ignores identified asymmetries.

4.1 PORTFOLIO OPTIMIZATION UNDER AN ASYMMETRIC CORRELATION

Let us consider an investor who allocates his wealth in N different stocks. At each period the investor is able to conveniently borrow and lend at his home risk-free rate. Asset allocation is based on the standard mean-variance portfolio optimisation framework. Furthermore, our investor is aware of the existence of bull/bear market regimes and estimates the inputs for his optimisation algorithm conditionally on his beliefs for the current period regime (provided by the econometrician). Let us formalise the story.

Assume that market returns may be drawn from one of the two different Gaussian distributions, conditionally on the current market regime.

$$m_t^e \mid (S_t = i) \sim N(\mu_i, \sigma_i^2), \text{ for } i = 1,2, t = 1,\ldots,T \tag{6}$$

or equivalently, in *regression form:*

$$m_t^e = \begin{cases} \mu_1 + \sigma_1 \varepsilon_t & \text{if } S_t = 1 \\ \mu_2 + \sigma_2 \varepsilon_t & \text{if } S_t = 2, \end{cases}$$

where $m_t^e = m_t - rf_t$ denotes excess return of the market, market S_t takes on the states 1 or 2 (bull or bear), ε_t denotes standard white noise and μ_i and σ_i regime parameters to be estimated.

As in Markose and Yang (2008), for each share (i = 1, 2,..., N) under consideration we link its (excess) returns $r_{it}^e = r_{it} - rf_t$ and markets excess return via CAPM-based regressions:

$$r_{it}^e = \alpha_i + \beta_i m_t^e + \varepsilon_{it}, \text{ for } t = 1, \ldots, T \qquad (7)$$

and, implicitly, individual shares are now also sensitive to changes in regime in the aggregate market. Expected returns for each share are now:

$$E(r_{it}^e) = \alpha_i + \beta_i E(m_t^e), \qquad (8)$$

and variances:

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \tilde{\sigma}_i^2, \qquad (9)$$

where σ_m^2 denote systematic – market variance and $\tilde{\sigma}_i^2$ idiosyncratic variance of asset i.

Our investor optimises his portfolio each month using Markowitz's mean-variance framework and, therefore, needs to supply his algorithm with the variance-covariance matrix of risky assets as well with individual expected returns. These moments now depend on the current world's market regime via (7) and (8) and therefore whole-sample moment statistics would be inappropriate inputs for the optimisation. Suppose however that the investor *knows* the current market regime (at time t). Let us further assume that he can also estimate the probability for the market to stay in either of the two regimes, as well the probability for market to change regimes. Now, he can easily estimate next period conditional moments, both for the world market and individual country returns as follows.

The market expected return at t+1 given the current market regime is estimated as:

$$e_1^w = E(w_{t+1}^e \mid S_t = 1) = P(S_{t+1} = 1 \mid S_t = 1)\mu_1 + P(S_{t+1} = 2 \mid S_t = 1)\mu_2$$
$$e_2^w = E(w_{t+1}^e \mid S_t = 2) = P(S_{t+1} = 1 \mid S_t = 2)\mu_1 + P(S_{t+1} = 2 \mid S_t = 2)\mu_2,$$

and conditional variances are:

$$\Sigma_1^w = Var(w_{t+1}^e \mid S_t = 1) = P(S_{t+1} = 1 \mid S_t = 1)\sigma_1^2 + P(S_{t+1} = 2 \mid S_t = 1)\sigma_2^2$$
$$+ P(S_{t+1} = 1 \mid S_t = 1)(1 - P(S_{t+1} = 1 \mid S_t = 1))(\mu_1 - \mu_2)^2$$

$$\Sigma_2^w = Var(w_{t+1}^e \mid S_t = 2) = P(S_{t+1} = 1 \mid S_t = 2)\sigma_1^2 + P(S_{t+1} = 2 \mid S_t = 2)\sigma_2^2 + P(S_{t+1} = 2 \mid S_t = 2)(1 - P(S_{t+1} = 2 \mid S_t = 2))(\mu_1 - \mu_2)^2$$

The moments for individual country returns may be estimated in the following way. First, from (7) for each country we have:

$$E(r_{it}^e \mid S_t = j) = \alpha_i + \beta_i E(w_t^e \mid S_t = j), \text{for } j = 1,2$$

and let $\alpha = (\alpha_{\mathrm{UK}}, \ldots, \alpha_{\mathrm{JPN}})^\tau$ and $\beta = (\beta_{\mathrm{UK}}, \ldots, \beta_{\mathrm{JPN}})^\tau$ be the vectors of estimated coefficients.

Now we can stack individual country expectations into a vector:

$$R_j = \begin{bmatrix} E(r_{UKt}^e \mid S_t = j) \\ \vdots \\ E(r_{JPNt}^e \mid S_t = j) \end{bmatrix} = \alpha + \beta \begin{bmatrix} E(w_t^e \mid S_t = j) \\ \vdots \\ E(w_t^e \mid S_t = j) \end{bmatrix}, \text{for } j = 1,2.$$

Conditional expectations for the following period are now:

$$e_1 = P(S_{t+1} = 1 \mid S_t = 1)R_1 + P(S_{t+1} = 2 \mid S_t = 1)R_2$$
$$e_2 = P(S_{t+1} = 1 \mid S_t = 2)R_1 + P(S_{t+1} = 2 \mid S_t = 2)R_2.$$

Conditional (regime-dependant) covariance matrix is:

$$\Omega_j = \beta\beta^\tau \sigma_j^2 + V^2, \text{ for } j = 1,2,$$

where $V = (\tilde{\sigma}_{UK}, \ldots, \tilde{\sigma}_{JPN})$. Finally, conditional covariance matrix for the period $t+1$ is:

$$\Sigma_1 = P(S_{t+1} = 1 \mid S_t = 1)\Omega_1 + P(S_{t+1} = 2 \mid S_t = 1)\Omega_2 + P(S_{t+1} = 1 \mid S_t = 1)(1 - P(S_{t+1} = 1 \mid S_t = 1))(R_1 - R_2)^2$$

$$\Sigma_2 = P(S_{t+1} = 1 \mid S_t = 2)\Omega_1 + P(S_{t+1} = 2 \mid S_t = 2)\Omega_2 + P(S_{t+1} = 2 \mid S_t = 2)(1 - P(S_{t+1} = 2 \mid S_t = 2))(R_1 - R_2)^2.$$

For details see Markose and Yang (2008), and Ang and Bekaert (2004).

4.2 RESULTS

The presented model is now tested in a portfolio optimization exercise applied to the three shares included in the Zagreb Stock Exchange – PBZ d.d. (PBZ-R-A), Dalekovod (DLKV-R-A) and Atlantska plovidba (ATPL-R-A). Transaction costs when re-optimizing a portfolio are ignored during the analysis. Although the model is illustrated for a small number of shares and all the transaction costs are ignored, this simple model is still sufficiently informative to present all the benefits of taking asymmetric correlation into account when re-optimizing a portfolio. This model therefore is not necessarily intended to yield super profit in practice, but only to highlight the costs of ignoring regimes in the equity markets, independent of the investment strategy.

The selection of the shares included in the analysis was based on two criteria. First, in order to diversify possible sectoral risks, only shares from different sectors of the economy are considered. After that it was necessary for a share to have available price data for a sufficiently long period, from 2001-2010.

One year Treasury bills are used as a risk free security in the analysis.

4.2.1 Regimes in the Croatian equity market and asymmetric correlations

The model assumes the returns on the market index to be drawn from one of the two Gaussian distributions, depending on the current market regime. Table 3 shows basic statistics of the market return for two regimes identified via Markov switching framework.

Table 3
The estimated parameters

	Calm (regime I)			**Turbulent (regime II)**		
	μ_1	σ_1	P	μ_2	σ_2	Q
Estimate	1.75	4.81	0.99	-4.06	10.93	0.94
Std. error	0.52	0.20	0.12	2.10	1.10	0.30

The bull (the calm) regime is characterized by monthly return of 1.75% and a standard deviation of 4.81%, while the bear (the turbulent) regime is characterized by a monthly loss of 4.06% followed by a standard deviation of 10.93%. Expected returns, standard deviations and co-variances of individual shares are implicitly dependant on the current market regime via CAPM relations to the market (6). Table 4 compares correlations among individual shares across the identified regimes.

Table 4
Asymmetric correlations

	PBZ	**DLKV**	**ATPL**
PBZ	**1.00**	0.69	0.55
DLKV	0.30	**1.00**	0.53
ATPL	0.20	0.18	**1.00**

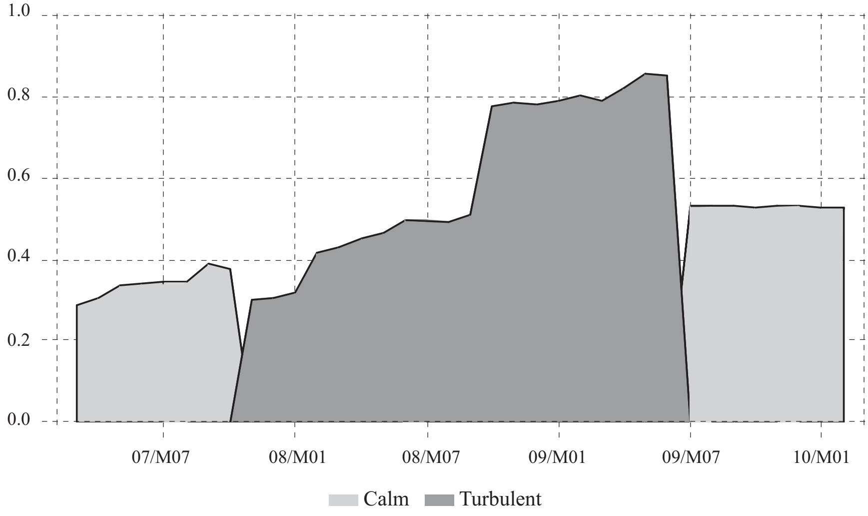
Note: Turbulent, calm.

For all the pairs of shares we found significant differences in correlations among the market regimes. During the falling, turbulent market periods, the links between returns appear to be over two times stronger compared to the rising markets. Taking into account that correlation among shares is an important input for a portfolio optimization algorithm, a procedure that ignores the regimes on the market can be expected to perform poorly in practice.

In order to get more insight into correlation dynamics between shares over time figure 5 shows time dependent correlations for the pair PBZ and Dalekovod. The figure illustrates the tighter links between shares during the bad market periods.

FIGURE 5

Dynamic correlations between Dalekovod and PBZ equity markets for two regimes



Note: For period t if $P(S_t = i \mid I_t) > 0.5$ the correlation under regime i is reported.

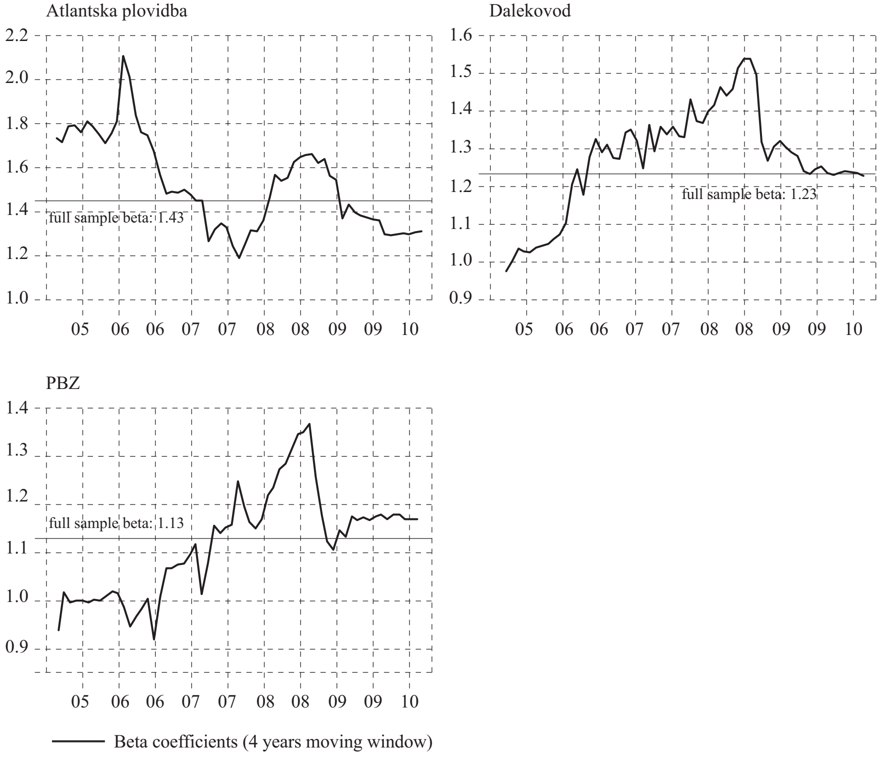
Only implicitly via the CAPM are individual shares also regime sensitive. When modelling asymmetric correlations on the world market and the UK equity market, Ang and Bekaert (2004), and Markose and Yang (2008) assume stable CAPM relations over time. However, Kunovac (2009) shows that these relations are not stable over time and that taking these instabilities into account when building a Markov switching portfolio model can significantly improve performances of the model in practice. For that reason, as well as the static CAPM equations we also test a model that uses coefficients calculated on a moving window of 48 months. In other words, at time t relation $r_{it} = \alpha_{ti} + \beta_{ti} m_t + \varepsilon_{it}$ is estimated where parameters α_{ti} and β_{ti} are estimated at period $t - 48{:}t$.

Table 5
CAPM coefficients

	PBZ	DLKV	ATPL
α	1.22	1.14	3.21
Std. error	0.61	0.72	1.50
β	1.14	1.20	1.46
Std. error	0.09	0.10	0.22

Table 5 compares the estimates of the CAPM coefficients for three shares on the whole sample. For all the shares under analysis the estimates of the α are larger than zero, but only marginally significant. On the other hand, the β coefficients are estimated as larger than unity. This suggests that all the shares yield returns that are higher than the market during the good periods and returns lower than CROBEX during the bad market periods.

Figure 6
Static and moving window estimates of CAPM betas



Note: Moving window estimates consider 48 month windows.

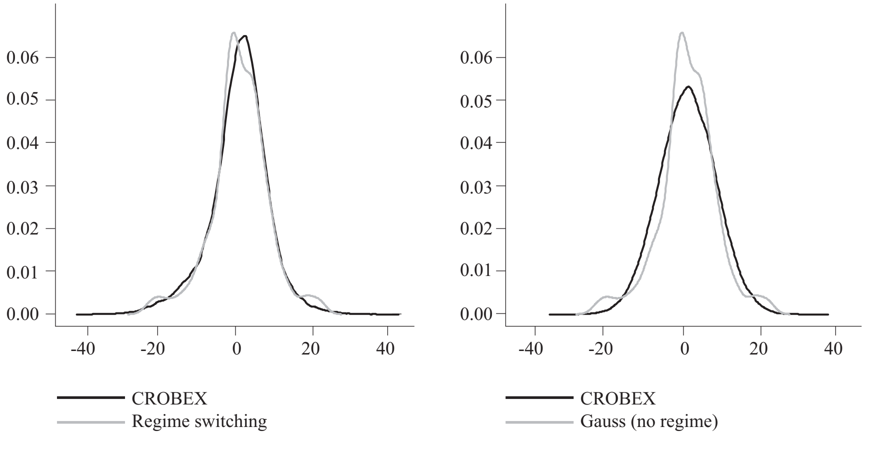
Figure 6 compares moving β coefficients with static coefficients. The figure suggest that the CAPM relations for the Croatian equity market are not stable over time and therefore static estimates can provide only a rough estimate of the link between individual share and the market. A direct consequence of this instability is the instability of the main inputs of the portfolio optimization algorithm – expected returns, standard deviations and correlations between shares. In order to estimate how costly it is to ignore this instability in practice, we implement both static and dynamic CAPM-based mean variance models.

4.2.2 How good is our Markov-Switching model for CROBEX? Evidence from Monte Carlo experiment

Here we discuss the reliability of the estimated Markov switching models. It appears that very few papers provide any evidence that the estimated MS model fits the data properly. On the other hand, it has been well documented that the maximisation of likelihood function in this context is not an easy task (Hamilton, 1994). Parameter spaces involved in the estimation are usually of large dimensions and there is no guarantee that the estimated parameters indeed maximise (*generally not concave*) the likelihood function globally. The main question is how we can be sure that the estimated parameters define distribution which is *close enough* to (empirical) distribution of actual, observed data, in our case to returns on CROBEX.

FIGURE 7

Densities implied by actual data on CROBEX and estimated two-regime Markov regime switching models



Note: For comparison purposes the figure also shows the density implied by the Gaussian distribution.

Motivated by an inspiring paper by Breunig and Pagan (2004), we adopt a simulation-based approach to test to what extent the Markov switching models estimated in this paper are capable of mimicking (empirical) distribution of returns on

CROBEX. The *test* only involves a graphical comparison of two density functions[6]. The first is the density estimated from observed return data and the second is the density of simulated (Monte Carlo) draws from return distribution(s) based on the maximum likelihood parameters (i.e. on the vector, $\theta = (\mu_1, \mu_2, \sigma_1, \sigma_2, p_{11}, p_{22})$). Although very simple, this method provides a powerful tool as a *goodness-of-fit* indicator.

In the first step I simulate a Markov chain via a Monte Carlo given estimated transition probabilities. Market return is defined conditionally on the realisation of the simulated regime process and can be easily drawn from appropriate normal density. In this way I simulate 100.000 draws implied by the estimated MS model and construct an approximate density function. Now comparing this density[7] to the density calculated directly from the series of the observed market return one can make an inference on the reliability of the MS model, i.e. of the reliability of estimated parameters.

Figure 7 compares the density implied by the estimated Markov switching model and that estimated from the observed data on CROBEX. For comparison purposes, the Gaussian density is also plotted. The figure suggests that the Markov switching model mimics the observed data much more accurately than the Gaussian density. Although the estimated models based on the two regime market mimic the observed data fairly well, this model is not capable of capturing extreme returns from the tails of the distribution. However this model is easily expanded to a three regime case, which is a model that easily captures the issue (Kunovac, 2009).

4.2.3 Asset allocation under regime switching

This section studies the benefits of taking the market regimes into account when optimizing an equity portfolio. In an out-of-sample exercise, performance of a standard mean variance model is compared to a Markov switching model that takes the asymmetric correlations into account. On the out-of-sample period, from March 2007 – March 2010, we compare the cumulative return on 1 monetary unit invested over the out-of-sample period to the cumulative return on 1 monetary unit invested under the standard mean-variance model that ignores regimes in equity markets. At each period the investor is able conveniently to borrow and lend at his home risk-free rate (1 year Treasury bills). In addition, there are short-selling constraints imposed, i.e. all the weights in the portfolio need to be non-negative. The model is evaluated in an out-of-sample as follows.

1) At time t estimate $\theta = (\mu_1, \mu_2, \sigma_1, \sigma_2, p_{11}, p_{22})$ using the information available up to t and make an inference about the current regime for CROBEX using the

[6] Probability density functions are approximated using *kernel density estimation* techniques (see Li and Racine, 2007).

[7] For details see Limić (2005).

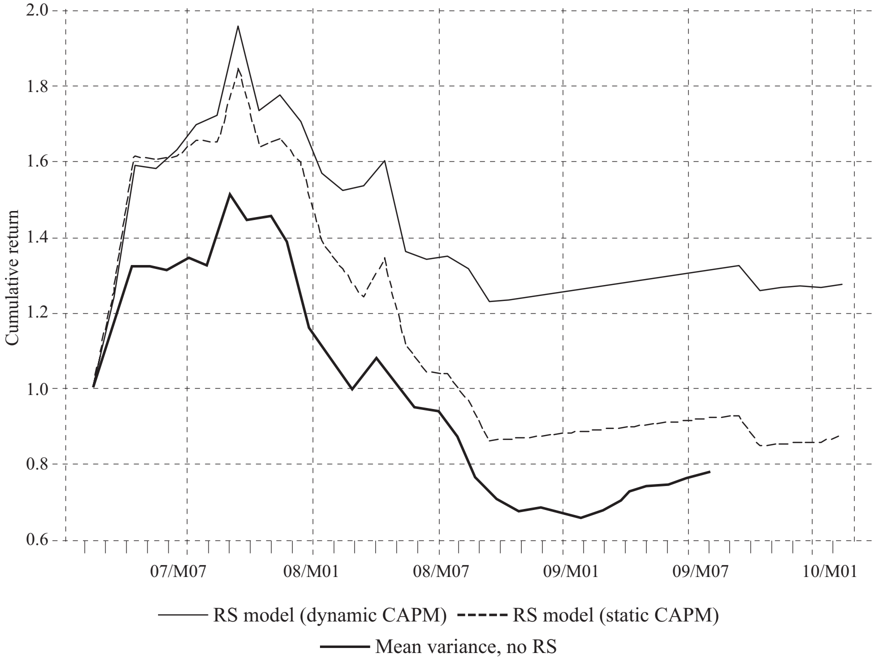
filtered probabilities. If $P(S_t = i \mid I_t) > 0.5$ for some i, the market is considered to be in state i at the moment.

2) Calculate conditional moments for CROBEX for the following period.
3) Estimate CAPM equations for individual shares and calculate regime-dependent moments for each share for the following period. Regression coefficients are estimated in two different ways. Firstly, using only in-sample period data, and secondly, using the moving window regression on the period $t-48{:}t$.
4) Given the risk aversion (3 was chosen arbitrarily here) optimise the portfolio.
5) Add another observation and go to step 1 (unless the sample is exhausted; in that case the procedure is over).

Figure 8 compares cumulative returns on one monetary unit implied by the three mean variance models – the first one is a standard MV model that ignores the market regimes and two Markov switching models.

Figure 8

The cumulative return on one monetary unit invested over the out-of-sample period under three strategies



The first MS model is static and envisages a stable relationship between individual shares and the CROBEX, while the second model takes the instability of the CAPM into account. At the very beginning of the period under analysis, Markov switching strategies based on the bull parameters outperform the standard model significantly. This is mainly due to a more aggressive investing strategy where, according to MS models, in good market periods, investors are even willing to borrow at a risk free rate in order to reinvest in risky assets. On the other hand, during the bad market periods, the MS models were able to anticipate the crisis and accordingly the investment was based on a different set of parameters. From the mid 2008 regime, switching models therefore follow a conservative strategy and invest exclusively in the risk free asset and therefore grow at the *T* bill rate. In contrast to that the standard MV model allocated the wealth according to an averaged set of parameters and therefore did not respond accurately to the crisis. As a consequence, during the recent crisis, the standard model held a large share of risky assets in its portfolio (over 30% on average) and therefore performed poorly.

Furthermore, our results suggest that the instability of the CAPM relations should be taken into account when building a MV model. Our dynamic model, based on fresh information, enables one to estimate a more accurate link between individual shares and the market, which might result in more accurate inputs for a portfolio optimization algorithm. A static Markov regime switching model did not outperform a standard MV model.

5 CONCLUSION

This paper studies the differences on the equity market in Croatia during bad and good market periods. In order to get a better insight into propagation mechanism of financial distress, links between the domestic market and several European markets are first studied in more detail. After that, the paper deals with the relationships among the shares included in CROBEX. The main findings of the paper may be summarised as follows.

Results of the analysis suggest that all the equity markets under analysis, including Croatia, operate under one of the two states of the world, i.e regimes, characterized by negatively related expected returns and standard deviations. The first regime appears to be characterised by high expected returns and low volatility and is therefore identified as a bull or calm regime. On the other hand, the second regime on average exhibits negative returns, followed by high volatility and is identified as a bear or turbulent regime. In addition, the increased synchronization of the market regimes in all the countries suggests the increased integration of the new and the old European countries. Croatia, the only non-EU member state, is not an exception in this context. Tight links between the markets are primarily a direct consequence of the broader economic integration process.

In line with related literature on developed markets (Ramchand and Susmel, 1997; Longin and Solnik, 2001; Ang and Chen, 2002) this paper also finds that correlations between the markets under analysis are asymmetric. During the bad periods these correlations are more than twice as large as in the good market periods. The result also holds for the links between new and the old European countries. These results are important in context of international diversification and suggest that there is only a limited benefit that foreign investors can count on when diversifying their portfolios by expanding to developing European markets.

A special attention is paid to the identification of the market regimes and testing the asymmetric correlations in Croatia. Moreover, a portfolio optimization model that incorporates the asymmetric nature of the market is introduced and finally, by using this model we illustrate potential losses that may arise if an investor ignores identified asymmetries. For all the pairs of shares, significant differences in correlations are found across the regimes. During the bad periods the links between the shares are more than twice as strong as in the good market periods. Since correlation between shares is an important input for a portfolio optimization algorithm, procedures that ignore the regimes on the market expectedly perform poorly in practice. In order to test this for the Croatian equity market, a simple out-of-sample portfolio optimization exercise is conducted where the performance of a standard mean variance model is compared to the results of a Markov switching version of a model. An important difference of the two models is that Markov switching model takes the asymmetry of the market into account. The results of the exercise suggest that a good model should take the asymmetric correlations into account in practice. During good periods, Markov switching portfolios, based on bull market inputs, outperform the standard model significantly. Similarly, during bad periods on the equity market, the standard model ignores the market regimes and therefore anticipates the crisis slowly. As a consequence, it holds a large share of risky assets during bad periods and finally performs poorly.

In the context of the methodology used, this paper builds on the existing framework with two important details. First, the model employed in this paper assumes that only the market index is explicitly regime sensitive and individual shares are regime sensitive only via CAPM relations with the market index. These relations are not stable on the Croatian market and therefore this paper tests whether correction for this instability improves the performances of the standard Markov switching model. The results obtained in their paper suggest that taking this instability into account can significantly improve the model. The second technical detail deals with the reliability of a Markov switching model. By using a simple Monte Carlo analysis it was shown that it is possible to replicate reliably monthly returns of CROBEX by a Markov model, which is not the case for a standard normal distribution.

This paper has established the existence of two regimes and asymmetric correlations in developing European equity markets. It has also highlighted the costs of ignoring these regimes while re-optimizing an equity portfolio on a daily basis. Established asymmetry is partially explained by common economic and financial shocks between the countries. To what extent these economic links are indeed responsible for the co-movements of the markets and what the role of potential irrational behaviour of the investors is raises an interesting question that needs answering.

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