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Comparison of Two Network-Theory-Based Methods for detecting Functional Regions

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Abstract

Background: Functional regions are abstract, uniformly defined territorial units that form an important basis for many development strategies of a country or a region. **Objectives:** This study analyses the application of network theory to the detection of such regions. **Methods/Approach:** Functional regions are analysed using two methods based on the graph theory: the Walktrap algorithm and the chain approach. The quality of the two regionalization methods is analysed using the fuzzy set theory with the revised method. Slovenia was used as a case study. **Results:** The Walktrap algorithm generated eight functional regions; seven of them corresponded to those identified in previous studies. The only difference occurred in the northwestern mountainous part of Slovenia. The chain approach led to similar results, although it resulted in a huge functional urban region of the capital Ljubljana. **Conclusions:** The results show that the Walktrap algorithm calculates regions that are more closed, where more workers find work in the home region, than the chain approach.

Keywords: functional regionalisation, graphs, Walktrap algorithm, chain method, fuzzy sets, Slovenia

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Introduction

Spatial organisation is a crucial factor in understanding and explaining various socio-economic phenomena. This knowledge is necessary for sustainable spatial development, spatial planning and the implementation of various spatial policies aimed at a more efficient organisation (Coombes et al., 2012; Erlebach et al., 2016; Halás et al., 2018). However, there are still administrative regions used by many governments for policymaking, resource allocation and research that are delimited for historical reasons (Ball, 1980; Casado-Diaz, 2000; Cörvers et al., 2009. Such regions, in many cases, do not reflect the actual conditions of that particular place. Consequently, in recent years efforts have been directed towards identifying and delimiting functional regions that are more meaningful than the regions currently in use.

Ullman (1980), Karlsson & Olsson (2006) and many other researchers defined a functional region (FR) as a territorial area with a high frequency of intraregional interactions. In this context, interactions may relate, for example, to trade exchanges, financial and information flows or the movement of people. A FR is therefore a group of basic spatial units (BSUs) grouped based on spatial flows or interactions between them. A process of grouping basic spatial units into functional regions with the aim of generalising the functional flows in the addressed area is called functional regionalization regionalisation. Drobne (2016: 13) describe FR as "an area with generalised patterns of flows and interactions in space".

A functional region is an abstract spatial concept (Pálóczi et al., 2016). Therefore, there is no single method for delimiting them and it is not straightforward interpreting the fact that the application of different methods, although using the same data, can lead to significantly different results (Laan & Schalke, 2001). In the literature, there are three general approaches to FR taxonomy (Halás et al., 2018): graph-theoretical methods (e.g. Benassi et al., 2015; Drobne et al., 2010; Holmes & Haggett, 1977; Karlsson & Olsson, 2006; Konjar et al., 2010; Nystuen & Dacey, 1961), methods of numerical taxonomy (e.g. Masser & Brown, 1975; Masser & Scheurwater, 1978, 1980) and rule-based methods (e.g. Coombes et al., 1986; Coombes & Bond, 2008; Halás et al., 2015). When FRs are modelled, they can be valued using the fuzzy set theory approach. Feng (2009) and Watts (2009, 2013) show how we can quantify the degree of uncertainty in individual BSUs. The same methodology can be applied to the FRs.

Network theory provides a suitable framework for the analysis of interactions in complex systems, which can be applied for the recognition of FRs. A network can be defined as a mathematical object consisting of vertices and edges (Newman, 2010). The vertices represent the analysed units, while the edges describe the relationships between them. Thanks to this general definition, a number of problems have been solved with network theory models, from the structure of the World Wide Web (Faloutsos et al., 1999) to food distribution systems (Garre et al., 2019).

One of the reasons for the widespread use of networks is also that the analysis of their topology can provide meaningful information about the modelled system (Newman, 2010). A wide range of indices has been defined, which describe different topological properties of the network and thus of the modelled system. Boss et al. (2004), for example, analysed the degree of distribution and clustering of a network describing the relationships between banks in Austria in order to assess the robustness of the system. However, the amount of information that can be obtained using these indices alone is limited. One of the network attributes that is difficult to understand with simple indexes is the existence of clusters. In this context, clusters are defined as a group of vertices that are highly interconnected. The identification of the clusters is very complex and (although many algorithms are available) not yet satisfactorily solved (Fortunato, 2010). Nevertheless, there are several numerical algorithms that can estimate cluster distribution that have been used in a variety of situations (e.g. Clauset et al., 2008; Lehmann et al. 2008; Ronhovde & Nussinov, 2009), including some case studies that relate to the spatial organisation of different areas (Ke et al., 2017; Khatoon & Banu, 2019).

In our study, we calculate functional regions for Slovenia using two methods based on graph theory: Walktrap algorithm (Pons & Latapy, 2006) and chain method (Drobne et al., 2010; Karlsson & Olsson, 2006; Konjar et al., 2010). The quality of both regionalization methods is analysed using fuzzy set theory (Feng, 2009; Watts, 2009, 2013) with the revised approach. The results for Slovenia show that the Walktrap algorithm calculates more self-contained regions, i.e. regions with a higher relative frequency of intraregional flows, than the chain approach. This is the first time that the Walktrap algorithm has been used to analyse functional regions by commuting data on the macro spatial level; and it is the first time that the functional region analysis algorithm has been used for Slovenia. A short report on the study was published in (Drobne et al., 2019).

Methodology

In a case study we used a data for Slovenia for the year 2017. The geodata for the 212 municipalities were taken from the "Free Access Database" from the Surveying and Mapping Authority of the Republic of Slovenia (GURS, 2018) and statistical data on inter- and intra-municipal labour commuting flows were obtained from the "SI - Stat Data Portal" of the Statistical Office of the Republic of Slovenia (SURS, 2018).

Gabrovec and Bole (2009) pointed out some methodological problems that may arise from the use of commuter statistics. Two potential problems are worth mentioning here: (a) the first is the incorrect registration of the place of residence or work and (b) the second is the supposed registration of commuting. Ad (a) When modelling FRs at the macro level, we consider in particular the largest relative interactions between the residential community and the working community. Due to the relativisation of the absolute flows of more than 400,000 commuters between Slovenian municipalities, this problem does not have a significant impact on the allocation of the municipality to a particular FR. Ad (b) The nature of work has changed considerably in recent decades. There are more and more professions in which workers can work at home, so that they only go to work a few times a week as needed. The problem mentioned does not concern the formation of FRs, since they are actually defined by the functional connections of the territory - which is particularly exposed in the case of telework.

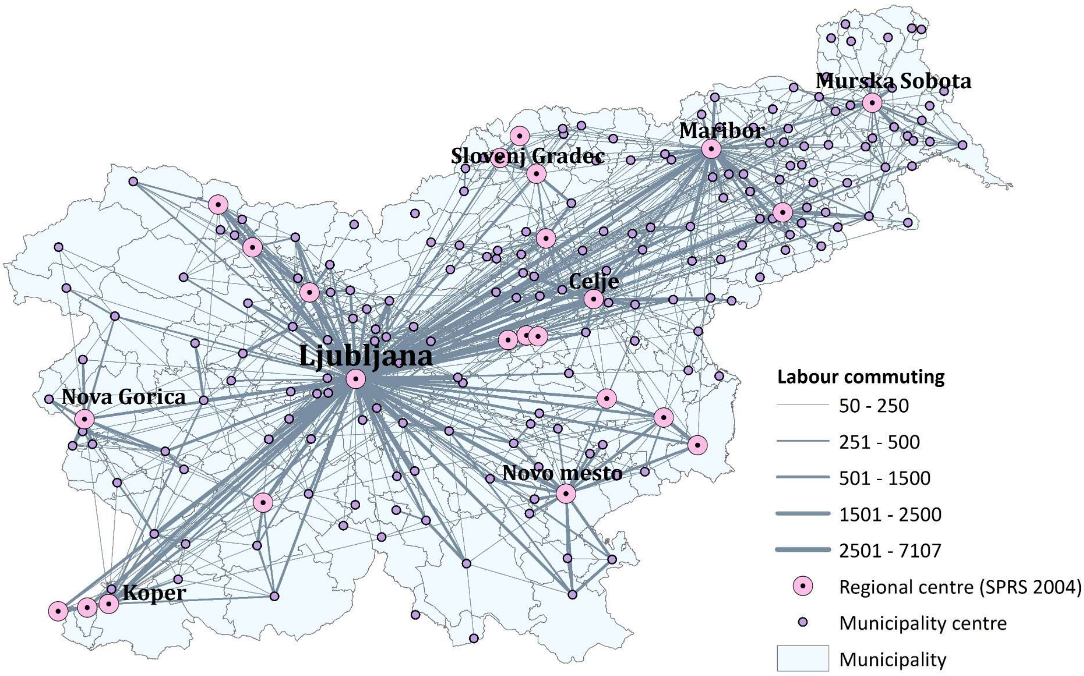
With data on the commuter statistics, we created an interaction matrix of a total of 44,944 (212x212) cells. The matrix consisted of 31,060 (69.1%) empty cells, 4764 (10.6%) cells that noted only 1 commuter, and only 283 (0.6%) cells that recorded 500 commuters or more. The interaction matrix recorded a total of 830,564 registered commuters (working population), but only 432,108 (52.2%) of them commuted between municipalities. The rest, i.e. 398,456 (47.8%) of the working population, thus lived and worked in the same municipality. The eleven largest inflows of 124,227 (28.7%) commuters ended in Slovenia's largest employment centre, i.e. the capital Ljubljana, while outflows from Ljubljana amounted to 20,402 (4.7%) commuters. Figure 1 shows the inter-municipal commuter flows for Slovenia in 2017; note that only flows with 50 and more commuters are shown on the map.

Regional centres in the Figure 1 were stated in (SPRS, 2004).

According to the Spatial Development Strategy of Slovenia (SPRS, 2004), there are fifteen urban centres of national significance in Slovenia: Ljubljana, Maribor, conurbation Koper–Izola–Piran, Celje, Kranj, Novo mesto, Nova Gorica, Murska

Sobota, Velenje, Postojna, Ptuj, and conurbations Slovenj Gradec–Ravne na Koroškem–Dravograd, Jesenice–Radovljica, Zagorje ob Savi–Trbovlje–Hrastnik, and Krško–Brežice–Sevnica. These centres are also known as regional centres of Slovenia. Despite the large number of regional centers in a relatively small country, most jobs and economic activities are concentrated in the functional urban areas of Ljubljana, Maribor, Celje, Koper - Izola - Piran, Kranj, Novo mesto, Velenje and Nova Gorica (Drobne & Bogataj, 2014).

Figure 1
Inter-municipal labour commuting flows in Slovenia in 2017



Source: Authors' work, GURS (2018), SURS (2018), SPRS (2004)
Note: Only flows with 50 and more commuters are shown.

We analysed the FRs in Slovenia using two methods based on network theory. For this purpose, the complex system of labour commuting between Slovenian municipalities in 2017 was described as a network. In our study, the municipal centroids are the vertexes of the network, which are connected by an edge if the inter-municipal commuter flows were recorded in the analysed period. Weights (w_{ij}) were assigned to each edge according to the number of commuters registered between the two municipalities. Therefore, w_{ij} represents the number of commuters from municipality i to j. Using the Walktrap algorithm (Pons & Latapy, 2006) and the chain approach (Drobne et al., 2010; Karlsson & Olsson, 2006; Konjar et al., 2010) we identified communities in the network - i.e. functional regions of the inter-municipal labour commuting flows.

The Walktrap algorithm is a heuristic algorithm that groups the vertices of the network on the basis of a distance, r, which measures the connectivity of two nodes, i.e. municipal centroids. The distance r_{ij} between the nodes i and j is as defined in Equation 1, where P_{ik}^{t} is the transition probability from node i to k in t steps, $d(k)$ is the degree of node k and n is the number of nodes in the network.

$$r_{ij} = \sqrt{\sum_{k=1}^{n} \frac{\left(P_{ik}^{t} - P_{jk}^{t}\right)^2}{d(k)}} \tag{1}$$

In the Walktrap algorithm, the transition probabilities are estimated using random walk. In short, Q random walks of length t are taken from randomly selected nodes. At each transition, the walker travels from node i to node j with the probability $w_{ij}/\sum_k w_{ik}$. Then the transition probabilities P_{ik}^{t} are calculated as the fraction of walkers that ended in node k after t steps. Once the distance matrix r_{ij} has been calculated, the vertices are aggregated using a hierarchical clustering algorithm. The Walktrap algorithm was applied using the implementation contained in the *igraph* R package (Csardi & Nepusz, 2006), with R version 3.4.3 (R Core Team, 2016). The parameter *t* was set to four, while *Q* was increased until the results converged (i.e. no variation of the functional regions for independent runs). When applied to the inter-municipal labour commuting data, we also tested *t* = 2, 3, 5 and 6, but only *t* = 4 generated realistic and compact FRs. To the authors' knowledge, this is the first time that Walktrap algorithm was used to analyse functional regions by commuting data at the macro spatial level.

The second method used to calculate FRs was the chain approach, which was introduced and applied by Karlsson and Olsson (2006) and later improved by Drobne et al. (2010) and Konjar et al. (2010). The first step in this approach is to identify the centres of the FRs. They are defined as the most important employment centres in the area studied, which are highly self-sufficient. Therefore, the centres can be defined nominally or calculated as the most self-sufficient centres. A municipality is highly self-sufficient if the majority of the active population works in the home municipality; this percentage is usually set at 66.67% or more (Drobne et al., 2010; Karlsson & Olsson, 2006; Konjar et al., 2010). Although methods for this step are described in the literature (Drobne et al., 2010; Konjar et al., 2010), in order to facilitate comparison with the Walktrap algorithm, the centres of the FRs calculated by the Walktrap algorithm were also used for the chain approach. In a second step, chains of nodes are formed by adding municipalities to self-sufficient municipalities, i.e. predefined centres of FRs, until the condition defined in Equation (2) is satisfied. This condition defines the boundary of FR_i, which is the break line, where the attraction theoretically corresponds is to the two nearest self-sufficient municipalities:

$$FR_i = \{x\colon w_i(x) \geq w_j(x)\} \tag{2}$$

In (2), i and j denote two FRs' centres connected by a line, where intermediate point between the end points i and j is denoted with x. At a location x, the commuting frequency to the centre i is $w_i(x)$. The chains were calculated in three different ways defined by three types of municipalities (nodes): (a) the chains of the municipalities that were directly connected to the centre by their maximum outflow; (b) the chains of municipalities that were indirectly connected with their maximum outflow to the centre via a non-self-sufficient municipality (such chains are determined iteratively); and (c) the chains of the pairs of municipalities, which presented to each other the destination of their maximum outflow, were connected to the functional region according to the second maximum flow. As proposed by Karlsson and Olsson (2006), the chain was allowed to have three links in our application for Slovenia. If more links existed, the link was broken at the weakest point. Moreover, we tested the approach by allowing three and four links, without any impact on the results. The chains were automatically calculated in our own program (Konjar et al., 2010) based on the Java platform.

After identifying the FRs with both algorithms, we compared the quality of both regionalization methods using Fuzzy Set Theory (FST), as proposed by Feng (2009) and Watts (2009, 2013). The FST extends Crisp Set Theory, by allowing one element (in this study BSU) to partially belong to a group (in this study to FR). Consequently, elements can belong to several groups at the same time. By using the FST approach, where each BSU can be partially allocated to a number of fuzzy FRs, we can identify potential misallocations of BSUs in the FRs by measuring a membership function. A membership function for BSU i in relation to fuzzy residential FR m is defined as

$$M'_{im} = \sum_{j \in (g)m} w_{ji} / w_{\cdot i} \ , \tag{3}$$

where BSU i belongs to FR m on the basis of a regionalisation method. And, the membership function with respect to fuzzy local employment FR m is defined as

$$M''_{im} = \sum_{j \in (g)m} w_{ij} / w_{i\cdot} \ . \tag{4}$$

Feng (2009) suggested to calculate the common membership function values with respect to a fuzzy FR, m, M_{im}, as an arithmetic mean of M'_{im} and M''_{im}. But M'_{im} and M''_{im} are relative values. For this reason, we suggest calculating the common membership function values as a geometric mean of the corresponding function values:

$$M_{im} = \sqrt{M'_{im} \cdot M''_{im}} \ . \tag{5}$$

To analyse the values of the membership function of each municipality, we mapped these values into the FRs, as proposed by Feng (2009). We also calculated the geometric mean membership values for each FR individual and for the whole system of FRs in Slovenia. This enabled us to compare the quality of the individual regionalisation.

In this study, we extended the classical approach of modelling and analysing FRs with manual reshaping of FRs. In the last step, we moved the municipalities with very low membership values to the neighbouring FRs when new membership values were higher.

The quality of functional regionalization - i.e. the calculation and modelling of functional regions and the analysis of the quality of functional regions - was performed in Mathematica 11.3 by using our programme code (Drobne, 2016; Drobne & Lakner, 2016).

Results

The results of the two applied regionalization methods, both based on network theory methods, are shown in Figure 2. The Walktrap algorithm (Figure 2a) generated eight FRs of Slovenia. Seven of them were expected and are consistent with previous research (see e.g. Drobne et al., 2010; Drobne, 2016; Konjar et al., 2010), these FRs are FR Murska Sobota, FR Maribor, FR Celje, FR Slovenj Gradec, FR Ljubljana, FR Novo mesto, and FR Nova Gorica. However, FR Tolmin, which consists of only three relatively large municipalities surrounded by high mountains, has never been modelled at the macro level of Slovenia – compare this result also with fifteen regional centres of national importance in Figure 1. Instead of FR Tolmin, previous studies in the scientific literature (Drobne, 2016; Drobne et al., 2010; Konjar et al., 2010) usually report FR Koper on the southwestern coastal region of Slovenia, while Koper is also one of the most important employment centres in Slovenia. Further analysis also revealed that the relationship of FR Tolmin to other FRs is much weaker than the relationships between FR with a more central location in Slovenia.

The result of the chain method (Figure 2b) is very similar to those of the Walktrap method. This is (partly) understandable since we used the same regional centres. The method estimates eight FRs, but there are differences between the sizes of the FRs calculated with each method. FR Ljubljana is much larger in the chain method, mostly at the expense of the FRs Celje and Novo mesto, which are consequently smaller than those calculated with the Walktrap algorithm. The influence of FR Ljubljana is even unexpected along the narrow strips of municipalities to the east. The remaining FRs are (almost) identical for both algorithms.

The quality analysis of the regionalisation procedures, i.e. the comparative analysis of the general membership values of FRs calculated with the FST approach, shows that the Walktrap algorithm generates FRs Ljubljana, Novo mesto, Celje and Murska Sobota with higher average membership values than the chain approach (see Figures 3a and 3b and Table 1). Furthermore, the mean membership values of almost all FRs generated by the Walktrap algorithm are higher or equal to the mean membership values of the FRs calculated by the chain method, the only exception being FR Slovenj Gradec.

In general, the municipalities with the highest membership values are in the centres of the FRs, while the municipalities with the lowest membership values are on the periphery of FRs. As far as FRs calculated according to the chain method are concerned, most of the municipalities with (very) low membership values are on the borderline between FRs. In most cases, these municipalities are also those that are assigned to a different FR by the Walktrap algorithm (i.e. in the border areas between the FRs Ljubljana, Celje and Novo mesto). Therefore, the chain algorithm may incorrectly assign some of these municipalities. On the other hand, in the FRs calculated by the Walktrap algorithm there are also some municipalities with low membership values. These municipalities are located on the border between the FRs Ljubljana and Novo mesto and Ljubljana and Nova Gorica and in the FR Slovenj Gradec, which is the only one with a lower average membership value compared to both FRs' systems. These results, together with the higher mean membership values of the Walktrap algorithm, indicate that the Walktrap algorithm provides a better classification for the case under investigation.

Figure 2
Eight functional regions in Slovenia in 2017 defined with the inter-municipal labour commuting flows and generated by the Walktrap algorithm (Figure 2a) and chain method (Figure 2b)

Figure 2a

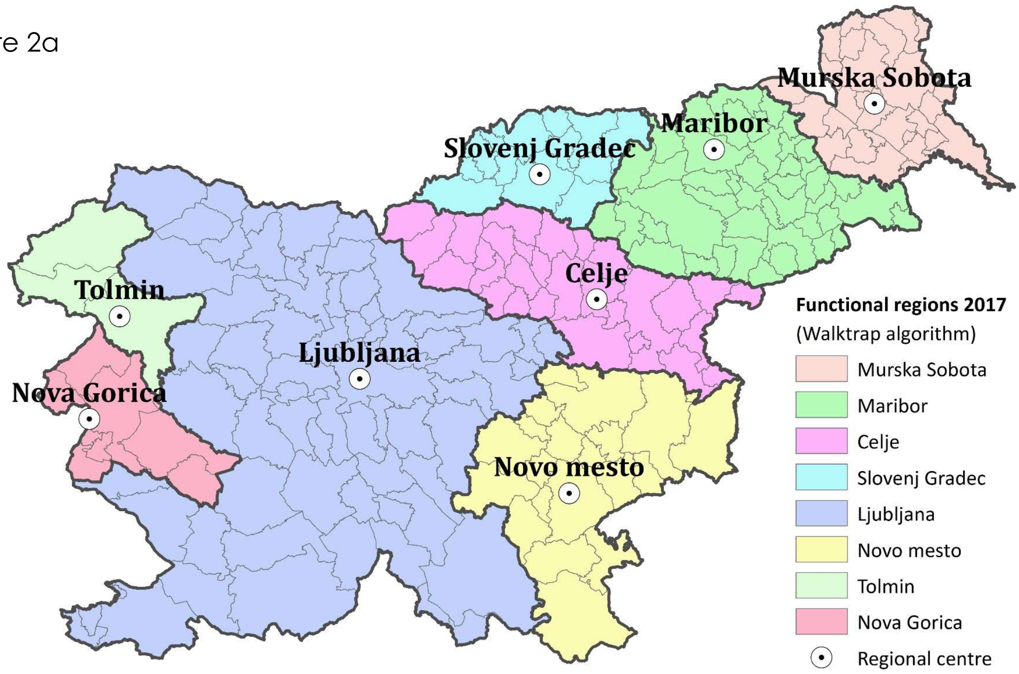
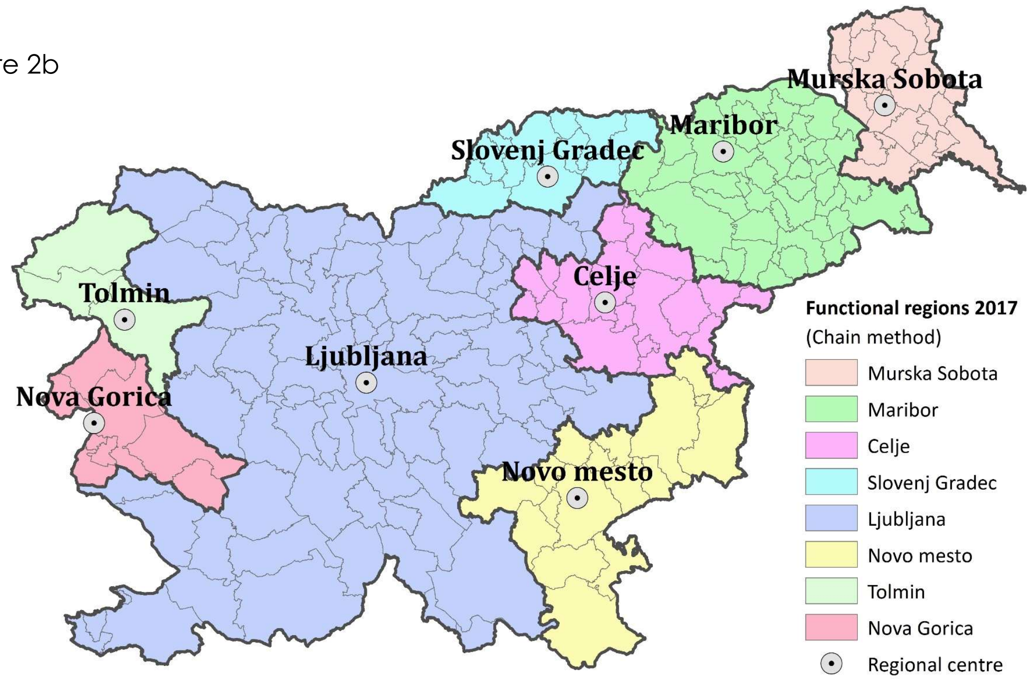


Figure 2b



Source: Authors' work.
Note: Basic spatial unit (BSU) is municipality.

Figure 3
Membership values of Slovenian municipalities in the functional region to which they were located by using Walktrap algorithm (Figure 3b) and chain method (Figure 3b)

Figure 3a

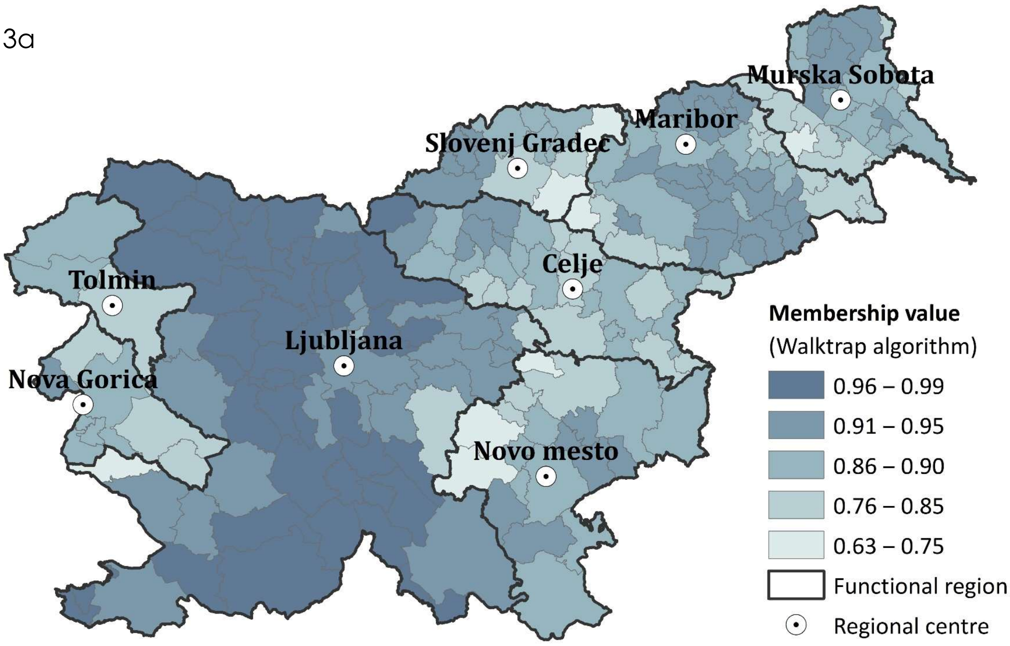
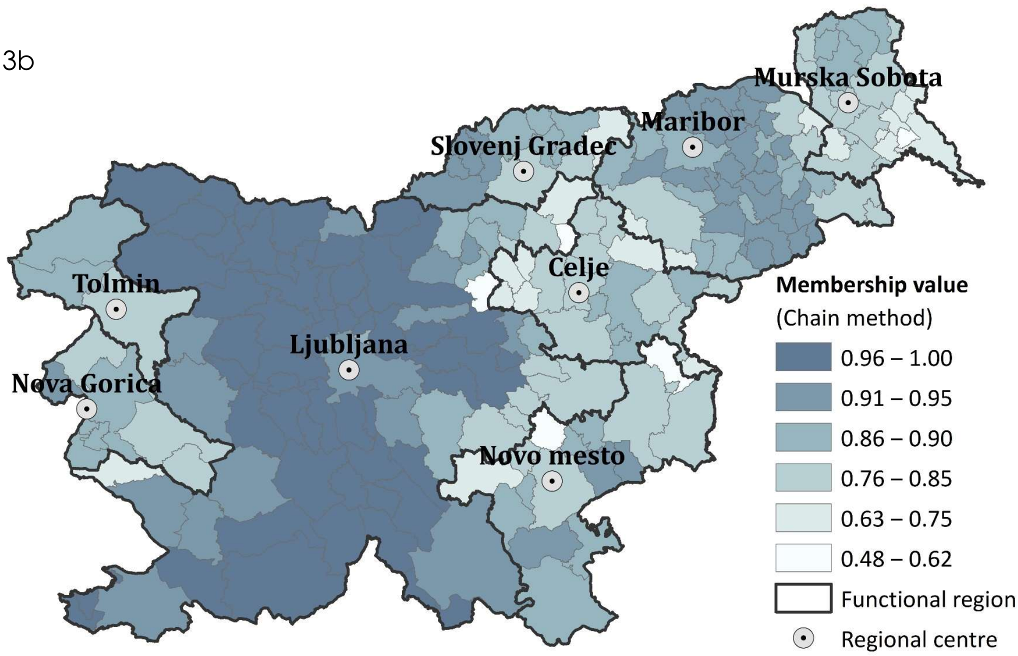


Figure 3b



Source: Authors' work.
Note: Basic spatial unit (BSU) is municipality.

Table 1
Mean membership values of the functional regionalization

Functional Region / Slovenia	Mean membership value	
	Walktrap algorithm	chain method
Slovenia	0.891	0.866
Murska Sobota	0.865	0.795
Maribor	0.884	0.880
Celje	0.854	0.782
Slovenj Gradec	0.836	0.849
Ljubljana	0.954	0.912
Novo mesto	0.849	0.794
Tolmin	0.839	0.839
Nova Gorica	0.860	0.860

Source: Authors' work.

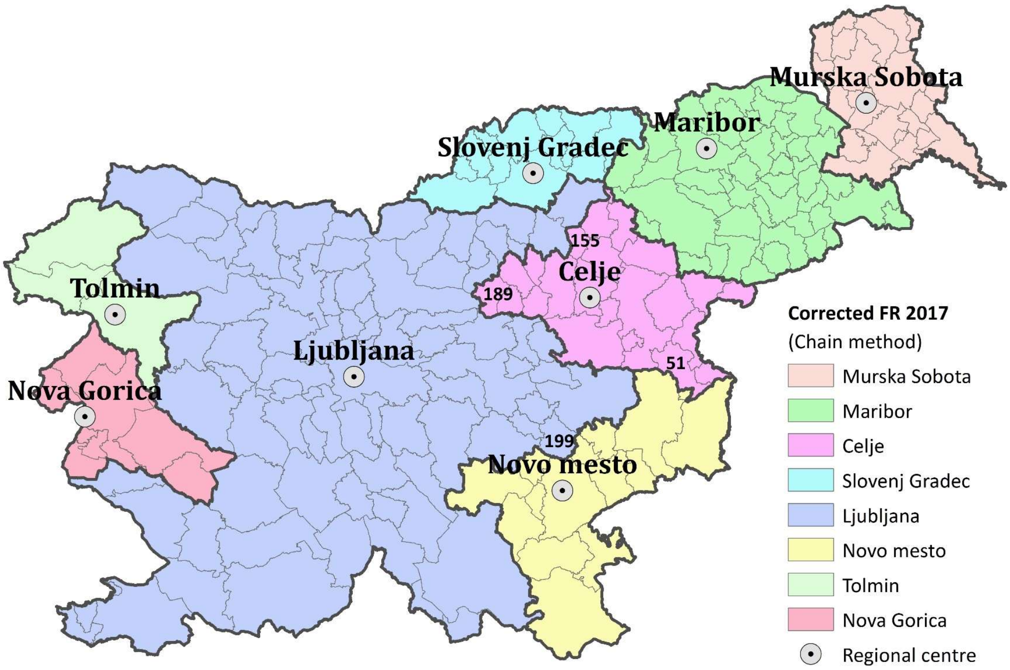
In the last step of our research, we analysed whether reshipment of FRs can improve the quality of regionalization results. For this reason, in our study we had used $M < 0.6$ to move the municipalities on the border of the functional region with a very low value of membership function to the neighbour FR. Such cases were only included in the system of FRs calculated using the chain method, where we moved four municipalities along the border lines between FR Ljubljana, Celje and Novo mesto, as shown in Figure 4. The relocation of these four municipalities - namely Kozje (51), Dobrna (155), Vransko (189) and Mokronog - Trebelno (199) - to the neighbouring FR increased the membership values for both: for individual municipalities and for the FR system as a whole. The results of the improvement in the quality of regionalisation for the whole system are shown in Table 2, while the improvement in membership values for individual municipalities is as follows: Kozje (51) from 0.482 to 0.782, Dobrna (155) from 0.546 to 0.689, Vransko (189) from 0.534 to 0.716 and Mokronog - Trebelno (199) from 0.530 to 0.779. We can conclude, that all individual results have improved significantly.

Table 2
Mean membership values of the corrected functional regionalisation with the chain method

Functional region / Slovenia	Moved municipalities	Mean membership value after reshaping FRs	
		average	improvement
Slovenia	4	0.874	+0.008
Murska Sobota		0.795	0
Maribor		0.880	0
Celje	+3	0.793	+0.011
Slovenj Gradec		0.849	0
Ljubljana	-2 And +1	0.922	+0.010
Novo mesto	-2	0.842	+0.048
Tolmin		0.839	0
Nova Gorica		0.860	0

Source: Authors' work.

Figure 4
Eight corrected functional regions in Slovenia in 2017 defined with the chain method



Source: Authors' work.
Notes: Basic spatial unit (BSU) is municipality. Municipalities that have been moved to the neighbour FR are denoted by the municipal code: (51) Kozje, (155) Dobrna, (189) Vransko, and (199) Mokronog - Trebelno.

Discussion and conclusions

Several aspects of governance, such as spatial development or territorial planning, require an understanding of the organisation of space in order to be effective. Therefore, identifying functional regions can be (in)valuable for governments and other authorities. In the paper, we analysed FRs using the Walktrap algorithm and the chain approach. The results of the two methods based on graph theory were compared and analysed in detail using fuzzy sets with the improved approach. In the case studied (inter-municipal commuter flows for Slovenia for the year 2017), the Walktrap algorithm identifies more meaningful FRs than the chain approach: the Walktrap algorithm calculates FRs with a lower level of potentially misallocated BSUs.

It should be stressed that FRs are abstract concepts, whose exact definition may vary from application to application. Therefore, FRs that may be valid for one application may not be suitable for the analysis of other case studies. In this sense, fuzzy set theory offers a more flexible framework than traditional set theory and allows the definition of partial membership relationships. In the case study analysed in this study, this refers to the uncertainty that a particular municipality belongs to a FR. In this study, we have applied this result to compare the FRs identified with the Walktrap algorithm and the chain method, concluding that the FRs identified with the Walktrap algorithm are more self-contained. However, the application of fuzzy set theory in this context is not limited to the comparison of regionalization methods. For example, it could be used to define different scenarios varying the position of the municipalities with the highest uncertainty. However, these applications are not the subject of this article and remain for future work. In this study, we have showed only a principle on how the reallocation of a single BSU at the border between FRs with a very low membership value could improve the results not only for this single unit, but also for the whole FRs' system.

In addition, network theory provides a good framework for the analysis of complex systems such as those analysed here. Although this study was limited to the identification of FRs based on community detection algorithms, a deeper analysis of network attributes could provide a meaningful insight into the structure of the system. For example, the topology of the commuter network in Slovenia could be compared with that of other countries to identify regions with a similar production system.

As a direction for future work, the algorithms analysed here could be compared with other graph-based methods of FRs identification (e.g. Benassi et al., 2015) as well as with other methods - e.g. with the most popular rule-based regionalisation method, i.e. CURDS's method (Coombes & Bond, 2008). Furthermore, network theory could be applied to analyse in detail the structure of functional regions at different spatial levels (micro, mezzo and macro). Furthermore, the results of the approach used in this study to assess functional regionalization should be compared with other approaches to quality assessment of functional regions, as recently proposed by Halás et al. (2019).

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References

1. Ball, R. M. (1980) "The use and definition of travel-to-work areas in Great Britain: some problems", Regional Studies, Vol. 14, No. 2, pp. 125-139.
2. Benassi, F. Deva, M., Zindato, D. (2015), "Graph regionalization with clustering and partitioning: an application for daily commuting flows in Albania", Regional Statistics, Vol. 5, No. 1, pp. 25-43.
3. Boss, M., Elsinger, H., Summer, M., Thurner, S. (2004), "Network topology of the interbank market", Quantitative Finance, Vol. 4, No. 6, pp. 677-684.
4. Casado-Diaz, J. M. (2000), "Local labour market areas in Spain: a case study", Regional Studies, Vol. 34, pp. 843-856.
5. Clauset, A., Moore, C., Newman, M. E. J. (2008), "Hierarchical structure and the prediction of missing links in networks", Nature, Vol. 453, No. 7191, pp. 98-101.
6. Coombes, M. G., Green, A. E., Openshaw, S. (1986), "An efficient algorithm to generate official statistical reporting areas: the case of the 1984 travel-to-work-areas revision in Britain", Journal of the Operational Research Society, Vol. 37, No. 10, pp. 943-953.
7. Coombes, M., Bond, S. (2008), "Travel-to-Work Areas: the 2007 review Office for National Statistics", London, available at: https://www.ncl.ac.uk/media/wwwnclacuk/curds/files/TTWA%20report.pdf (25 February 2016)
8. Coombes, M., Casado-Díaz, J. M., Martínez-Bernabeu, L., Carausu, F. (2012), "Study on comparable labour market areas - Final research report", available at: http://www.istat.it/it/files/2014/12/Final-Report_LMA-v1-0-17102012.pdf (25 March 2016).
9. Cörvers, F., Hensen, M., Bongaerts, D. (2009), "Delimitation and coherence of functional and administrative regions", Regional Studies, Vol. 43, No. 1, pp. 19-31.

10. Csardi, G., Nepusz, T., (2006), "The igraph software package for complex network research", InterJournal Complex Systems, Vol. 1695, No. 5, pp. 1-9.
11. Drobne, A., Lakner, M. (2016), "Use of constraints in the hierarchical aggregation procedure Intramax", Business Systems Research, Vol. 7, No. 2, pp. 5-22.
12. Drobne, S. (2016), "Model vrednotenja števila in območij funkcionalnih regij" (A model evaluating the number and areas of functional regions), PhD thesis, University of Ljubljana, available at: https://repozitorij.uni-lj.si/Dokument.php?id=97829&lang=slv (7 November 2019).
13. Drobne, S., Bogataj, M. (2014), "Regions for servicing old people: case study of Slovenia", Business Systems Research, Vol. 5 No. 3, pp. 19-36.
14. Drobne, S., Garre, A., Hontoria, E., Konjar, M. (2019), "Functional regions detection by Walktrap and chains´ methods", in Zadnik Stirn, L., Kljajić Borštnar, M. Žerovnik, J., Drobne, S., Povh, J. (Eds.), 15th International Symposium on Operational Research, 25-27 September, Slovenian Society Informatika, Bled, pp. 449-454.
15. Drobne, S., Konjar, M., Lisec, A. (2010), "Razmejitev funkcionalnih regij Slovenije na podlagi analize trga dela" (Delimitation of functional regions of Slovenia based on labour market analysis), Geodetski vestnik, Vol. 54, No. 3, pp. 481-500.
16. Erlebach, M., Tomáš, M., Tonev, P. (2016), "A functional interaction approach to the definition of meso regions: the case of the Czech Republic", Moravian Geographical Reports, Vol. 24, No. 2, pp. 37-46.
17. Faloutsos, M., Faloutsos, P., Faloutsos, C. (1999), "On power-law relationships of the internet topology", ACM SIGCOMM Computer Communication Review, Vol. 29, No. 4, 251-262.
18. Feng, Z. (2009), "Fuzziness of travel to work areas", Regional Studies, Vol. 43, No. 5, pp. 707-720.
19. Fortunato, S. (2010), "Community detection in graphs", Physics Reports, Vol. 486, No. 3-5, pp. 75-174.
20. Gabrovec, M., Bole, D. (2009), "Dnevna mobilnost v Sloveniji" (Daily mobility in Slovenia), available at: https://giam.zrc-sazu.si/sites/default/files/9789612541187.pdf (25 February 2020).
21. Garre, A., Frenandez, P.S., Brereton, P., Elliott, C., Mojtahed, V. (2019), "The use of trade data to predict the source and spread of food safety outbreaks: An innovative mathematical modelling approach", Food Research International, Vol. 123, pp. 717-721.
22. GURS. (2018), "Spatial data on municipalities, 2011", available at: https://egp.gu.gov.si/egp/ (15 August 2018).
23. Halás, M., Klapka, P., Erlebach, M. (2019), "Unveiling spatial uncertainty: a method to evaluate the fuzzy nature of functional regions", Regional Studies, Vol. 53, No. 7, pp 1029-1041.
24. Halás, M., Klapka, P., Hurbánek, P., Bleha, B., Pénzes, J., Pálóczi, G. (2018), "A definition of relevant functional regions for international comparisons: The case of Central Europe", Area, Vol. 51, pp. 489-499.
25. Halás, M., Klapka, P., Tonev, P., Bednář, M. (2015), "An alternative definition and use for the constraint function for rule-based methods of functional regionalisation", Environment and Planning A, Vol. 47, pp. 1175-1191.
26. Holmes, J. H., Haggett, P. (1977), "Graph theory interpretation of flow matrices: a note on maximization procedures for identifying significant links", Geographical Analysis, Vol. 9, pp. 388-399.
27. Karlsson, C., Olsson, M. (2006), "The identification of functional regions: theory, methods, and applications", The Annals of Regional Science, Vol. 40, No. 1, pp. 1-18.
28. Ke, W., Chen, W., Yu, Z. (2017), "Uncovering spatial structures of regional city networks from expressway traffic flow data: A case study from Jiangsu province, China", Sustainability, Vol. 9, No. 9, pp. 1541.
29. Khatoon, M., Banu, W. A. (2019), "An efficient method to detect communities in social networks using DBSCAN algorithm", Social Network Analysis and Mining, Vol. 9, No. 1, 9.
30. Konjar, M., Lisec, A., Drobne, S. (2010), "Methods for delineation of functional regions using data on commuters", in Painho, M., Santos, M. Y., Pundt, H. (Eds.), 13th AGILE International

Conference on Geographic Information Science, May 10-14, Guimarães, Springer-Verlag, pp. 1-10.
31. Laan van der, L., Schalke, R. (2001), "Reality versus policy: the delineation and testing of local labour market and spatial policy areas", European Planning Studies, Vol. 9, No. 2, pp. 201-221.
32. Lehmann, S., Schwartz, M., Hansen, L.K., (2008), "Biclique communities", Physical Review E, Vol. 9, No. 1, 016108.
33. Masser, I., Brown, P. J. B. (1975), "Hierarchical aggregation procedures for interaction data", Environment and Planning A, Vol. 7, No. 5, pp. 509-523.
34. Masser, I., Scheurwater, J. (1978), "The specification of multi-level systems for spatial analysis", in: Masser, I., Brown, P. J. B. (Eds.), Spatial representation and spatial interaction, Springer US, Leiden and Boston, pp. 151-172.
35. Masser, I., Scheurwater, J. (1980), "Functional regionalisation of spatial interaction data: an evaluation of some suggested strategies", Environment and Planning A, Vol. 12, No. 12, pp. 1357-1382.
36. Newman, M. (2010), Networks: An introduction, Oxford University Press, New York.
37. Nystuen, J. D., Dacey, M. F. (1961), "A graph theory interpretation of nodal regions", Papers of the Regional Science Association, Vol. 7, pp. 29-42.
38. Pálóczi, G., Pénzes, J., Hurbánek, P., Halás, M., Klapka, P. (2016), "Attempts to delineate functional regions in Hungary based on commuting data", Regional Statistics, Vol. 6, No. 1, pp. 23-41.
39. Pons, P., Latapy, M. (2006), "Computing communities in large networks using random walks", Journal of Graph Algorithms and Applications, Vol. 10, No. 2, pp. 191-218.
40. R Core Team. (2016), "R: a language and environment for statistical computing", available at: https://www.R-project.org/ (7 November 2019).
41. Ronhovde, P., Nussinov, Z. (2009), "Multiresolution community detection for megascale networks by information-based replica correlations", Physical Reviews E, Vol. 80, No. 1, 016109.
42. SPRS. (2004), "Spatial development strategy of Slovenia", available at: http://www.mop.gov.si/fileadmin/mop.gov.si/pageuploads/podrocja/prostorski_razvoj/SPRS_angleska_verzija.pdf (6 January 2017).
43. SURS. (2018), "SI-Stat data portal", available at: http://pxweb.stat.si/pxweb/dialog/statfile1.asp (29 November 2018).
44. Ullman, E. L. (1980), Geography as spatial interaction, University of Washington Press, Seattle.
45. Watts, M. (2009), "Rules versus hierarchy: an application of fuzzy set theory to the assessment of spatial grouping techniques", in Kolehmainen, M., Toivanen, P., Beliczynski, B. (eds), 9th International Conference on Adaptive and Natural Computing Algorithms, 23-25 April, Springer-Verlag, Berlin Heidelberg, pp. 517-526.
46. Watts, M. (2013), "Different spatial grouping algorithms: an application to the design of Australia's new statistical geography", Spatial Economic Analysis, Vol. 8, No. 1, pp. 92-112.

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