Perceptionization of FM/FD/1 queuing model under various fuzzy numbers

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Abstract. We present a FM/FD/1 queuing model with unbounded limit under different fuzzy numbers. The arrival (landing) rate and service (administration) rate are thought to be fuzzy numbers such as triangular, trapezoidal and pentagonal fuzzy numbers. Because random event can only be observed in an uncertain manner, the fuzzy result of an uncertainty mapping is a fuzzy random variable. Consequently, it is conceivable to characterize the specific connection between randomness and fuzziness. The execution proportions of this lining miniature are fuzzified after that examined by utilizing α–cut estimations and DSW algorithm (Dong, Shah and Wong). Relating to different fuzzy numbers, the numerical precedents are delineated to test the attainability of this model (miniature). A comparative illustration corresponding to each fuzzy number is accomplished for various estimations of α.

Keywords: DSW algorithm, fuzzy number, perceptionization, queuing model, α – cut

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1. Introduction


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to pursue certain appropriations. In common practice the landing rate, administration rate is much of the time portrayed by etymological terms, for example, high, low, extremely low and moderate can be best depicted by the fuzzy sets. A FM/FD/1 queue, the simplest queue with deterministic service time, has a variety of applications in the performance evaluation of production management, telecommunications networks, and other areas. The primary thought of this paper is to acquire the exact crisp values from the fuzzy values and then applying within the queuing performance formulas to 3 sorts of participation capacities, i.e. triangular, trapezoidal and pentagonal enrolment capacities. Fuzzy Queuing models are also studied by Mueen [16], Julia Rose Mary [11], Rakesh Kumar [19], Vasanth Kumar [9, 27], Usha Prameela and Pavan [25, 26], Aria [1, 2], Wagner [28], Hajipour [8], Fazzolari [6], Kobayashi [13], Kumar [14], Sushil [23] and Novak [18]. In Section 2, some basic ideas and definitions are presented. In Section 3, the presumptions and notations are described. In Section 4 the proposed queuing miniature is given. In Section 5, the result approach to the present model is described. In Section 6, three numerical precedents are solved. In Section 7 the results and discussions are presented. In Section 8 the model is concluded.

2. Basic definitions

2.1. Fuzzy number

A fuzzy set \( \tilde{A} \) is characterized on the set of real numbers \( R \) is said to be fuzzy number if it has the accompanying qualities such as \( \tilde{A} \) is normal, convex and the support of it is closed and bounded [12].

2.2. \( \alpha \)–cut

An \( \alpha \)–cut of a fuzzy set [12] is a crisp set \( A \) that contains all the elements of the universal set \( X \) that have a participation grade in \( \tilde{A} \) greater than or equal to determined estimation of \( \alpha \), thus

\[
\alpha = \{ x \in X : \mu_{\tilde{A}}(x) \geq \alpha, 0 \leq \alpha \leq 1 \}.
\]  

2.3. Arithmetic for interval analysis

Let the two interval numbers designated by ordered pairs of real numbers with lower and upper limits be \( G = [a_1, a_2], a_1 \leq a_2 \) and \( H = [b_1, b_2], b_1 \leq b_2 \), with following properties:

\[
[ a_1, a_2 ] + [ b_1, b_2 ] = [ a_1 + b_1, a_2 + b_2 ]
\]

\[
[ a_1, a_2 ] - [ b_1, b_2 ] = [ a_1 - b_1, a_2 - b_2 ]
\]

\[
[ a_1, a_2 ] \times [ b_1, b_2 ] = [ \min(a_1 b_1, a_1 b_2, a_2 b_1, a_2 b_2), \max(a_1 b_1, a_1 b_2, a_2 b_1, a_2 b_2) ]
\]

\[
[ a_1, a_2 ] \div [ b_1, b_2 ] = [ a_1, a_2 ] \times [ 1/b_1, 1/b_2 ] \text{ provided that } 0 \text{ does not belong to } [ b_1, b_2 ]
\]

\[
\alpha \times [ a_1, a_2 ] = [ \alpha a_1, \alpha a_2 ] \text{ for } \alpha > 0 \text{ and } [ \alpha a_2, \alpha a_1 ] \text{ for } \alpha < 0.
\]

3. Presumptions and notations

In the present model, accompanying presumptions are utilized similar as in [13]:

i) Unbounded limit of FM/FD/1/∞/FCFS queuing model with one server

ii) Exponentially disseminated arrival times
Perceptionization of FM/FD/1 queuing model under various fuzzy numbers

iii) Deterministic service appropriation, i.e. fixed

iv) Landing rate, administration rate are fuzzy numbers.

Notations are given as:

$\mu =$ average number of clients being overhauled per unit of time
$\lambda =$ average number of clients arriving per unit of time
$L_s =$ average number of clients in the framework
$L_q =$ average number of clients holding up in the line
$W_s =$ average holding up time of a client in the framework
$W_q =$ average holding up time of a client in the line
$X =$ set of the inter entry time
$Y =$ set of the administration time
$A =$ inter entry time
$S =$ administration times

4. Proposed queuing model

We propose a single server queuing model, with first come first served (FCFS) regulation \[13, 18\]. It is indicated in Kendall’s notation as (FM/FD/1):($\infty$/FCFS). Here FM denotes fuzzified exponential dispersion with landing rate $\lambda$ and FD denotes fuzzified steady (constant) dispersion with administration rate $\mu$. This process is stochastic whose state space is the set \{0, 1, 2, 3, \ldots\} where the value indicates the number of customers present in the system, incorporating any entity at present in administration. It is of infinite size, so there is no restriction on the number of customers it can contain. The execution proportions of the proposed model are given as similar to \[23\]. Namely, expected range of consumers within the system is:

$$L_s = \rho + \frac{\rho^2}{2(1 - \rho)}, \quad \rho = \frac{\lambda}{\mu}. \quad (2)$$

The expected range of consumers within the queue is:

$$L_q = \frac{\rho^2}{2(1 - \rho)}. \quad (3)$$

The expected time a client spends within the queue is:

$$W_q = \frac{\rho}{2(1 - \rho)\mu}. \quad (4)$$

The expected time a client spends within the system is:

$$W_s = \frac{1}{\mu} + \frac{\rho}{2(1 - \rho)\mu}. \quad (5)$$
5. Solution procedure

DSW (Dong, Shah and Wong) is a rough technique utilizes intervals at various $\alpha$–cut dimensions in characterizing execution proportions [15]. It avoids variation from the output membership function because of use of the segregation reaching on the fuzzy variable area. Any persistent participation capacity can be spoken to by ceaseless scope of $\alpha$–cut in term from $\alpha = 0$ to $\alpha = 1$. Let $\mu_\tilde{A}(a)$ and $\mu_\hat{S}(s)$ be membership functions of the inter entry time and the overhauled time, separately. The inter arrival time and service times are fuzzy sets, depicted as:

$$\tilde{A} = \{(a, \mu_\tilde{A}(a)), \ a \in X\} \text{ and } \hat{S} = \{(s, \mu_\hat{S}(s)), \ s \in Y\}. \quad (6)$$

The $\alpha$–cuts of inter entry time, overhauled time are represented as:

$$\tilde{A}(\alpha) = \{\mu_\tilde{A}(a) \geq 0, \ a \in X\} \text{ and } \hat{S}(\alpha) = \{\mu_\hat{S}(s) \geq a, \ s \in Y\}. \quad (7)$$

The DSW calculation contains the accompanying steps:

**Step 1.** Stipulate $\alpha$–cut esteem where $0 \leq \alpha \leq 1$

**Step 2.** Discover the intervals in the input membership functions that compare to this $\alpha$

**Step 3.** Utilizing standard binary interval operations, process the interval for the output membership function for the chosen $\alpha$–cut dimension

**Step 4.** Iterate stages 1–3 for various estimations of $\alpha$ to finish $\alpha$–cut portrayal of the arrangement

6. Numerical precedents

We explain the accompanying numerical models, thinking about different fuzzy numbers.

6.1. Precedent 1

Consider an FM/FD/1/$\infty$/FIFO model where both the entry rate and overhauled rate are triangular fuzzy numbers represented by $\lambda = [1, 2, 3]$ and $\mu = [11, 12, 13]$. The interval of certainty at probability level $\alpha$ as $[1 + \alpha, 3 - \alpha]$ and $[11 + \alpha, 13 - \alpha]$. By taking $\alpha$ values from 0, 0.1, ..., 1 the execution measures are shown in the Table 1.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$L_q$</th>
<th>$L_s$</th>
<th>$W_s$</th>
<th>$W_q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0032, 0.0511</td>
<td>0.0801, 0.3238</td>
<td>0.0801, 0.1079</td>
<td>0.0032, 0.0170</td>
</tr>
<tr>
<td>0.1</td>
<td>0.0039, 0.0461</td>
<td>0.0892, 0.3074</td>
<td>0.0811, 0.1060</td>
<td>0.0036, 0.0159</td>
</tr>
<tr>
<td>0.2</td>
<td>0.0048, 0.0416</td>
<td>0.0985, 0.2916</td>
<td>0.0821, 0.1041</td>
<td>0.0040, 0.0148</td>
</tr>
<tr>
<td>0.3</td>
<td>0.0058, 0.0375</td>
<td>0.1082, 0.2764</td>
<td>0.0832, 0.1023</td>
<td>0.0044, 0.0138</td>
</tr>
<tr>
<td>0.4</td>
<td>0.0069, 0.0336</td>
<td>0.1180, 0.2617</td>
<td>0.0843, 0.1006</td>
<td>0.0049, 0.0129</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0081, 0.0301</td>
<td>0.1281, 0.2475</td>
<td>0.0854, 0.0990</td>
<td>0.0054, 0.0120</td>
</tr>
<tr>
<td>0.6</td>
<td>0.0095, 0.0269</td>
<td>0.1385, 0.2338</td>
<td>0.0866, 0.0974</td>
<td>0.0059, 0.0112</td>
</tr>
<tr>
<td>0.7</td>
<td>0.0110, 0.0240</td>
<td>0.1492, 0.2202</td>
<td>0.0878, 0.0959</td>
<td>0.0065, 0.0104</td>
</tr>
<tr>
<td>0.8</td>
<td>0.0127, 0.0213</td>
<td>0.1603, 0.2078</td>
<td>0.0890, 0.0944</td>
<td>0.0070, 0.0097</td>
</tr>
<tr>
<td>0.9</td>
<td>0.0146, 0.0189</td>
<td>0.1716, 0.1953</td>
<td>0.0903, 0.0930</td>
<td>0.0076, 0.0090</td>
</tr>
<tr>
<td>1</td>
<td>0.0166, 0.0166</td>
<td>0.1833, 0.1833</td>
<td>0.0916, 0.0916</td>
<td>0.0083, 0.0083</td>
</tr>
</tbody>
</table>

Table 1: The $\alpha$–cuts of $L_s$, $L_q$, $W_s$ and $W_q$ at $\alpha$ values (precedent 1)
6.2. Precedent 2

Acknowledge that both landing rate and administration rate are trapezoidal fuzzy numbers represented by \( \lambda = [1, 2, 3, 4] \) and \( \mu = [11, 12, 13, 14] \). The interval of certainty at probability level \( \alpha \) as \([1 + \alpha, 4 - \alpha]\) and \([11 + \alpha, 14 - \alpha]\), where \( x = [1 + \alpha, 4 - \alpha] \) and \( y = [11 + \alpha, 14 - \alpha] \). By taking \( \alpha \) values from 0, 0.1, \ldots, 1 the execution measures are shown in the Table 2.

<table>
<thead>
<tr>
<th>( \alpha )</th>
<th>( L_q )</th>
<th>( L_s )</th>
<th>( W_s )</th>
<th>( W_q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>[0.0027, 0.1039]</td>
<td>[0.0741, 0.4675]</td>
<td>[0.0741, 0.1168]</td>
<td>[0.0027, 0.0259]</td>
</tr>
<tr>
<td>0.1</td>
<td>[0.0034, 0.0951]</td>
<td>[0.0825, 0.4465]</td>
<td>[0.0750, 0.1144]</td>
<td>[0.0030, 0.0243]</td>
</tr>
<tr>
<td>0.2</td>
<td>[0.0041, 0.0871]</td>
<td>[0.0910, 0.4264]</td>
<td>[0.0759, 0.1122]</td>
<td>[0.0034, 0.0229]</td>
</tr>
<tr>
<td>0.3</td>
<td>[0.0049, 0.0797]</td>
<td>[0.0998, 0.4071]</td>
<td>[0.0768, 0.1100]</td>
<td>[0.0038, 0.0215]</td>
</tr>
<tr>
<td>0.4</td>
<td>[0.0059, 0.0728]</td>
<td>[0.1088, 0.3886]</td>
<td>[0.0777, 0.1079]</td>
<td>[0.0042, 0.0202]</td>
</tr>
<tr>
<td>0.5</td>
<td>[0.0069, 0.0665]</td>
<td>[0.1180, 0.3709]</td>
<td>[0.0787, 0.1059]</td>
<td>[0.0046, 0.0190]</td>
</tr>
<tr>
<td>0.6</td>
<td>[0.0080, 0.0607]</td>
<td>[0.1275, 0.3538]</td>
<td>[0.0796, 0.1040]</td>
<td>[0.0050, 0.0178]</td>
</tr>
<tr>
<td>0.7</td>
<td>[0.0093, 0.0554]</td>
<td>[0.1371, 0.3374]</td>
<td>[0.0806, 0.1022]</td>
<td>[0.0055, 0.0167]</td>
</tr>
<tr>
<td>0.8</td>
<td>[0.0107, 0.0504]</td>
<td>[0.1471, 0.3216]</td>
<td>[0.0817, 0.1005]</td>
<td>[0.0059, 0.0157]</td>
</tr>
<tr>
<td>0.9</td>
<td>[0.0123, 0.0458]</td>
<td>[0.1573, 0.3063]</td>
<td>[0.0828, 0.0988]</td>
<td>[0.0064, 0.0148]</td>
</tr>
<tr>
<td>1</td>
<td>[0.0139, 0.0416]</td>
<td>[0.1678, 0.2916]</td>
<td>[0.0839, 0.0972]</td>
<td>[0.0069, 0.0138]</td>
</tr>
</tbody>
</table>

Table 2: The \( \alpha \)-cuts of \( L_s, L_q, W_s \) and \( W_q \) at \( \alpha \) values (precedent 2)
6.3. Precedent 3

Recognize both the entry and overhauled rate are pentagonal fuzzy numbers represented by $\lambda = [1, 2, 3, 4, 5]$ and $\mu = [11, 12, 13, 14, 15]$. The interval of certainty at probability level $\alpha$ as $[1 + \alpha, 5 - \alpha]$ and $[11 + \alpha, 15 - \alpha]$, where $x = [1 + \alpha, 5 - \alpha]$ and $y = [11 + \alpha, 15 - \alpha]$. By taking $\alpha$ values from 0, 0.1, . . . , 1 the execution measures are shown in the Table 3.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$L_q$</th>
<th>$L_s$</th>
<th>$W_s$</th>
<th>$W_q$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.0023, 0.1893</td>
<td>0.0690, 0.6439</td>
<td>0.0690, 0.1287</td>
<td>0.0023, 0.0378</td>
</tr>
<tr>
<td>0.1</td>
<td>0.0035, 0.1607</td>
<td>0.0846, 0.5892</td>
<td>0.0705, 0.1227</td>
<td>0.0029, 0.0334</td>
</tr>
<tr>
<td>0.2</td>
<td>0.0050, 0.1364</td>
<td>0.1009, 0.5399</td>
<td>0.0721, 0.1173</td>
<td>0.0036, 0.0296</td>
</tr>
<tr>
<td>0.3</td>
<td>0.0069, 0.1159</td>
<td>0.1180, 0.4952</td>
<td>0.0737, 0.1125</td>
<td>0.0043, 0.0263</td>
</tr>
<tr>
<td>0.4</td>
<td>0.0092, 0.0983</td>
<td>0.1359, 0.4542</td>
<td>0.0755, 0.1081</td>
<td>0.0051, 0.0234</td>
</tr>
<tr>
<td>0.5</td>
<td>0.0119, 0.0833</td>
<td>0.1547, 0.4166</td>
<td>0.0773, 0.1041</td>
<td>0.0059, 0.0208</td>
</tr>
<tr>
<td>0.6</td>
<td>0.0151, 0.0704</td>
<td>0.1745, 0.3819</td>
<td>0.0793, 0.1005</td>
<td>0.0068, 0.0185</td>
</tr>
<tr>
<td>0.7</td>
<td>0.0189, 0.0593</td>
<td>0.1953, 0.3497</td>
<td>0.0814, 0.0971</td>
<td>0.0078, 0.0164</td>
</tr>
<tr>
<td>0.8</td>
<td>0.0233, 0.0498</td>
<td>0.2173, 0.3197</td>
<td>0.0836, 0.0940</td>
<td>0.0089, 0.0146</td>
</tr>
<tr>
<td>0.9</td>
<td>0.0285, 0.0416</td>
<td>0.2406, 0.2916</td>
<td>0.0859, 0.0911</td>
<td>0.0101, 0.0130</td>
</tr>
<tr>
<td>1</td>
<td>0.0346, 0.0346</td>
<td>0.2653, 0.2653</td>
<td>0.0884, 0.0884</td>
<td>0.0115, 0.0115</td>
</tr>
</tbody>
</table>

Table 3: The $\alpha$–cuts of $L_s$, $L_q$, $W_s$ and $W_q$ at $\alpha$ values (precedent 3)
7. Results and discussions

Utilizing MATLAB, we achieve $\alpha$–cuts of entry rate, administration rate and fuzzy anticipated number of occupations in queue just as system at eleven patent dimensions: 0, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9 and 1. Crisp intervals for fuzzy anticipated number of employments in queue and system at various probability $\alpha$ levels are exhibited in Tables 1–3. The execution proportions for example anticipated number of clients in the system $L_a$, anticipated length of the line $L_q$, the average holding up time of a client in the system $W_s$ and the average holding up time of a client in the queue $W_q$ likewise inferred in Tables 1–3.

From the Table 1 we can extract following information:

i) Anticipated number of clients in the line is 0.0166 and inconceivable falls outside $[0.0032, 0.0511]$

ii) Anticipated number of clients in the system is 0.1833 and inconceivable falls outside $[0.0801, 0.3238]$

iii) Average holding up time of a client in the line is 0.0083 and inconceivable falls outside $[0.0032, 0.0170]$

iv) Average holding up time of a client in the system is 0.0916 and inconceivable falls outside $[0.0801, 0.1079]$
From the Table 2 we can extract following information:

\textbf{v)} Expected number of clients in the line is 0.0416 and inconceivable falls outside [0.0027, 0.1039]

\textbf{vi)} Anticipated number of clients in the framework is 0.2916 and inconceivable falls outside [0.0741, 0.4675]

\textbf{vii)} Average holding up time of a client in the queue is 0.0138 and inconceivable falls outside [0.0027, 0.0259]

\textbf{viii)} Average holding up time of a client in the system is 0.0972 and inconceivable falls outside [0.0741, 0.1168]

From the Table 3 we can extract following information:

\textbf{ix)} Anticipated number of clients in the line is 0.0346 and inconceivable falls outside [0.0023, 0.1893]

\textbf{x)} Anticipated number of clients in the framework is 0.2653 and inconceivable falls outside [0.0690, 0.6439]

\textbf{xi)} Average holding up time of a client in the line is 0.0115 and inconceivable falls outside [0.0023, 0.0378]

\textbf{xii)} Average holding up time of a customer in the system is 0.0884 and inconceivable falls outside [0.0690, 0.1287]

Finally, comparison of FM/FD/1 queuing model without fuzzy number and the FM/FD/1 queuing model with various fuzzy numbers is given in Table 4.

<table>
<thead>
<tr>
<th>FM/FD/1 without fuzzy number</th>
<th>FM/FD/1 with fuzzy number</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\lambda = 2, \mu = 12)</td>
<td></td>
</tr>
<tr>
<td>\textbf{Triangular fuzzy number}</td>
<td>\textbf{Trapezoidal fuzzy number}</td>
</tr>
<tr>
<td>(\lambda = [1, 2, 3])</td>
<td>(\lambda = [1, 2, 3, 4])</td>
</tr>
<tr>
<td>(\mu = [11, 12, 13])</td>
<td>(\mu = [11, 12, 13, 14])</td>
</tr>
<tr>
<td>(L_q)</td>
<td>0.0166</td>
</tr>
<tr>
<td>(L_s)</td>
<td>0.1832</td>
</tr>
<tr>
<td>(W_s)</td>
<td>0.0916</td>
</tr>
<tr>
<td>(W_q)</td>
<td>0.0083</td>
</tr>
</tbody>
</table>

\textbf{Table 4: Comparison of FM/FD/1 queuing model without and with fuzzy number}

In this investigation, the outcome shows that the exhibition measures \(L_q, L_s, W_s\) and \(W_q\) for both queuing theory model and fuzzy queuing model were processed and analysed. In view of the outcome, the fuzzy queuing model is substantially more successful and proficient to measure the exhibition of FM/FD/1 lining framework since the fuzzy set hypothesis is more effectively versatile. Applying the fuzzy lining model gives more extensive data, which will be exceptionally valuable in characterizing a lining framework. Hence, this investigation infers that fuzzy lining is one of the elective approaches to register the exhibition measures since the data acquired from the application is a lot clearer to understand and interpret.
8. Conclusion

Here, we surmise that fuzzy set hypothesis has been connected to lining hypothesis more over the inter landing time and administration time are fuzzy identity. The execution proportions for example entity length, line length, entity time, line time and so forth are likewise fuzzy in nature. Numerical precedents demonstrate the proficiency of DSW calculation. Here it is noticed that the achievement of queuing model can be upgraded by expanding the number of variables. The proposed model can help the businesses, wholesalers and retailers in precisely deciding the ideal execution of the queuing system. There are different viewpoints that the paper can be extended. One of them is to think about random variable, or fuzzy random variable to landing rate and administration rate. Another conceivable dimensional to stretch out this paper is to consider intuitionistic fuzzy numbers.

References


