Determining the Optimal Location of Substations for Electric Distribution: A Real Application on a Regional Basis

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\textbf{Abstract.} Substation optimization involves positioning a substation(s) at the most suitable point with minimum cost. In this study, multiple data were collected from the Turkish Electric Distribution Incorporated Company (TEIAS) for the substation in the Eryaman region. The substations will be placed at straight points. Using the capacity of the substations, power, voltage drops, and service interruptions were calculated for a 10-year planning period. Each criterion represents one constraint. The established model serves the following objectives: determining the optimal point and size of the substations in the given planning period by minimizing costs and satisfying the demand of the sectors without service interruption.

The model is solved in GAMS and optimal results are obtained. Our results demonstrate that by installing the two substations at the given locations the energy demand of the Eryaman region will be satisfied. To test the model’s applicability for larger systems, the number and the capacity of the substations and the transformers are increased. The results show that optimality has been maintained and that the model remains valid.

\textbf{Keywords:} energy, optimization, power distribution, substation

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\section{1. Introduction}

Energy is vital for human beings to avoid extinction. Many studies have been performed regarding the daily production, location, and timing of electrical energy [1]. As the population grows, the demand for energy simultaneously grows. To satisfy the electrical energy demand without harming nature and to minimally affect the natural balance, new technologies and methodologies are continuously used. Because electrical energy cannot be stored for a long period, the production cost remains high. To depend less on electrical energy, new low-energy technologies are invented [7].

Power distribution can be regarded as the final stage of electric power delivery. It gained significance in the 1880’s when electricity started being generated in power stations. In order to distribute electrical energy to individual consumers, power stations were installed all over the world to transmit the electricity produced. Until that time, the electricity had been consumed at the same point where it was produced [19].

Undoubtedly, there is a large body of literature on energy; therefore, we only deal with some studies below that can be thought of as most related to our study. Salyani et al. [17] performed

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simultaneous optimization of substations, feeders, and renewable and nonrenewable distributed
generations in a distribution network. Wang et al. [23] used an optimization method based
on random fork tree coding for the electrical networks of offshore wind farms. Li et al. [9]
combined a technique with a labeling-bus-set approach to configure the substations in power
systems. Mak et al. [10] used sensitivity analysis of volt-VAR optimization in order to deter-
mine the data changes in distribution networks with distributed energy resources. Mikulović et
al. [13] rationalized the operation for industrial networks. Roig and Segundo [18] reduced the
low voltage using power cables in electromagnetic field emission. Giassi et al. [6] performed an
economical layout optimization of wave energy parks clustered in electrical subsystems. Khodr
et al. [8] designed grounding systems in substations using a Mixed-Integer Linear Program-
ning (MILP) formulation. Aydin et al. [3] determined an optimum route for the electrical
energy transmission line using multicriteria with Q-learning. Jing et al. [21] studied the results
of the multicriteria decision analysis aid in sustainable energy decision-making. Ekren and
Ekren [4] studied the simulation-based optimization of a PV/wind hybrid energy conversion
system. Ortmann et al. [14] used experimental validation of feedback optimization in power
distribution grids. Alcayde et al. [12] used Pareto optimization methods to minimize voltage
development and power losses. Jing et al. [22] maximized the system to save energy and reduce
environmental impact. Maestre et al. [15] studied the energy flow in HVAC systems. Chojnacki
[2] optimized the time periods of MV/LV transformer-distribution substations. Mancarella et
[16] planned electrical distribution systems based on a probabilistic model using multiobjective
particle swarm optimization. Singh et al. [20] used multicriteria decision-making with monarch
butterfly optimization to optimize the distributed energy resources in distribution networks.
El-Fouly et al. [5] developed a new optimization model for substation siting, sizing, and
timing. For the proposed model, linear functions are used to generate the total cost function.
In the developed model, the voltage drops, the capacity of the substations and the transformers,
and power flows are used as the electric constraints. To prevent nonlinearity and avoid local
solutions, the model is formulated as a Mixed Integer Linear Programming (MILP) model. To
calculate its efficiency, the model is operated with a numerical sample.

In the present study, we implement a case study for which substations are to be positioned
in the Eryaman region in Ankara, the capital of Turkey. For this purpose, substantive data were
collected for the substation(s) that is (are) going to be positioned and balanced distribution was
integrated into the cost minimization. However, a balanced distribution does not significantly
affect the objective function. Thus, the integrated component is considered the background
and it provides a balanced distribution for the generated power. In addition to the constraints
that are considered in the literature, especially in El-Fouly et al. [5], there is another constraint
in case a short circuit occurs, which is unlikely to appear in the related literature, to the best
of our knowledge. The purpose of this additional constraint is to distribute balanced power
to prevent system defects that may occur except in the case of external factors (bad weather
conditions, acts of terrorism, etc.).

The rest of the paper is organized as follows: after the problem is defined in Section 2, we
propose our mathematical model in Section 3. Section 4 includes computational results and their
analysis. We present sensitivity analysis of our model in Section 5 and finally our conclusion
and future research directions are revealed in Section 6.

2. Problem Definition

The explored area consists of nine sectors. The area is estimated as a square and the distances
among the power distributing cables are assumed to be equal. When the cable length increases,
the cost increases and the efficiency decreases. The long cable lines to be placed also require
mid points, which increases total cost. The area of each sector is 6 km². The fourth and sixth
sectors are chosen to place the substations. The chosen positions are the most suitable because they can easily distribute power to all nine sectors. Each substation (TM) has capacity of 50 MW and has two transformers (TR) with a capacity of 25 MW. Figure 1 shows the sectors and positions of the placed substations.

![Figure 1: Points of placed substations.](image)

<table>
<thead>
<tr>
<th>Substation/Transformer</th>
<th>Capacity (MW)</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>Substation 1</td>
<td>50</td>
<td>Sector 4</td>
</tr>
<tr>
<td>Transformer 1</td>
<td>25</td>
<td>Sector 4</td>
</tr>
<tr>
<td>Transformer 2</td>
<td>25</td>
<td>Sector 4</td>
</tr>
<tr>
<td>Substation 2</td>
<td>50</td>
<td>Sector 6</td>
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<tr>
<td>Transformer 1</td>
<td>25</td>
<td>Sector 6</td>
</tr>
<tr>
<td>Transformer 2</td>
<td>25</td>
<td>Sector 6</td>
</tr>
</tbody>
</table>

**Table 1: Substation and Transformers Unit Capacities**

As observed in Table 1, the unit capacities of the substations and the transformers are 50 MW and 25 MW, respectively. The planning period is 10 years; each period consists of 2 years. Thus, the demands will be studied for 5 periods. The 10-year demands are shown in Table 2. The feeder lines of the substations are shown in Table 3.

<table>
<thead>
<tr>
<th>(D_{n,p} ) (MW)</th>
<th>Sectors (p)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period (n)</td>
<td>1 2 3 4 5 6 7 8 9</td>
</tr>
<tr>
<td>1</td>
<td>5 5 5 5 4 7 5 7 4 9</td>
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<td>6 7 7 5 6 6 8 7 8</td>
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<tr>
<td>5</td>
<td>8 7 8 6 5 9 7 9 9</td>
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</tbody>
</table>

**Table 2: 10-year Demands of All Regions**
3. Mathematical Model

The main objective of planning the position to place a substation and its components is to minimize the cost and the energy loss. The cost involves determining a substation location, placing period, loading the transformers, etc. While planning a substation location, if more substations are placed than required, the installation cost will significantly increase. In this case, the placing period will lengthen, and the interest rate, the taxes, the inflation rate, and the insurance rates will be affected. The energy loss is directly connected with loading the power equipment. If the loading values increase, the total cost will uniformly increase.

This problem is formulated to achieve the following objective components:

- Determining the optimal placing locations in the planned period and
- Satisfying the electric demand of the sectors without service interruption.

The notations used in the model are as follows:

**Index Set:**

- \( N \) : the number of periods in 10 years (because one period consists of 2 years, this planned period is chosen to supply the necessary time to install the substations)
- \( I \) : the number of substations; \( I = 1, 2 \) shows that two substations are planned to be placed.
- \( J \) : the number of transformers. \( J = 1, 2 \) shows that there are two transformers per substation planned to be placed.

**Parameters:**

- \( TM_{SM_{I,N}} \) : fixed cost of substation \( I \), installed in the \( N^{th} \) period
- \( TR_{SM_{I,J,N}} \) : fixed cost of the transformers \( J \) from substation \( I \), installed in the \( N^{th} \) period
- \( K_{cu} \) : copper loss of the transformer in nominal power (kW)
- \( C \) : energy cost ($/kWh)
- \( H_{tr} \) : unit efficiency of the transformer (MW)
- \( R \) : resistance of the feeders, the value is 0.047
- \( V_{nom} \) : system’s nominal voltage; the feeder line types are 154 kV
- \( V_{max} \) : system’s maximum permitted voltage drop value, which is $\pm 1.25\%$ of the nominal voltage value. If the nominal voltage is 154 kV, the maximal permitted voltage drop value is 155.925 kV

<table>
<thead>
<tr>
<th>Sectors</th>
<th>Sectors</th>
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<tr>
<td><strong>TM 1 - feeders</strong></td>
<td><strong>TM 2 - feeders</strong></td>
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<td><strong>Ending Point</strong></td>
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<td><strong>Starting Point</strong></td>
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<tr>
<td>9</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 3: Feeder lines of the substations
Decision Variables:

\[ S_{I,N} : \begin{cases} 1 & \text{if substation } I \text{ is installed in the } N^{th} \text{ period.} \\ 0 & \text{o.w.} \end{cases} \]

\[ X_{I,J,N} : \begin{cases} 1 & \text{if transformer } J \text{ of substation } I \text{ is installed in the } N^{th} \text{ period.} \\ 0 & \text{o.w.} \end{cases} \]

\[ Z_{I,J,N,P} : \begin{cases} 1 & \text{transformer } J \text{ of substation } I \text{ supplies energy to sec. } P \text{ in the } N^{th} \text{ period.} \\ 0 & \text{o.w.} \end{cases} \]

\[ L_{I,J,N} : \begin{cases} 1 & \text{if a malfunction of transformer } J \text{ of substation } I \text{ occurs in period } N. \\ 0 & \text{o.w.} \end{cases} \]

\[ F_{I,J,N,P} : \text{the amount of transmitted power to sector } P \text{ from transformer } J \text{ of substation } I \text{ in the } N^{th} \text{ period.} \]

\[ D_{N,P} : \text{the demand of sector } P \text{ in the } N^{th} \text{ period.} \]

\[ MaxL : \text{balanced distribution of the service interruption period in the feeders.} \]

Mathematical Model:

Objective function:

\[
\min Z = \sum_{I=1}^{2} \sum_{N=1}^{5} (TM_{SMI,N} \times S_{I,N}) + \sum_{I=1}^{2} \sum_{J=1}^{2} \sum_{N=1}^{5} TR_{SMI,J,N} \times X_{I,J,N} + \left( \frac{K_{cu} \times C}{H_{tr}} \times 8760 \times X_{I,J,N} \right) + 0.1 \times \min L
\]  \hspace{1cm} (1)

Constraints:

\[
S_{I,N} \leq S_{I,N+1} \quad I = 1,2; N = 1,\ldots,4 \tag{2}
\]

\[
X_{I,J,N} \leq X_{I,J,N+1} \quad I = 1,2; J = 1,2; N = 1,\ldots,4 \tag{3}
\]

\[
\sum_{i=1}^{2} \sum_{j=1}^{2} Z_{I,J,N,P} = 1 \quad N = 1,\ldots,5; P = 1,\ldots,9 \tag{4}
\]

\[
\sum_{i=1}^{2} \sum_{j=1}^{2} F_{I,J,N,P} = D_{N,P} \quad N = 1,\ldots,5; P = 1,\ldots,9 \tag{5}
\]

\[
F_{I,J,N,P} = D_{N,P} \times Z_{I,J,N,P} \quad I = 1,2; J = 1,2; N = 1,\ldots,5; P = 1,\ldots,9 \tag{6}
\]

\[
Z_{I,J,N,P} \leq Z_{I,J,N+1,P} \quad I = 1,2; J = 1,2; N = 1,\ldots,5; P = 1,\ldots,9 \tag{7}
\]

\[
\sum_{i=1}^{2} \sum_{j=1}^{2} Z_{I,J,N,P} \leq \text{CARD}(P) \times X_{I,J,N} \quad N = 1,\ldots,5; P = 1,\ldots,9 \tag{8}
\]

\[
\sum_{i=1}^{2} \sum_{j=1}^{2} X_{I,J,N} \leq \text{CARD}(J) \times S_{I,N} \quad J = 1,2; N = 1,\ldots,5 \tag{9}
\]

\[
\sum_{j=1}^{9} \sum_{P=1}^{2} F_{I,J,N,P} \leq 40 \quad I = 1,2; N = 1,\ldots,5 \tag{10}
\]
\[
\sum_{P=1}^{9} F_{I,J,N,P} \leq 20 \quad I = 1, 2; J = 1, 2; N = 1, \ldots, 5 \quad (11)
\]
\[
((1000 \cdot D_{N,P} \cdot R) / V_{nom}) \cdot Z_{I,J,N,P} \leq V_{\text{max}} \quad I = 1, 2; J = 1, 2; N = 1, \ldots, 5; P = 1, \ldots, 9 \quad (12)
\]
\[
L_{I,J,N} = \sum_{P=1}^{9} Z_{I,J,N,P} \quad I = 1, 2; J = 1, 2; N = 1, \ldots, 5 \quad (13)
\]
\[
\max L \leq L_{I,J,N} \quad I = 1, 2; J = 1, 2; N = 1, \ldots, 5 \quad (14)
\]
\[
S_{I,N}, X_{I,J,N}, Z_{I,J,N,P}, L_{I,J,N} \in \{0, 1\} \quad I = 1, 2; J = 1, 2; N = 1, \ldots, 5; P = 1, \ldots, 9 \quad (15)
\]
\[
F_{I,J,N,P}, D_{N,P}, \max L \geq 0 \quad I = 1, 2; J = 1, 2; N = 1, \ldots, 5; P = 1, \ldots, 9 \quad (16)
\]

The objective function (1) is determined in a 10-year planning period. Even if there is a load connected to the primary bobbins of the transformer, specific losses occur because of the magnetic field from the bobbin resistances. These losses are named copper losses and appear as heat.

To not significantly affect the total cost, the value is multiplied by a 0.1 scaled multiplier and added to the total cost function. Because of service interruptions, the electric demands will not be satisfied; thus, significant financial losses will occur. When service interruptions cannot be prevented (bad weather conditions, terrorist incidents, etc.), the power cuts must be limited to a short period. The service interruption of each feeder indicates a total power cut for each substation. In addition to the external factors, when a malfunction occurs due to internal factors, distribution balancing is considered an option.

Fixed cost constraint (2) is a decision variable and represents whether substation \(I\) is installed in the \(N^{\text{th}}\) period. The substation cost considers when the substation is installed. If the power is transmitted in the first period, then \(S_{I,N} = 1\) and the substation is installed. In addition, if the substation is installed in the first period, it will remain functional for the remaining periods. The substation cannot be uninstalled for the remaining periods.

The other fixed cost constraint, (3) \(X_{I,J,N}\), is also a \(0 - 1\) integer decision variable. This variable depends on the first variable, which shows the transformers in substation \(I\). If transformer \(J\) is placed in substation \(I\) in the \(N^{\text{th}}\) period, the value is 1. Similar to the first variable, if the second variable \(J\) is installed in the \(N^{\text{th}}\) period, it cannot be uninstalled in the \(N + 1^{\text{st}}\) period.

Each sector demand can be fulfilled from only one feeder. In other words, each sector will supply energy from only one substation to satisfy the radial flow constraint (4). The radial flow is the current that flows through the bobbin diameter of the transformers. Thus, the current reaches the feeders and the sectors via the distributing lines.

\(F_{I,J,N,P}\) (5) and \(D_{N,P}\) (6) represent the amount of transmitted power and demand of the sector \(P\) from transformer \(J\) of substation \(I\) in the \(N^{\text{th}}\) period, respectively.

\(Z_{I,J,N,P}\) (7) represents the continuity of energy distribution to sector \(P\) in period \(N\); it will continue to distribute energy in the remaining periods if the value is 1.

The power flow constraint represents the law of energy preservation. The substation loading and the transformer loading of period \(N\) are equal, which must be simultaneously equal to or greater than the energy demand of each sector.

When transformer \(J\) of substation \(I\) distributes energy to sector \(P\) in period \(N\), the amount must be equal to or greater than the demand of the distributed sector. If the substation or one of the transformers of the substation is not functional or not installed, the energy distribution of sector \(P\) is interrupted. If the substation or the transformers are installed, the energy is transmitted to the determined sector for period \(N\) and remains transmitted for the remainder of the period to the same sector from the same substation or its transformer. This state can be provided using the \(\text{CARD}(P)\) command in GAMS (8).

The loading amount of substation \(I\) must be equal to or greater than the loading amount of transformer \(J\) in period \(N\) to ensure that transformer \(J\) can satisfy the energy demand of the sectors. If substation \(I\) is not functional or not installed, the energy distribution to transformer \(J\) will be interrupted. If substation \(I\) is operational or installed, transformer \(J\) will distribute energy from the
period that the substation is installed and continue distributing energy for the remaining periods. This state can also be provided using the \texttt{CARD}\(_{(P,J)}\) command (9).

The capacities of each substation and each transformer are 50 MW and 25 MW, respectively. The transformers can operate with max. 80\% efficiency. Thus, the usage capacities of each substation and each transformer are 40 MW (10) and 20 MW (11), respectively.

The voltage constraint (12) enables the feeders’ voltage to remain within the permissible standard values. Each feeder requires the constraint to determine and not to exceed the voltage drop maximum limit value. If the voltage drop value exceeds the maximum permissible limit, the feeders may become overloaded and malfunction. Because each feeder distributes energy to only one sector, the demand will not be satisfied.

A service interruption is a defect that occurs mostly because of the conditions that are irrelevant to the internal system. Bad weather conditions (snow, lightning, etc.), terrorist incidents, and technical and suddenly occurring defects (isolator defect, phase conductor defect) are some of the reasons that the feeders cannot transmit energy, which results in the substation being out of order.

Service interruptions are mostly inevitable and can cause a serious problem. Although technical defects can be prevented, for malfunctions caused by external defects, the effective methods are not sufficiently applied. The transformer remains out of order until the defect is rectified. To determine the service interruption number for the substation to be placed, retrospective data are gathered from several provinces. The calculation is based on the cumulative annual service interruption periods. The inoperative period of the substation is equal to the repairing period of the feeders. By determining the maximal service interruption number (13), each feeder of the substation will be balance-distributed by shortening the repair periods and re-operating the substations.

If \(L_{I,J,N}\) is 1, transformer \(J\) remains operating and the feeder distributes the energy to sector \(P\). Otherwise, if the value is 0, service interruption occurs, and the feeder cannot distribute energy to sector \(P\).

\(\text{max}L\) represents the maximum assigned sector number; if the number exceeds the \(\text{max}L\) value, service interruption occurs (14).

4. Computational Analysis

The model is solved using GAMS and the optimal result is found. The solution is shown in Table 4. The optimal cost for the 10-year planning period is $4,250,249.00. The cost obtained is near the value that Turkish Electric Distribution Inc. (TEIAS) intends to spend, which shows that the resulting cost is meaningful.

<table>
<thead>
<tr>
<th>Period ((N))</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
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\textbf{Table 4: Sector energy distribution}

As observed in Figure 2, the first transformer (TR) of the first substation (TM) distributes energy to sectors 1 and 3, and the second transformer distributes energy to sectors 2, 5, and 8. The first transformer of the second substation distributes energy to sectors 4 and 7, whereas the second transformer distributes energy to sectors 6 and 9. From another viewpoint, sectors 1, 2, 3, 5, and 8 receive their energy from the first substation, whereas sectors 4, 6, 7, and 9 receive their energy from the second substation. The distribution assignment shows that each sector is supplied with energy from only one substation, i.e., the electric energy demand of one sector cannot be satisfied by more than one substation.
Table 5 shows the satisfied demands of the sectors in a periodical base. As observed from the table, the values are equal to the electric energy demand of the sectors.

<table>
<thead>
<tr>
<th>$D_{n,p}$ (MW)</th>
<th>Period (N)</th>
<th>1</th>
<th>2</th>
<th>3</th>
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</table>

Table 5: Satisfied energy demand of the sectors

In Table 5, the energy distributions in all five periods to each sector are shown in detail. The distribution to a specific sector starting from the first period for all five periods is supplied by one substation, which results from the $CARD_{(P)}$ and $CARD_{(J)}$ commands of the power flow constraint. If a substation and its transformers are installed in a specific period and distribute energy to sector $P$, they will continue distributing energy to the same sector for the remaining periods. For example, if the energy demand of sector 1 is satisfied by the first transformer of the first substation in period 1, this transformer will continue supplying energy to sector 1 for the remaining periods. If the energy demand of sector 7 is satisfied by the first transformer of the second substation in period 1, this transformer will continue supplying energy to sector 7 for the remaining periods.

The capacities of the substations and the transformers for five periods are shown in Tables 6 and 7. As observed from the tables, the substations and the transformers produce and distribute energy without exceeding the 80% efficiency and capacity limit. The second transformer of each substation produces and distributes energy near the capacity limit.

<table>
<thead>
<tr>
<th>Period (N)</th>
<th>Sectors (I)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>26</td>
</tr>
<tr>
<td>2</td>
<td>32</td>
</tr>
<tr>
<td>3</td>
<td>31</td>
</tr>
<tr>
<td>4</td>
<td>33</td>
</tr>
<tr>
<td>5</td>
<td>36</td>
</tr>
</tbody>
</table>

Table 6: Usage capacity of the substations

As shown in Table 6, the maximum usage capacity of each substation is 40 MW, neither substation exceeds the capacity, and both substations operate with optimum efficiency.
Determining the Optimal Location of Substations for Electric Distribution: A Real Application on a Regional Basis

<table>
<thead>
<tr>
<th>Period (N)</th>
<th>Substation 1</th>
<th>Substation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Transformer 1</td>
<td>Transformer 2</td>
</tr>
<tr>
<td>2</td>
<td>Transformer 1</td>
<td>Transformer 2</td>
</tr>
<tr>
<td>3</td>
<td>Transformer 1</td>
<td>Transformer 2</td>
</tr>
<tr>
<td>4</td>
<td>Transformer 1</td>
<td>Transformer 2</td>
</tr>
<tr>
<td>5</td>
<td>Transformer 1</td>
<td>Transformer 2</td>
</tr>
</tbody>
</table>

Table 7: Usage capacity of the transformer

As shown in Table 7, the maximum usage capacity of each transformer is 20 MW, neither transformer exceeds the maximum usage limit, and both operate with optimum efficiency. The second transformer of each substation is operating at nearly full capacity for all five periods.

<table>
<thead>
<tr>
<th>Period (N)</th>
<th>Sectors (P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1, 2, 3, 4, 5, 6, 7, 8, 9</td>
</tr>
<tr>
<td>2</td>
<td>1, 2, 3, 4, 5, 6, 7, 8, 9</td>
</tr>
<tr>
<td>3</td>
<td>1, 2, 3, 4, 5, 6, 7, 8, 9</td>
</tr>
<tr>
<td>4</td>
<td>1, 2, 3, 4, 5, 6, 7, 8, 9</td>
</tr>
<tr>
<td>5</td>
<td>1, 2, 3, 4, 5, 6, 7, 8, 9</td>
</tr>
</tbody>
</table>

Table 8: Voltage drop values

Table 8 shows the voltage drop values of the feeders. Because all entries in the table are nonzero, no feeder has exceeded the maximum voltage drop limit. Thus, the system operates/distributes energy without interruption. Increasing the power also increases the voltage drops.

As observed from all tables, each decision variable takes a value of 1 from the \{0, 1\} space. In other words, each substation and each transformer are operating fully and each is assigned for specific sectors.

Balanced distribution, which helps avoid service interruptions, is shown in Table 9.

<table>
<thead>
<tr>
<th>Period (N)</th>
<th>Substation 1</th>
<th>Substation 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 9: Balanced distribution

To better understand the table entries, the first transformer of the first substation simultaneously distributes energy to two sectors in period 1 and continues simultaneously distributing energy to two sectors for all five periods. The second transformer of the second substation simultaneously distributes energy to three sectors in period 1 and continues simultaneously distributing energy to three sectors for the remaining periods.

The max. L value is 3, which indicates that a transformer can simultaneously distribute energy to, at most, three sectors. If this value is exceeded, service interruption will occur.
5. Sensitivity Analysis

The 10-year retrospective data are gathered and formed into five periods to calculate the energy demand and determine the optimal points for the substation(s) to be placed in the Eryaman region. Two substations with a capacity of 50 MW, two transformers for each substation, and nine feeders are sufficient to meet the electric demand of the Eryaman region. The cost gained from the calculations is near the cost that TEIAS intends to incur.

The present study, which was applied for the Eryaman region, can also be easily applied to other regions using identical or different constraints. Table 10 represents a realistic example for this study. The model is tested by increasing the substation capacities using the same constraints.

<table>
<thead>
<tr>
<th>Capacity (MW)</th>
<th>Solving Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.031 sec.</td>
</tr>
<tr>
<td>200</td>
<td>0.032 sec.</td>
</tr>
<tr>
<td>300</td>
<td>0.024 sec.</td>
</tr>
<tr>
<td>400</td>
<td>0.036 sec.</td>
</tr>
</tbody>
</table>

Table 10: Solution time of the model by increasing the capacities of the two substations

For the Eryaman region, two substations with a total capacity of 100 MW were sufficient to satisfy the energy demand of all sectors. By increasing the capacities to 200, 300, and 400 MW, the solutions show that the model can be easily solved and that optimality is not affected. It also shows that the model can be used for larger regions with higher energy demands.

Another testing method is to increase the capacities and the number of substations. The substation capacities are increased to 200, 300, and 500 MW, and the number of substations is increased to 3, 4, and 10, respectively. Thus, the number of transformers will be uniformly increased.

The results are given in Table 11.

<table>
<thead>
<tr>
<th>Capacity (MW)</th>
<th>Substation</th>
<th>Transformer</th>
<th>Solving Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>4</td>
<td>8</td>
<td>0.047 sec.</td>
</tr>
<tr>
<td>200</td>
<td>4</td>
<td>16</td>
<td>0.110 sec.</td>
</tr>
<tr>
<td>300</td>
<td>3</td>
<td>18</td>
<td>0.109 sec.</td>
</tr>
<tr>
<td>500</td>
<td>10</td>
<td>50</td>
<td>0.202 sec.</td>
</tr>
</tbody>
</table>

Table 11: Solving time with increased capacities

As observed from both tables, the model can be used for larger areas and higher energy demands. Optimality will not be affected and the problem will be solved without difficulty.

6. Conclusion and Future Research Perspectives

The model is operated using retrospective data and the energy demand is calculated for the Eryaman region. The results show that if two substations and nine feeders are placed, the energy demand of the region is satisfied. The results obtained demonstrate that the solution is optimal and by using alternative inputs optimality is not affected. In other words, our model can also serve for larger systems with higher energy demands.

For further studies, the model can be used for one specific region, more than one sector in one town, and for the entire country. In the case of modification, the installation and the fixed cost will most likely be changed. The feeder lines to be placed will lose linearity because of the geographical position (field incompatibilities, building location points, etc.). Placing mid points for the feeders and using other technical electric components will introduce different points and visions to the study. Another expansion of the model is to add alternative current circuits or to change the network tensions based on the circuit currency degrees, etc. Corporations use reactive energy in addition to active energy. The reactive energy is calculated and paid for by TEDAŞ (Turkish Electricity Distribution Inc.). The reactive power is unused power and can be reduced. To prevent power loss, the power factor can be calculated and used for further studies as an alternative.
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References


