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A control rule for planning promotion in a Nigerian university setting

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Abstract. This paper examines the attainability problem in a graded manpower system, where the objective is to maximise the total throughput. The problem is modelled as a linear programming problem (LPP) and the evolution of structures in the system is described using the Markov chain model. The decision variables are the promotion rates. Results from the LPP provide a guide to the administrative authority of the system on how promotion and retrenchment should be implemented. The utility of the model is demonstrated using a university setting in Nigeria.

Keywords: attainability, linear programming, manpower system, Markov chain, promotion control

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1. Introduction

This paper concentrates on the attainability problem under promotion control, where retrenchment is allowed. The paper considers a manpower system stratified into k categories according to the grades. Let S be the set of categories and let each staff belong to one and only one category in S. Let the number of staff in each category of the system at a period of time t be denoted by the $k-$tuple stock vector $\mathbf{x}(t) = [x_1(t), \cdots, x_k(t)]$. The vector $\mathbf{x}(t)$ is also referred to as the structure of the system at time t. The stock $x_i(t)$ changes after a period of time in such fashion that: a proportion p_{ii} is still in category i, a proportion p_{ij} is promoted to the next higher category $i+1$, $i, j = i+1 \in S$, a proportion p_{i0} leaves the system in category i, $0 \notin S$, or a proportion of recruits, r_i, enter category i. The problem of attaining a desired structure $\mathbf{x}^*$ with the structure $\mathbf{x}$ as the starting point involves finding either the transition matrix $\mathbf{P} = (p_{ij})_{i,j \in S}$ or the recruitment distribution $\mathbf{r}$, such that a structure $\tilde{\mathbf{x}}$ is attained which is

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as close as possible to $\mathbf{x}^*$ [4]. When the task is to find $\mathbf{r}$, the strategy is called recruitment control; otherwise, it is called promotion control. In either case, one of the policy matrices is given. Works on recruitment control are popular in the literature, whereas little has been said about promotion control [4, 5, 9]. Nilakantan [21] considered a manpower system which operates under certain recruitment restrictions as a means to protect the career prospects of its members. Later on, Nilakantan [22] evaluated the manpower system which seeks to control the blend of internal and outsource manpower. Udom [29] studied manpower control for a hierarchical system based on promotion and interdepartmental transfers. The study assumed that several policy matrices are available and that the task of the manpower planner was to determine the policy matrix that will minimise the total cost. The present study relaxes this assumption and seeks to determine the policy matrix that maximises the throughput per staff.

This study was originally motivated by an examination of the model for stationarity with promotion control [4]. The model was constructed on the assumption that recruitment is done to replace wastage and to achieve the desired growth at a rate of expansion, α. The model for the attainability problem under promotion control is expressed as

$$\mathbf{q}(t)\mathbf{P} = (1+\alpha)\mathbf{D} - \mathbf{q}(t)(\mathbf{w}' + \mathbf{e}'\alpha)\mathbf{r} \tag{1}$$

where $\mathbf{q}(t)$ is the relative structure of the system at time t, $\mathbf{w}$ is a $1\times k$ vector of wastage probabilities, and $\mathbf{r}$ is a $1\times k$ vector of the distribution of recruits. The prime is used to denote a matrix-vector transposition. The distribution of the attainable structure for the system is given by the $1\times k$ vector, $\mathbf{D} = (d_i)$. The vector $\mathbf{D}$ is such that $\mathbf{De} = 1$, where $\mathbf{e}$ is a $k\times 1$ vector of ones. As at the time of writing this paper, the only promotion control model in discrete-time known to the author that is similar to this study design is that of Bartholomew *et al.* [4].

Mathematical models for manpower planning abound in the literature [2, 3, 4, 11, 12, 14, 24, 30, 33]. Among these models is the Markov chain model, which is commonly used for systems with countable state space [10, 13, 17, 18, 27, 28, 31, 32]. The Markov chain model provides a means of unifying the states of the manpower system. More so, the Markov models have been used as a theoretical framework to assess the underlying nature of the flows of students through educational systems [8, 19, 20].

This study complements the existing literature on the mathematics of manpower planning by focusing on the aspect of promotion control. The study attempts to answer the following questions: ‘Given that the administrative authority of a manpower system is considering recruiting a certain number of staff such that the staff structure closely follows a certain attainability requirement, what then should be the promotion rates in the system in order to achieve the requirement? Will this lead to retrenchment? If it does, how many staff should be retrenched?’

The study assumes that transitions in the manpower system take place at discrete times [4, 16] and that the objective is to maximise the throughput per staff [23]. Another assumption is that the recruitment is carried out gradually so much so that the new recruits in the system at any given period do not exceed the existing staff strength. For convenience, the recruitment is assumed to take place at the end of the period according to the distribution $\mathbf{r} = [r_1, \cdots, r_k]$. The attainability problem of the manpower system is formulated as a linear programming problem [15] within the Markov chain framework. The basic idea of this study is to find the one-step transition matrix $\mathbf{P} = (p_{ij})_{i,j \in S}$, which maximises the throughput without violating the admissible conditions on the p_{ij}

$$\text{(i.e., } 0 \le p_{ij} \le 1 \text{ and } \sum_{j=1}^{k} p_{ij} \le 1\text{)},$$

given the number of new recruits and the attainable structure. The task of finding the one-step transition matrix is a non-trivial problem. This is because of the possibility of attrition of staff in the system. Davis *et al.* [6] had earlier provided a way to obtain estimates of the one-step transition probabilities from irregularly spaced data based on partial odds. This study adopts the linear programming (LP) approach so as to incorporate the objective function of the system, as well as the constraints. Dantzig's rule is used as the pivoting rule [26]. The structure derived from the solution for the constrained optimization problem is the optimal structure. The results obtained from the model proposed in this study are compared with that obtained using the model of Bartholomew *et al.* [4]. The optimal structure obtained by our model is denoted as $\mathbf{x}^{opt}$. Generally, we use the superscript *opt* to denote the optimal value and the superscript *Barth* to denote value obtained using the model of Bartholomew *et al.* [4].
This study highlights: (1) how to find a one-step optimal transition matrix under promotion control for a manpower system; (2) how to determine the specific category in a manpower system where retrenchment should be done whenever it is carried out; (3) the use of throughput as a major goal in the manpower system, rather than utilising the profit-seeking motive and the cost reduction approach [1, 23, 29]; (4) a normative approach based on linear programming as a solution technique to the attainability problem in manpower systems; and (5) the decision making under uncertainty with the aid of the Markov chain framework.

2. Model description

The evolution of structures in a manpower system is commonly modelled using the Markov chain framework as [4, 9]:

$$\mathbf{x}(t+1) = \mathbf{x}(t)\mathbf{P} + h(t+1)\mathbf{r}, \quad t = 0,1,2,\cdots \tag{2}$$

where $h(t+1)$ is the number of new recruits, which, oftentimes, is decided by the administrative authority of the system. In the absence of attrition such as retrenchment, voluntary withdrawal, dismissal, retirement, etc., the desired structure is achieved when

$$\mathbf{x}(t+1) = \left(\mathbf{x}(t)\mathbf{e} + h(t+1)\right)\mathbf{D} \tag{3}$$

Since wastage occurs in practice, we have

$$\mathbf{x}(t+1) \leq \left(\mathbf{x}(t)\mathbf{e} + h(t+1)\right)\mathbf{D} \tag{4}$$

Therefore

$$\mathbf{x}(t)\mathbf{P} \leq \mathbf{b} \tag{5}$$

where $\mathbf{b} = (\mathbf{x}(t)\mathbf{e} + h(t+1))\mathbf{D} - h(t+1)\mathbf{r}$. This study assumes that the right hand side (RHS) values of equation (5), i.e., the entries in vector $\mathbf{b}$, are non-negative. This is because a negative entry would imply that $\mathbf{x}(t)\mathbf{e} < h(t+1)$ for some $d_i < r_i$. Thus, if any of the RHS values is negative, then it is set equal to zero.

The structure of the transition matrix $\mathbf{P}$ is determined from the promotion policy of the manpower system. For instance, in a $3-$graded manpower system where the policy is that both demotion and double promotion are not allowed, the transition matrix is

$$\mathbf{P} = \begin{bmatrix} p_{11} & p_{12} & 0 \\ 0 & p_{22} & p_{23} \\ 0 & 0 & p_{33} \end{bmatrix} \tag{6}$$

The admissible conditions for the matrix $\mathbf{P} = (p_{ij})_{i,j \in S}$ are that $\mathbf{P}$ is sub-stochastic and that $p_{ij} \geq 0$ for $i, j \in S$. Considering equation (1) for the $3-$graded manpower system, we have

$$\begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix} \begin{bmatrix} p_{11} & p_{12} & 0 \\ 0 & p_{22} & p_{23} \\ 0 & 0 & p_{33} \end{bmatrix} = \begin{bmatrix} f_1 & f_2 & f_3 \end{bmatrix} \tag{7}$$

where q_i is the i th entry in the vector $\mathbf{q}(t)$ and f_i is the i th entry in the vector obtained by simplifying the expression $(1+\alpha)\mathbf{D}-\mathbf{q}(t)(\mathbf{w}'+\mathbf{e}'\alpha)\mathbf{r}$. Since the matrix $\mathbf{P}$ is being sought for under promotion control, we equate both sides of equation (7) and get

$$p_{11} = \frac{f_1}{q_1} \tag{8}$$

$$q_1 p_{12} + q_2 p_{22} = f_2 \tag{9}$$

$$q_2 p_{23} + q_3 p_{33} = f_3 \tag{10}$$

There are infinitely many solutions to the simultaneous equations (9) and (10). To ensure that the solutions does not violate the stochastic property that

$$\sum_{j=1}^{3} p_{ij} + p_{i0} = 1 \tag{11}$$

with p_{i0} being the i th entry in $\mathbf{w}$, equation (11) is introduced as a constraint. It turns out that the use of this constraint gives

$$p_{12} = 1 - \frac{1}{q_1}(f_1 + q_1 p_{10}) \tag{12}$$

$$p_{22} = \frac{1}{q_2}(f_1 + f_2 + q_1 p_{10} - q_1) \tag{13}$$

$$p_{23} = 1 - \frac{1}{q_2}(f_1 + f_2 + q_1 p_{10} + q_2 p_{20} - q_1) \tag{14}$$

and

$$p_{33} = \frac{1}{q_3}(f_1 + f_2 + f_3 + q_1 p_{10} + q_2 p_{20} - q_1 - q_2) \tag{15}$$

However, there is no guarantee that p_{33} in equation (15) will be equal to $1 - p_{30}$. Thus p_{33} may not be uniquely determined. Another limitation of the model [4] is that it may give a non-admissible solution for $\mathbf{P}$. This is the case whenever

$\mathbf{q}(t)\mathbf{P} < (1+\alpha)\mathbf{D} - \mathbf{q}(t)(\mathbf{w}' + \mathbf{e}'\alpha)\mathbf{r}$ for at least one element in the matrix-vector product. These limitations became a driving force to consider an alternative formulation for the attainability problem under promotion control. More specifically, the problem is formulated as a linear programming problem within the Markov chain framework with a view to circumventing the possibility of obtaining a non-admissible solution and uniquely determining the promotion rates. This new formulation is a more advanced model than the one in the literature [4].

Let $\Pi = (\pi_i)$ be a $k \times 1$ vector of the throughput per employee and z the total throughput. Then, the objective function for the manpower system is specified as: $\max \ z = \mathbf{x}(t)\mathbf{P}\Pi$. Notice that the new recruits $h(t+1)$ are not included. This is because new recruits are treated as if they all came into the system at the end of the period. Thus, the attainability problem is formulated as a linear programming problem (LPP) of the form

LPP 1:

$$\max \ z = \mathbf{x}(t)\mathbf{P}\Pi$$

subject to

$$\mathbf{x}(t)\mathbf{P} \le \mathbf{b}$$
$$\mathbf{P}\mathbf{e} \le \mathbf{e}$$
$$p_{ij} \ge 0 \ \text{ for } \ i, j \in S .$$

Rearranging **LPP 1** gives

LPP 2:

$$\max \ z = \Phi\mathbf{Y}$$

subject to

$$\mathbf{A}\mathbf{Y} \le \Delta$$

where Φ is a row vector containing the product $x_i(t)\pi_j$; $\mathbf{A}$ is a matrix of the functional constraints with entries $0, 1$ and $x_i(t)$; $\mathbf{Y}$ is a column vector of the p_{ij}; and Δ is a column vector made up of the transpose of vector $\mathbf{b}$ and 1. Returning to the policy matrix of the $3-$graded system in equation (6), we have

$$\Phi = \left[x_1(t)\pi_1 \quad x_1(t)\pi_2 \quad x_2(t)\pi_2 \quad x_2(t)\pi_3 \quad x_3(t)\pi_3\right],$$

$$\mathbf{A}=\begin{bmatrix} x_1(t) & 0 & 0 & 0 & 0 \\ 0 & x_1(t) & x_2(t) & 0 & 0 \\ 0 & 0 & 0 & x_2(t) & x_3(t) \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix},\ \mathbf{Y}=\begin{bmatrix} p_{11} \\ p_{12} \\ p_{22} \\ p_{23} \\ p_{33} \end{bmatrix} \text{ and } \Delta=\begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ 1 \\ 1 \\ 1 \end{bmatrix}.$$

The solution technique to **LPP 2** involves introducing a $2k \times 1$ column vector of slack variables, $\mathbf{Y}_s$, so that the problem becomes

$$z - \Phi\mathbf{Y} = 0$$

$$[\mathbf{A}, \mathbf{I}]\begin{bmatrix} \mathbf{Y} \\ \mathbf{Y}_s \end{bmatrix} = \Delta$$

or, equivalently,

$$\begin{bmatrix} 1 & -\Phi & \mathbf{0}' \\ \mathbf{0} & \mathbf{A} & \mathbf{I} \end{bmatrix}\begin{bmatrix} z \\ \mathbf{Y} \\ \mathbf{Y}_s \end{bmatrix} = \begin{bmatrix} 0 \\ \Delta \end{bmatrix} \tag{16}$$

where $\mathbf{0}$ is a $2k \times 1$ null vector and $\mathbf{I}$ is a $2k \times 2k$ identity matrix. The entries in $\mathbf{Y}_s$ are the initial basic variables with the inverse basis matrix $\mathbf{B}_{(0)}^{-1} = \mathbf{I}$. Thereafter, the inverse basis matrix $\mathbf{B}_{(v)}^{-1}$ for the vth iteration as well as the simplex multiplier $\Phi_{\mathbf{B}_v}\mathbf{B}_{(v)}^{-1}$ is determined using the Dantzig's rule [15, 26]. $\Phi_{\mathbf{B}_v}$ is the coefficient of the initial non-basic variables of the objective function at the vth iteration. The new set of equations at any iteration is obtained by evaluating

$$\begin{bmatrix} 1 & \Phi_{\mathbf{B}_v}\mathbf{B}_{(v)}^{-1} \\ \mathbf{0} & \mathbf{B}_{(v)}^{-1} \end{bmatrix}\begin{bmatrix} 1 & -\Phi & \mathbf{0}' \\ \mathbf{0} & \mathbf{A} & \mathbf{I} \end{bmatrix}\begin{bmatrix} z \\ \mathbf{Y} \\ \mathbf{Y}_s \end{bmatrix} = \begin{bmatrix} 1 & \Phi_{\mathbf{B}_v}\mathbf{B}_{(v)}^{-1} \\ \mathbf{0} & \mathbf{B}_{(v)}^{-1} \end{bmatrix}\begin{bmatrix} 0 \\ \Delta \end{bmatrix} \tag{17}$$

The iteration continues until the difference $\Phi_{\mathbf{B}_v}\mathbf{B}_{(v)}^{-1}\mathbf{A} - \Phi$ contains only non-negative entries. This is the stopping rule. The entries in $\mathbf{Y}$ at which the stopping rule is attained are referred to as the optimal solution. Let $\mathbf{Y}^{opt}$ denote the optimal solution. The vector $\mathbf{Y}^{opt}$ is used to construct the optimal transition matrix. For instance, in the $3-$graded system, $y_1^{opt} = p_{11}^{opt}$, $y_2^{opt} = p_{12}^{opt}$, $y_3^{opt} = p_{22}^{opt}$, $y_4^{opt} = p_{23}^{opt}$ and $y_5^{opt} = p_{33}^{opt}$. Let $\mathbf{P}^{opt}$ be the optimal transition matrix formed from $\mathbf{Y}^{opt}$. Then the optimal structure at $t+1$ is

$$\mathbf{x}^{opt}(t+1) = \mathbf{x}(t)\mathbf{P}^{opt} + h(t+1)\mathbf{r} \tag{18}$$

When $p_{ii}^{opt} = 1$, the optimal policy is that the staff in category i should be stagnated. On the other hand, when $0 \leq p_{ii}^{opt} < 1$, the optimal policy recommends that a proportion of $(1 - p_{ii}^{opt})$ staff should leave category i. The extent to which $\mathbf{x}^{opt}(t+1)$ closely follows the desired distribution is determined using the Euclidean norm:

$$E = \left|\mathbf{x}^{opt}(t+1)(\mathbf{x}^{opt}(t+1)\mathbf{e})^{-1} - \mathbf{D}\right| \tag{19}$$

Finally, the question as to whether implementation of the optimal policy on the manpower system leads to retrenchment is addressed. This is done by introducing a vector $\mathbf{R}(t+1)$ as the refinement vector. The refinement vector is defined as:

$$\mathbf{R}(t+1) = \left[\sum_{j=1}^{k} p_{1j}^{opt} x_1(t) \quad \sum_{j=1}^{k} p_{2j}^{opt} x_2(t) \quad \cdots \quad \sum_{j=1}^{k} p_{kj}^{opt} x_k(t)\right] - \mathbf{x}(t) \tag{20}$$

The entries in $\mathbf{R}(t+1)$ are either zero or negative. This is because

$$\sum_{j=1}^{k} p_{ij}^{opt} \leq 1.$$

A negative entry in $\mathbf{R}(t+1)$ indicates retrenchment in the category corresponding to the position of the entry, while a zero entry implies no retrenchment. The magnitude of the entry in $\mathbf{R}(t+1)$ gives the number of staff to be retrenched. The change in the total manpower stock at the one-step period $t+1$ as a result of implementing the optimal policy is

$$\delta\mathbf{x}(t+1) = \text{total inflow of new recruits} - \text{total outflow due to retrenchment}$$

where $\delta\mathbf{x}(t+1)$ denotes the change in the total manpower stock at the one-step period $t+1$. Therefore

$$\delta\mathbf{x}(t+1) = h(t+1) + \mathbf{R}(t+1)\mathbf{e} \tag{21}$$

When $\delta\mathbf{x}(t+1) > 0$, the optimum policy leads to an expansion in the total manpower stock and $\delta\mathbf{x}(t+1) < 0$ indicates that the optimal policy creates a contraction in the total stock. The total manpower stock is unchanged when $\delta\mathbf{x}(t+1) = 0$.

3. Application

The utility of the model in this paper is illustrated using data on a university-faculty setting in Nigeria. The university system in Nigeria is regulated by the National Universities Commission (NUC). The commission provides guidelines for program evaluation in the university system. Among the guidelines is the academic staff-mix by rank, which states that the existing staff structure for academic staff should closely follow the structure 20:35:45 for Professors/Readers: Senior Lecturers: Lecturer I and below (excluding the position of Graduate Assistant), respectively [25]. The study utilises data from the Faculty of Physical Sciences at the University of Benin, Nigeria [7]. The Faculty of Physical Sciences consists of five departments. We denote the departments as D1, D2, D3, D4, D5. The structure, wastage and the recruitment distribution over time for each department in the faculty as well as the initial Euclidean norms are given as follows:

D1:

$$\mathbf{x}(t) = \begin{bmatrix} 9 & 4 & 12 \end{bmatrix},\ \mathbf{w}(t) = \begin{bmatrix} \frac{1}{43} & \frac{1}{41} & \frac{5}{57} \end{bmatrix},\ \mathbf{r}(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},\ E_0 = 0.1226.$$

D2:

$$\mathbf{x}(t) = \begin{bmatrix} 16 & 9 & 2 \end{bmatrix},\ \mathbf{w}(t) = \begin{bmatrix} \frac{2}{109} & 0 & 0 \end{bmatrix},\ \mathbf{r}(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},\ E_0 = 0.0365.$$

D3:

$$\mathbf{x}(t) = \begin{bmatrix} 4 & 4 & 5 \end{bmatrix},\ \mathbf{w}(t) = \begin{bmatrix} 0 & \frac{1}{25} & \frac{2}{26} \end{bmatrix},\ \mathbf{r}(t) = \begin{bmatrix} 7/8 & 1/8 & 0 \end{bmatrix},$$
$$E_0 = 0.0561.$$

D4:

$$\mathbf{x}(t) = [14 \quad 6 \quad 12],\ \mathbf{w}(t) = \begin{bmatrix} \frac{1}{83} & 0 & \frac{1}{34} \end{bmatrix},\ \mathbf{r}(t) = [1 \quad 0 \quad 0],\ E_0 = 0.0572 .$$

D5:

$$\mathbf{x}(t) = [8 \quad 4 \quad 3],\ \mathbf{w}(t) = \begin{bmatrix} \frac{2}{41} & \frac{1}{26} & \frac{3}{13} \end{bmatrix},\ \mathbf{r}(t) = [8/9 \quad 1/9 \quad 0],$$
$$E_0 = 0.0139 .$$

The university's Annual Performance Evaluation Report (APER) form stipulates a minimum of 17 points and 20 points from publications for consideration to the rank of Senior Lecturer and Associate Professor, respectively. Apart from the points assigned to publications, there are other tasks required from the academic staff without incurring any points. Such tasks require a subjective allocation of points. Examples of such tasks are teaching and supervision of projects, external examination, administrative duties, etc. Arbitrary points are assigned to these tasks. The throughput per staff for each category is quantified by assigning some points to the task performed in each category as follows:

- Lecturer I and below (excluding the position of Graduate Assistant) – To be considered into this category, a minimum of a masters' degree is required. Apart from research, staff members in this category are to be engaged in teaching and supervision of projects at the undergraduate level. Throughput per staff in this category is allocated 5 points.
- Senior Lecturer – The throughput per staff in this category is allocated 28 points. This comprises a minimum of 17 points from publications, as well as teaching and supervision of projects at both the undergraduate and postgraduate levels (11 points).
- Professors/Associate Professor – The throughput per staff in this category is allocated 38 points. This comprises a minimum of 20 points from publications to be eligible for promotion to the rank of Associate Professor, the teaching and supervision of projects at both the undergraduate and postgraduate levels (11 points), external examination (2 points) and administrative duties (5 points).

The model proposed in this study and the model in the literature [4] are both illustrated for the case where the university management seeks to recruit 8 new academic staff into each department. At first, the optimal policy is determined using the proposed model (**LPP 2**) in this study.

Using the aforementioned throughput points and data, the **LPP 2** model is implemented in the MATLAB environment. The implementation involves introducing six slack variables to augment the LPP in such fashion that the initial inverse basis matrix is

$$\mathbf{B}_{(0)}^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The results for D1 are presented in detail and comments are provided for the others. For the instances where integer values are required, the results are rounded to the nearest whole number. For D1, the coefficient vector of the objective function is

$$\Phi = \begin{bmatrix} 45 & 252 & 112 & 152 & 456 \end{bmatrix}$$

and the coefficient matrix of the functional constraints is

$$\mathbf{A} = \begin{bmatrix} 9 & 0 & 0 & 0 & 0 \\ 0 & 9 & 4 & 0 & 0 \\ 0 & 0 & 0 & 4 & 12 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The right hand side vector is

$$\Delta = \begin{bmatrix} 6 & 11 & 6 & 1 & 1 & 1 \end{bmatrix}'.$$

Using these matrices and vectors as inputs in the MATLAB environment, we get the following optimal solutions:

$$\mathbf{B}_{(6)}^{-1} = \begin{bmatrix} 0 & -0.1111 & 0 & 1.0000 & 0.4444 & 0 \\ 1.0000 & 1.0000 & 0 & -9.0000 & -4.0000 & 0 \\ 0 & 0 & 0.0833 & 0 & 0 & 0 \\ 0 & 0.1111 & 0 & 0 & -0.4444 & 0 \\ 0 & 0 & 0 & 0 & 1.0000 & 0 \\ 0 & 0 & -0.0833 & 0 & 0 & 1.0000 \end{bmatrix},$$

$$\mathbf{P}^{opt} = \begin{bmatrix} 0.2222 & 0.7778 & 0 \\ 0 & 1.0000 & 0 \\ 0 & 0 & 0.5000 \end{bmatrix},$$

$$z1^{opt} = 546\,,\ \mathbf{x}^{opt}(t) = \begin{bmatrix} 10 & 11 & 6 \end{bmatrix},\ \mathbf{R}(t+1) = \begin{bmatrix} 0 & 0 & -6 \end{bmatrix},$$

and the Euclidean norm is 0.0101. The optimal policy is to retrench 6 staff in category 3 and that no one should be promoted in category 2, whereas about 78% of the staff in category 1 should be promoted to category 2.
For D2, we obtain

$$\mathbf{P}^{opt} = \begin{bmatrix} 0.4375 & 0.5625 & 0 \\ 0 & 0.3333 & 0.6667 \\ 0 & 0 & 0.5000 \end{bmatrix},$$

$$z2^{opt} = 637\,,\ \mathbf{x}^{opt}(t) = \begin{bmatrix} 15 & 12 & 7 \end{bmatrix},\ \mathbf{R}(t+1) = \begin{bmatrix} 0 & 0 & -1 \end{bmatrix},$$

and the Euclidean norm is 1.2111×10^{-4}. The results show that the optimal policy is to retrench 1 staff in category 3, while 56% and 67% of the staff in categories 1 and 2 should be promoted, respectively.
For D3,

$$\mathbf{P}^{opt} = \begin{bmatrix} 0.5000 & 0.5000 & 0 \\ 0 & 1.0000 & 0 \\ 0 & 0 & 0.8000 \end{bmatrix},$$

$$z3^{opt} = 330\,,\ \mathbf{x}^{opt}(t) = \begin{bmatrix} 9 & 7 & 4 \end{bmatrix},\ \mathbf{R}(t+1) = \begin{bmatrix} 0 & 0 & -1 \end{bmatrix},$$

and the Euclidean norm is 0. Here the optimal staff-mix is exactly as specified by the NUC. The optimal policy is that 1 staff should be retrenched from category 3, no staff should be promoted in category 2 and 50% of the staff in category 1 should be promoted.
For D4,

$$\mathbf{P}^{opt} = \begin{bmatrix} 0.4286 & 0.5714 & 0 \\ 0 & 1.0000 & 0 \\ 0 & 0 & 0.6667 \end{bmatrix},$$

$$z4^{opt} = 726,\ \mathbf{x}^{opt}(t) = \begin{bmatrix} 14 & 14 & 8 \end{bmatrix},\ \mathbf{R}(t+1) = \begin{bmatrix} 0 & 0 & -4 \end{bmatrix},$$

and the Euclidean norm is 0.0057. For this department, 4 staff in category 3 should be retrenched. Even though no staff should be promoted in category 2, the results reveal that 57% of the staff in category 1 should be promoted.
For D5,

$$\mathbf{P}^{opt} = \begin{bmatrix} 0.3750 & 0.6250 & 0 \\ 0 & 0.5000 & 0.5000 \\ 0 & 0 & 0.6667 \end{bmatrix},$$

$$z5^{opt} = 363,\ \mathbf{x}^{opt}(t) = \begin{bmatrix} 10 & 8 & 4 \end{bmatrix},\ \mathbf{R}(t+1) = \begin{bmatrix} 0 & 0 & -1 \end{bmatrix},$$

and the Euclidean norm is 5.3719×10^{-4}. The optimal policy is to retrench 1 staff in category 3 and that 63% and 50% of the staff in categories 1 and 2 should be promoted, respectively.
Next the attainability problem is solved for the university-faculty using the model of Bartholomew *et al.* [4] and then the obtained results are compared with that obtained by using the proposed model in this paper. Using equations (8), (12) – (15), the matrix $\mathbf{P}$ is computed and thereafter the discrepancy between the desired structure and the structure obtained by the model of Bartholomew *et al.* [4] is measured. The following results are obtained.
For D1,

$$\mathbf{P}^{Barth} = \begin{bmatrix} 0.6101 & 0.3667 & 0 \\ 0 & 0.3300 & 0.6456 \\ 0 & 0 & 0.3348 \end{bmatrix},\ E = 0.0338.$$

For D2,

$$\mathbf{P}^{Barth} = \begin{bmatrix} 0.4660 & 0.5156 & 0 \\ 0 & 0.1481 & 0.8519 \\ 0 & 0 & -0.3333 \end{bmatrix}, \ E = 0.0021.$$

For D3,

$$\mathbf{P}^{Barth} = \begin{bmatrix} 0.4934 & 0.5066 & 0 \\ 0 & 0.3273 & 0.6327 \\ 0 & 0 & 0.3339 \end{bmatrix}, \ E = 0.0207.$$

For D4,

$$\mathbf{P}^{Barth} = \begin{bmatrix} 0.6770 & 0.3109 & 0 \\ 0 & 0.3015 & 0.6985 \\ 0 & 0 & 0.3174 \end{bmatrix}, \ E = 0.0375.$$

For D5,

$$\mathbf{P}^{Barth} = \begin{bmatrix} 0.2675 & 0.6837 & 0 \\ 0 & 0.1036 & 0.8579 \\ 0 & 0 & 0.3894 \end{bmatrix}, \ E = 0.0022.$$

Comparing these results and that obtained using the proposed model, it is easy to see that the model proposed in this study is better than that of Bartholomew *et al.* [4] owing to the fact that the model in this paper gives admissible solutions (unlike that of Bartholomew *et al.* wherein p_{33} of D2 is negative) and the proposed model produces smaller discrepancies according to the Euclidean norm. Furthermore, this paper ascertains whether the optimal admissible solution obtained by the proposed model is contained in the infinitely many solutions of the model of Bartholomew *et al.* [4]. This is achieved by working out the product $\mathbf{q}(t)\mathbf{P}^{opt}$ and simplifying the expression $(1+\alpha)\mathbf{D} - \mathbf{q}(t)(\mathbf{w}' + \mathbf{e}'\alpha)\mathbf{r}$. It is found that

$$\mathbf{q}(t)\mathbf{P}^{opt} \neq (1+\alpha)\mathbf{D} - \mathbf{q}(t)(\mathbf{w}' + \mathbf{e}'\alpha)\mathbf{r}$$

This result leads to the conclusion that an optimal solution may not be guaranteed by the model in the literature [4]. In all, the implementation of the optimal policy leads to an expansion in the manpower stock. Comparing the norms before and after the use of the optimal promotion control strategy, it is found that there is a reduction in the Euclidean norms. Therefore, the promotion strategy proposed in this paper is capable of generating a structure which is close to the desired staff-mix. The proposed model therefore is a significant contribution to the manpower planning literature on attainable structures under promotion control.

4. Concluding remarks

This paper presents a linear programming model for determining the one-step promotion rates that tends to equalise the attainability benchmark for a manpower system. The model characterises the evolution of structures in the system by a Markov chain. The closest rivalry to the present study is the model for promotion control proposed by Bartholomew *et al.* [4]. However, the findings in this study reveal that the model proposed in this paper is better than that of Bartholomew *et al.* [4] in the sense that: the new model gives smaller discrepancies according to the Euclidean norm, it is optimal given that it maximises the throughput, and it yields admissible solution. Our model is based on the aggregation of staff in each category of the system. Nonetheless, there are some practical issues that may limit the use of the model proposed here. These issues include, but not limited to, the psychological effect of being denied promotion, the failure of staff to meet the promotion criteria, and the yardstick to discriminate among staff to be stepped down for promotion. More so, the application of our model shows that, even though no recruitment is allowed into category 3, at least one staff in the rank of either an Associate Professor or a Professor should be retrenched. This seems reasonable as it will give room to those in the lower ranks to progress to category 3. However, the study does not consider the subjective cost of laying-off the staff in the professorial cadre as well as the burden of severance pay to the system. Furthermore, the study utilises a simplifying assumption of equal throughput per staff per category as the performance rating per individual was not considered. In reality, some staff may be exceptional in the performance of their duties. Though these are useful areas for future research, the model proposed in this study should therefore be applied with caution.

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