Discrete self–organizing migration algorithm and p–location problems

Jaroslav Janáček[1,*] **and** **Marek Kvet**[1]

[1] *Faculty of Management Science and Informatics, University of Žilina*
Univerzitná 8215/1, 010 26 Žilina, Slovakia
E-mail: ⟨{*jaroslav.janacek, marek.kvet*}*@fri.uniza.sk*⟩

Abstract. Mathematical modelling, and integer programming generally, has many practical applications in different areas of human life. Effective and fast solving approaches for various optimization problems play an important role in the decision-making process and therefore, big attention is paid to the development of many exact and approximate algorithms. This paper deals only with a special class of location problems in which given number of facilities are to be chosen to minimize the objective function value. Since the exact methods are not suitable for their unpredictable computational time or memory demands, we focus here on possible usage of a special type of a particle swarm optimization algorithm transformed by discretization and meme usage into so-called discrete self-organizing migrating algorithm. In the paper, there is confirmed that it is possible to suggest a sophisticated heuristic for zero-one programming problem, which can produce near-to-optimal solution in much smaller time than the time demanded by exact methods. We introduce a special adaptation of the discrete self-organizing migration algorithm to the p-location problem making use of the path-relinking method. In the theoretical part of this paper, we introduce several strategies of the migration process. To verify their features and effectiveness, a computational study with real-sized benchmarks was performed. The main goal of the experiments was to find the most efficient version of the suggested solving tool.

Keywords: discrete self–organizing migration algorithm, large location problems, meme usage, path–relinking method

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1. Introduction

Many kinds of public service systems are usually designed to satisfy their users by providing necessary service in case of emergency. Since the resources, from which the service can be provided, are limited mostly due to economic or technological restrictions, the mathematical model of the problem may often take the form of a weighted p-median problem [3, 7, 18, 21] or any similar one. To make the mathematical model applicable in wider spectrum of research problems, the concept of so-called generalized disutility has been introduced [8, 13, 15, 19, 26]. This model extension enables to cover more demands of system users, which can arise almost simultaneously, and furthermore, the nearest located service center does not need to have sufficient capacity to solve all the assigned demands at the same time. Of course, mentioned model generalization brings some difficulties into the solving process of the problem.

The necessity to solve large problem instances has led to the development of several effective solving techniques. The first group of approaches contains the exact methods based on the branch and bound principle. Many of them use a specific model reformulation, in which so-called radial approach is applied [1, 3, 9, 10, 24]. One of the biggest disadvantages of the

[*] Corresponding author.

exact methods consists in their almost unpredictable demands, which usually make the solving process extremely time consuming. Simultaneously, many different approximate methods have been developed. Currently, the main attention is paid to various metaheuristic approaches, i.e. genetic algorithms, scatter search, path-relinking method and many others [11, 12, 14, 23], the aim of which can be specified as a task of obtaining a good solution in acceptably short time.

In this paper, we focus on study of possible usage of a special type of a particle swarm optimization algorithm [2] transformed by discretization and meme usage into so-called discrete self-organizing migrating algorithm [5, 22, 30].

The principle of the self-organizing migrating algorithm has proved to be very hopeful tool for solving real life problems from the field of engineering [6, 25]. The discrete version of the self-organizing migrating algorithm has been successful in solving complicated problems described as zero-one programming problems in the field of scheduling and routing [4]. The successful results in solving real complex discrete problems indicate that the principle can be also used to solve other scheduling problems referenced in [27, 28, 29]. Nevertheless, it must be taken into account that a successful application of any metaheuristic must be paid by very thoroughly performed implementation of the solving algorithm making use of individual characteristics of the problem.

In this paper, we deal with the hypotheses that stochastic characteristic of service of randomly emerging demands including impact of limited capacity of service centers can be described using generalized disutility model and, furthermore, that it is possible to suggest a sophisticated heuristic for a specific problem, which can produce near-to-optimal solution in much smaller time than the time demanded by an exact method.

The study is aimed at application of the algorithm to efficient solving of the min-sum p-location problem. We introduce a special adaptation of the discrete self-organizing migration algorithm to the p-location problem making use of the path-relinking method. We study various strategies of the migration process and compare them to find the most efficient version of the solving tool.

2. Strategies of discrete self-organizing migrating algorithm

The standard general particle swarm optimization algorithm repeatedly updates a population (swarm) of problem solutions (particle positions) by performing a perturbation with each solution of the population taking into account the general best-found solution and also the best solutions in trajectories traversed by the individual particles. The way of perturbation depends on the range of the optimized objective function and the perturbation operations include weights of the general and local best-found solutions and some random changes. The derived discrete self-organizing migrating algorithm is also based on the idea of updating particle positions corresponding to problem solutions, nevertheless updating of an individual particle is performed by moving its current position to the best position found on the trajectory connecting the current position of the updated particle to some other particle. The choice of the other particle is specified by the used migration strategy. Based on the way of population updating, various strategies can be distinguished. We will study the following often referred ones [30].

The strategy AllToOne (ATO): Each particle of the current population is evaluated and the best one is denoted as a leader. Then the trajectories from the individual particles to the leader are inspected and the particle positions are updated with exception of the leader. Updated particle positions and the leader position represent the new population.

The strategy AllToAll (ATA): This strategy does not distinguish any special particle, e.g. the leader. Each trajectory from an updated particle to other particle locations is inspected and the best found positions enter the new population.

The above mentioned strategies have also their adaptive versions, which follow.

The strategy AllToOne-Adaptive ($ATO - A$): This strategy starts similarly to ATO by population evaluation and the leader determination. Then, trajectories from particles to the leader are subsequently inspected and the position of the leader is updated immediately after each trajectory inspection.

The strategy AllToAll-Adaptive ($ATA - A$): This strategy differs from the above-described ATA only in the start of a single trajectory inspection. When trajectories from an updated particle are inspected, the starting position corresponding to the particle is updated after each trajectory inspection. A decision on the most efficient strategy for the studied p-location problem is matter of experiments, which will be referred in the remainder of the paper.

3. Implementations of the discrete self-organizing migrating algorithm for the min-sum p-location problem

The solved p-location problem defined for m possible facility locations and p facilities to be located can be described by (1) in general.

$$min\left\{f\left(P\right), P \subset \{1, 2, \ldots, m\}, |P| = p\right\} \tag{1}$$

The generalized min-sum version of the p-location problem is determined by the objective function f specification (2).

$$f\left(P\right) = \sum_{j=1}^{n} b_j \sum_{k=1}^{r} q_k min_k \left\{d_{ij} : i \in P\right\} \tag{2}$$

The objective function value $f\left(P\right)$ for given subset P of facility locations is computed for n users with weights b_j for $j = 1, \ldots, n$ and for probability values q_k for $k = 1, \ldots, r$, where q_k is probability that the k-th nearest facility is the closest available one. The symbol d_{ij} stays for the distance between locations i and j and operation $min_k\left\{\right\}$ denotes the k-th minimal value of the list of values in the brackets.

```
Path-relinking(P, Q)
begin
  0. Initialize $B$ by $argmin\{f(P), f(Q)\}$, determine $U = P - Q$ and $V = Q - P$.
  1. Determine facility locations $i^* \in U$ and $g^* \in V$ by
     $(i^*, g^*) = argmin\{f((P - \{i\}) \cup \{g\}) : i \in U, g \in V\}$.
  2. Update $P$, $U$, $V$, and $B$ according to
     $P = (P - \{i^*\}) \cup \{g^*\}$
     $U = U - \{i^*\}$
     $V = V - \{g^*\}$
     $B = argmin\{f(P), f(P^*)\}$
  3. If —U— = —V— = 1, then terminate and return $B$, otherwise perform
     exchange $P$ for $U$ and $Q$ for $V$ and go to 1.
end.
```

Table 1: *Algorithm Path-relinking (P, Q)*

The presented implementations of the discrete self-organizing migrating algorithms are based on the path-relinking method, implementation of which makes use of a special structure of the set of all feasible solutions of (1). The method inspects vertices of an m-dimensional hypercube lying on the shortest path in the hypercube surface between elements of a pair of input vertices corresponding with a pair of the p-location problem solutions. The path-relinking method

[16, 20] skips the hypercube vertices, which do not correspond to feasible solutions of (1), i.e. the vertices, where $|P| \neq p$. Let P and Q denote a pair of input solutions. Then, the used path-relinking method is described by the following steps.
Let $S = \left\{P^1, \dots, P^{|S|}\right\}$ denote a starting population of feasible solutions of the problem (1). Descriptions of the studied implementations associated with the individual strategies follow.

```
ATO(S, $t^{max}$)
begin
  0. Compute $f\left(P^k\right)$, for each $P^k \in S$. Determine the leader
     $P^l = argmin\left\{f\left(P^k\right) : k = 1, \dots, |S|\right\}$.
  1. Initialize $S^{new} = \left\{P^l\right\}$.
  2. For each $P^k \in S - \left\{P^l\right\}$ compute $B^k = Path - relinking\left(P^k, P^l\right)$
     and perform $S^{new} = S^{new} \cup \left\{B^k\right\}$.
  3. Determine $P^l = argmin\left\{f\left(B^k\right) : k = 1, \dots, |S|\right\}$.
  4. If $(currentTime > t^{max})$, then terminate and return $P^l$,
     otherwise set $S = S^{new}$ and go to 1.
end.
```

Table 2: *ATO (S, t^{max})*

```
ATA(S, $t^{max}$)
begin
  0. Initialize $S^{new} = \emptyset$.
  1. For each $P^k \in S$ initialize $B^k = P^k$
     and perform steps 2 and 3 and then to 4.
  2. For each $P^t \in S - \left\{P^k\right\}$ compute
     $B^k = argmin\left\{f\left(B^k\right), f\left(Path - relinking\left(P^k, P^t\right)\right)\right\}$.
  3. Update $S^{new} = S^{new} \cup \left\{B^k\right\}$.
  4. If $(currentTime > t^{max})$, then terminate and return
     $argmin\left\{f\left(B^k\right) : k = 1, \dots, |S|\right\}$,
     otherwise set $S = S^{new}$ and go to 0.
end.
```

Table 3: *ATA (S, t^{max})*

```
ATO-A(S, $t^{max}$)
begin
  0. Compute $f\left(P^k\right)$ for each $P^k \in S$. Determine the leader
     $P^l = argmin\{f(Pk) : k = 1, \dots, |S|\}$.
  1. For each $P^k \in S - \left\{P^l\right\}$ update
     $P^l = argmin\left\{f\left(P^l\right), f\left(Path - relinking\left(P^k, P^l\right)\right)\right\}$.
  2. If $(currentTime > t^{max})$, then terminate and return $P^l$,
     otherwise go to 1.
end.
```

Table 4: *ATO-A (S, t^{max})*

```
ATA-A(S, $t^{max}$)
begin
  0. Initialize $S^{new} = \emptyset$.
  1. For each $P^k \in S$ initialize $B^k = P^k$ and perform steps 2 and 3 and then to 4.
  2. For each $P^t \in S - \{P^k\}$ compute
     $B^k = argmin\left\{f\left(B^k\right), f\left(Path - relinking\left(B^k, P^t\right)\right)\right\}$.
  3. Update $S^{new} = S^{new} \cup \{B^k\}$.
  4. If ($currentTime > t^{max}$), then terminate and return
     $argmin\left\{f\left(B^k\right) : k = 1, \dots, |S|\right\}$,
     otherwise set $S = S^{new}$ and go to 0.
end.
```

Table 5: *ATA-A (S, t^{max})*

4. Numerical experiments

The main goal of the following computational study is a comparison of the presented strategies of the discrete self-organizing migrating algorithm (DSOMA) with the goal to find the most efficient implementation of the algorithm for solving the generalized weighted p-median problem. The initial study has been performed using the benchmarks, which were derived from some of the Slovak self-governing regions. The individual instances of the benchmarks are denoted by the following names: Nitra (NR), Trenčín (TN), Trnava (TT), Žilina (ZA), Banská Bystrica (BB), Košice (KE) and Prešov (PO). The sizes of the individual benchmarks are determined by integers m and p. The number m gives the number of possible center locations and p gives the number of service centers to be located. The generalized disutility objective function was computed for $r = 3$. The coefficients q_k for $k = 1, \dots, r$ have been obtained from statistics presented in [17] and their values are: $q_1 = 77.063$, $q_2 = 16.476$ and $q_3 = 100{-}q_1{-}q_2$.

The starting population for each solved instance was established as a uniformly deployed set of p-location problem solutions according to [14] and [15]. To increase robustness of the comparison, we made use of the uniformly deployed set property that each permutation of m subscripts of locations generates a different uniformly deployed set. Thus, we generated randomly ten different uniformly deployed sets for each benchmark.

The sizes m and p and uniformly deployed set parameters $|S|$ and h are reported in Table 1 together with the optimal solution values of (1) taken from [14]. The optimal solution values are reported in the column denoted by *OptSol*. The symbol $|S|$ denotes the cardinality of the set S and h is the minimal Hamming distance between each two problem solutions of the set.

Region	m	p	$\|S\|$	h	*OptSol*
NR	350	27	83	50	48940
TN	276	21	137	38	35275
TT	249	18	212	32	41338
ZA	315	29	112	52	42110
BB	515	36	172	66	44752
KE	460	32	60	60	45588
PO	664	32	232	60	56704

Table 6: *Benchmark parameters*

Experiments with the individual strategies were organized so that each implementation was applied to each uniformly deployed set for given value of t^{max} and then the results were summarized in the table associated with the time threshold t^{max}. A table for given t^{max}

	$t^{max} = 10s.$				$t^{max} = 5s.$			
	ATO		***ATO-A***		***ATO***		***ATO-A***	
Region	F^{*avg}	CT [s]	F^{*avg}	CT [s]	F^{*avg}	CT [s]	F^{*avg}	CT [s]
NR	49320	10.0	48965	10.7	50911	5.1	48965	7.4
TN	35533	10.0	35277	11.5	35533	5.0	35277	7.0
TT	41521	10.0	41386	10.8	41521	5.0	41386	6.6
ZA	42314	10.0	42121	14.4	49807	5.2	42121	5.2
BB	56510	25.0	44752	25.3	56510	26.1	44752	26.1
KE	46070	10.0	45661	10.9	55502	5.9	45678	6.0
PO	72673	33.4	56736	33.3	72673	35.2	56736	34.9

Table 8: *Results of numerical experiments with all-to-one strategies*

contains sections denoted by ***ATO***, ***ATA***, ***ATO-A***, ***ATA-A***, in which the results obtained by the associated strategies are plotted. Each section contains two columns denoted by F^{*avg} and CT [s]. The column denoted by F^{*avg} reports the average minimum of the objective function of resulting solutions obtained by the approach application to ten uniformly deployed sets. The average computational times of the studied approaches are reported in the columns denoted by CT [s]. These values are given in seconds. As concerns the threshold t^{max} and reported computational time CT, we have to notice that the termination rule acts only after the new population is completed. Taking into account the computational complexity of the population update under the individual strategies, the threshold t^{max} can be several times exceeded in all-to-all strategies. That is why computational time is reported regardless of the threshold setting. The experiments were performed for $t^{max} = 10$ seconds. The numerical experiments were run on a PC equipped with the Intel® Core™ i7 3610QM processor with the parameters: 2.3 GHz and 8 GB RAM. The algorithms were implemented in the Java language and run in the NetBeans IDE 8.2 environment.

	ATO		***ATA***		***ATO-A***		***ATA-A***	
Region	F^{*avg}	CT [s]	F^{*avg}	CT [s]	F^{*avg}	CT [s]	F^{*avg}	CT [s]
NR	49320	10.0	57556	300.1	48965	10.7	48961	149.8
TN	35533	10.0	42473	316.3	35277	11.5	35276	159.4
TT	41521	10.0	46528	456.0	41386	10.8	41386	230.6
ZA	42314	10.0	49025	562.3	42121	14.4	42120	283.6

Table 7: *Results of numerical experiments for the threshold $t^{max} = 10$ seconds*

Having compared the preliminary results reported in Table 7, we have found that the both all-to-all strategies are very inefficient concerning the type of studied problem and the way of creating the starting population. In the prescribed time $t^{max} = 10$ seconds, these strategies were not able to complete a single population update. The update takes an order of magnitude higher time than the threshold. Despite this extension, the results of ATA differ significantly from the optimal solution, while the other strategies reach almost the optimal solutions with accuracy better than one percent of the optimal value. It can be noticed that the adaptive mode considerably contributes to the resulting solution quality. As the all-to-all approach represents such a burden in a strategy performance that it demands for inacceptable computational time, we restricted the remainder of the computational study only to all-to-one strategies. We applied them to a broader set of benchmarks and, furthermore, we reduced the time threshold to estimate differences in their convergence speed. The obtained results are plotted in Table 8.

The results reported in Table 8 confirm advantage of the adaptive mode over the simple all-to one strategy. Even if the ATO strategy is able to repeat the population update more times in the given threshold compared to $ATO - A$, the $ATO - A$ results are better even for the harder instances BB and PO.

5. Conclusion

The paper deals with possible strategies of the self-organizing migrating algorithm and follows the goal to find the most suitable strategy for the generalized weighted p-median problem using a uniformly deployed set of solutions as the starting population. We have performed a series of numerical experiments and found that the most suitable strategy for the given type of problem and the special way of starting population determination is the adaptive all-to one strategy. We confirmed the hypothesis that it is possible to suggest a sophisticated heuristic for a specific problem, which can produce near-to-optimal solution in much smaller time than the time demanded by an exact method.

Further research in this area may be focused on exploitation the preliminary mapping of the feasible solution set in the range of objective function values yielded by the uniformly deployed set for accelerating the self-organizing migrating algorithm performance. The next objective of the future research will be aimed at applications of the suggested method to obtaining series of non-dominated solutions of the bi-criterial p-location problems.

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