Croatian Operational Research Review
CRORR **13**(2022), 203-215

PVAR model with collapsed instruments in the real exchange rates misalignment's analysis

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Abstract. The causes and the consequences of the real exchange rates misalignment's of European Union (EU) members were examined in this paper by implementing stationary panel vector autoregression (PVAR) model with fixed effects. PVAR methodology was recognized as the most appropriate in line with data structure and the objectives of the research. For estimation purpose, the generalized method of moments (GMM) in first differences, with a reduced number of instruments, was applied. Primarily objective was to find whether a collapsed matrix of instruments helps in reducing the dynamic panel bias within the two–step estimation of PVAR model when employing the first difference GMM estimator. Even though, the benefits of collapsed instrument matrix have been documented in rare simulation studies, this paper empirically demonstrates it's utility considering balanced panel data. In that context, recommendations to potential users are given and supported by open source codes in the RStudio environment. Besides, auxiliary findings contribute to a better understanding of influential channels through which EU policy makers should reduce a real exchange rates misalignment's.

Keywords: collapsed instruments, European Union, generalized method of moments, panel vector autoregression, real exchange rate misalignment

Received: February 8, 2022; accepted: November 3, 2022; available online: December 22, 2022

DOI: 10.17535/crorr.2022.0015

1. Introduction

Recent studies have confirmed real exchange rates misalignment's issue regarding a different productivity and growth among EU members, resulting in economic divergence instead of the preferred convergence [8, 9, 10]. It is generally accepted attitude that imbalance of the real exchange rate and it's volatility affect economic variables such as imports, exports, the balance of payments, inflation and production [12, 13, 14], which directly affect the economic performance of a particular country. Moreover, [16] pointed out that intense exchange rate fluctuations are the main cause of uncertainty in the export and import of competitive goods and services, and thus discourages investment in these sectors. A real exchange rate misalignment can also affect domestic and foreign investments through the gross capital formation [27], while in other instances, it can affect the tradable goods sector and its competitiveness vis-á-vis the trading partners. A firm's trade cannot be returned to the original level after extreme exchange rate changes and real exchange rate appreciation serves as a subsidy for imports, while [2] observe the negative effect of high and frequent volatility on the productivity growth of developing

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countries as well as countries with poorly developed financial markets. Higher real exchange rate volatility can induce increased profit uncertainty on foreign currency–denominated contracts, which causes a greater reduction in economic growth. Further, higher real exchange rate volatility often leads to higher prices of goods traded internationally causing traders to add a risk premium to these goods to cover unexpected exchange rate fluctuations.

Hence, the paper intends to analyze macroeconomic variables that cause real exchange rate misalignment's of EU members using panel vector autoregression methodology (PVAR) which is a great challenge due to the most common, but usually overlooked and neglected issue of too many instruments used for model estimation [22, 28]. By employing the PVAR methodology to a real life problem, this paper empirically demonstrates whether the collapsed matrix of instruments is more convenient in short panel with limited number of cross–sectional units N, which is a primary objective. The reason for employing PVAR at first is dealing with a historical data (observed annually) of EU members ($N = 27$) and keeping in mind that real exchange rate misalignment can be the cause and the consequence of economic variables simultaneously. The second, more relevant reason, is utilizing a decreased number of instruments in the GMM estimation to reduce the exposure to overfitting endogenous variables when cross–sectional dimension of panel data is not large even it's greater than time dimension. As far as we know from the previous studies, several authors have already found interactions between real exchange rates misalignment's and various macroeconomic variables, see e.g. [21, 27, 30], but there is no similar empirical study that considers the PVAR methodology within EU members and handles the instrument proliferation.

This paper primarily contributes to the existing studies of PVAR methodology by empirical demonstration of the benefits from collapsed matrix of instruments. It also enriches the literature on real exchange rates misalignment's, i.e. it offers a more comprehensive analysis of EU real exchange rate behavior due to impulses in other variables by employing stationary panel vector autoregression model with fixed effects, and contributes to a better understanding of the explained variance of real exchange rates misalignment's in the log–run. Finally, a less formal purpose of this research is to encourage potential users to implement open source codes in RStudio within panelvar package, and to popularize the improved PVAR GMM estimator proposed by [31].

After the introduction section, Section 2 reviews the literature about real exchange rate misalignment's with a special attention to models and variables employed in previous studies. In Section 3 the data and PVAR methodology are given. Section 4 presents empirical results along with interpretation and discussion, while Section 5 concludes.

2. Literature review

The real exchange rate misalignment is well–known as a discrepancy of the actual real exchange rate from its long–run equilibrium, see e.g. [25], but the issue of establishing an equilibrium real exchange rate remains unsolved. Conforming to [8] several approaches of the equilibrium exchange rate exist: 1) purchasing power parity (PPP), 2) fundamental equilibrium exchange rate (FEER), 3) behavioral equilibrium exchange rate (BEER), and 4) permanent equilibrium exchange rate (PEER). PPP most common criticisms are focused on real exchange rate oscillations that theory holds constant or approximately equal in the medium and longer period. FEER approach engages with price elasticities that are not easy to estimate, as well as current account targets that should be achieved, which in turn is its main drawback [25]. Due to [11], the critique of the most commonly employed BEER and PEER approaches suggests that economic fundamentals used in equilibrium exchange rate estimation do not necessarily have to be at equilibrium level, which consequently results in a poor estimation.

Various authors have been studied interactions among current account balance and real exchange rate [8, 9]. In particular [9] has applied a panel VAR analysis to 27 EU members over the period 1994–2012 and has found that the increase in a current account imbalance causes an expansion of real exchange rate misalignment and thus lowering competitiveness. However, author [9] does not report the number of instruments, which in case of $T = 19$ can not be less than $N = 27$ when applying standard GMM–style estimator to the homogeneous PVAR model [1]. The same author also applies a Bayesian PVAR to overcome cross–sectional dependence, heterogeneity of groups and the bias in the small sample, but gives no explanation of it's settings or any appropriateness of it's use. Further, authors [26, 29] have recognized public debt as a significant explanatory variable of the real exchange rate misalignment, i.e. the higher public debt of a specific country is associated with the real exchange rate depreciation. Using the Johansen cointegration approach limited to a single country [24] come to a similar conclusions about the negative impact of exchange rate depreciation which stimulates an external indebtedness in Croatia. Conventional economic theory also assumes that government spending affects the real exchange rate, e.g. [7] within structural VAR for the period 2002–2012 concluded that a positive shock to government spending in Turkey induces real exchange rate appreciation. Likewise, price levels have been recognized as an important source of exchange rate misalignment. Using a dynamic panel data (DPD) model [32] on the Middle East and North Africa countries (MENA) showed that inflation along with the institutions quality and financial development are important factors that drift the real exchange rate from its steady–state. [19] using the structural vector autoregressive model (SVAR) on the example of Egypt found that nominal shock attributed to consumer price index (CPI) leads to the real exchange rate depreciation, but only in the short–run. It is explained that higher inflation rates cause real exchange rate depreciation. Contrary, a depreciated real exchange rate stimulates inflation.

Most of the studies involving panel analysis of the real exchange rates misalignment's are concentrated on a single equation approach [21, 27], such as panel ARDL (panel autoregressive distributed lag model) or DOLS (dynamic OLS panel model), while others neglect panel structure of the data by using euro area aggregated variables, see e.g. [30]. Even though, dynamic panel models have experienced a great popularity among users, the main drawback is the bias of the coefficient with respect to the lagged endogenous variable, which cannot be easily annulated for small T and large N [23]. Some bias corrected estimators were proposed in the literature, but unbiased estimators of dynamic panels considering generalized method of moments (GMM), which are in the focus of practitioners as well as academics, still suffer from overfitting problem and instrument proliferation [18]. This means that the number of instruments increase quadratically with respect to T and GMM estimator is not consistent any more.

From the methodological point of view it is advisable to address the number of instruments when using GMM dynamic panel models, having in mind that inclusion of all available instruments can cause above mentioned problems. This is a case when the number of instruments surpasses the number of cross–sectional units [28]. Consequently, it weakens the power of the Sargan–Hansen–J test for instrument validity [17]. Therefore, indisputable indicator of too many instruments is a high p–value of the J–statistic, i.e. close or equal to 1, which misleadingly forces the researchers to never reject the null hypothesis of instruments validity. Keeping the number of instruments small when cross–sectional dimension is small should not be an exception, particularly when dealing with PVAR models. [22] suggests simply to increase the number of cross-sectional units whenever is possible, to limit the lag length of instruments, or to reduce too many instruments by principal component analysis. Factorization of the instruments can be helpful when employing a single DPD model, but not in case of a system of DPD models such as PVAR. An elegant solution is to collapse the instruments.

This paper offers several contributions to the existing literature. It empirically demonstrates that collapsed matrix of instruments helps in avoiding instrument proliferation and ensures GMM unbiasedness and consistency when PVAR model is implemented. Namely, in case when N is limited and small it clearly suggests the optimal number of time units T as well as the number of endogenous variables m, for which the collapsed number of instruments is less than the number of cross–sectional units. Moreover, in a given settings, results notably confirm that Hansen overidentification test is not weakened any more. From the practical point of view this paper provides a straightforward guidance how to get the most from the short panel data when users decide to utilize PVAR in their studies. Further, it collects economics variables as the leading causes of real exchange rates misalignment's. Previous studies on this topic have analyzed misalignment's only in one particular country or in more countries with different currencies in a given year, but this paper focuses on understanding of misalignment's in all EU countries over more years. By employing PVAR methodology with collapsed instruments, an unbiased and consistent estimates with all endogenous variables are achieved. Finally, the most effective channels for misalignment's reduction are suggested.

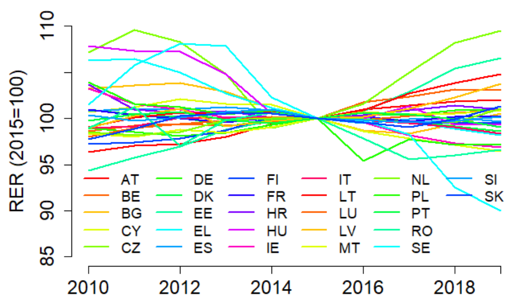
3. Data and methodology

Current study focuses on real exchange rates misalignment's determinants of EU member countries ($N = 27$) from 2010 to 2019 ($T = 10$) by using a PVAR model. Apart from the real exchange rate misalignment (MIS), two additional endogenous variables were selected as the most relevant and prevailing in the ongoing literature: current account balance (CUR) and public debt (PD). CUR variable is given as percentage of the gross domestic product (GDP) considering EU countries and rest of the world as trading partners, PD represents a general government net lending or borrowing as a percentage of GDP likewise, while RER was originally reported as index (2015=100) of real effective exchange rate towards 27 EU members. The sample of panel data was downloaded from EUROSTAT public source. A limitation to three endogenous variables was imposed due to relatively small number of observations even though N is greater than T. Otherwise, estimates will not be unbiased and consistent. For the same reason predetermined or any exogenous variables are not contemplated in this paper. Still, PVAR model includes two or more endogenous variables, which is the minimum for its use. With three endogenous variables ($m = 3$) and one time lag ($p = 1$) nine parameters should be estimated. However, the optimal lag length procedure of [3] is applied in the following section. Imposed restrictions cause PVAR usage more challenging in getting the most we can achieve from the EU members when historical data are observed annually. Moreover, annual data are not contaminated by seasonality. Because of the large number of endogenous variables m and the number of lags p, the PVAR model can be oversized, i.e. too many parameters have to be estimated, hence there is a great chance that stability condition of the model is not met. This makes even more difficult to identify valid instruments when employing GMM estimator in the first differences.

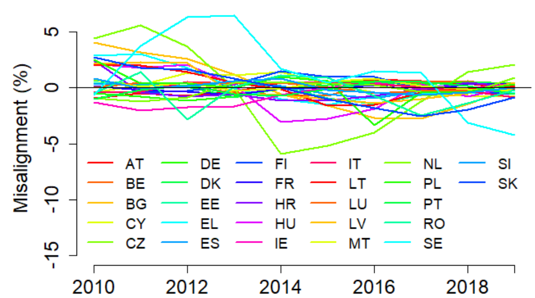
To obtain historical data of real exchange rate misalignment (MIS) for each member of the EU, an equilibrium real exchange rate (ERER) was initially calculated directly from the actual real exchange rate time–series (RER) using Hodrick–Prescott (HP) filter with adjustment parameter $\lambda = 100$. In this way short–term fluctuations are eliminated from yearly data, and EU members' real exchange rate misalignment's were computed in the next step as:

$$MIS = \frac{RER - ERER}{ERER} \times 100. \tag{1}$$

A visual inspection of the real exchange rate and it's misalignment across EU members over 10 years period is provided (Figure 1). The right panel of Figure 1 presents RER positive and negative misalignment's (MIS), that is, currency is overvalued or undervalued relative to it's long–run, while zero misalignment indicates equilibrium state. In general, RER fluctuations induced by changes in transportation costs or tariffs don't necessarily demonstrate fundamental misalignment's, but play a key role in equalizing the prices of tradables across countries. RER misalignment ranges from −5.89% to +6.47%. The minimum is observed for Czechia (country code CZ) and the maximum is observed for Sweden (country code SE). Although both countries have experienced extreme values of RER misalignment's, and therefore can be identified as outliers among other EU countries, they are not omitted from PVAR analysis due to requirement of cross–sectional dimension.



(a) *Time–series of RER indices*



(b) *Time–series of RER misalignment's*

Figure 1: *EU real exchange rates indices and misalignment's from 2010 to 2019.*

Prior to the PVAR model estimation, various panel unit root tests were conducted for 10, 9 and 8 years to examine stationarity of panel time–series over three periods of time (Table 1). First one is a complete sample period ($T = 10$), while the other two are reduced for a one year successively, keeping the most recent year fixed. Decreasing a number of years is particularly relevant for instruments reduction. Test of Im, Pesaran and Shin as well as Levin, Lin and Chu test were solely used as unformal tests, without heavy conclusion about stationarity when dealing with very short time–series. In spite of that, authors intention was to check stationarity of endogenous variables as an indicator of PVAR model stability in the post–estimation stage. Stability condition plays important role when applying first–difference GMM with lagged level of endogenous variables as instruments, that is, the same method fails if unit roots exist in short PVAR model [5].

Unit root statistic	Im, Pesaran and Shin $T = 10$	$T = 9$	$T = 8$	Levin, Lin and Chu $T = 10$	$T = 9$	$T = 8$
MIS	-5^{***}	-4^{***}	-8^{***}	-9^{***}	-6^{***}	-9^{***}
CUR	-8^{***}	-7^{***}	-8^{***}	-11^{***}	-10^{***}	-12^{***}
PD	-4^{***}	-10^{***}	-12^{***}	-6^{***}	-16^{***}	-18^{***}

*Note: statistical significance at 10%, 5% and 1% level is denoted by *, **, ****

Table 1: *Panel unit root test results for three time periods.*

Results of panel unit root tests imply the rejection of null hypothesis in favor of alternative indicating that all variables are stationary in the levels for all time periods considering 0.01 significance level. These results, although unofficial, are acceptable as they suggest that PVAR model might be stable.

Since the seminal paper of [18] who have first formalized the vector autoregressive model with panel data, it has not been extensively studied in empirical papers. A PVAR model become popular after the breakthrough of [1] who provided an pvar code for Stata. However, [31] extended the PVAR model of [18] to allow for p lags of m endogenous variables, k predetermined variables, n strictly exogenous variables and provided an panelvar package for RStudio users. The same package incorporates an Hansen overidenification test, the model selection procedure and Windmeijer corrected standard errors adopted to both extended estimators, the first difference and the system PVAR GMM estimator.

Due to a low and limited number of cross–sectional units of EU countries ($N = 27$) this paper considers only endogenous variables within stationary PVAR(m, p) with fixed effects:

$$y_{i,t} = \mu_i + \sum_{l=1}^{p} A_l y_{i,t-l} + \varepsilon_{i,t} \qquad i = 1, 2, ..., N \qquad t = 1, 2, ..., T, \tag{2}$$

where $y_{i,t}$ is m–dimensional vector of endogenous variables for the i-th cross–sectional unit at time t, terms μ_i represent time invariant fixed effects, and A_l are the $m \times m$ companion matrices of the coefficients with respect to the vector of lagged endogenous variables $y_{i,t-l}$. Subscript l denotes the time lag $l = 1, 2, ..., p$ and all unit roots of companion matrices A_l should lie inside the unit circle [31]. The disturbances $\varepsilon_{i,t}$ which are assumed to be independently and identically distributed for all i and t with zero expectation and positive semidefinite covariance matrix, should be independent from μ_i. Likewise, μ_i are supposed to be independent from lagged endogenous variables $y_{i,t-l}$, otherwise diagonal coefficients of companion matrix could be upward biased, causing the endogeneity problem. The first difference transformation is used to eliminate fixed effects:

$$\Delta y_{i,t} = \sum_{l=1}^{p} A_l \Delta y_{i,t-l} + \Delta\varepsilon_{i,t} \qquad i = 1, 2, ..., N \qquad t = p+2,\ p+3, ..., T, \tag{3}$$

but lagged endogenous variables might be potentially correlated with disturbances $\varepsilon_{i,t}$ and instrumenting is required [4, 18, 28]. The first difference GMM moment conditions for each cross–sectional unit emerge from orthogonality conditions:

$$\mathbb{E}\big[Q_i \Delta\varepsilon_i\big] = 0, \tag{4}$$

where Q_i is a block diagonal matrix of instruments stacked over $t = p+2,\ p+3, ..., T$. This means that matrix Q_i has $T - p - 1$ columns with instruments generated for each time period and available lag. Standard instruments are lagged levels of endogenous variables which are orthogonal to the disturbances of first–differenced PVAR(m, p) model in Eq. (3). Unavailable instruments cause of missing lags of endogenous variables for certain time periods are set to zero. For simplicity of exposition lets consider standard DPD model as a special case of PVAR$(1, 1)$ with single endogenous variable $m = 1$ and one time lag $p = 1$. All available instruments for periods $t = 3, 4, ..., T$ are stacked into blocks of diagonal matrix with increasing dimension of each subsequent block by one, which generates uncollapsed matrix of instruments:

$$Q_i = \begin{array}{c} \begin{array}{ccccc} t=p+2 & t=p+3 & t=p+4 & \cdots & t=T \end{array} \\ \begin{bmatrix} y_{i,t-p-1} & 0 & 0 & 0 & 0 \\ 0 & y_{i,t-p-1} & 0 & 0 & 0 \\ 0 & y_{i,t-p-2} & 0 & 0 & 0 \\ 0 & 0 & y_{i,t-p-1} & 0 & 0 \\ 0 & 0 & y_{i,t-p-2} & 0 & 0 \\ 0 & 0 & y_{i,t-p-3} & 0 & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & 0 & 0 & y_{i,t-p-1} \\ 0 & 0 & 0 & 0 & y_{i,t-p-2} \\ 0 & 0 & 0 & 0 & y_{i,t-p-3} \\ 0 & 0 & 0 & 0 & y_{i,t-p-4} \\ 0 & 0 & 0 & 0 & \vdots \\ 0 & 0 & 0 & 0 & y_{i,t-p-(T-p-1)} \end{bmatrix} \end{array} = \begin{bmatrix} y_{i,1} & 0 & 0 & 0 & 0 \\ 0 & y_{i,2} & 0 & 0 & 0 \\ 0 & y_{i,1} & 0 & 0 & 0 \\ 0 & 0 & y_{i,3} & 0 & 0 \\ 0 & 0 & y_{i,2} & 0 & 0 \\ 0 & 0 & y_{i,1} & 0 & 0 \\ \vdots & & & \ddots & \vdots \\ 0 & 0 & 0 & 0 & y_{i,T-2} \\ 0 & 0 & 0 & 0 & y_{i,T-3} \\ 0 & 0 & 0 & 0 & y_{i,T-4} \\ 0 & 0 & 0 & 0 & y_{i,T-5} \\ 0 & 0 & 0 & 0 & \vdots \\ 0 & 0 & 0 & 0 & y_{i,1} \end{bmatrix}. \tag{5}$$

Uncollapsed matrix in Eq. (5) has $(T-p-1)(T-p)/2$ instruments, that is, for $p=1$ it equals $(T-2)(T-1)/2$. Every row of matrix Q_i is orthogonal to the vector of differenced disturbances according to the GMM moment conditions in Eq. (4). To clarify a distinction in generating instruments between a single equation DPD and a system of two equations with two endogenous variables, e.g. y and x, lets consider the PVAR$(2,1)$. Uncollapsed matrix Q_i then has $m(T-2)(T-1)/2$ unique instruments per equation, and after the Kronecker product with m–dimensional identity matrix it results in total of $m^2(T-2)(T-1)/2$ instruments:

$$
Q_i \otimes I_{m\times m} = \begin{bmatrix}
y_{i,1} & 0 & 0 & 0 \\
x_{i,1} & 0 & 0 & 0 \\
0 & y_{i,2} & 0 & 0 \\
0 & x_{i,2} & 0 & 0 \\
0 & y_{i,1} & 0 & 0 \\
0 & x_{i,1} & 0 & 0 \\
\vdots & & \ddots & \vdots \\
0 & 0 & 0 & y_{i,T-2} \\
0 & 0 & 0 & x_{i,T-2} \\
0 & 0 & 0 & y_{i,T-3} \\
0 & 0 & 0 & x_{i,T-3} \\
0 & 0 & 0 & y_{i,T-4} \\
0 & 0 & 0 & x_{i,T-4} \\
0 & 0 & 0 & \vdots \\
0 & 0 & 0 & y_{i,1} \\
0 & 0 & 0 & x_{i,1}
\end{bmatrix} \otimes \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix}
y_{i,1} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & y_{i,1} & 0 & 0 & 0 & 0 & 0 \\
x_{i,1} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & x_{i,1} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & y_{i,2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & y_{i,2} & 0 & 0 & 0 \\
0 & 0 & x_{i,2} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & x_{i,2} & 0 & 0 & 0 \\
0 & 0 & y_{i1} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & y_{i1} & 0 & 0 & 0 \\
0 & 0 & x_{i1} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & x_{i1} & 0 & 0 & 0 \\
\vdots & & & & \ddots & & \vdots \\
0 & 0 & 0 & 0 & 0 & y_{i,T-2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & y_{i,T-2} \\
0 & 0 & 0 & 0 & 0 & x_{i,T-2} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & x_{i,T-2} \\
0 & 0 & 0 & 0 & 0 & y_{i,T-3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & y_{i,T-3} \\
0 & 0 & 0 & 0 & 0 & x_{i,T-3} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & x_{i,T-3} \\
0 & 0 & 0 & 0 & 0 & y_{i,T-4} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & y_{i,T-4} \\
0 & 0 & 0 & 0 & 0 & x_{i,T-4} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & x_{i,T-4} \\
0 & 0 & 0 & 0 & 0 & \vdots & \vdots \\
0 & 0 & 0 & 0 & 0 & y_{i,1} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & y_{i,1} \\
0 & 0 & 0 & 0 & 0 & x_{i,1} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & x_{i,1}
\end{bmatrix}. \tag{6}
$$

The number of uncollapsed instruments reduces with respect to p for unchanged time dimension T and the number of endogenous variables m. For example, there would be $m^2p(p-1)/2$ less instruments by cutting the first 4 rows and the first 2 columns from a full instrument matrix in Eq. (6) for time lag $p=2$. Therefore, a general expression for counting the number of uncollapsed instruments in PVAR(m,p) model, as long as all variables are endogenous, is:

$$
nrow(Q_i) = m^2 \left[\frac{(T-2)(T-1) - p^2 + p}{2} \right] \tag{7}
$$

It is evident that the number of uncollapsed instruments is equal to the number of rows of matrix Q_i. Given expression in Eq. (7) is valid for PVAR(m,p) model with $m \geq 2$ and without any predetermined or exogenous variables. To reduce the number of instruments the blocks of matrix Eq. (6) can be collapsed vertically to the top of the matrix:

$$Q_i^* = \begin{bmatrix} y_{i,1} & 0 & y_{i,2} & 0 & \cdots & y_{i,T-2} & 0 \\ 0 & y_{i,1} & 0 & y_{i,2} & & 0 & y_{i,T-2} \\ x_{i,1} & 0 & x_{i,2} & 0 & & x_{i,T-2} & 0 \\ 0 & x_{i,1} & 0 & x_{i,2} & & 0 & x_{i,T-2} \\ 0 & 0 & y_{i,1} & 0 & & y_{i,T-3} & 0 \\ 0 & 0 & 0 & y_{i,1} & & 0 & y_{i,T-3} \\ 0 & 0 & x_{i,1} & 0 & & x_{i,T-3} & 0 \\ 0 & 0 & 0 & x_{i,1} & & 0 & x_{i,T-3} \\ 0 & 0 & 0 & 0 & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & & y_{i,1} & 0 \\ 0 & 0 & 0 & 0 & & 0 & y_{i,1} \\ 0 & 0 & 0 & 0 & & x_{i,1} & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & x_{i,1} \end{bmatrix}. \tag{8}$$

Collapsed matrix Q_i^* imposes instruments which number is invariant of lag p:

$$nrow(Q_i^*) = m^2(T-2). \tag{9}$$

Thus, if researcher decides to use PVAR in short panels then collapsing of instrument matrix is recommended as the number of instruments can be reduced bellow the number of cross–sectional units with respect to the number of endogenous variables m and time period T.

4. Results and discussion

Results from estimated PVAR models are presented and discussed in this section. In short panels ($N = 27$) uncollapsed and collapsed number of instruments should be reported before employing the first difference GMM estimator to determine possible m and T over multiple lags p for which PVAR(m,p) meets the necessary constraints for it's use. This study allows different number of endogenous variables m (2 or 3), different number of time periods T (10, 9 or 8) and different time lags p (1, 2 or 3). Considering these settings and according to Eq. (7) and Eq. (9) the number of uncollapsed and collapsed instruments are given in Table 2.

$T=10$	Uncollapsed instruments		Collapsed instruments	
	$m=2$	$m=3$	$m=2$	$m=3$
$p=1$	144	324	32	72
$p=2$	140	315	32	72
$p=3$	132	297	32	72
$T=9$	Uncollapsed instruments		Collapsed instruments	
$p=1$	112	252	28	63
$p=2$	108	243	28	63
$p=3$	100	225	28	63
$T=8$	Uncollapsed instruments		Collapsed instruments	
$p=1$	84	189	24	54
$p=2$	80	180	24	54
$p=3$	72	162	24	54

Table 2: *Number of uncollapsed and collapsed instruments with respect to T, m and p.*

It is not surprising that among all combinations in Table 2 only one is feasible for running PVAR which indisputably confirms it's very restrictive usage in practical applications. These feasible settings refer to 24 collapsed instruments with only 2 endogenous variables and 8 years of observation at any time lag, that is, the number of collapsed instruments is less then the number of cross–sectional units (27 EU members). Although uncollapsed instruments are decreasing with respect to the time lag p it is not acceptable to have a large number of estimated parameters for computational reasons. This feature is not an issue when dealing with collapsed instruments as they are invariant from p, leaving some flexibility to appropriate lag selection according to commonly used information criteria.

Two PVAR models are estimated which combine different pairs of endogenous variables as collapsed number of instruments 24 is only feasible for PVAR$(2, p)$ with $N = 27$ and $T = 8$. Thus, the Model 1 analyzes the interdependency between real exchange rate misalignment (MIS) and current account balance (CUR), while in the Model 2 current account balance is replaced with the public debt (PD) keeping real exchange rate misalignment as a crucial variable in this study. For estimation purpose panelvar package was used within RStudio programming environment (link to the reproducible commands is in the appendix). Since the aim of the paper is to determine the variables that most influence real exchange rate misalignments of EU members, in Tables 3-4 estimation results are presented along with lags $p = 1,\ 2,\ 3$.

Time lag	$p = 1$		$p = 2$		$p = 3$	
Endo. variable	$\Delta MIS_{i,t}$	$\Delta CUR_{i,t}$	$\Delta MIS_{i,t}$	$\Delta CUR_{i,t}$	$\Delta MIS_{i,t}$	$\Delta CUR_{i,t}$
$\Delta MIS_{i,t-1}$	0.7665***	−0.2089**	0.8000***	−0.1340**	0.8509***	0.6294
	(0.0451)	(0.1046)	(0.0925)	(0.0475)	(0.1479)	(0.4970)
$\Delta CUR_{i,t-1}$	0.1221***	0.0723	0.2266**	0.4049	0.1807**	0.5898
	(0.0298)	(0.2352)	(0.1198)	(0.4606)	(0.0686)	(0.4261)
$\Delta MIS_{i,t-2}$			0.2561**	0.0873	0.2313**	0.1699
			(0.0783)	(0.1524)	(0.0417)	(0.2917)
$\Delta CUR_{i,t-2}$			−0.0401	0.4126	0.0522	0.4928*
			(0.0560)	(0.2255)	(0.0686)	(0.2439)
$\Delta MIS_{i,t-3}$					−0.0858	−0.4232
					(0.1570)	(0.2642)
$\Delta CUR_{i,t-3}$					0.0871	0.1958
					(0.0896)	(0.2015)
Observations	162		135		108	
Groups	27		27		27	
Obs per group	6		5		4	
Instruments	24		24		24	
Hansen $p-$value	0.198		0.292		0.244	
MMSC–BIC	−88.9522		−77.6045		−69.3243	
MMSC–AIC	−21.0251		−19.4991		−21.0460	
MMSC–HQIC	−52.1835		−46.2920		−43.3998	
Stability	Yes		Yes		Yes	

*Note: statistical significance at 10%, 5% and 1% level is denoted by *, **, ***, while Windmeijer robust standard errors are in the parentheses*

Table 3: *PVAR(2,p) estimation results from Model 1.*

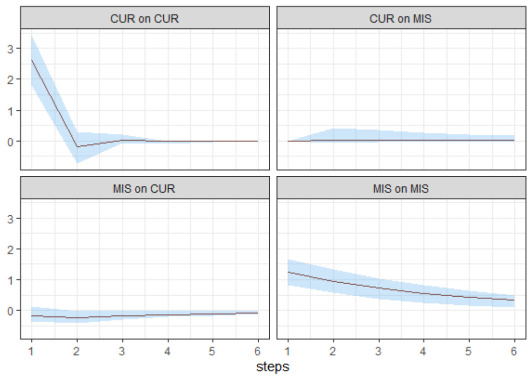
All three types of model and moment selection criteria (MMSC) indicate the appropriateness of Model 1 with one lag over the models with two and three lags. These criteria are analogous to the already known information criteria for deciding between alternative models (Akaike, Bayesian and Hannan–Quinn information criteria). Each PVAR$(2, p)$ model satisfies stability condition as all the eigenvalues lie inside the unit circle. Hansen test is not weakened due to collapsed number of instruments, and the null hypothesis of joint instrument validity can not be rejected, that is, model is correctly specified as the $p-$value of $J-$statistic, with degrees of freedom equal to the number of instruments corrected for the number of estimates, is greater than any significance level. These findings confirm that overfitting of endogenous variables is successfully avoided and there is no risk of instrument proliferation. Another issue in short panels concerns biased standard errors [20]. For the same reason a Windmeijer finite sample correction of the two–step GMM standard errors are obtained [6, 33]. According to model PVAR$(2,1)$ in Table 3 current account balance is positively significant at the 1% level. This result is contrary to [9], but expected as the current account imbalance leads to the RER misalignment. Likewise, autoregressive coefficient is significant at 1% level in the first equation and it is significant at 5% level in the second equation although negative. However, results from Table 4 indicate that RER misalignment effects public debt significantly but not otherwise, while autoregressive coefficient is still persistent and significant at 1% level. The same conclusion about adequacy of the Model 1 with one lag can be carried as for Model 2 with respect to Hansen test, MMSC information criteria and stability condition. Regarding both models with lags $p = 3$ it is common that most estimates are not significant cause of the many parameters.

Time lag	$p=1$		$p=2$		$p=3$	
Endo. variable	$\Delta MIS_{i,t}$	$\Delta PD_{i,t}$	$\Delta MIS_{i,t}$	$\Delta PD_{i,t}$	$\Delta MIS_{i,t}$	$\Delta PD_{i,t}$
$\Delta MIS_{i,t-1}$	0.7653***	0.5921**	0.7848***	0.6072*	0.8886***	0.3530
	(0.0439)	(0.2204)	(0.0968)	(0.2724)	(0.1641)	(0.3531)
$\Delta PD_{i,t-1}$	−0.0020	0.4111***	0.0096	0.3168***	−0.0299	0.0866
	(0.0149)	(0.1172)	(0.0165)	(0.0901)	(0.0212)	(0.1437)
$\Delta MIS_{i,t-2}$			−0.3028**	−0.1853	−0.1663	−0.0483
			(0.0995)	(0.1869)	(0.1453)	(0.1910)
$\Delta PD_{i,t-2}$			0.0094	0.0690	0.0167	0.1026
			(0.0101)	(0.0759)	(0.0166)	(0.1137)
$\Delta MIS_{i,t-3}$					−0.1057	−0.0539
					(0.1388)	(0.2172)
$\Delta PD_{i,t-3}$					−0.0047	−0.0357
					(0.0171)	(0.0599)
Observations	162		135		108	
Groups	27		27		27	
Obs per group	6		5		4	
Instruments	24		24		24	
Hansen $p-$value	0.424		0.390		0.257	
MMSC–BIC	−91.3771		−81.1799		−69.5541	
MMSC–AIC	−23.4499		−23.0745		−21.2757	
MMSC–HQIC	−54.6084		−49.8675		−43.6296	
Stability	Yes		Yes		Yes	

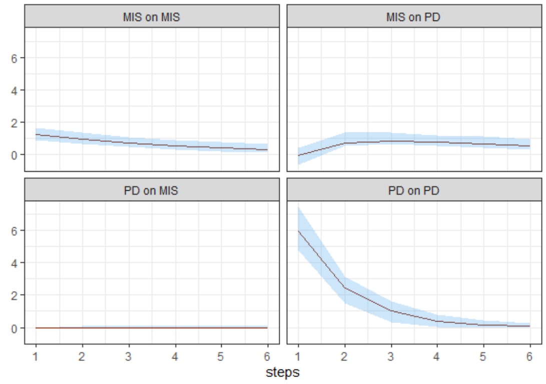
*Note: statistical significance at 10%, 5% and 1% level is denoted by *, **, ***, while Windmeijer robust standard errors are in the parentheses*

Table 4: *PVAR(2,p) estimation results from Model 2.*

A particular advantage of PVAR is the ability to estimate the responses to the orthogonal shocks, i.e. the impact of a shock of one variable on another variable, holding the shocks of the other variables equal to zero [15]. Orthogonal impulse response function (OIRF) shows the size and duration of the shock response with a confidence limits in order to determine the uncertainty of the shock response using bootstrapping method (Figure 2).



(a) *OIRF from Model 1*



(b) *OIRF from Model 2*

Figure 2: *Orthogonalized impulse response functions and 95% confidence limits.*

The forecast error variance decomposition (FEVD) is another tool for interpreting the PVAR model that determines how much of the forecast error variance of each variable can be explained by shocks of other variables. As seen in Table 5, variable current account can explain 2.29% of the variation in real exchange rate misalignments up to 6th year while public debt can account for more than 5% of the variation in the MIS. Contrary, results indicate that variable MIS account more explained variation in CUR then on PD.

Forecast	PVAR(2,1) Model 1		PVAR(2,1) Model 2	
horizon	MIS $\to$ CUR	CUR $\to$ MIS	MIS $\to$ PD	PD $\to$ MIS
1	0.0000	0.00476	0.0000	0.0005
3	0.0051	0.0178	0.0001	0.0283
6	0.0059	0.0229	0.0002	0.0559

Table 5: *Forecast error variance decomposition from Model 1 and Model 2.*

5. Conclusion

Although PVAR models are suitable for capturing both static and dynamic interdependencies in the panel data and treating the causality between variables symmetrically, a great causation should be given do the number of instruments when dealing with short panels. Contrary, in case of long panels (large N) it is advised to report the number of instruments and to check the robustness of results to instrument reduction, as too many instruments might cause overfitting problem. When N is small the only feasible solution is to collapse the matrix of instruments and accordingly adopt the number of endogenous variables and time periods for which PVAR can be applied. This makes PVAR utility very restrictive, particularly as the first difference two–step GMM estimator requires all eigenvalues of companion matrices inside the unit circle and all valid instruments. Indisputable indicator of too many instruments issue is a high p–value of the Hansen $J-$statistic, i.e. close or equal to 1, which misleadingly forces the researchers to never reject the null hypothesis of instruments validity. Keeping the number of instruments small when cross–sectional dimension is small should not be an exception, particularly when dealing with PVAR models.

This paper primarily contributes to the existing studies of PVAR methodology by empirical demonstration of the benefits from collapsed matrix of instruments, i.e. in a given settings of 24 collapsed instruments results notably confirm that Hansen overidentification test is not weakened any more. It also enriches the literature on real exchange rates misalignment's, that is, it offers a more comprehensive analysis of EU real exchange rate behavior due to the shocks in other variables by employing stationary panel vector autoregression model with fixed effects, and contributes to a profound understanding of the explained variance of real exchange rates misalignment in the log–run. Summarized findings imply that real exchange rate misalignment effects more current account balance than public debt, while public debt effects more real exchange rate misalignment then otherwise. This paper is limited to a PVAR(2,1) model, but from the practical point of view it provides a straightforward guidance how to get the most from the short panel data when users decide to utilize PVAR in their studies.

Appendix

Implementation of the codes in RStudio requires installation of three additional packages with accompanying dependencies: `pdfetch` for loading the data directly into RStudio from EUROSTAT web source, `mFilter` for employing Hodrick–Prescott filter, and `panelvar` package for PVAR estimation by `pvargmm()` command. Codes are available at this link.

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