

Stock assessment using cumulative prospect theory in DEA cross-efficiency model: A case study of the Indian stock market

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Abstract. Market volatility is becoming increasingly common as numerous factors are implemented in the financial system. As a result, portfolio managers and individual investors require reliable methods to assess stock performance. This study examines stock assessments using cross-efficiency evaluations in cases where negative data is present. An alternative approach to achieve this goal is to use an RDM DDF-based cross-efficiency model which oversees the negative data. We expand the RDM-based cross-efficiency analysis, which uses row and column average values to select portfolios and identify different groups for stock management. To explore the psychological factors that influence the choices made by stock market investors, we incorporate the cumulative prospect theory value for each stock as an output and the variance as an input to evaluate the overall efficiency of the assets. For the empirical analysis, our study focuses on a sample of 30 stocks listed on the Nifty-50 on India's National Stock Exchange. The results of our empirical study verify that the proposed method can serve as an effective tool for stock selection. This demonstrates how the chosen portfolio gives companies a more diversified and well-balanced approach for selecting stocks, thus improving logical decision-making.

Keywords: cross-efficiency, cumulative prospect theory, data envelopment analysis, diversification, stock selection

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1. Introduction

After Markowitz [14] introduced the mean-variance optimization (MVO) model in the 1950s, the financial market grew exponentially and consistently. The Mean-Variance Optimization (MVO) model represents a crucial element in modern finance, offering a balance between portfolio returns and risks, thereby quantifying portfolio performance. With rapid advancements in artificial intelligence, the MVO selection process has now been integrated into diverse methodological frameworks within contemporary investment science. In the global market, asset assessment and ranking are vital for investors. In this study, we employ data envelopment analysis (DEA) with cross-efficiency analysis in the presence of negative input-output to improve portfolio selection by enhancing the stock evaluation and management process.

DEA stands out as one of the distinguished methodologies for examining the performance of decision-making units (DMUs) that manage a variety of inputs and outputs. After the

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groundbreaking research of Charnes et al. [3], who introduced non-parametric programming to approximate the production frontier, there has been a notable increase in publications concerning DEA principles and their practical implications in recent years. Pioneering studies, in this field, such as by Murthi et al. [15], were instrumental in applying DEA and crafting the DEA portfolio efficiency index (DPEI). Similarly, Chen et al. [4] introduced three distinct DEA models that incorporate various risk indicators for assessing portfolio efficiency in a possibilistic environment. Gupta et al. [7] developed an integrated approach for efficient portfolio evaluation, featuring a dynamic re-balancing strategy to empower investors in constructing optimal portfolios within a credibilistic framework. Kumar et al. [10] presented an innovative approach to multiperiod efficient portfolio selection (MPEPS), which involves a two-stage process. Additionally, Zhou et al. [26] proposed a portfolio re-balancing strategy that capitalizes on DEA frontier improvement within the mean-variance (MV) framework.

In DEA, a Decision Making Unit (DMU) evaluates itself by optimizing input and output weights to attain maximum relative efficiency. However, DEA has been criticized for its flexible weight assignment and reliance on self-assessment. To overcome this challenge, Sexton et al. [20] introduced a cross-efficiency method that integrates peer evaluation, shifting focus from self-assessment. Oukil and Amin [16] employed the concept of maximum cross-efficiency to evaluate the performance of baseball team players, presenting an alternative approach to team management. Wu [24] applied cross-efficiency to stock organization and portfolio optimization, and Lim et al. [11] introduced a novel methodology for portfolio selection on the Korean stock exchange using DEA cross-efficiency. Amin and Hajjani [1] demonstrated the effect of alternative optimal solutions in the construction of a cross-efficiency matrix (CEM) and validated that the performance of the MV portfolio selection method is enhanced. Deng and Fang [6] incorporated a DEA prospect cross-efficiency evaluation into an innovative MVM (mean-variance-maverick) framework for fuzzy portfolio selection. Amin and Oukil [2] and Sanei and Banihashemi [19] effectively applied the cross-efficiency evaluation for portfolio section.

Instead of utilizing DEA methods, researchers have also employed multi-criteria decision-making (MCDM) methods that focus on analyzing and ranking decision alternatives based on a set of criteria. Kovač and Podrug [9] provide practical perspectives on utilizing MCDM and the MPT theory for share selection and portfolio optimization. In addition, Poklepovi and Babi [17] proposed a hybrid MCDM technique to evaluate the shares of the Zagreb stock exchange. DEA offers an advantage over MCDM because MCDM relies on a uniform set of weights, whereas DEA utilizes a linear programming model to determine individual weights for each DMU, along with an enhancement in efficiency.

Cross-efficiency analysis is frequently employed in scenarios where the production frontier exhibits constant returns to scale (CRS), as occurrences of negative efficiencies are uncommon across different situations. Lim et al. [12] and Wu et al. [25] introduced two notable methods for evaluations of the cross-efficiencies in cases involving variable returns to scale (VRS) production frontier. Their primary aims are either to reduce negative efficiencies or transform them into positive values. Recognizing the limitations in prior research, Kao and Liu [8] proposed a slack-based measure model for assessing the cross-efficiency value of DMUs without eliciting negative efficiencies. Their concept provides reasonable efficiencies for only weakly efficient DMUs. Soltanifar and Sharafi [21] presented a non-radial cross-efficiency model integrated with a hybrid MCDM technique. In contrast, Lin [13] proposed an approach for cross-efficiency evaluation based on the Directional Distance Function (DDF), utilizing the Range Directional Measure (RDM) model introduced by Portela et al. [18] and duality theory within the framework of VRS technology. The model introduced by Lin [13] can handle situations with negative input-output variables and successfully addresses the issue of negative cross-efficiency. Shrivastava et al. [22] introduced a portfolio selection approach that leverages cross-efficiency, by employing the model from [13] and Cumulative Prospect Theory (CPT). They calculated the CPT value of the assets from their weekly returns using the Cumulative probability (uniform

probability) weighting function and the utility function in CPT. The CPT-based model then constructs effective portfolios by utilizing the CEM as the basis for return calculations. Chen et al. [5] explored a portfolio selection challenge that incorporated fuzzy DEA cross-efficiency evaluation, considering both undesirable fuzzy inputs and outputs.

Typically, the mean values of the CEM columns are the standard for evaluating the performance of a specific DMU. The simultaneous assessment of both the conventional cross-efficiency score (from the CEM column average) and the performance diversity score (from the CEM row average) offers a holistic method to identify stocks that outperform their peers and possess distinctive capabilities absent in others.

This study addresses the issue of robust stock selection using a cross-efficiency evaluation, that accommodates both positive and negative data. The CA and RA are used for robust portfolio selection and the identification of various types of assets. When implemented in a strategic stock selection process, this approach enables managers to pinpoint stocks/assets that not only deliver strong overall performance but also offer diverse capabilities to set them apart from other alternatives. The notable contributions of this research are as follows:

- Firstly, only a limited number of studies in the current body of literature make use of the DEA cross-efficiency model to handle negative input-output data in portfolio selection, as seen in the works of Chen et al. [5] Talluri et al.[23] and Shrivastava et al. [22], and among others.
- Additionally this study addresses asset cross-efficiency by incorporating the CPT value and return of each stock as outputs, and variance as an input. Our method combines cross-efficiency with CPT, exploring the psychological aspects that affect decision-makers when shaping portfolios. It is important to highlight that only a few studies (e.g., Shrivastava et al. [22] have employed CPT values as an output.
- Finally, in contrast to the methods employed by Talluri et al. [23], this study employs a DDF-based RDM cross-efficiency model to classify the assets. In contrast to, the classification employed by Shrivastava et al.[22], this study classifies stocks into distinct categories, to enable the inclusion of well-performing stocks with both similarity and variation in their performance. This approach offers financial experts a more comprehensive and detailed understanding of the subject.

The rest of the study comprises four sections, including the preliminary section which discusses the DEA cross-efficiency techniques to deal with negative values in input/output. The proposed work methodology is described in section 3. Next, the practical application of the proposed approach is demonstrated to highlight its validity and effectiveness in section 4. The paper concludes with Section 5, which examines the results and proposes potential avenues for future research.

2. Preliminaries

This section provides an overview of the key studies that motivated us to initiate this research. Let a group of n homogeneous DMUs be evaluated. Assume that each of these DMUs consumes m different inputs x_{ij} ($i = 1, 2, \dots, m$) to generate s different outputs y_{rj} ($r = 1, 2, \dots, s$). Specifically, j^{th} DMU consumes amount x_{ij} of input i and generates amount y_{rj} of output r . Cross-efficiency is a concept employed in ranking to tackle the issue of inconsistency among the efficiencies of a set of DMUs assessed using different weightings in DEA. The traditional cross-efficiency method relies on the CCR model and cannot handle the negative input-output data.

To address this challenge, Lin [13] introduces a VRS cross-efficiency evaluation approach based on DDF, allowing input-output variables to take on negative values in line with the approach proposed by Portela et al. [18], as outlined below:

$$\begin{aligned}
\text{Model I} \quad & \text{Min} \sum_{i=1}^m v_{ik}(x_{ik} + d_{ik}^-) - \sum_{r=1}^s \mu_{rk}(y_{rk} - d_{rk}^+) + \xi \\
& \text{s.t.} \\
& \sum_{i=1}^m v_{ik}x_{ij} - \sum_{r=1}^s \mu_{rk}y_{rj} + \xi \geq 0, \\
& \sum_{i=1}^m v_{ik}d_{ik}^- + \sum_{r=1}^s \mu_{rk}d_{rk}^+ = 1, \quad j = 1, 2, \dots, n, \\
& v_{ik}, \mu_{rk} \geq 0, \quad r = 1, 2, \dots, s \quad i = 1, 2, \dots, m.
\end{aligned}$$

where the directional vectors are computed as

$$\begin{aligned}
d_{rk}^+ &= \max_{j=1,2,\dots,n} y_{rj} - y_{rk}, \quad r = 1, 2, \dots, s, \\
d_{ik}^- &= x_{ik} - \min_{j=1,2,\dots,n} x_{ij}, \quad i = 1, 2, \dots, m.
\end{aligned}$$

are the possible non-negative improvements for inputs and outputs. Using the weights of DMU_k derived from M2, established the cross-efficiency of DMU_j, in the following manner:

$$\beta_{kj} = \frac{\sum_{i=1}^m v_{ik}^*(x_{ij} + d_{ik}^-) - \sum_{r=1}^s \mu_{rk}^*(y_{rj} + d_{rk}^+) - \xi^*}{\sum_{i=1}^m v_{ik}^*d_{ik}^- + \sum_{r=1}^s \mu_{rk}^*d_{rk}^+}. \quad (1)$$

Thus, the cross-efficiency score of jth DMU is given by $\beta_j = \frac{1}{n} \sum_{k=1}^n \beta_{kj}$ which is the simple column average of the cross-efficiency matrix. Similarly, we obtain the row average cross-efficiency as $\eta_j = \frac{1}{n} \sum_{k=1}^n \beta_{jk}$. The self-efficiency of DMU_k will be obtained by putting $j = k$ in the equation (1) and denoted as β_{kk}^* . The row average cross-efficiency provides insights into the individual characteristics of each DMU. When the value of a DMU's row mean is relatively lower compared to other DMUs, it distinguishes that DMU from the rest, indicating a high level of performance diversity.

Calculation of the "Maverick index" (MI)

The cross-efficiency scores can be utilized even further by defining the extent to which there is variation between the peer appraisal and the self-appraisal efficiency of DMUs. In finance, the Maverick index serves as a valuable tool for assessing the false positives among financial assets. This measure of concordance between the two efficiencies is called the Maverick index [?] and it is defined as follows:

$$MI_k = \frac{\beta_{kk}^* - \beta_k}{\beta_k} \quad (2)$$

MI quantifies the extent of disparity between a self-evaluation score and a peer-evaluation score. A higher MI value suggests a higher likelihood of the DMU being mistakenly perceived as efficient. Conversely, a lower index value indicates a more favorable outcome.

Calculation of The “Performance Diversity Index” (PDI)

The calculation of the PDI follows a methodology similar to that of MI. To determine the performance of an evaluated DMU (stock), its basic efficiency is compared with the average cross-efficiency of all other DMUs, using the weights specific to the evaluated DMU. Consequently, all DMUs involved in the assessment procedure are subject to the same criteria. The PDI is computed as follows:

$$PDI_k = \frac{\beta_{kk}^* - \eta_k}{\eta_k}. \quad (3)$$

3. Proposed Methodology

DEA cross-efficiency is the most appropriate approach for portfolio selection because it rationalizes equities more thoroughly and analyses stocks based on efficiency rather than absolute output. As financial decision-making often involves risks and uncertainties, decision analysis should consider the behavioral aspects of decisions. From this perspective, this study introduces a resilient approach to stock management, which encompasses the following stages:

Choice of input and output variables: Input: The variance of each asset is computed to assess its associated risk, which will serve as an input in the proposed method.

Output: To account for the irrational behaviors of decision-makers (DMs) in stock selection, we incorporate the CPT value of stocks using the procedures outlined by Shrivastava et al. [22] as an output. The second output is defined as the mean return of the stocks and is computed using the following formula: $R_{jt} = \frac{C_{j(t+1)} - C_{jt}}{C_{jt}}$ where C_{jt} and $C_{j(t+1)}$ are the closing price of the j^{th} stock at time t and $t + 1$ respectively.

Assessing the cross-efficiency of the stocks: Model (M2) is utilized to calculate the cross-efficiency scores for the chosen stocks. Subsequently, we determine the self-efficiency, the row average efficiency, and the column average efficiency from the obtained Cross-Efficiency Matrix (CEM).

Categorizing the Stocks: Based on the CA and RA scores obtained from M2 and equation, we categorize the stocks into the following groups: Group G1: High Column Average/High Row Average (HC/HR)

Group G2: High Column Average/Low Row Average (HC/LR)

Group G3: Low Column Average/High Row Average (LC/HR)

Group G4: Low Column Average/Low Row Average (LC/LR)

The RA value describes the unique characteristics of each stock. When a stock’s row average value is comparatively lower than that of other stocks, it stands out from the rest, indicating a higher degree of performance diversity. When we depict the means of the rows and columns derived from the cross-efficiency analysis in a quadrant format, stocks in G2 exhibit superior efficiency than the other stocks, setting them apart. Stocks originally in G1 and G3 should be relocated to G2 to improve their performance. In contrast, stocks situated in G4 demonstrate subpar efficiency and fail to distinguish themselves from other stocks.

4. Empirical illustration

To demonstrate the effectiveness of the proposed DEA cross-efficiency evaluation approach addressing negative data, we employ it to analyze real data from the Indian stock market. We use data from Shrivastava et al. [22] focusing on the performance of the top 30 assets included in the Nifty 50 index. For each asset, one input variable variance, and two output variables, CPT value and long mean return are considered. For the long mean return, we consider the data from 05-03-2018 to 25-02-2019. To calculate the CPT value and variance, we select the weekly return of stocks for the period from 05-03-2018 to 02-08-2018. We select the outputs

for stock evaluation because of their direct relevance to portfolio assessment, and, importantly, their ability to account for the DM's behavioral characteristics.

To evaluate the performance of the listed stocks, the M1 model is used to evaluate the assessment of the stocks. Table 1 shows the CPT value, mean return, and variance with self-efficiency obtained by M1 of the thirty stocks.

S.No	Stocks	Variance	CPT value	Return	Self-efficiency
S_1	BAJAJ-AUTO	0.0013	-0.0273	0.0001	0.4933
S_2	BRITANNIA	0.0007	-0.0065	0.0043	0.8914
S_3	HEROMOTOCO	0.0008	-0.0248	-0.0049	0.4717
S_4	ICICIBANK	0.0024	-0.0221	0.006	0.4848
S_5	INDUSINDBK	0.0004	-0.0067	0.0013	0.8383
S_6	KOTAK BANK	0.0007	-0.0080	0.0043	0.8914
S_7	L & T	0.0007	-0.0122	0.0016	0.7138
S_8	NESTLEIND	0.0009	-0.0041	0.007	1.0000
S_9	NTPC	0.0005	-0.0159	0.0001	0.7125
S_{10}	ULTRACEMCO	0.0009	-0.0176	0.0011	0.6200
S_{11}	WIPRO	0.0011	-0.0135	0.0041	0.6902
S_{12}	TCS	0.0009	0.0008	0.006	0.9569
S_{13}	ITC	0.0008	0.0042	0.0027	0.8092
S_{14}	TITAN	0.001	-0.0175	0.0059	0.8333
S_{15}	BAJFINANCE	0.002	0.0227	0.0125	1.0000
S_{16}	BAJAJFINSV	0.0011	-0.0038	0.0078	0.9756
S_{17}	SHREECEM	0.0009	-0.0134	0.0037	0.7399
S_{18}	TATASTEEL	0.0017	-0.0291	-0.0011	0.4177
S_{19}	BHARTIARTL	0.0011	-0.0267	-0.0013	0.4959
S_{20}	GRASIM	0.0011	-0.0213	-0.0037	0.4594
S_{21}	TATACONSUM	0.002	-0.0581	-0.0033	0.3361
S_{22}	M&M	0.0006	0.0086	-0.0010	1.0000
S_{23}	ONGC	0.0009	-0.0226	-0.0016	0.5309
S_{24}	MARUTI	0.0004	-0.0115	-0.0028	0.6319
S_{25}	RELIANCE	0.0016	0.0039	0.008	0.7770
S_{26}	TECHM	0.0012	-0.0059	0.0046	0.7109
S_{27}	CIPLA	0.0012	-0.0100	-0.0007	0.5287
S_{28}	COALINDIA	0.0014	-0.0324	-0.0040	0.4031
S_{29}	HINDALCO	0.0034	-0.0344	0.0008	0.2996
S_{30}	HDFCLIFE	0.001	-0.0180	-0.0010	0.5384

Table 1: *Input Output variable with Self-efficiency after solving Model I.*

4.1. Stock management using the proposed method

The model M1 computes the cross-efficiency of the stocks. The cross-efficiency matrix (CEM) is generated using the input and output weights from M1 and Equation (1). The resulting CEM offers unique insights pertinent to this analysis. A widely recognized use of CEM involves averaging the efficiency scores down a column and designating that average as the peer appraisal score, commonly known as the CA efficiency.

Stocks	S_1	S_1	S_3	S_4	S_5	S_6	S_7	S_8	S_9	S_{10}
CA efficiency	0.4816	0.8650	0.4521	0.4550	0.8027	0.8588	0.6902	0.9871	0.6750	0.6022
RA efficiency	0.6614	0.6332	0.6614	0.6182	0.6332	0.6332	0.6614	0.6182	0.6332	0.6614
Stocks	S_{11}	S_{12}	S_{13}	S_{14}	S_{15}	S_{16}	S_{17}	S_{18}	S_{19}	S_{20}
CA efficiency	0.6759	0.9237	0.7644	0.7953	0.9591	0.9386	0.7219	0.4053	0.4816	0.4396
RA efficiency	0.6614	0.6614	0.6614	0.6182	0.6182	0.6182	0.6332	0.6614	0.6614	0.6614
Stocks	S_{21}	S_{22}	S_{23}	S_{24}	S_{25}	S_{26}	S_{27}	S_{28}	S_{29}	S_{30}
CA efficiency	0.3290	0.6737	0.5130	0.6007	0.7510	0.6877	0.5037	0.3894	0.2915	0.5179
RA efficiency	0.6182	0.4965	0.6614	0.6614	0.6614	0.6614	0.6614	0.6614	0.6182	0.6614

Table 2: The column and row average efficiency obtained from CEM.

A stock with a high CA performs well because it outperforms the other units. Also, the low Row average (RA) value, indicates that the specific stock possesses distinctive capabilities and expertise that other stocks lack. The CA efficiency and RA efficiency obtained from CEM are presented in Table 2. We categorize the stocks based on their CA and RA values. Figure 1 shows the grouping of stocks based on the CA and RA scores for all 30 stocks. The data is categorized as either high or low for both CA and RA. The mean of these scores is employed to split the categories into high and low (in this instance, the average resulted in a value of 0.641). Although the classification thresholds are subjective, they can also be determined by decision makers or managers engaged in stock evaluation..

Discussion on the different groups of stocks The suggested stock classification aids investors in formulating portfolio optimization approaches. Supreme or exceptional performers are prime candidates for long-term investments because of their outstanding performance in various aspects and unique skills that contribute to building a robust and diversified stock selection, capable of meeting the current and future demands of the financial market. However, all-round good performers and niche performers exhibit differences. While the former excel overall compared to their peers, they lack the specific distinctive skills that distinguish them. Detailed descriptions of these groups are as follows.

1. All-round good-performing stocks (HC/HR): Six stocks are assigned to the HC/HR quadrant (G1). Stocks in this group are referred to as all-round good performers because they possess strengths that align with the performance of other assets. A portfolio generated with these stocks may perform well, but it may not be stable. The RA and CA efficiencies along with MI and PDI are presented in Table 3. The average MI and PDI for this portfolio are 0.0364 and 0.1739, respectively.

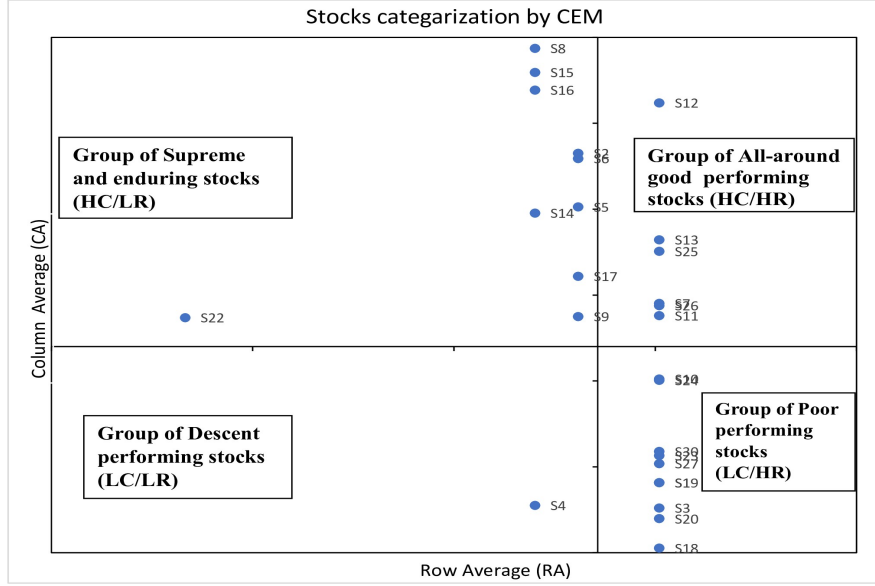


Figure 1: Different group of stocks based on CA & RA.

Stocks	CA	RA	MI	PDI
S_{12}	0.9237	0.6614	0.0359	0.4469
S_{13}	0.7644	0.6614	0.0587	0.2236
S_{25}	0.7510	0.6614	0.0346	0.1749
S_7	0.6902	0.6614	0.0341	0.0793
S_{26}	0.6877	0.6614	0.0337	0.0749
S_{11}	0.6759	0.6614	0.0211	0.0436
Average	0.7488	0.6614	0.0364	0.1739

Table 3: Statistics of group HC/HR.

2. Supreme and enduring stocks: Table 4, describes the list of stocks in the HC/LR (G2) group. The stocks within this category possess distinctive resources and strengths to set them apart or align them with the performance of other stocks. Consequently, assets from this group can be regarded as long-term stability portfolios. The portfolio obtained from this group is well-diversified with a PDI value of 0.4609.

3. Descent-performing stocks: Only stock 4 comes under the category G3. The stock in this group has unique resources that set them apart from other stocks but lack the strength to match the performance of their peers. But these are less diversified.

4. Poor performing stocks: Within the high LC/HR (G4) quadrant, there are thirteen assets, which are typically considered subpar performers. These assets lack distinctive resources or strengths that can outperform their market counterparts. Consequently, portfolios comprising assets from this group may not exhibit strong performance, and the overall diversification of such portfolios is notably low. The results are shown in Table 6.

After examining the tables, it is evident that G2 (HC/LR) assets have the best performance

Stocks	CA	RA	MI	PDI
S_8	0.98713	0.6182	0.01304	0.61758
S_{15}	0.95908	0.6182	0.04266	0.61758
S_{16}	0.93862	0.6182	0.039398	0.57811
S_2	0.86501	0.6332	0.03050	0.40787
S_6	0.85878	0.6332	0.03798	0.40787
S_5	0.80265	0.6332	0.04441	0.32400
S_{14}	0.79534	0.6182	0.04772	0.34793
S_{17}	0.72189	0.6332	0.02495	0.16859
S_9	0.67497	0.6332	0.05560	0.12532
S_{22}	0.6737	0.4965	0.48431	1.0141
Average	0.8277	0.6135	0.0820	0.4609

Table 4: *Statistics of group HC/LR.*

Stocks	CA	RA	MI	PDI
S_4	0.4550	0.618	0.06541	-0.2158

Table 5: *Statistics of group LC/LR.*

Stocks	CA	RA	MI	PDI
S_{10}	0.6022	0.6614	0.0296	-0.06253
S_{24}	0.6007	0.6614	0.05200	-0.04454
S_{30}	0.51788	0.6614	0.03962	-0.1859
S_{23}	0.5130	0.6614	0.034913	-0.19725
S_{27}	0.5037	0.6614	0.049605	-0.20058
S_{19}	0.4816	0.6614	0.02968	-0.25017
S_1	0.4816	0.6614	0.02435	-0.25411
S_3	0.45208	0.6614	0.04339	-0.28677
S_{20}	0.43963	0.6614	0.04498	-0.30536
S_{18}	0.40534	0.6614	0.03049	-0.368416
S_{28}	0.38944	0.6614	0.035076	-0.39049
S_{21}	0.32902	0.61821	0.02151	-0.45633
S_{29}	0.29147	0.61821	0.027893	-0.51537
Average	0.46212	0.65472	0.035624	-0.27060

Table 6: *Statistics of group LC/HR.*

with an average MI score of 0.0820 and a higher average PDI score of 0.4609. G3 and G4 have significantly lower PDI scores when assessing overall diversification with less MI. It should be

noted that the portfolio can also be chosen from a combination of the above groups of stocks or from the entire group. Similar to each category the stocks with low MI or high PDI can be selected.

To the best of our knowledge and based on our extensive literature review, there is currently no available approach to directly compare the outcomes of the method proposed in this research. Existing studies that utilize DEA cross-evaluations for portfolio selection, stocks are typically ranked in descending sequence of their cross-efficiency values, and the top n stocks are selected, where n corresponds to the number of stocks desired in portfolio. In contrast, our method involves the categorization of stocks into distinct groups based on the average values of the rows and columns in the Cross-Efficiency Matrix (CEM). Therefore, a direct comparison between these two methods is not meaningful.

5. Conclusion

We used the RDM DDF-based cross-efficiency analysis to categorize stocks with negative data. Although various approaches are available for categorizing and grouping stocks, the proposed classification methodology stands out as distinctive and holds significance in the context of portfolio construction. CA values in CEM are commonly employed in the standard practice of evaluating a unit's overall comparative performance. We established an alternative approach that involves the RA values of CEM to determine the breadth of performance diversity. These two ideas are merged to categorize financial assets into different groups, which further enables a more effective stock rationalization process that considers comparative performance and diversity of performance concurrently.

The limitations and potential future directions of this study include the Indian stock market with a restricted dataset. To enhance the significance of efficiency comparisons and benchmark selection, it would be valuable to encompass major markets in other countries. In future research, broadening the modeling scope to include ordinal and interval data values would enable the consideration of the impact of fuzzy input-output data.

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