Quantifying the effects of expectation variability on economic dynamics: Insights from the Dornbusch overshooting model

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Abstract. The article applies the famous Dornbusch "overshooting" model to investigate the impact of different types of expectations on economic model stability. We tested the well-known Dornbusch model with discrete variables. Initially, we established a foundational model, employing simulations to illustrate the impact of each parameter within the model on the overall solution, starting from an initial value. Subsequently, we explored the influence of different expectation types on the stability of the steady state vector, considering simple, static, adaptive, and rational expectations (Muth-type rational expectation and perfect foresight). It was observed that static expectations and perfect foresight did not contribute to stability. In the case of simple, adaptive, and rational expectations, the stability conditions (parameter combinations) are the same as the stability conditions for the steady state vector of the baseline model. This condition is restricted to four parameters: two related to the interest rate, one to the foreign-domestic price level and one to the adjustment speed. We also run a simulation to show how the inclusion of each type of expectation leads to a change in the global solution of the model for a given initial value.

Keywords: expectation, learning process, macroeconomic modelling, rational expectation

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1. Introduction

A fundamental feature of any economic process is the question of stability. We can define stability properties using strict mathematical rules, but the variables that play a role are sometimes based on different economic theories. In addition to different economic theories, the expectations of economic agents also have an impact on stability. This is a slight extension of the basic model by considering a factor that is the apparent driving force behind everyday economic processes: expectations. The structure of the study is as follows: the next section contains the literature review. In Section 3, we present the discrete version of the Dornbusch model and the impact of the model parameters on the evolution of the exchange rate and the price level over time. Then in Section 4, the model is further tested by incorporating different types of expectations into the discrete version of the original model. In Section 5 we show the evolution of the solution curves when incorporating different types of expectations. Section 6 draws conclusions, while Section 7 sums up and suggests future research directions.

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2. Literature review

It is important to note at the outset that there is very limited academic work in the literature that tests different types of expectations when using the same economic model. The discovery that this is so came as a surprise to us, because in the debate among modelling economists over how to take expectations into account, it has been acknowledged that it can be useful to show the impact of economists' different views on how economic agents might think. Moreover, no literature directly addresses the question of the impact of expectations on the actual evolution of processes. The number of econometric-based studies is extensive, with the most relevant ones in the field being [4, 8, 14, 9, 2, 5]. Comprehensive empirical literature reviews of econometric approaches are thoroughly covered in the following works: [18, 13, 10]. The analytical examination of exchange rate expectations, generally encompassing economic agents' anticipations, is not a widely explored topic in the scholarly literature (for example, see [19]). We believe that this can be attributed to the context-dependence inherent in expectations, hindering the proliferation of general, aggregated models in the literature. In this study, we operate with aggregated societal expectations derived from individual expectations, refraining from investigating the mechanisms through which societal expectations emerge from individual anticipations.

Our study is based on Dornbusch's overshooting model [6]. However, in contrast to the original model, we shift our focus to examining the impact of the expectation type utilised in the model from the overshooting. Four main types of expectations are used in theoretical research, which we consider in this study. In the case of *static expectation*, economic agents expect always to see the same value for the variable under study, regardless of circumstances. In the case of *simple expectation*, economic agents observe the variable's current value and expect it for the next period. In the case of *adaptive expectation*, economic agents undergo a learning process in which they also use the error of the previous period in the expectation formation process. The first application of this type of expectation is well known: adaptive expectations were implicitly introduced by Cagan in his paper titled The Monetary Dynamics of Hyperinflation [3]. Cagan looked for a relationship between the money supply and the price level in hyperinflation. He was the first to describe which is now known in economic literature as the adaptive expectation formula. The most widely used expectation, in line with mainstream economics, is the *rational expectation* type, for which the first definition comes from Muth [15]. A rational expectation is, in fact, a conditional expected value of a probability variable for which the amount of information available to the individual is the key factor (to be more precise: Muth focused his analysis to the conditional distribution of economic variables, but most of his followers have reduced this often to the much more simple conditional expected value (Muth, 1961, p. 316)). The distribution of the probability variable is always determined in the current model. But the question arises: What does the probability value truly signify? The expected value of the variable of interest is the variable of probability.

The expectation types used in the economic models are formalised in Table 1.

Expectation	Context
Simple	$X_t^{exp} = X_{t-1}$
Static	$X_t^{exp} = X_{t-1}^{exp} = X$
Adaptive	$X_t^{exp} = X_{t-1}^{exp} + \beta\left(X_{t-1} - X_{t-1}^{exp}\right), \qquad 0 < \beta < 1$
Rational	$X_t^{exp} = E_{t-1}\left(X_t \vert I_{t-1}\right)$

Table 1: *Types of expectations most commonly used in economic modelling.*

The variable X_t^{exp} means the expected value of the X variable in the t^{th} time-period. The duration of the time period depends on the characteristics of the examined model. Focusing on the rational expectation, E refers to the expected value, I_{t-1} is the information set in the $(t-1)^{\mathrm{th}}$ period. Thus X_t^{exp} is the conditional expected value of X_{t-1} as a probability variable.

In public and in the literature, there is often a problem of different or even identical interpretations of two concepts or types of expectations: rational expectations and perfect foresight. In the case of rational expectation, based on Muth's study, the economic agent estimates the value of the variable in consideration for the following period. Since systematic error is ruled out in Muth's theory, i.e. the aggregate individual estimates coincide with the values from the model. We use the following formula in this article: $X_t^{exp} = X_t$. This is one of the theoretical approaches to rational expectations that we are also working with in this article. A similar concept is used by [11], who examines the impact of market reactions on the exchange rate using a list of possible types of expectations. In the case of rational expectations, he uses the same methodology as in our paper. The other theoretical approach is perfect foresight, considered as one special form of rational expectations. In the case of perfect foresight, it is assumed that the agent knows the long-run equilibrium value of the variable in consideration and expects it for each period. The explanation is that it assumes that the current value of the variable is around the long-run equilibrium value so that the expected value is not worth changing; i.e. $X_t^{exp} = \overline{X}$, where $\overline{X}$ is the long-run equilibrium value of X in the model. In this sense, perfect foresight is, in fact, a "perfect static expectation" in the sense that the expected value for each period is not independent of the model but is rather the long-run equilibrium value of the variable under consideration. The original Dornbusch model works with perfect foresight. According to Rüdiger Dornbusch, the perfect foresight is "the deterministic equivalent of rational expectation" ([6, p. 1167, 10^{th} footnote]). Shone, whose study we use as a basis for examining a discrete version of the Dornbusch model, also used the concept of perfect foresight in a special case [17]: according to Shone, the optimal form of incorporating expectation into the model is that $e_t^{exp} - e_{t-1}^{exp} = v(\overline{e} - e_t)$. Where e^{exp} means the expected exchange rate, e_t is the current exchange rate, $\overline{e}$ is the equilibrium rate of exchange rate and $v > 0$ parameter. The perfect foresight case is $e_t^{exp} - e_{t-1}^{exp} = e_t - e_{t-1}$.

In this paper, we consider both approaches (Muth-type rational expectation and Shone-type perfect foresight) and incorporate them into the basic model: first, Shone's perfect foresight, and then our approach, which is $E_t^{exp} = E_t$.

In the following paragraphs, we present our perspectives on the issue of rational expectation theory and its applicability within models. There is a large literature on different interpretations of rational expectation and the challenge for the modeller is to select the most relevant version of it. Rational expectations theory assumes that economic agents use the information available to them rationally and optimally to forecast future outcomes. As stated in King's article, expectations about the future require that the long run and the short run are treated jointly [12, p. 75]. In our opinion, this time horizon aspect represents the weak point in the interpretation and use of rational expectations. In the case of the long run and in economic terms, the concept of perfect foresight best fits rational expectations (in line with Dornbusch's claim that perfect foresight is the deterministic equivalent of rational expectations ([6, p. 1167]). Viewed thus, perfect foresight turns out to be a special case where economic agents are aware of the long-term equilibrium value of the variable under consideration. However, this does not imply that they will also expect it for every period.

3. Dornbusch overshooting model in 'discrete time'

Transforming the Dornbusch model into a discrete version is not a new idea, for example, see [1, 16]. Our discrete model's conditions are the same as in Dornbusch's model: *i.* Small open economy; *ii.* Perfect capital mobility; *iii.* Goods markets adjust more slowly than capital markets; *iv.* The world import price in the goods market is an exogenous variable. Absolute and relative prices are determined by aggregate demand for domestic goods so that domestic production and imports are imperfect substitutes. The endogenous variables are the exchange rate, E, and the domestic price level, P. In some cases, the expected exchange rate also be an endogenous variable. All endogenous variables depend on time. The subscript of the variables indicates the period considered. The other time-dependent variables are exogenous, such as R^*, M and Y, i.e. the foreign nominal rate of interest, the domestic nominal stock of money and the domestic commodity supply. All three factors vary at a constant growth rate in each period. Data prior to the period under consideration are also obtained externally. In solving the model and stability analysis, we keep these variables constant for simplicity, following Dornbusch's model and Azariadis' additions. We consider the model in its original and logarithmic form in the following. Thus, the logarithmic form of relations (1), (2), and (3) are (4), (5), (6):

$$R_t^* = (1+\Psi)R_{t-1}^* \text{ where } 0 < \Psi < 1 \tag{1}$$

$$M_t = (1+\mathrm{T})M_{t-1} \text{ where } 0 < \mathrm{T} < 1 \tag{2}$$

$$Y_t = (1+\mathrm{H})Y_{t-1} \text{ where } 0 < \mathrm{H} < 1 \tag{3}$$

$$r_t^* = \varphi + r_{t-1}^* \text{ where } \varphi = ln(1+\Psi) \tag{4}$$

$$m_t = \tau + m_{t-1} \text{ where } \tau = ln(1+\mathrm{T}) \tag{5}$$

$$y_t = \vartheta + y_{t-1} \text{ where } \vartheta = ln(1+\mathrm{H}) \tag{6}$$

The constant values are denoted as follows: r^*, m, y. Consequently,

$$r_t = r^* + x_t \tag{7}$$

$$x_t = e_t^{exp} - e_{t-1} \tag{8}$$

Where e_t^{exp} is the value of the natural logarithm of the expected exchange rate at time t and e_{t-1} is the current exchange rate observed at the time $(t-1)$. In other words, the expected rate of appreciation or depreciation is equal to the difference (in logarithmic form) between the expected exchange rate in period t and the current exchange rate in the previous period. Consequently, the rate of appreciation or depreciation also depends on the exchange rate expectations of economic agents, so the value of the domestic interest rate depends on a subjective assessment of the exchange rate expectations of economic agents. We now turn to the money market. We assume a Cagan money demand function (9), also in the original model and in logarithmic form (10):

$$\frac{M_t}{P_t} = \mathrm{Y}_t^{\phi}(\exp)^{-\lambda r_t} \tag{9}$$

$$-\lambda r_t + \phi y = m - p_t \tag{10}$$

Price level over time, based on the original Dornbusch model [6, p. 1164, equation (8)]

$$\Delta p_t = p_t - p_{t-1} = \pi[u + \delta(e_{t-1} - p_{t-1}) + (\gamma - 1)y - \sigma r_{t-1}] \tag{11}$$

The definition of each parameter and variable is in Table 2.

Parameter	Definition
π	Speed of adjustment from the original Dornbusch model, $\pi > 0$
u	Shift parameter from the original Dornbusch model, $u > 0$
δ and σ	Sensitivities of aggregate demand with respect to the foreign-to domestic price level and ratio and the domestic real interest rate $\delta > 0$, $\sigma > 0$
γ	Sensitivity of the income in the demand function of the commodity market, $\gamma > 0$
λ and ϕ	Sensitivities of the liquidity preference (price-deflated demand for money) schedule with respect to real income and the nominal rate of interest, $\lambda > 0$, $\phi > 0$

Table 2: *The definition of the parameters.*

We assume that the money market is always in equilibrium, if the relationship at time t holds for the money market equilibrium, it also held in the previous period, i.e.

$$-\lambda r_{t-1} + \phi y = m - p_{t-1}. \tag{12}$$

As a result, the domestic interest rate in the period prior to the period under consideration has been used:

$$r_{t-1} = -\frac{1}{\lambda} m + \frac{1}{\lambda} p_{t-1} + \frac{\phi}{\lambda} y \tag{13}$$

The price level in period t is as follows from the original model:

$$p_t = \left(1 - \pi\delta - \frac{\pi\sigma}{\lambda}\right) p_{t-1} + \pi\delta e_{t-1} + \pi\left(\gamma - 1 - \frac{\pi\sigma}{\lambda}\right) y + \frac{\pi\sigma}{\lambda} m + \pi u \tag{14}$$

The following defines the exchange rate as an extension of the Dornbusch model. Changes in the exchange rate are determined by the supply and demand for foreign exchange in the market. The change in the exchange rate of an economy can be described as follows:

$$\Delta e_t = e_t - e_{t-1} = \mu(D_t^D - D_t^S) \tag{15}$$

Where D_t^D is the demand for foreign exchange, D_t^S is the supply for foreign exchange, $0 < \mu < 1$. The demand for foreign exchange at time t is a function of the exchange rate at time t and national income at time t. Since foreign currency exchange supply and demand are determined by foreign trade activity, it is worth considering whether it is worth introducing some time lag as follows. The foreign currency exchange supply at time t is a function of the exchange rate at time t and the national income abroad at time $t-1$. All this is formal:

$$D_t^D = f(e_t, y_{t-1}) = -\kappa e_t + \zeta y \tag{16}$$

$$D_t^S = g(e_t, y^*) = \omega e_t + \xi y^* \tag{17}$$

where $\kappa, \zeta, \omega, \xi > 0$ are parameters. As a result of the above

$$e_t - e_{t-1} = \mu(-\kappa e_t + \zeta y - \omega e_t - \xi y^*) \tag{18}$$

By transforming the above equation, we can obtain a formula for the exchange rate at time t:

$$e_t = \frac{\mu}{1 + \mu(\kappa + \omega)}[\zeta y - \xi y^*] + \frac{\mu}{1 + \mu(\kappa + \omega)} e_{t-1} \tag{19}$$

In summary, the model is:

- Interest rate

$$r_t = r^* + x_t \tag{20}$$

- The rate of appreciation or depreciation:

$$x_t = e_t^{exp} - e_{t-1} \tag{21}$$

- The exchange rate equation for the t^{th} period:

$$e_t = \frac{\mu}{1+\mu(\kappa+\omega)}[\zeta y - \xi y^*] + \frac{1}{1+\mu(\kappa+\omega)} e_{t-1} \tag{22}$$

- Price level of the t^{th} period:

$$p_t = \left(1 - \pi\delta - \frac{\pi\sigma}{\lambda}\right) p_{t-1} + \pi\delta e_{t-1} + \pi\left(\gamma - 1 - \frac{\phi}{\lambda}\right) y + \frac{\pi\sigma}{\lambda} m + \pi u \tag{23}$$

- The money market equilibrium:

$$-\lambda r_t + \phi y = m - p_t \tag{24}$$

In the case the above model is in fact a simple system of two difference equations. The endogenous variables are the exchange rate and the price level.

$$\begin{bmatrix} e_t \\ p_t \end{bmatrix} = \begin{bmatrix} \frac{1}{1+\mu(\kappa+\omega)} & 0 \\ \pi\delta & 1-\pi\delta-\frac{\pi\sigma}{\lambda} \end{bmatrix} \begin{bmatrix} e_{t-1} \\ p_{t-1} \end{bmatrix} + \begin{bmatrix} \frac{\mu}{1+\mu(\kappa+\omega)}[\zeta y - \xi y^*] \\ \left(\gamma - 1 - \frac{\phi}{\lambda}\right) y + \frac{\sigma}{\lambda} m + \pi u \end{bmatrix} \tag{25}$$

With a view of the system of difference equations, we highlight two main areas for analysis: its solution and steady state equilibrium stability. For the basic case, we show the solution with a given initial value vector. Let the initial value vector be $\overline{G}_0 := \begin{bmatrix} e_0 \\ p_0 \end{bmatrix}$. The solution of the system is

$$\begin{aligned} \bar{G}_t = {} & \begin{bmatrix} \frac{1}{1+\mu(\kappa+\omega)} & 0 \\ \pi\delta & 1-\pi\delta-\frac{\pi\sigma}{\lambda} \end{bmatrix}^t \begin{bmatrix} e_0 \\ p_0 \end{bmatrix} + \\ & + \sum_{i=0}^{t-1} \begin{bmatrix} \frac{1}{1+\mu(\kappa+\omega)} & 0 \\ \pi\delta & 1-\pi\delta-\frac{\pi\sigma}{\lambda} \end{bmatrix}^{t-i-1} \begin{bmatrix} \frac{\mu}{1+\mu(\kappa+\omega)}[\zeta y - \xi y^*] \\ \left(\gamma - 1 - \frac{\phi}{\lambda}\right) y + \frac{\phi}{\lambda} m + \pi u \end{bmatrix}_i . \end{aligned} \tag{26}$$

Using Matlab software, running a simulation, we can plot the solution for the exchange rate and the price level. The parameters and exogenous variables were given specific values: $\mu = 0.5, \kappa = 0.3, \omega = 0.7, \pi = 0.8, \delta = 0.6, \sigma = 0.4, \lambda = 0.9, \zeta = 0.2, u = 0.8, \xi = 0.8, \gamma = 0.5, \phi = 0.5, \begin{bmatrix} e_0 \\ p_0 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} y^* = 15, y = 3, m = 50$. The effect of the coefficient matrix parameters on the solution curves is next investigated. The solution curves are shown in Figure 1.

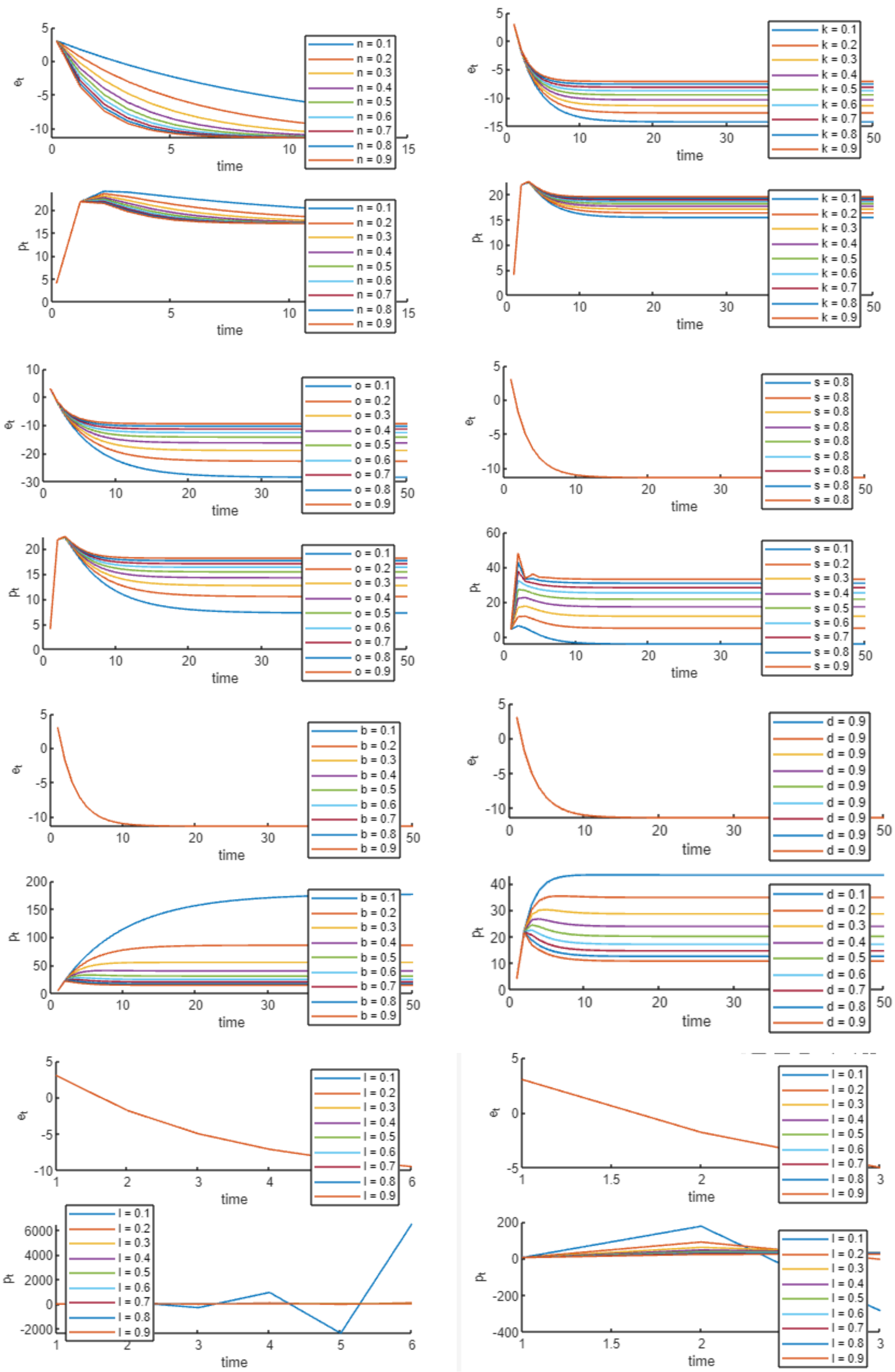

Figure 1: *The evolution over time of the price level and the exchange rate in a discrete model based on the Dornbusch model; given initial values, parameters and exogenous variables. The figure was created using Matlab software. an explanation of the letters in the figures:* $n = \mu, k = \kappa, b = \pi, d = \delta, l = \lambda, o = \omega, s = \sigma$.

Changing the values of the parameters $n = \mu, k = \kappa, o = \omega$, and $s = \sigma$ do not cause a significant alteration. Specifically, the solution curves converge to a steady state over time, even though there are differences in their values. While variations exist, both in direction and magnitude, these alterations do not bring about substantial changes. The role of time is to smooth out any short-term oscillations and outlier values as time progresses. In the case of $b = \pi$ and $d = \delta$, the situation is somewhat different: long-term adaptation is still a characteristic of these parameters; however, there is an impact on the dynamics of short-term values based on the specific values of these parameters. This is not surprising given the definitions of the parameters. The interest rate sensitivity of money demand (λ), exhibits distinct behavior compared to the other parameters. Examining both short-term and long-term scenarios reveals that a specific value for this parameter ($\lambda = 0.1$) results in a notably unstable solution. In the short term, it is evident that the values of the other parameters also influence the solution curve, but over the long term, their impact becomes negligible. The explanation for this phenomenon will be detailed in subsequent stability analyses, reflecting the stability properties of the steady-state vector of the differential equation system.

In the following, we look for the steady state conditions and continue our study by examining it. The steady state equilibrium vector of the model is

$$\bar{\mathcal{E}} = \left[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{1+\mu(\kappa+\omega)} & 0 \\ \pi\delta & 1 - \pi\delta - \frac{\pi\sigma}{\lambda} \end{bmatrix}\right]^{-1} \begin{bmatrix} \frac{\mu}{1+\mu(\kappa+\omega)}\left[\zeta y - \xi y^*\right] \\ \left(\gamma - 1 - \frac{\phi}{\lambda}\right) y + \frac{\sigma}{\lambda} m + \pi u \end{bmatrix}$$

$$\bar{\mathcal{E}} = \begin{bmatrix} \frac{1+\mu(\kappa+\omega)}{\mu(\kappa+\omega)} & 0 \\ \frac{(\delta\lambda\pi(\mu(\kappa+\omega)+1))}{\mu(\kappa+\omega)(\sigma+\delta\lambda)} & \frac{\lambda}{(\sigma+\delta\lambda)\pi} \end{bmatrix} \begin{bmatrix} \frac{\mu}{1+\mu*(\kappa+\omega)}\left[\zeta y - \xi y^*\right] \\ \left(\gamma - 1 - \frac{\phi}{\lambda}\right) y + \frac{\sigma}{\lambda} m + \pi u \end{bmatrix}$$

$$\bar{\mathcal{E}} = \begin{bmatrix} \frac{\zeta y - \xi y^*}{\kappa+\omega} \\ \frac{\lambda}{(\sigma+\delta\lambda)\pi}\left[\left(\gamma - 1 + \frac{\phi}{\lambda}\right) y + \frac{\sigma}{\lambda} m + \pi u\right] + \frac{\delta\lambda\pi[\zeta y - \xi y^*]}{(\kappa+\omega)(\sigma+\delta\lambda)} \end{bmatrix} \tag{27}$$

Next, we present the stability analysis. The steady state equilibrium of the difference equation system is stable if and only if $|I - A| \neq 0$, and if the eigenvalues of the coefficient matrix, A, are less than 1 in absolute value [7].

First condition: For proofing the first condition,

$$|I - A| = \frac{\mu(\kappa+\omega)}{1+\mu(\kappa+\omega)}\left(\frac{\pi\sigma}{\lambda} + \pi\delta\right).$$

Given the property of the parameters that each parameter is positive, it can be concluded that the value of the determinant is non-zero in all cases.

Second condition: To prove the second condition, we have to examine the eigenvalues of the coefficient matrix, which are: $\tau_1 = \frac{1}{\mu(k+\omega)+1}$, $\tau_2 = 1 - \frac{\sigma\pi}{\lambda} - \delta\pi$. For the first eigenvalue, the condition is satisfied for all parameter values. For the second eigenvalue the conditions are $-\frac{\sigma}{\lambda} < \delta$ and $2 > \frac{\sigma\pi}{\lambda} + \delta\pi$. The first condition for the second eigenvalue is satisfied because of the parameters' value. The second condition for the second eigenvalue is based on the sensitivity of interest rate from (9) and (11). All conditions satisfy during the simulation above.

Considering the stability condition $|\tau_2| < 1$, the upper bound is satisfied for all possible parameters since $\frac{\sigma\pi}{\lambda} + \delta\pi > 0$. For the lower bound we must have $-1 < 1 - \frac{\sigma\pi}{\lambda} - \delta\pi$, i.e. $\pi\left(\frac{\sigma}{\lambda} + \delta\right) < 2$, implying that only certain combinations of these parameters will fulfil the inequality. Therefore, it is hard to formulate general stability conditions, as e.g. $\pi < \frac{2}{\frac{\sigma}{\lambda}+\delta}$, the lower (higher) are δ and σ, the higher can (lower must) be π to ensure stability; the lower (higher) is λ, the lower must (higher could) be π. If $-1 < \tau_2 < 0$, i.e. if $1 < \pi\left(\frac{\sigma}{\lambda} + \delta\right) < 2$, the equilibrium will be attended cyclically, otherwise we will have a smooth motion of the price level.

Hence, the graphics in connection with changing λ parameter should be interpreted in the following way:

Cyclical development of the price level can be observed for small λ and for high σ. Taking the condition for stability and substituting the values for parameters π, δ, σ, and for π, δ, λ, resp., as given in the 2[nd] footnote, we have $1 < 0.8\left(\frac{0.4}{\lambda} + 0.6\right) < 2$ and $1 < 0.8\left(\frac{\sigma}{0.9} + 0.6\right) < 2$, therefore $1 < \frac{0.32}{\lambda} + 0.48 < 2$ and $1 < \frac{0.8}{0.9}\sigma + 0.48 < 2$, thus price level moves cyclically if $0.21 < \lambda < 0.615$, and $0.58 < \sigma < 1,71$. In the case of these parameters asymptotical stability exists for λ if $0.8\left(\frac{0.4}{\lambda} + 0.6\right) < 2$, i.e. if $0.27 < \lambda$, and for σ if $\sigma < 1,37$. Finally, the price level is stable for all possible σ parameters, and for all $0.58 < \sigma$ the price level moves cyclically. Due to parameter λ, the price level is stable for all $0.27 < \lambda$, and therefore unstable for $\lambda < 0.27$. Combing this with cyclical and non-cyclical development, we have the following classification (Table 3).

Value of λ	**Price level's development**
$0 < \lambda < 0.21$	noncyclical unstable
$0.21 < \lambda < 0.27$	cyclical unstable
$0.27 < \lambda < 0.615$	cyclical stable
$0.615 < \lambda < 1$	noncyclical stable

Table 3: *Stability conditions for interest rate sensitivity* (λ).

In summary, the solution of the discrete model, based on the Dornbusch model, plays a significant role for each parameter. The stability analysis of the steady state vector shows that the stability depends on the relationship between the σ, π, λ and δ parameters, i.e. the key factors are the sensitivities in connection with the rate if interest and the speed of adjustment of the price level's evolution in time. To test whether this condition is met, a simulation was carried out in which the value of each parameter was set between 0.1 and 0.9, with a step size of 0.1, in line with the economic interpretation. The results were tested with the Matlab software for all possible parameter combinations (6561 cases). The result is that the system is stable in 88.5% of the cases.

4. Expectation formation variability and its impact on the Dornbusch overshooting model

The summary table (Tables 4-5) below shows how the inclusion of each type of expectation changes the basic model. The purpose of modifying the basic model is to show that expectations, which are often misunderstood, play a fundamental role in the stability of the equilibrium state. In the case of the expected exchange rate, the expectations relation is incorporated into the system of difference equations in the form shown in Table 1. In the subsequent sections of this paper, we will thoroughly analyse the relationships outlined in the table.

Expectation	The system	Steady state equilibrium
Simple: $e^{exp} = e_{t-1}$	$\begin{bmatrix} e_t^{exp} \\ e_t \\ p_t \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & \frac{1}{1+\mu(\kappa+\omega)} & 0 \\ 0 & \pi\delta & 1-\pi\delta-\frac{\pi\sigma}{\lambda} \end{bmatrix} \begin{bmatrix} e_{t-1}^{exp} \\ e_{t-1} \\ p_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\mu}{1+\mu(\kappa+\omega)}[\zeta y-\xi y^*] \\ \left(\gamma-1-\frac{\phi}{\lambda}\right) y+\frac{\sigma}{\lambda}m+\pi u \end{bmatrix}$	$\bar{\mathcal{E}} = [I-A]^{-1} b = \begin{bmatrix} \frac{[\zeta y-\xi y^*]}{\kappa+\omega} \\ \frac{[\zeta y-\xi y^*]}{\kappa+\omega} \\ \left[\left(\gamma-1-\frac{\phi}{\lambda}\right) y+\frac{\sigma}{\lambda}m+\pi u\right]\frac{\delta\lambda(\mu(\kappa+\omega)+1)}{\mu(\kappa+\omega)(\sigma+\delta\lambda)} \end{bmatrix}$
Static: $e_t^{exp} = e_{t-1}^{exp} = e^{exp}$	$\begin{bmatrix} e_t^{exp} \\ e_t \\ p_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{1+\mu(\kappa+\omega)} & 0 \\ 0 & \pi\delta & 1-\pi\delta-\frac{\pi\sigma}{\lambda} \end{bmatrix} \begin{bmatrix} e_{t-1}^{exp} \\ e_{t-1} \\ p_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\mu}{1+\mu(\kappa+\omega)}[\zeta y-\xi y^*] \\ \left(\gamma-1-\frac{\phi}{\lambda}\right) y+\frac{\sigma}{\lambda}m+\pi u \end{bmatrix}$	$\bar{\mathcal{E}} := [I-A]^{-1} b$ cannot be determined because the condition that $\lvert I-A \rvert = 0$.
Adaptive: $e_t^{exp} = e_{t-1}^{exp} + \beta\left(e_{t-1} - e_{t-1}^{exp}\right)$, $0<\beta<1$	$\begin{bmatrix} e_t^{exp} \\ e_t \\ p_t \end{bmatrix} = \begin{bmatrix} \beta & 1-\beta & 0 \\ 0 & \frac{1}{1+\mu(\kappa+\omega)} & 0 \\ 0 & \pi\delta & 1-\pi\delta-\frac{\pi\sigma}{\lambda} \end{bmatrix} \begin{bmatrix} e_{t-1}^{exp} \\ e_{t-1} \\ p_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{\mu}{1+\mu(\kappa+\omega)}[\zeta y-\xi y^*] \\ \left(\gamma-1-\frac{\phi}{\lambda}\right) y+\frac{\sigma}{\lambda}m+\pi u \end{bmatrix}$	$\bar{\mathcal{E}} := [I-A]^{-1} b = \begin{bmatrix} \frac{[\zeta y-\xi y^*]}{\kappa+\omega} \\ \frac{[\zeta y-\xi y^*]}{\kappa+\omega} \\ \frac{[\zeta y-\xi y^*]\pi\delta\lambda+(\kappa+\omega)\left[\left(\gamma-1-\frac{\phi}{\lambda}\right)y+\frac{\sigma}{\lambda}m+\pi u\right]}{(\kappa+\omega)(\sigma+\delta\lambda\pi)} \end{bmatrix}$
Rational: $e_t^{exp} = e_t$	$\begin{bmatrix} e_t^{exp} \\ e_t \\ p_t \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{1+\mu(\kappa+\omega)} & 0 \\ 0 & \frac{1}{1+\mu(\kappa+\omega)} & 0 \\ 0 & \pi\delta & 1-\pi\delta-\frac{\pi\sigma}{\lambda} \end{bmatrix} \begin{bmatrix} e_{t-1}^{exp} \\ e_{t-1} \\ p_{t-1} \end{bmatrix} + \begin{bmatrix} \frac{\mu}{1+\mu(\kappa+\omega)}[\zeta y-\xi y^*] \\ \frac{\mu}{1+\mu(\kappa+\omega)}[\zeta y-\xi y^*] \\ \left(\gamma-1-\frac{\phi}{\lambda}\right) y+\frac{\sigma}{\lambda}m+\pi u \end{bmatrix}$	$\bar{\mathcal{E}} := [I-A]^{-1} b = \begin{bmatrix} \frac{\zeta y-\xi y^*}{\kappa+\omega} \\ \frac{\zeta y-\xi y^*}{\kappa+\omega} \\ \frac{\lambda}{\sigma+\delta\lambda\pi}\left[\left(\gamma-1+\frac{\phi}{\lambda}\right) y+\frac{\sigma}{\lambda}m+\pi u\right]+\frac{\delta\lambda\mu[\zeta y-\xi y^*]}{(\kappa+\omega)(\pi\delta\lambda+\sigma)} \end{bmatrix}$

Table 4: *Different types of expectations in the model: simple, static, adaptive and rational and their effects for the stability conditions.*

Expectation	The system	Conditions for the stability	Key factors (eigenvalues of the coefficient matrix)
Simple: $e^{exp} = e_{t-1}$	$\begin{bmatrix} e_t^{exp} \\ e_t \\ p_t \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & \frac{1}{1+\mu(\kappa+\omega)} & 0 \\ 0 & \pi\delta & 1-\pi\delta-\frac{\pi\sigma}{\lambda} \end{bmatrix} \begin{bmatrix} e_{t-1}^{exp} \\ e_{t-1} \\ p_{t-1} \end{bmatrix} +$ $+ \begin{bmatrix} 0 \\ \frac{\mu}{1+\mu(\kappa+\omega)} [\zeta y - \xi y^*] \\ \left(\gamma - 1 - \frac{\phi}{\lambda}\right) y + \frac{\sigma}{\lambda} m + \pi u \end{bmatrix}$	$\lvert I-A \rvert =$ $= \frac{\mu(\kappa+\omega)}{\mu(\kappa+\omega)+1} \left(\pi\delta + \frac{\pi\sigma}{\lambda}\right)$	$\tau_1 = 0$ $\tau_2 = \frac{1}{1+\mu(\kappa+\omega)}$ $\tau_3 = 1 - \pi\delta - \frac{\pi\sigma}{\lambda}$
Static: $e_t^{exp} = e_{t-1}^{exp} =$ $= e^{exp}$	$\begin{bmatrix} e_t^{exp} \\ e_t \\ p_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{1+\mu(\kappa+\omega)} & 0 \\ 0 & \pi\delta & 1-\pi\delta-\frac{\pi\sigma}{\lambda} \end{bmatrix} \begin{bmatrix} e_{t-1}^{exp} \\ e_{t-1} \\ p_{t-1} \end{bmatrix} +$ $+ \begin{bmatrix} 0 \\ \frac{\mu}{1+\mu(\kappa+\omega)} [\zeta y - \xi y^*] \\ \left(\gamma - 1 - \frac{\phi}{\lambda}\right) y + \frac{\sigma}{\lambda} m + \pi u \end{bmatrix}$	–	–
Adaptive: $e_t^{exp} = e_{t-1}^{exp} +$ $+ \beta \left(e_{t-1} - e_{t-1}^{exp}\right),$ $0 < \beta < 1$	$\begin{bmatrix} e_t^{exp} \\ e_t \\ p_t \end{bmatrix} = \begin{bmatrix} \beta & 1-\beta & 0 \\ 0 & \frac{1}{1+\mu(\kappa+\omega)} & 0 \\ 0 & \pi\delta & 1-\pi\delta-\frac{\pi\sigma}{\lambda} \end{bmatrix} \begin{bmatrix} e_{t-1}^{exp} \\ e_{t-1} \\ p_{t-1} \end{bmatrix} +$ $+ \begin{bmatrix} 0 \\ \frac{\mu}{1+\mu(\kappa+\omega)} [\zeta y - \xi y^*] \\ \left(\gamma - 1 - \frac{\phi}{\lambda}\right) y + \frac{\sigma}{\lambda} m + \pi u \end{bmatrix}$	$\lvert I-A \rvert =$ $= (1-\beta) \left(\frac{\mu(\kappa+\omega)}{1+\mu(\kappa+\omega)}\right) =$ $= \left(\pi\delta + \frac{\sigma}{\lambda}\right)$	$\tau_1 = \beta$ $\tau_2 = \frac{1}{1+\mu(\kappa+\omega)}$ $\tau_3 = 1 - \pi\delta - \frac{\sigma}{\lambda}$
Rational: $e_t^{exp} = e_t$	$\begin{bmatrix} e_t^{exp} \\ e_t \\ p_t \end{bmatrix} = \begin{bmatrix} 0 & \frac{1}{1+\mu(\kappa+\omega)} & 0 \\ 0 & \frac{1}{1+\mu(\kappa+\omega)} & 0 \\ 0 & \pi\delta & 1-\pi\delta-\frac{\pi\sigma}{\lambda} \end{bmatrix} \begin{bmatrix} e_{t-1}^{exp} \\ e_{t-1} \\ p_{t-1} \end{bmatrix} +$ $+ \begin{bmatrix} \frac{\mu}{1+\mu(\kappa+\omega)} [\zeta y - \xi y^*] \\ \frac{\mu}{1+\mu(\kappa+\omega)} [\zeta y - \xi y^*] \\ \left(\gamma - 1 - \frac{\phi}{\lambda}\right) y + \frac{\sigma}{\lambda} m + \pi u \end{bmatrix}$	$\lvert I-A \rvert =$ $= \left(\frac{\mu(\kappa+\omega)}{1+\mu(\kappa+\omega)}\right) \left(\pi\delta + \frac{\sigma}{\lambda}\right)$	$\tau_1 = 0$ $\tau_2 = \frac{1}{\mu(k+\omega)+1}$ $\tau_3 = 1 - \frac{\sigma}{\lambda} - \delta\pi$

Table 5: *Different types of expectations in the model: simple, static, adaptive and rational and their effects for the stability conditions.*

In the specific cases, the stability properties are examined in detail. At first glance at the table, it is noticeable that the stability conditions of the steady state vectors for each expectation are very similar. So similar, in fact, that the stability of the steady state vector is determined by the same four parameters in the basic case.

In the case of a **simple expectation** case, by studying the values in the steady state vector, it can be concluded that in the steady state, the expected exchange rate and the actual realised exchange rate are the same as each other. This implies that the inclusion of simple expectations in the model leads to efficient expectation formation, i.e. the expected exchange rate and the current exchange rate are the same in steady state equilibrium. Next, we present the stability analysis. The stability conditions are in Tables 4–5.

First condition: $|I - A|$ cannot be zero, because all parameters are positive.

Second condition: It can be seen that all three eigenvalues satisfy the condition that $|\tau_i| < 1$, $i = 1, \dots, 3$. For τ_1 and τ_2 the conditions satisfy. In case of τ_3, the situation is more complicated, just as in the basic case.

The **static case** is not relevant because the necessary condition for the existence of the stationary vector of the system of difference equations is not satisfied, since the determinant of the difference between the unit matrix and the coefficient matrix is zero. Failure to include a static expectation sends the message that considering an exogenous, constant value as an expectation, independent of the variables and parameters of the model, is not appropriate and will not produce relevant results. The message we can give to the modelling economist is to reject this case.

The third option is the **adaptive expectation**. Suppose that economic agents go through a learning process and that in each period they take into account how much they were wrong about the value of the exchange rate in the previous period. The system of equations, the steady state vector and the stability conditions are given in the table above. In the case of adaptive expectation, it should be remembered that the model is given in log-linear form, so the relationship in the cell is also log-linear. This does not logically change the learning process, i.e. adaptive expectation theory. In summary, this means nothing more than that in the non-log-linearised model we are working with the following relationship: $E_t^{exp} = E_{t-1}^{exp}\left(\frac{E_{t-1}}{E_{t-1}^{exp}}\right)$, where the variables are the original value of the exchange rate and the expected exchange rate. Looking at the elements of the steady state vector, we conclude that the adaptive expectation method works because the elements in the vector are the same for the expected exchange rate and the current exchange rate. Applying adaptive expectations, exchange rate expectations, on the other hand, have no effect on the long-run equilibrium state of the current exchange rate. Thus, the long-run equilibrium exchange rate is not affected by expectations in this form. The long-run equilibrium price level is determined by the money market, the foreign exchange market and, of course, the commodity market; formally arguing: simply because their parameters are included in the vector. It is surprising that the beta parameter of adaptive expectations does not play a role in this case. That is, in the case of steady state equilibrium, the extent to which economic agents take into account their error from the previous period in the expected and current exchange rate does not play a role. We continue with a stability analysis of the steady state equilibrium:

First condition: the determinant of the difference between the unit matrix and the coefficient matrix is detailed in Tables 4-5. Taking the values of the parameters into consideration, it can be concluded that the condition is fulfilled.

Second condition: The first eigenvalue is the parameter defined in adaptive expectations. It is specified as $0 < \beta < 1$. In case of the second and third eigenvalue, the conditions are identical to the one calculated for the base case without expectations. The properties of adaptive expectation are discussed in more detail during the simulation.

The case of rational expectation can be approached in different ways, as we have already mentioned in the study. Since Dornbusch also assumed perfect foresight in his model. We were interested in Shone's approach to perfect foresight [17]: $e_t^{exp} - e_{t-1}^{exp} = e_t - e_{t-1}$. Using this equation, previous calculation implies the model as follows:

$$\begin{bmatrix} e_t^{exp} \\ e_t \\ p_t \end{bmatrix} = \begin{bmatrix} 1 & \frac{-\mu(\kappa+\omega)}{1+\mu(\kappa+\omega)} & 0 \\ 0 & \frac{1}{1+\mu(\kappa+\omega)} & 0 \\ 0 & \pi\delta & 1-\pi\delta-\frac{\sigma}{\lambda} \end{bmatrix} \begin{bmatrix} e_{t-1}^{exp} \\ e_{t-1} \\ p_{t-1} \end{bmatrix} + \begin{bmatrix} \frac{\mu}{1+\mu(\kappa+\omega)}\left[\zeta y - \xi y^*\right] \\ \frac{\mu}{1+\mu(\kappa+\omega)}\left[\zeta y - \xi y^*\right] \\ \left(\gamma - 1 - \frac{\phi}{\lambda}\right) y + \frac{\sigma}{\lambda} m + \pi u \end{bmatrix} \tag{28}$$

In this variant, the current exchange rate in the expected exchange rate context is replaced by the current exchange rate context as a function of the previous time interval. Here, however, the stability of the steady state vector is not satisfied, because $|I - A| = 0$. In the other approach to perfect foresight, where we face a "perfect static expectation", this version of the model simply reproduces the static case.

In the other approach, the expected exchange rate and the current exchange rate are the same for the same time interval, when we apply the Muth-type definition of the rational expectation, i.e. $e_t^{exp} = e_t$. This alternative is also shown in Tables 4-5. Looking at the steady state vector, we can see that the expected and current exchange rates coincide in this variant as well. In addition, the stability properties are comparable to the previous cases.

In this section we showed analytically how different types of expectations can modify the steady state stability conditions of the same model. It is shown that static expectations and perfect foresight do not yield results, whereas in the simple, adaptive and Muth-type rational cases, all three types of expectations work well in the sense that the expected and current exchange rates are the same in the long-run equilibrium state based on the steady state vector.

5. Simulation tests on the solution curves of the modified discrete model for different expectation types

We have seen in the previous section that the inclusion of different types of expectations in the model does not change the congruence of the expected and current exchange rate in the steady state vector (except the static expectation). In this section we focus on the time evolution of the expected exchange rate, the current exchange rate, and the price-level during simulations. The solutions are defined and presented as the following cases: simple expectation, adaptive expectation, rational expectation. The parameters and values are the same as in the base case, the following $\mu = 0.5, \kappa = 0.3, \omega = 0.7, \pi = 0.8, \delta = 0.6, \sigma = 0.4, \lambda = 0.9, \zeta = 0.2, u = 0.8, \xi = 0.8, \gamma = 0.5, \phi = 0.5, \begin{bmatrix} e_0 \\ p_0 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}, y^* = 15, y = 3, m = 50$. During the simulations, suppose that the initial values of the expected and current exchange rates are the same in case of simple and adaptive expectatios. In case of rational expectation, the initial values of the expected and current exchange rates differ, to illustrate the rapid adjustment process of the expectation type. The order of the figures is as follows: general model, simple case, adaptive case ($\beta = 0.4, 0.6, 0.8, 0.99$), rational case.

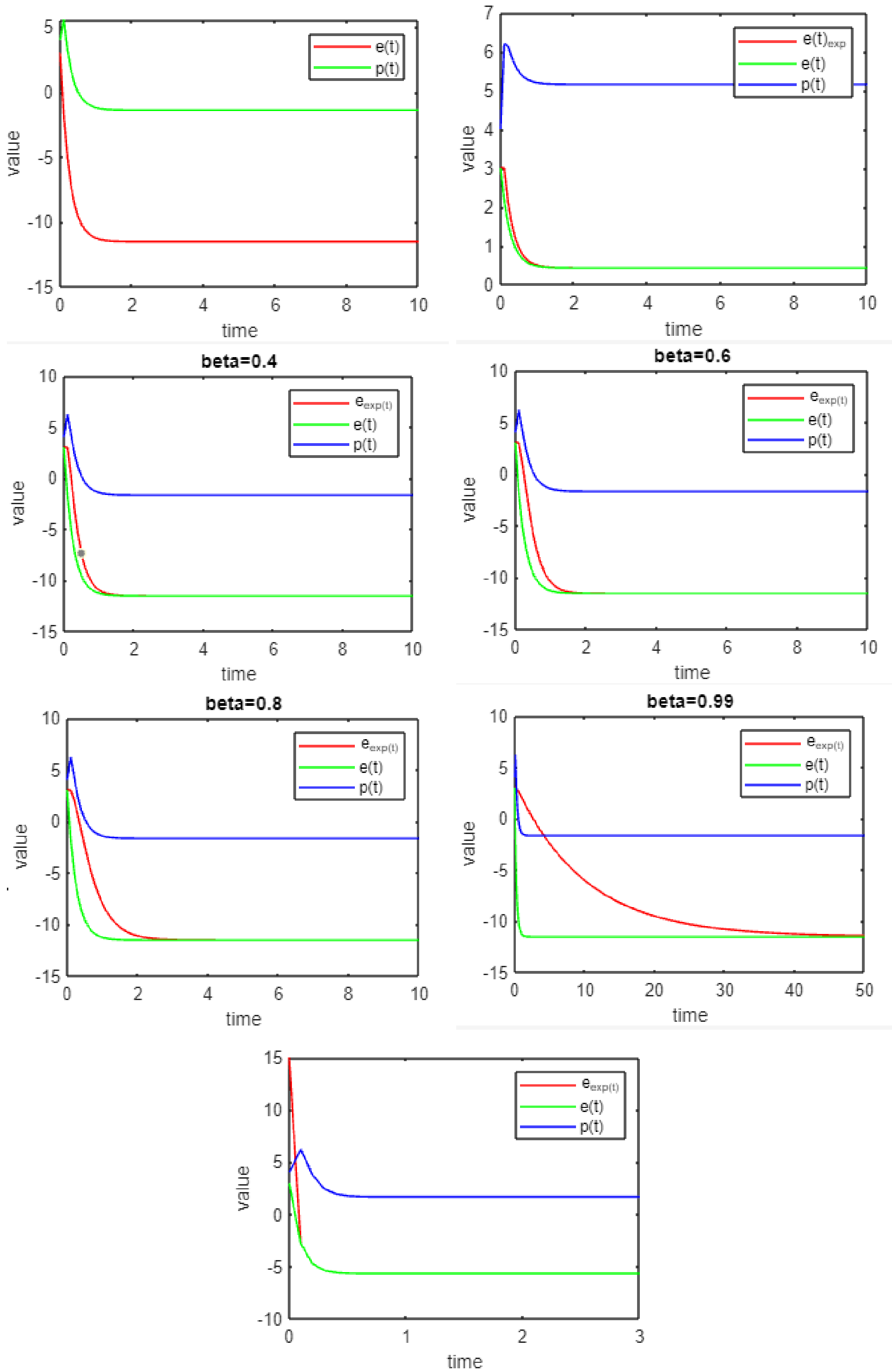

Figure 2: *Solution curves for e_t, p_t, e_t^{exp} for the basic model, the simple expectation case, the adaptive expectation case ($\beta = 0.4, 0.6, 0.8, 0.99$) and the rational case. The figures were created using Matlab software.*

In the baseline model, the initial, short-lived decline is replaced by a steady state for both the exchange rate and the price level. By including simple expectations before the steady state, a periodic lag between the expected and the observed exchange rate is clearly visible. This version of the model introduces an interesting phenomenon for the price level: short-term overshooting. The price level starts to rise significantly away from the initial value and then, after reaching its maximum, starts to fall until it reaches a constant level.

In the case of adaptive expectations, the β parameter indicates the weight that the operator gives to the previous period's error (in our case, this is done in aggregate). The higher the β, the greater the weight of the previous period's error in the estimate. Looking at the solution curves from this point of view, an interesting phenomenon emerges: the larger the β, the longer the learning process: the later the expected exchange rate curve catches up with the observed exchange rate curve. It is interesting to note that if we allowed $\beta = 1$ in the context of adaptive expectation, we would get static expectation. In the case of rational expectations, the simulation clearly shows that the expected value and the current value coincide from the beginning I have changed the expected rate to 15 in this case to show the immediate one-step adjustment to the expected rate.

In summary, the steady state is established in all examined cases, both for the price level and the exchange rate, in a short period. The use of different expectation types results in the correspondence of the expected exchange rate to the current exchange rate occurring at different times depending on which expectation type we worked with. The most rapidly adapted is the rational expectation, and the slowest is the adaptive expectation, where the beta value is close to 1 (i.e. close to the static expectation pattern).

6. Conclusion

In our study, we investigated the effect of the types of expectations used in the literature on a specific model, the Dornbusch overshooting model (Dornbusch 1976). We focused on defining the steady state equilibrium, investigating its stability properties and the global solution with given parameters, exogenous variables and initial value. There is no consensus in the literature on the use of different types of expectations: for example, the interpretation of perfect foresight and rational expectations as one concept is widespread. From our research in the case of the discrete Dornbusch model, perfect foresight case does not work in this context. For simple, adaptive, and Muth-style rational expectations, short-term divergences and oscillations gradually converge over time, leading to the establishment of a steady state. In the case of adaptive expectations, the expectation parameter plays a significant role in the adjustment process. A higher expectation parameter delays the convergence of expected and current exchange rates.

7. Summary

One of the main problems for modelling economists is to model the behaviour of economic agents. Given that the last decade has seen the emergence of several types of expectations that initially proved useful, later proved more complicated, or raised concerns (e.g. adaptive expectations lacking predictive power, as opposed to rational expectations, etc.), the question arises as to how the types of expectations adopted in the literature can modify the properties of models. An effective way to do this is to test the same model with achieving expectation types. The key message of the study is that, in the literature, commonly considered types of expectations show short-term effects, but in the long run, the concepts of simple, adaptive, and rational expectations all hold true: the expected exchange rate aligns with the current exchange rate. Additionally, each expectation type has unique characteristics among stability conditions, but the stability is determined by the interplay of fundamental parameters.

As a further research objective, we would like to highlight the value of testing the theoretical model and examining alternative expectation types.

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