A cost analysis of single-server discouraged arrivals with differentiated vacation queueing model

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Abstract. This paper investigates an M/M/1 queueing system with differentiated vacations and discouraged arrivals, focusing on two types of vacations. The server switches to type I vacation with rate γ_1 when the system is empty during an active state. If no customers are waiting when it returns from a type I vacation, it then switches to a type II vacation with a rate of γ_2. Both vacation times and service duration follow exponential distributions. The study utilises the Probability Generating Function (PGF) technique to derive steady-state solutions for both vacation policies. Furthermore, the research explores relevant performance metrics and provides numerical examples to illustrate the system's behaviour under various conditions. The cost analysis of the M/M/1 differentiated vacation system with discouraged arrival queueing and various aspects of the system's behaviour under different arrival rates (λ, λ_1, λ_2) are discussed.

Keywords: cost analysis, differentiated vacations, discouraged arrivals, PSO algorithm, steady state solution.

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1. Introduction

Queueing theory is a branch of applied probability theory with numerous applications, including communication networks, manufacturing facilities, and computer systems. In the traditional queueing strategy, the server is always accessible; however, real-world scenarios may arise where the server becomes unreachable.

When the server finishes serving a unit and finds that the system is empty, it is known as a vacation. Queueing systems with server vacations have garnered significant interest from researchers. In a survey, the primary goal, as described by Doshi [10], is to provide a comprehensive understanding of vacations. The paper demonstrates how analysing alternative vacation models becomes more manageable by comprehending the behaviour of these queueing models. Additionally, the available results are applied to a few selected real-life applications. In another study Levy and Yechiali [21], the utilisation of idle time in an M/G/1 queueing system is analyzed. To prevent the server from being completely inactive, additional work is performed during the vacation. Upon completion of the vacation, the server rejoins the main network. The survey Haviv [15] discusses the strategic timing of arrivals. The book Tian and Zhang [23], have explored various vacation model categories due to their wide range of applications in interaction, technological networks, and manufacturing facilities.

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Servers can take two types of vacations. If type I vacation is complete and no more customers are present, the server switches to type II vacation; this is defined as a differentiated vacation. The study considers differentiated vacations, vacation interruptions, and impatient customers (balking and reneging). Recursive methods are used to obtain the explicit expression of the probability, followed by sensitivity analysis which are analysed in Bouchentouf and Guendouzi [5]. An infinite single-server Markovian queueing model with both single and multiple vacation policies, as well as working breakdowns, repairs, balking, and reneging, is analysed by Chettouf et al [9] for a customer care centre. A finite capacity multi-server Markovian queueing model with Bernoulli feedback, synchronous multiple vacation policies, and impatient customers is discussed by Bouchentouf et al [4]. Numerous real-world systems, such as contact centres, manufacturing processes, and contemporary information and communication technology networks, have implemented the proposed queueing model. A single-server Markovian feedback queue with variations of different vacation policies, balking, the server's state-dependent reneging, and retention of reneged customers was examined by Bouchentouf et al [3]. In an analysis of a multi-server queue with impatient customers and Bernoulli feedback, Bouchentouf et al [7] considered a variation with multiple vacations. The probability-generating function approach is used to solve differential equations and derive steady-state probabilities using the Chapman-Kolmogorov equations. In Afroun et al [2], an M/M/1/N queueing system with various vacations, Bernoulli feedback, balking, reneging, and retention of the impatient customers, as well as the potential for a server failure and repair, are examined. Using the Q-matrix (infinitesimal generator matrix) approach, the system's steady-state probabilities are determined. A feedback queueing system featuring a form of multiple vacation policy, balking, the server's state-dependent reneging, and the retention of reneged customers were discussed by Cherfaoui et al [8]. Working vacations and vacation interruptions are covered by threshold policies. The system's performance metrics and steady-state probability are derived from the application of the Successive over-Relaxation (SoR) technique. Additionally, a quasi-Newton optimisation technique is used for an optimum analysis. Ultimately, a conclusion, several numerical examples, and a discussion of the future's potential are provided by Kumar et al [19]. Bouchentouf et al [6] establishes a cost optimisation analysis for an M/M/1/N queueing system with differentiated working vacations, Bernoulli schedule vacation interruption, balking and reneging. Suranga sampath and Liu [22] used various analytical tools, including the Laplace transform, probability-generating functions, and explicit mean and variance systems. Transient state probabilities are calculated, and the study explores the system's behaviour.

Vijayashree and Janani [27] evaluates the transient solution of an M/M/1 queue with differentiated vacation. Vijayashree and Ambika [26] analyses the concept of an M/M/1 queue with differentiated vacation, vacation interruption, and customer impatience. The study examines the mean and variance, presenting a comprehensive analysis.

In a separate study Ebenesar and Chandrika [12], a single-server retrial queueing model with Markovian arrival processes for customer arrivals is discussed. The M/G/1 retrial queueing system has two simultaneous vacation modes. Performance metrics and numerical results are discussed. An analysis is carried out by Ebenesar et al [11] to examine the steady-state behaviour of a single-server retrial queueing model that includes server breakdown and frequent vacation. Performance metrics are considered using supplemental variable approaches.

Admission management is discussed in Ebenesar and Chandrika [13] to balance effective system utilisation by providing acceptable performance metrics. The server's condition determines whether each customer may access the system. Accepted customers receive the first necessary service, with the option for a second service or to exit after the service is rendered. During certain periods, arrivals are restricted due to an extended queue, known as discouraged arrivals. The concept is particularly relevant during the pandemic (COVID), when restrictions are imposed on arrivals from other countries and crowded places. Kumar and Sharma [20] explains a finite Markovian single-server queuing model with discouraged arrivals, reneging, and retention of reneged customers. The steady-state solution is derived. Performance measures are obtained, and special cases of the model are explored.

Hur and Paik [16] explores an M/G/1 queue subject to a regulatory policy with a general server setup time. The arrival rate fluctuates based on the idle, setup, and busy states of the server. The steady-state queue length distribution function and the Laplace-Stieltjes transform of waiting time are derived. In Hassin et al [14], the RASTA phenomenon is explored, where customers decide whether to join or block queues based on the implications of their entry-level choices. In Tian et al [24], a repairable M/M/1 retrial queueing model with setup delays is reviewed. The server is closed down to reduce operating costs after the system is empty, and the system won't start up until a new customers arrives. Steady-state probability, performance measures, and the effect of some parameter costs are evaluated.

Rasheed and Manoharan [1] examines the concept of discouraged arrival in Markovian queuing systems, where the rapid rate of service is controlled based on the number of customers within the system. The study determines the steady-state probability and other performance metrics for this adaptive queueing system.

The differentiated vacation concepts are discussed by various authors. Ibe and Isijola [17] initially proposed differentiated vacations, which are categorised into longer and shorter durations in this paper. The paper then provides numerical examples with different arrival rates. In another study Isijola and Ibe [18], differentiated vacation with vacation interruption is described. According to Vadivukarasi and Kalidass [25], differentiated vacations lead to bulky entry queues. The matrix geometry approach is used to determine stability criteria, and the probability-generating function is employed to determine system size. The PSO approach is used to examine optimal service rates. Our investigation in this article focuses on the new concept of discouraged arrival rates. This study examines the total cost of the suggested model. The PSO method was also used to determine the system's cost-effectiveness.The findings of this article are displayed in the table together with the different arrival rates and discouraged arrival rates of the cost values and expected number of customers.

This paragraph discusses two kinds of vacations, type I and type II, where the type I vacation rate is lower than the type II vacation rate. Section 2 provides a detailed explanation of this model. In Section 3, the transition diagram, the local balance equation, and the probability for the busy state are obtained. Performance measures are described in Section 4, with numerical examples provided in Section 5. A cost analysis is presented in Section 6, and Section 7 introduces the PSO algorithm for the model. Special cases are provided in Section 8. Practical application is explained in Section 9. Conclusion of this model are described in section 10.

2. The System Descriptions

The proposed model operates under the following assumptions:

Arrival Process:

- The queue has an infinite capacity to accommodate customers.
- The customer arrival process follows a Poisson distribution with a rate of λ.

Service Operation:

- Customers are admitted into the system based on the First-Come-First-Serve (FCFS) principle.
- Service times are modelled to follow an exponential distribution with the parameter μ.

Vacation Types:

- In this model, vacations are incorporated into two categories: type I vacation, denoted by γ_1, and type II vacation, denoted by γ_2. As indicated by the relationship $\gamma_2 \leq \gamma_1$, type II vacation occurs less than type I vacation.

- Customers are served by the server when it is in an active state. Upon completing service during an active state, the server immediately transitions to type I vacation.

- After finishing the type I vacation, the server returns to an active state. If a new customer is present, the server provides immediate service; otherwise, the server switches to a type II vacation.

Arrival Rate Restriction during Vacations:

- During vacation periods, customer arrivals are restricted based on the following rates: $\lambda_2 \leq \lambda_1 \leq \lambda$.

- Here, λ_1 represents the discouraged arrival rate during type I vacation, and λ_2 represents the discouraged arrival rate during type II vacation.

3. Steady-State Solution

Let $N(t)$ be the number of customers in the system at time t and $J(t)$ be the state of the service provider at time t as

$$J(t) = \begin{cases} 0, \text{ while the server are in active state,} \\ 1, \text{ if the server is on first vacation,} \\ 2, \text{ if server is on second vacation.} \end{cases} \tag{1}$$

Then $\{(J(t), N(t)), t \geq 0\}$ is a state-space Markov process.

Let $p_{i,j}$ be the probability that the service provider be in the i^{th} state $(i = 0, 1, 2)$ with $j(\geq 0)$ number of customers.

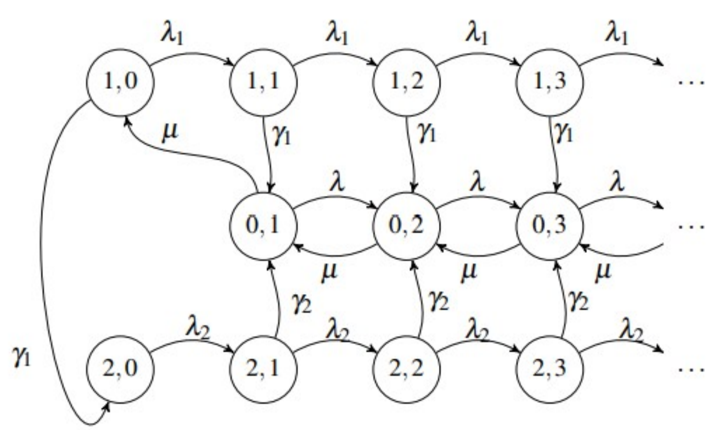

Figure 1: *State transition diagram.*

The steady-state balancing flow equations of the proposed model are as follows:

$$(\lambda+\mu)p_{0,1} = \mu p_{0,2}+\gamma_1 p_{1,1}+\gamma_2 p_{2,1} \quad (2)$$
$$(\lambda+\mu)p_{0,n} = \mu p_{0,n+1}+\gamma_1 p_{1,n}+\gamma_2 p_{2,n}+\lambda p_{0,n-1}, n\geq 2 \quad (3)$$
$$(\lambda_1+\gamma_1)p_{1,0} = \mu p_{0,1} \quad (4)$$
$$(\lambda_1+\gamma_1)p_{1,n} = \lambda_1 p_{1,n-1}, n\geq 1 \quad (5)$$
$$\lambda_2 p_{2,0} = \gamma_1 p_{1,0} \quad (6)$$
$$(\lambda_2+\gamma_2)p_{2,n} = \lambda_2 p_{2,n-1}, n\geq 1 \quad (7)$$

Let,

$$P_i(z)=\sum_{n=1}^{\infty} p_{i,n}z^n, \quad i=0,1,2$$

be the function that generates probabilities of active state and the vacations states.

By summing up all the possible values of n and by multiplying by z^n to equations (1) to (6), we the probability generating functions of active state and vacations states (type I and type II) respectively,

$$P_0(z) = \frac{\mu z p_{0,1}-\gamma_1 z P_1(z)-\gamma_2 z P_2(z)}{\lambda z^2-(\lambda+\mu)z+\mu} \quad (8)$$
$$P_1(z) = \frac{\lambda_1 z}{\lambda_1(1-z)+\gamma_1}p_{1,0} \quad (9)$$
$$P_2(z) = \frac{\lambda_2 z}{\lambda_2(1-z)+\gamma_2}p_{2,0} \quad (10)$$

Substitute $z=1$ in equations (7), (8) and (9), we obtain,

$$P_0(1) = \frac{\mu((\gamma_1^2(\lambda_2+\gamma_2))+(\lambda_1\gamma_2(\lambda_1+\gamma_1)))}{\gamma_1\gamma_2(\lambda_1+\gamma_1)(\mu-\lambda)}p_{0,1} \quad (11)$$
$$P_1(1) = \frac{\lambda_1\mu}{\gamma_1(\lambda_1+\gamma_1)}p_{0,1} \quad (12)$$
$$P_2(1) = \frac{\mu\gamma_1}{\gamma_2(\lambda_1+\gamma_1)}p_{0,1} \quad (13)$$

Finally, by using the rule of total probability,

$$P_0(1)+P_1(1)+p_{1,0}+P_2(1)+p_{2,0}=1 \quad (14)$$

where

$$p_{1,0} = \frac{\mu}{\lambda_1+\gamma_1}p_{0,1} \quad (15)$$
$$p_{2,0} = \frac{\mu\gamma_1}{\lambda_2(\lambda_1+\gamma_1)}p_{0,1} \quad (16)$$

The probability that the server is in an active state is as follows,

$$p_{0,1} = \frac{\lambda_2\gamma_1\gamma_2(\lambda_1+\gamma_1)(\mu-\lambda)}{\mu\gamma_1^2(\lambda_2+\gamma_2)(\mu+\lambda_2-\lambda)+\mu\lambda_2\gamma_2(\lambda_1+\gamma_1)(\mu+\lambda_1-\lambda)} \quad (17)$$

4. Performance Measures

The expected number of customers when a service provider is in an active state is:

$$E(L_B) = P_0'(1) \tag{18}$$

$$= \frac{\mu^2(((\gamma_1^3)(\lambda_2+\gamma_2)(\lambda_2+\gamma_2)) + (\gamma_2^2(\lambda_1+\gamma_1)(\lambda_2+(\lambda_1\gamma_1)))) - \mu(((\lambda\lambda_2\gamma_1^3)(\lambda_2+\gamma_2)) + (\lambda_3\gamma_2^2)(\lambda_1+\gamma_1))}{\gamma_1^2\gamma_2^2(\lambda_1+\gamma_1)(\mu-\lambda)^2} p_{0,1} \tag{19}$$

The expected number of customers during type I vacation is:

$$E(L_{v_1}) = P_1'(1) \tag{20}$$

$$= \frac{\lambda_1\mu}{\gamma_1^2} p_{0,1} \tag{21}$$

The expected number of customers in the system during type II vacation is:

$$E(L_{v_2}) = P_2'(1) \tag{22}$$

$$= \frac{\mu\gamma_1(\lambda_2+\gamma_2)}{\gamma_2^2(\lambda_1+\gamma_1)} p_{0,1} \tag{23}$$

The total average number of customers in the system is

$$E(L) = E(L_B) + E(L_{v_1}) + E(L_{v_2}) \tag{24}$$

Expected waiting time:

$$E(W) = \frac{E(L)}{\lambda} \tag{25}$$

5. Numerical Analysis

This section presents various numerical examples to illustrate the influence of different parameters, including arrival rate, service rate, and vacation rate, on performance measures.

The relationship between the likelihood of one customer being in an active condition is depicted in Figure 2. With a fixed service rate, $p_{0,1}$ falls whenever λ rises. This figures indicate varying service rates in this instance. Additionally, it emphasised that when service rates rise, $p_{0,1}$ falls. Depending on the service rate, it demonstrates that $p_{0,1}$ increases with λ. That means that whenever the arrival rates increase, the probability that the server will take a type I vacation when the system is empty also increases. Figures 3 and 4 show the impact on $p_{0,1}$ of the discouraging arrival rate during type I and type II vacations, respectively. As the discouraged arrival rate increases during type I vacation, Figure 3 illustrates a decrease in $p_{0,1}$. A decrease in $p_{0,1}$ is also shown in Figure 4 when the rate of discouraged arrivals increases during type II vacation.

The expected number of customers in the system under various situations was evaluated in Figures 5, 6, and 7. As average arrival rates increase, Figure 5 illustrates an increase in the expected number of customers, indicating that arrival rates result in a larger number of customers in the system. The number of customers in the system appears to be decreased by the server's vacation under this policy, as Figure 6 shows the decrease in the expected number of customers during type I vacation. As the discouraged arrival rates during type II vacation increase, Figure 7 shows an increase in the expected number of customers, suggesting that a higher rate of discouraged arrivals during type II vacation results in higher numbers of customers in the system.

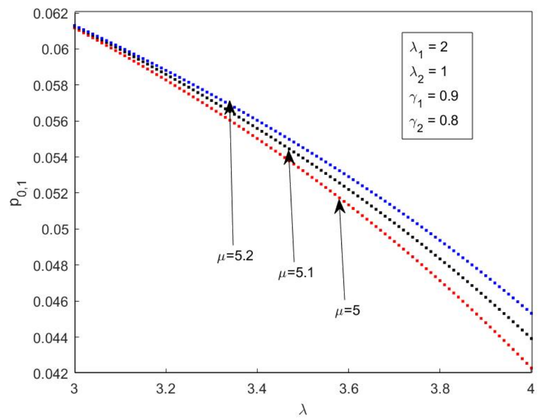

Figure 2: $p_{0,1}$ *against* λ.

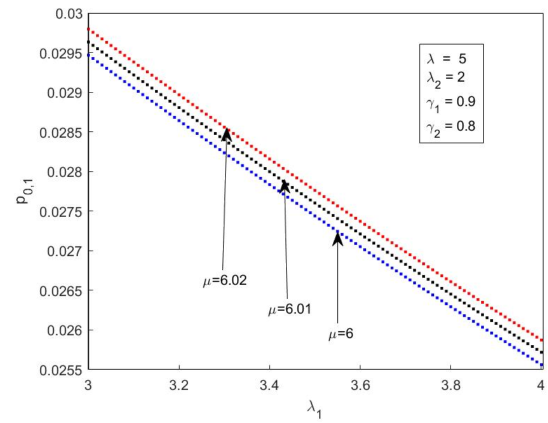

Figure 3: $p_{0,1}$ *against* λ_1.

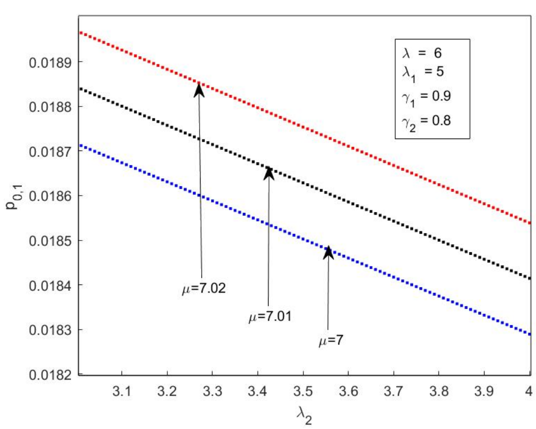

Figure 4: $p_{0,1}$ *against* λ_2.

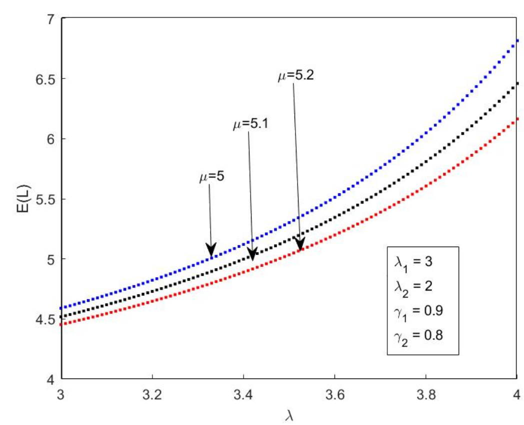

Figure 5: $E(L)$ *against* λ.

The expected waiting time in the system under various situations was evaluated in Figures 8, 9, and 10. As arrival rates increase, Figure 8 illustrates an increase in the expected waiting time with a distinct service rate. As the discouraged arrival rate (during type I vacation) increases, Figure 9 indicates that expected waiting time increases with different service rates. Likewise, as the discouraged arrival rates (during type II vacation) increase, Figure 10 shows expected waiting time increasing with different service rates in the system.

Figure 11 shows that the expected number of customers in the system decreases as the service rate increases with differnet λ. According to this, an increased service rate results in a decrease in customers in the system, which may decrease the waiting time of customers and increase overall effectiveness. Figure 12 shows that the estimated waiting time in the queue decreases for varying arrival rates as the service rate increases. Thus it suggest that longer waiting times for customers are achieved with higher service rates.

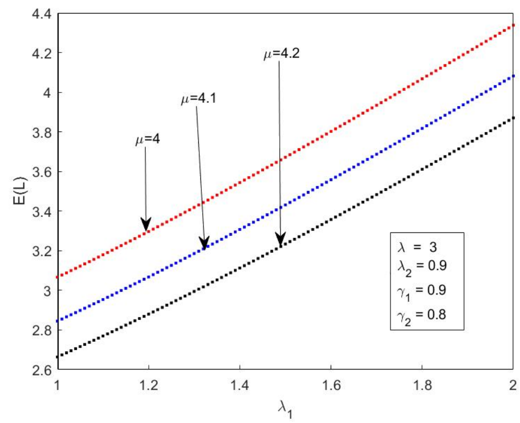

Figure 6: $E(L)$ *against* λ_1.

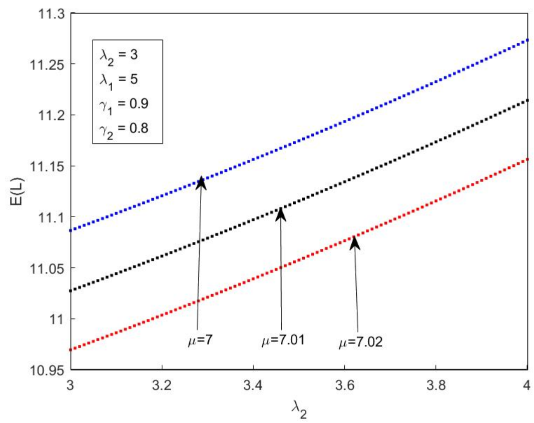

Figure 7: $E(L)$ *against* λ_2.

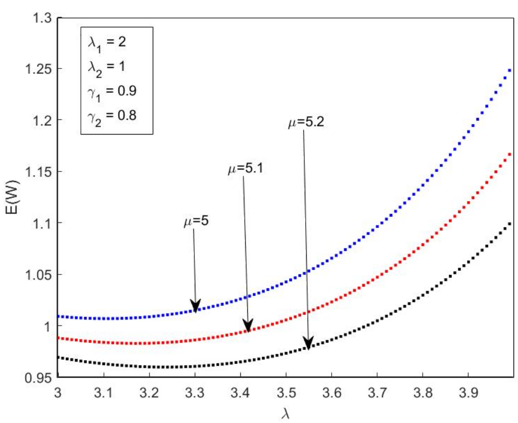

Figure 8: $E(W)$ *against* λ.

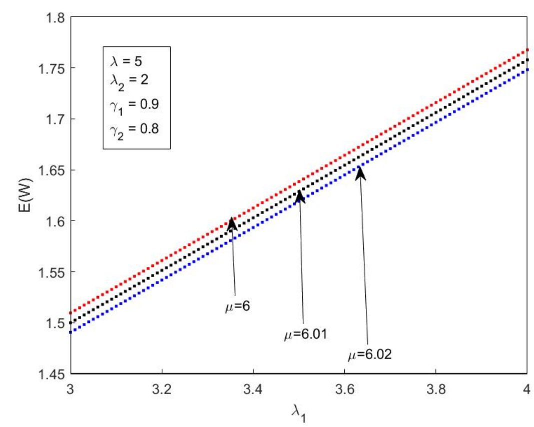

Figure 9: $E(W)$ *against* λ_1.

6. Cost Analysis

Define the cost function TC as

$$TC = C_N E(L) + C_W E(W) + C_0 P_0 + \sum_{i=1}^{2} C_i P_i + C_\mu \tag{26}$$

where,
C_N = holding cost for each customers seen in the system;
C_W = waiting cost if one customers is to receive the service;
C_0 = cost for the period the server handling service process;
C_i = cost when the server is on i^{th} type vacations (i=1,2)
C_μ = cost for service.
$p_{0,n}$ = probability when the server is on active state,
$p_{i,n}$ = probability when the server is on the i^{th} type vacations (i=1,2)
E(L) = the expected number of customers in the system,
E(W) = the expected waiting time of a customers in the system, respectively.

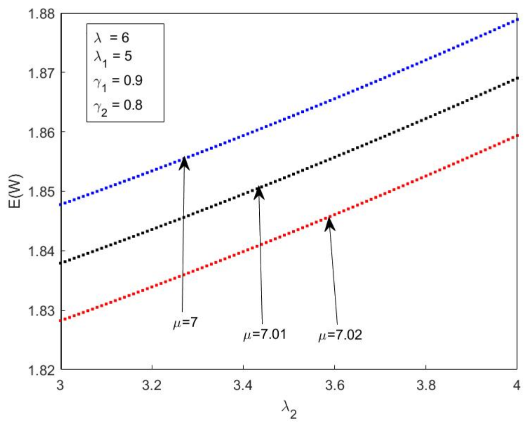

Figure 10: $E(W)$ *against* λ_2.

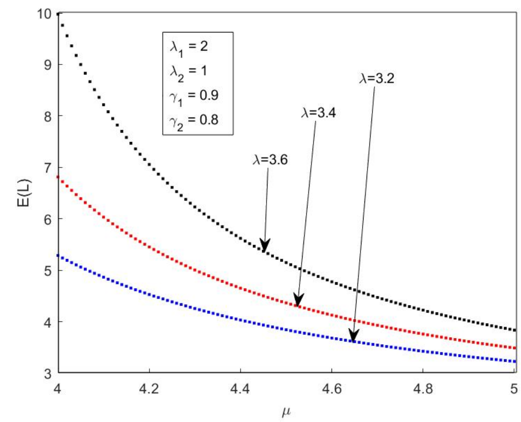

Figure 11: $E(L)$ *against* μ.

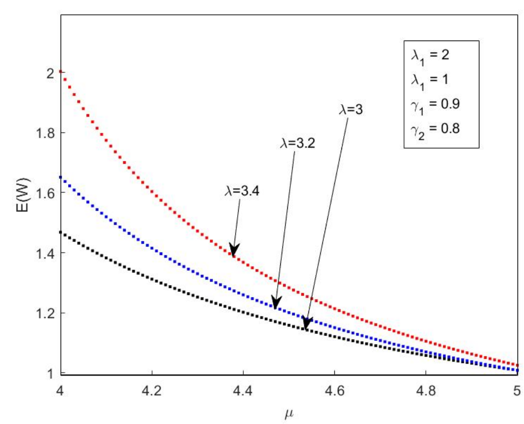

Figure 12: $E(W)$ *against* μ.

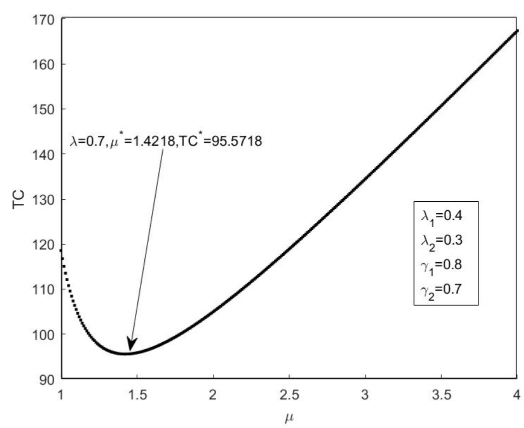

Figure 13: TC *against* λ.

In this section, the cost analysis of the model is studied, as it is very important in designing, improving, and maintaining the model from a cost-benefit point of view. The section 6 Total Cost function (TC) is nonlinear and too complicated to calculate analytically. Numerical optimisation techniques then make it simple to find the ideal value of the service rate. The lowest costs and service rate are shown in Figure 13 using a convex graphical representation. From the figure, it is observed that if the service rate increases, the total cost becomes convex. The expected cost function is optimised by using the PSO method. Figures 14 and 15 show the optimum total cost and service rate with different λ. Since the curves are convex, the presence of ideal values are required. Furthermore, it is noted that, as would be expected intuitively, with λ, both the ideal service in concern and its expected expenses grow.

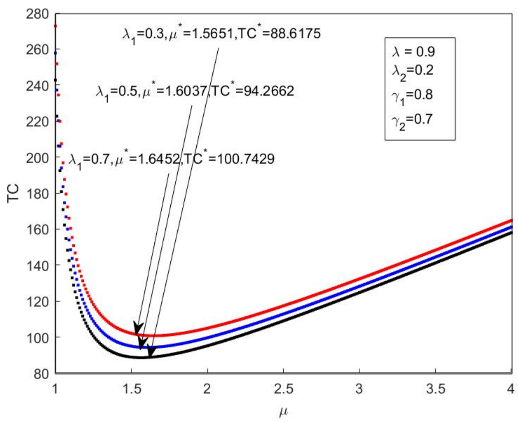

Figure 14: TC *against* λ_1.

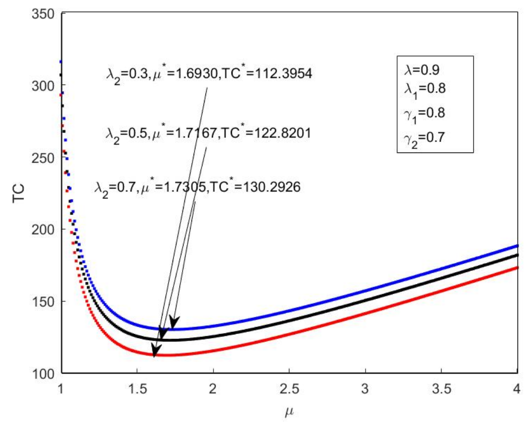

Figure 15: TC *against* λ_2.

7. PSO Algorithm

PSO is a computational method for optimising the service and total cost. One effective technique for figuring out a complicated function's optimal value is the PSO algorithm. It uses location, position and velocity control to find the optimal path based on particle movement. That means minimising the cost corresponding to the best service rate.

When using the PSO algorithm, one can evaluate the service rate behaviour, the expected number of customers, and the cost analysis, as shown in Table 1. The service time, expected number of customers, and cost do increase whenever the arrival rate increases. We consider the arrival rate and discouraged arrival rate on type II vacations to be constant. In type I vacations, the discouraged arrival rate increases, followed by the service rate, the expected number of customers, and the total cost, which are also shown in Table 2. Table 3 shows that in the case that the discouraged arrival rate in type II vacation grows, the corresponding service rate, projected number of customers, and overall cost would all increase. The preceding three tables were analysed using the Particle Swarm Optimisation (PSO) algorithm. The results show that the service rate, expected customer count, and overall cost of the queueing system all increase in tandem with increases in arrival rates and discouraged arrival rates (types I and II). Consider vacation rates and discouraged arrival rates as constants. This observation highlights the sensitivity of these performance metrices to changes in arrival patterns and vacation types, emphasising the importance of accurately modelling and managing these factors in queueing systems to optimise system efficiency and cost-effectiveness. These findings contribute valuable insights to the field of queueing theory. Articulately in understanding the dynamic nature of queueing systems and the implications of varying arrival and vacation rates on system performance.

λ	μ^*	$E(L)$	$E(W)$	TC^*
0.5	1.2272	0.9750	1.9499	94.2419
0.6	1.3224	1.0221	1.7035	94.3628
0.7	1.4218	1.0659	1.5227	95.5718
0.8	1.5238	1.1069	1.3837	97.4617
0.9	1.6275	1.1457	1.2730	99.8042

Table 1: *PSO values for different* λ.

λ_1	μ^*	$E(L)$	$E(W)$	TC^*
0.3	1.56	0.89	0.99	88.61
0.4	1.58	0.95	1.05	91.32
0.5	1.60	1.02	1.13	94.26
0.6	1.62	1.09	1.21	97.41
0.7	1.64	1.17	1.31	100.74

Table 2: *PSO values for different* λ_1.

λ_2	μ^*	$E(L)$	$E(W)$	TC^*
0.3	1.69	1.47	1.63	112.39
0.4	1.70	1.63	1.81	118.20
0.5	1.71	1.76	1.95	122.82
0.6	1.71	1.87	2.08	126.76
0.7	1.73	1.98	2.20	130.29

Table 3: *PSO values for different* λ_2.

8. Special Cases

- While substituting, λ_1, λ_2=λ then the probability coincide with Ibe[2014] as,

$$p_{0,1} = \frac{\lambda\gamma_1\gamma_2(\mu-\lambda)(\lambda+\gamma_1)}{\mu^2(\lambda\gamma_2(\lambda+\gamma_1)+\gamma_1^2(\lambda+\gamma_2))} \tag{27}$$

- Putting λ_2=λ and $\gamma_1\rightarrow\infty$ the the PGF becomes,

$$P_2(z) = \frac{\mu z}{\lambda(1-z)+\gamma_2}p_{0,1} \tag{28}$$

$$P_0(z) = \frac{\mu z(z-1)(\lambda+\gamma_2)}{(\lambda(1-z)+\gamma_2)(-\lambda z^2+(\lambda+\mu)z)}p_{0,1} \tag{29}$$

$$p_{0,1} = \frac{\lambda\gamma_2(\mu-\lambda)}{\mu^2(\lambda+\gamma_2)} \tag{30}$$

The above results coincides with M/M/1 single vacation queueing system.

- Putting $\gamma_2 \to \infty$ and $\lambda_2 \to \infty$ the the PGF becomes,

$$P_0(z) = \frac{\mu z(\lambda_1(z-1)(2\gamma_1+\lambda_1)-\gamma_1^2)}{(1-z)((\lambda z(\lambda+\gamma_1)(\gamma_1+\lambda_1(1-z)))+(\mu(\lambda_1+\gamma_1)(\lambda(z-1)-\gamma_1)))} p_{0,1} \quad (31)$$

$$P_1(z) = \frac{\lambda_1 \mu z}{(\lambda_1+\gamma_1)(-\lambda_1 z+\lambda_1+\gamma_1)} p_{0,1} \quad (32)$$

$$p_{0,1} = \frac{\gamma_1(\lambda_1+\gamma_1)(\mu-\lambda)}{\mu(\gamma_1^2+(\lambda_1+\gamma_1)(\mu+\lambda_1-\lambda))} \quad (33)$$

The above results coincides with M/M/1 single vacation with discouraged arrival queueing system.

9. Practical Application

In many real-world situations, the service facility has been safeguarded from having to wait a long time. Long wait times might discourage potential customers and force servers to improve the quality of their services. Thus, while bearing in mind that queueing systems are state-dependent, it is beneficial to do studies on them. To prevent lengthy queues from accumulating in computer and communication systems, for instance, the congestion management mechanism adjusts packet transmission rates based on the length of the queue at the source or destination. In this case, the arrival rate will vary according to the status of the server. It is more useful for waiting length reduced and provide more convenient.

10. Conclusion

This paper has examined the M/M/1 differentiated vacation queue with a discouraged arrival rate, utilising the probability-generating function approach to formulate the queueing model. Steady-State Probabilities and Performance Measures are also derived in this paper. The investigation into total cost for arrival rate λ, along with discouraged arrival rates λ_1 and λ_2, has provided valuable insights.

Future research will expand upon these findings by investigating an M/M/C discouraged arrival queueing model with varying arrival rates. This extension will further enhance the understanding of queueing systems with differentiated vacations and discouraged arrivals, offering potential applications in various real-world scenarios.

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