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Wind curl vs variable eddy viscosity: A Northern Adriatic related modelling study

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The influence of different magnitudes of the vertically constant eddy viscosity, as well as heterogeneity in the wind field, on the wind induced motions in the Northern Adriatic has been considered in several recent modelling studies. Those studies, incorporating partial field data verification, have suggested several lines of improving the Northern Adriatic model predictions, among them more adequate treatment of the vertical eddy viscosity. This paper presents preliminary results of the Northern Adriatic model that allows for the vertically variable eddy viscosity. The results indicate an improvement in current field prediction which has been called for by previous model to data comparisons.

Vrtložnost u polju vjetra i varijabilna turbulentna viskoznost: modelska studija na primjeru Sjevernog Jadrana

U nekoliko nedavnih modelskih studija analiziran je utjecaj različitih iznosa vertikalno konstantnog koeficijenta turbulentne viskoznosti, te nehomogenosti u polju vjetra, na vjetrom uzrokovane struje u Sjevernoj Jadranu. Spomenute studije, koje uključuju i djelomičnu verifikaciju modela empirijskim podacima, sugeriraju nekoliko smjerova mogućih poboljšanja sjevernojadranskog modela. Jedno od mogućih poboljšanja je i primjerenija formulacija vertikalne turbulentne viskoznosti. U ovom radu su izloženi preliminarni rezultati primjene modela Sjevernog Jadrana koji dopušta vertikalnu promjenljivost spomenutog koeficijenta. Analiza predikcija strujnog polja ukazuje da se uvođenjem varijabilne vertikalne turbulentne viskoznosti mogu umanjiti, pa i odstraniti neki nedostaci uočeni prilikom ranijih usporedbi s empirijskim podacima.

1. Introduction

In our recent modelling studies the influence of different magnitudes of the vertically constant eddy viscosity coefficient (Kuzmić et al., 1985), as well as heterogeneity in the wind field (Orlić et al., 1986) on the wind-induced motions in the Northern Adriatic has been considered. The magnitude of the eddy viscosity coefficient have proved to be of considerable influence on the magnitude and particularly direction of the current vectors at different depths. The wind-field heterogeneity studies have suggested that the wind curl is the most energetic source of variability in the fields commonly considered in the analysis (the elevation of sea surface and different velocity fields). Comparisons of model predictions to available and appropriately processed field data (multi-level current meter measurements at several locations) have shown considerable similarity between the measured and model-generated vectors in terms of magnitude, direction and relative position of different-depth vectors, but all three aspects have left room for improvements. The improvements can be achieved by more complete sources of field data (e.g. better assessment of wind field characteristics over the sea) or more appropriate model formulations. A possible contribution to the latter could be more adequate treatment of vertical eddy viscosity, e.g. by making it depth dependent. Therefore, a three-dimensional, hydrodynamical numerical model has been developed that allows for vertically variable eddy viscosity.

This paper presents preliminary results of a modelling study of combined influences of the wind curl and variable vertical eddy viscosity, based on the new model. The model formulation is briefly presented in the second section. Its application to the Northern Adriatic, assuming realistic coastal geometry but flat bottom in order to exclude the topographic effect, is the subject of the third section. Four basic, reference cases are presented and discussed in the same section.

2. Model formulation

A three-dimensional, linear, hydrodynamical, numerical model has been used in this study of the wind field heterogeneity and eddy viscosity variability and their influence on the motions in the Northern Adriatic. The governing equations are derived assuming homogeneous and incompressible water, hydrostatic motion and constant Coriolis parameter. Furthermore, neglecting the advective terms and lateral shear, the equations of continuity and motion may be written as:

$$\frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial \sigma}\int_{0}^{1} u h \, d\sigma + \frac{\partial}{\partial y}\int_{0}^{1} v h \, d\sigma = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} - fv = -g\frac{\partial \zeta}{\partial \sigma} + \frac{1}{h}\frac{\partial}{\partial \sigma}\left(\frac{N(\sigma)}{h}\frac{\partial u}{\partial \sigma}\right) \tag{2}$$

$$\frac{\partial v}{\partial t} + fu = -g\frac{\partial \zeta}{\partial y} + \frac{1}{h}\frac{\partial}{\partial \sigma}\left(\frac{N(\sigma)}{h}\frac{\partial v}{\partial \sigma}\right) \tag{3}$$

where t denotes time;

ζ	is the elevation of the water surface;
u, v	are the horizontal components of currents at depth z;
h	is the undisturbed depth of water;
σ	is the transformed vertical coordinate, positive downwards;
$N(\sigma)$	is the coefficient of vertical eddy viscosity;
f	is the Coriolis parameter; and
g	is the acceleration of gravity.

The equations (1) – (3) are written in coordinate system (x, y, σ, t) in which the horizontal coordinates are laid at the undisturbed sea surface and the vertical coordinate is stretched using the transformation $\sigma = z/h\,(x, y)$. The transformation is similar to the one proposed by Phillips (1957) and is often used when both variable topography and vertical resolution is important (e.g. Paul and Lick, 1974).

The usual condition of rest is assumed initially ($\zeta = u = v = 0$). Conditions prescribed at the surface and bottom boundaries are:

$$\frac{N(\sigma)}{h}\frac{\partial u}{\partial \sigma} = -\frac{\tau_{xs}}{\rho}, \quad \frac{N(\sigma)}{h}\frac{\partial v}{\partial \sigma} = -\frac{\tau_{ys}}{\rho} \quad \text{at } \sigma = 0 \tag{4}$$

and

$$\frac{N(\sigma)}{h}\frac{\partial u}{\partial \sigma} = -\frac{\tau_{xb}}{\rho}, \quad \frac{N(\sigma)}{h}\frac{\partial v}{\partial \sigma} = -\frac{\tau_{yb}}{\rho} \quad \text{at } \sigma = 1 \tag{5}$$

The stresses are defined as:

$$\tau_{xs} = C_D\,\rho_a u_a \sqrt{u_a^2 + v_a^2}\,, \quad \tau_{ys} = C_D\,\rho_a v_a \sqrt{u_a^2 + v_a^2} \tag{6}$$

and

$$\tau_{xb} = k\rho u(\sigma{=}1)\;, \qquad \tau_{yb} = k\rho v(\sigma{=}1) \tag{7}$$

In the above relations u_a and v_a denote the wind components, ρ_a is the density of air, ρ is the density of sea water, C_D is nondimensional drag coefficient and k is the coefficient of bottom friction.

Along the solid boundary zero normal horizontal flow is assumed:

$$(u, v)_n = w_n = 0 \tag{8}$$

while a radiation condition of the form:

$$\overline{w_n} + \sqrt{gh}\,\frac{\zeta}{h} = 0 \tag{9}$$

where overbar denotes the vertical averaging, is postulated at the open boundary.

Before the numerical solution is attempted the equations (1) – (3) are transformed using an eigenfunction, integral method pioneered by Heaps (1972). The transformed equations read:

$$\frac{\partial \zeta}{\partial t} + \sum_{r=1}^{M} [\frac{\partial}{\partial x}(h a_r \varphi_r u_r) + \frac{\partial}{\partial y}(h a_r \varphi_r v_r)] = 0 \tag{10}$$

$$\frac{\partial u_r}{\partial t} + \lambda_r u_r - f v_r = -g a_r \frac{\partial \zeta}{\partial x} + \frac{\tau_{xs}}{\rho h}, r = 1, M \tag{11}$$

$$\frac{\partial v_r}{\partial t} + \lambda_r v_r + f u_r = -g a_r \frac{\partial \zeta}{\partial y} + \frac{\tau_{ys}}{\rho h}, r = 1, M \tag{12}$$

where

u_r, v_r are transformed velocity components;

λ_r is the r-th eigenvalue of the vertical eddy viscosity operator;

φ_r, a_r are coefficients depending on the r-th eigenvalue;

M is number of modes.

At each time step one can recover the u and v components of velocity by a finite inverse transformation:

$$u(\sigma) = \sum_{r=1}^{M} \varphi_r u_r \psi_r(\sigma) \quad , \quad v(\sigma) = \sum_{r=1}^{M} \varphi_r v_r \psi_r(\sigma) \tag{13}$$

where $\psi_r(\sigma)$ is the r-th eigenfunction of the vertical eddy viscosity operator.

In this model the formulation of Heaps is generalized in a sense that the eigenproblem is solved numerically in a separate program which allows for rather general formulation of the vertical eddy viscosity. Consequently, the eigenvalues λ_r, eigenfunctions ψ_r, and coefficients a_r and φ_r are precomputed and treated as an input to the model. Details of this model formulation can be found in Kuzmić (1986). Equations (10)–(12) are solved numerically using forward-time staggered-space finite difference scheme.

3. Numerical experiments and results

The model described in the previous section has been applied to the Northern Adriatic. The part of the Adriatic considered in the model is shown in Figure 1. As can be seen from the figure, the coastal geometry is reasonably well represented. The real topography, however, has not been considered in order to exclude the topographic effect. Boundaries of the modelled area are schematized to fit a field of 31 x 24 rectangular boxes of 7.5 km in both x (northeastward) and y (northwestward) directions. With this grid size and depth of 40 m the CFL criterion was satisfied, with significant margin, with the time step of 2 min. Actually, the step has been retained from real-topography runs (maximum depth of 60 m). The parameters f, ρ, and g were set equal to 1.031 x 10^{-4} s^{-1}, 1025 kg m^{-3} and 9.81 m s^{-2} respectively. The drag coefficient C_D = 2.5 10^{-3} and ρ_a = 1.247 kg m^{-3} have been used in all calculations of the wind stress. The wind velocity has been of southwestward direction, and magnitude of 10 m s^{-1} in the homogeneous case. In the heterogeneous case the velocity has been allowed to

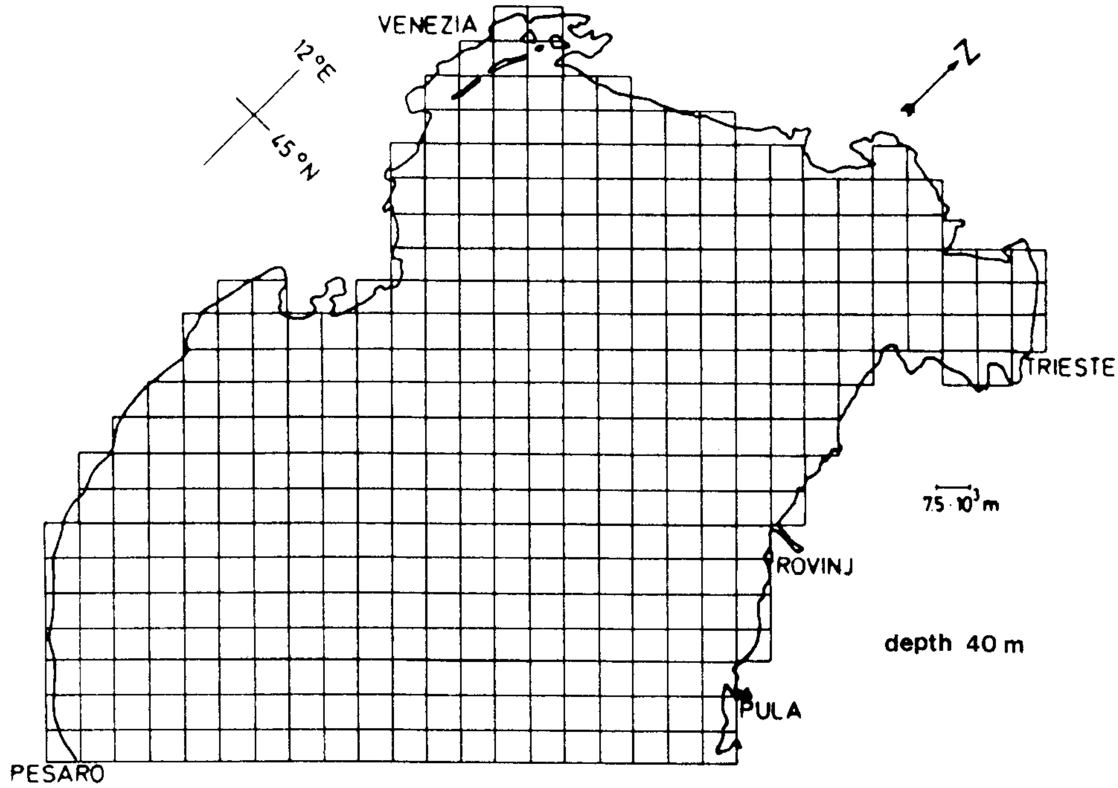

Figure 1. Coastal geometry and finite-difference grid of the Northern Adriatic model.

vary from 10 m s^{-1} at both ends towards zero in the middle (see Figures 2–5). This schematized form of wind curl was selected to reflect the evidence of bura wind being weaker near Rovinj than near Pula and Trieste (Orlić et al., 1986). The vertical eddy viscosity coefficient, N, has been kept at 0.01 m^2 s^{-1} in the constant case and allowed to linearly decrease by an order of magnitude in the variable case (see Figures 2–5). The coefficient of linear bottom friction has been kept equal to 2.5 x 10^{-3} in all runs.

With all the necessary parameters set, four runs have been performed to assess the influence of wind heterogeneity and vertically variable eddy viscosity. The runs represent four cases: a) uniform wind – constant viscosity, b) uniform wind – decreasing viscosity, c) wind curl – constant viscosity, and d) wind curl – decreasing viscosity. Each time the model was run for 48 simulated hours. Assuming that the transient behaviour is well characterized by this two-day period, four output fields were then analysed. The fields were those of the sea level displacement, vertically averaged current, surface

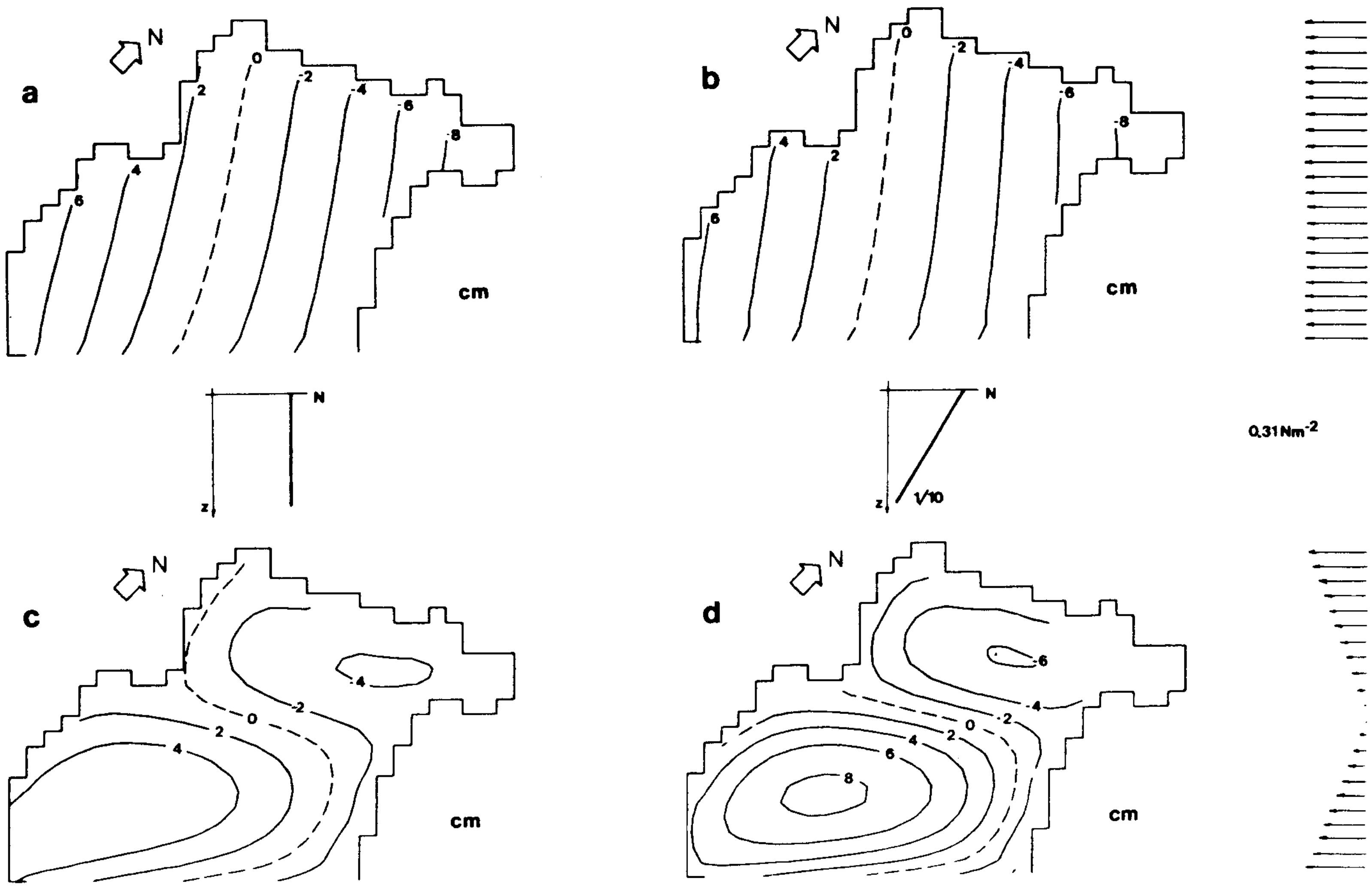

Figure 2. Sea level distribution predicted by the model.
(a) uniform wind – constant viscosity, (b) uniform wind – variable viscosity, (c) wind curl – constant viscosity.
(d) wind curl – variable viscosity.

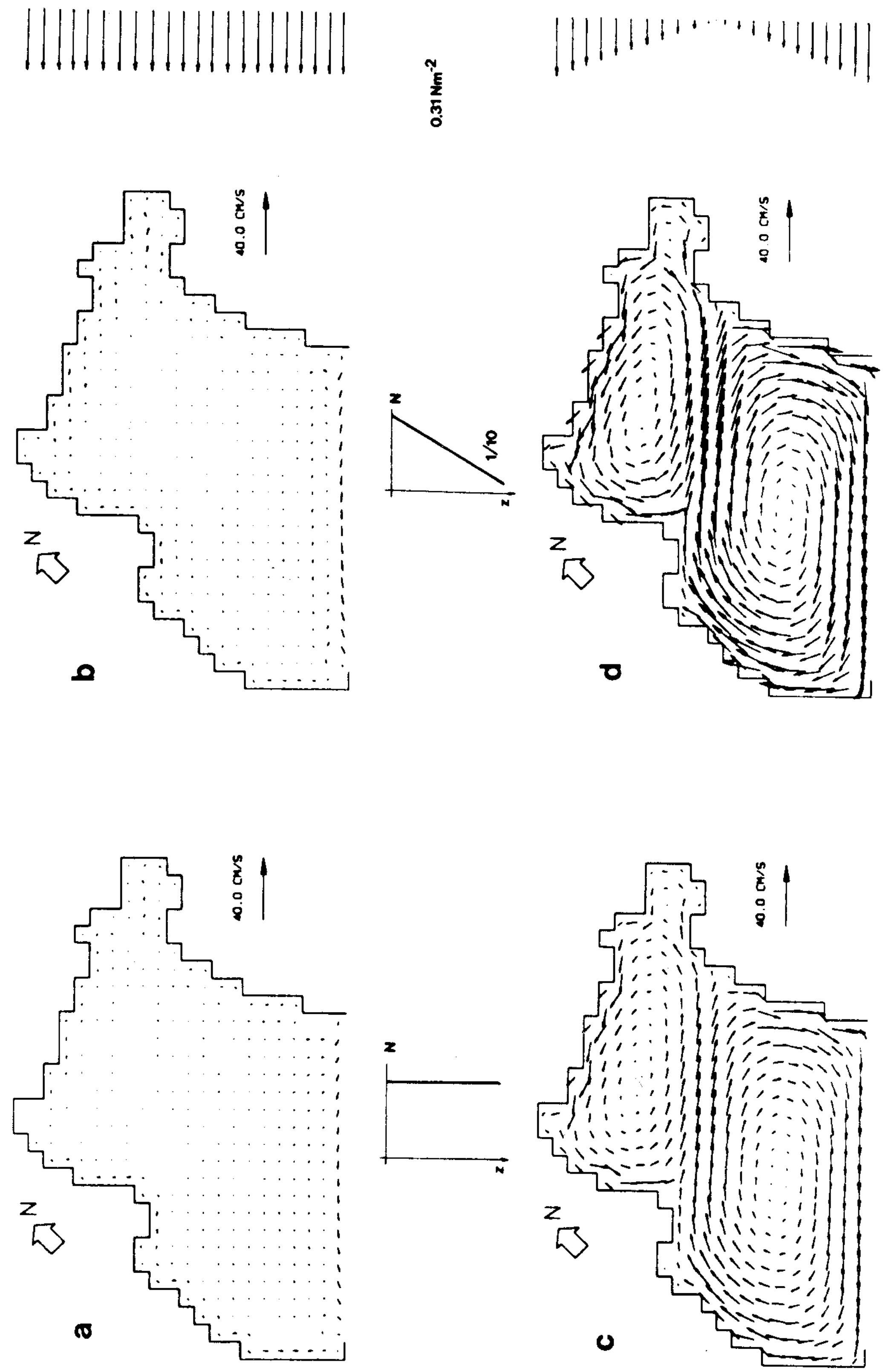

Figure 3. Vertically averaged currents predicted by the model. (a) – (d) as in Figure 2.

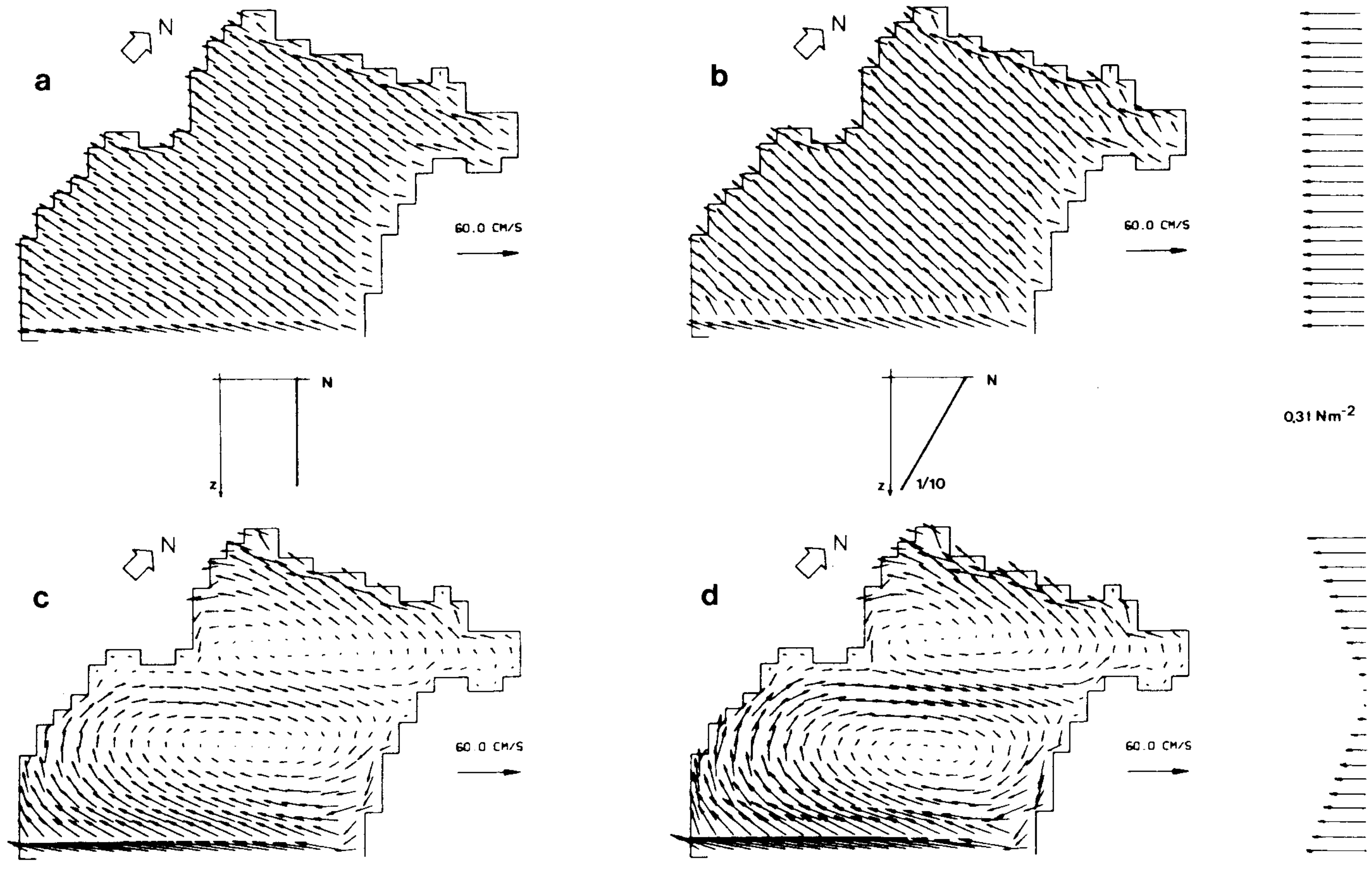

Figure 4. Surface currents predicted by the model. (a) – (d) as in Figure 2.

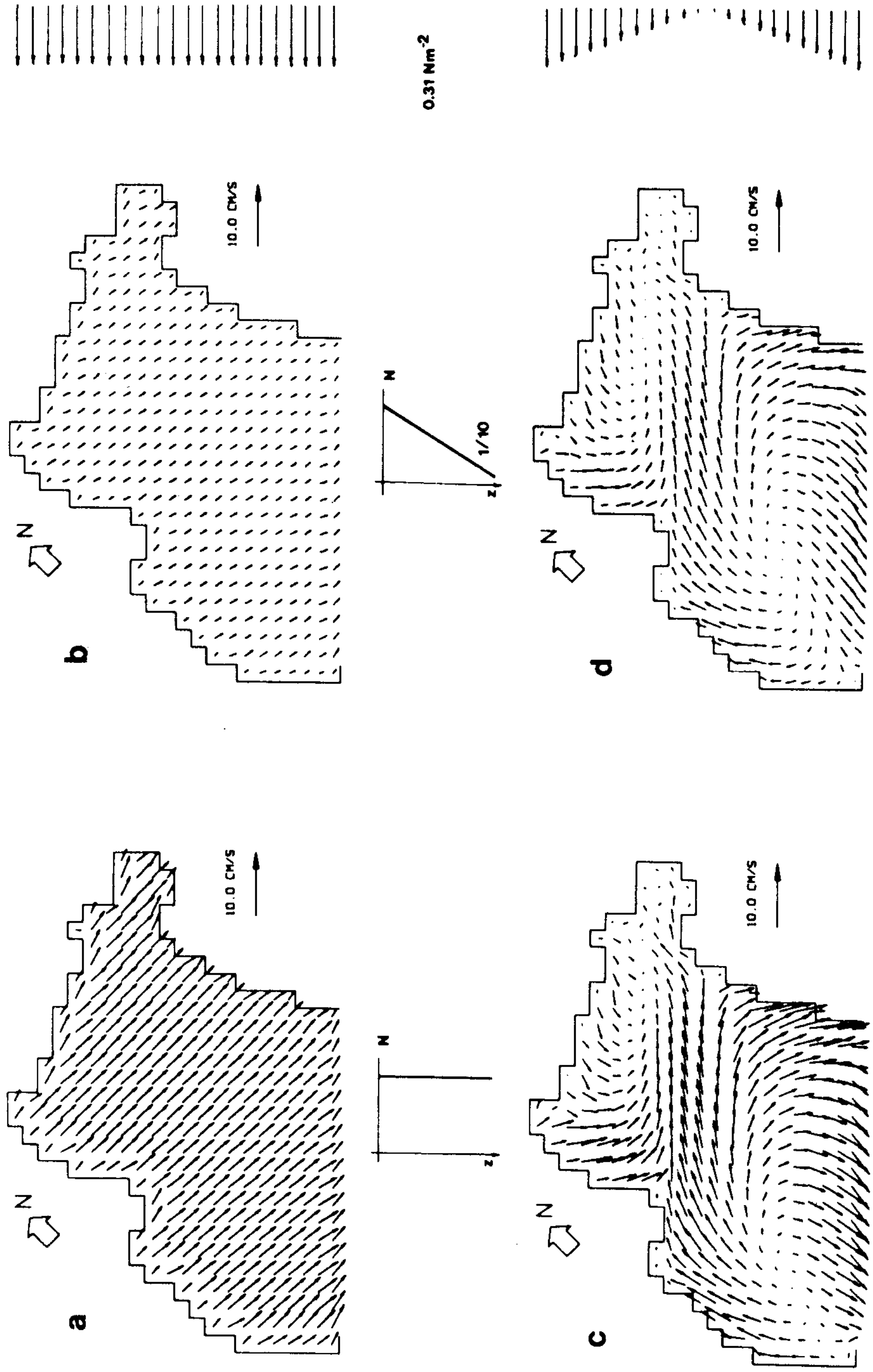

Figure 5. Bottom currents predicted by the model. (a) – (d) as in Figure 2.

current and bottom current. These four fields are presented in Figures 2–5 for each of the four runs. Each figure has the same structure and refers to one of the fields. We will briefly consider each of them.

The combination of homogeneous wind, flat bottom and constant eddy viscosity results in a rather regular distribution of the sea level heights (Figure 2a). The effect of variable eddy viscosity (shift in position and deflection of isolines) is visible in Figure 2b. The effect of the wind curl is more dramatic (Figure 2c). Superposition of the effects of wind curl and variable viscosity results in a wider range of sea level changes, asymmetrical as in previous cases due to particular coastal geometry (Figure 2d). Absence of the topographic effect in all predictions and uniformity of the wind field in cases a) and b) predictably produces rather insignificant vertically averaged currents (Figure 3a and 3b). Two gyres are formed when the wind curl is applied (Figure 3c) and they get intensified when the viscosity is variable (Figure 3d). The wind curl is effective enough to impose gyres on the surface fields and cause the water to flow in upwind direction in the middle of the basin (Figures 4c and 4d). The variable viscosity reduces the bottom currents regardless of the wind (Figure 5b and 5d).

To appreciate those differences better it is perhaps more appropriate to look at the vertical distribution of currents. Such distributions are presented in Figure 6 for two selected locations (of low (L) and high (H) wind-curl influence) and the four previously described cases. Eleven current vectors (one every 4 meters) are plotted for each case and each point. Although some vectors are necessarily overshadowed when current spiral is projected onto the horizontal plane, closer inspection of this figure reveals several interesting points. Comparison of L and H points in the case c) clearly demonstrates the kind of change the wind curl per se can introduce. We should remember that the applied wind curl, although highly schematized, is basically supported by the field data. A decrease of vertical eddy viscosity, although reducing the current at the bottom, tends to increase currents in the bottom layer. However, when the wind-curl influence is low the effect is more pronounced in the homogeneous case (Figure 6La vs Figure 6Lb). High wind-curl influence is accompanied by more dramatic influence of the decreasing eddy viscosity (Figure 6Hc vs Figure 6Hd). Similar interplay of wind-curl and variable viscosity influences can be observed at other points, not presented in the figure. The effect of increased near-bottom current, visible in Figure 6Hd, is particularly important because previous, constant vertical eddy viscosity studies and model to data comparisons suggest the need for precisely that kind of improvement.

To summarize, the presented preliminary analysis of the four rather schematized cases offers encouraging results and suggests the importance of both the wind heterogeneity and vertically variable eddy viscosity for the wind-induced motions in the Northern Adriatic. The work on other, more realistic cases is in progress.

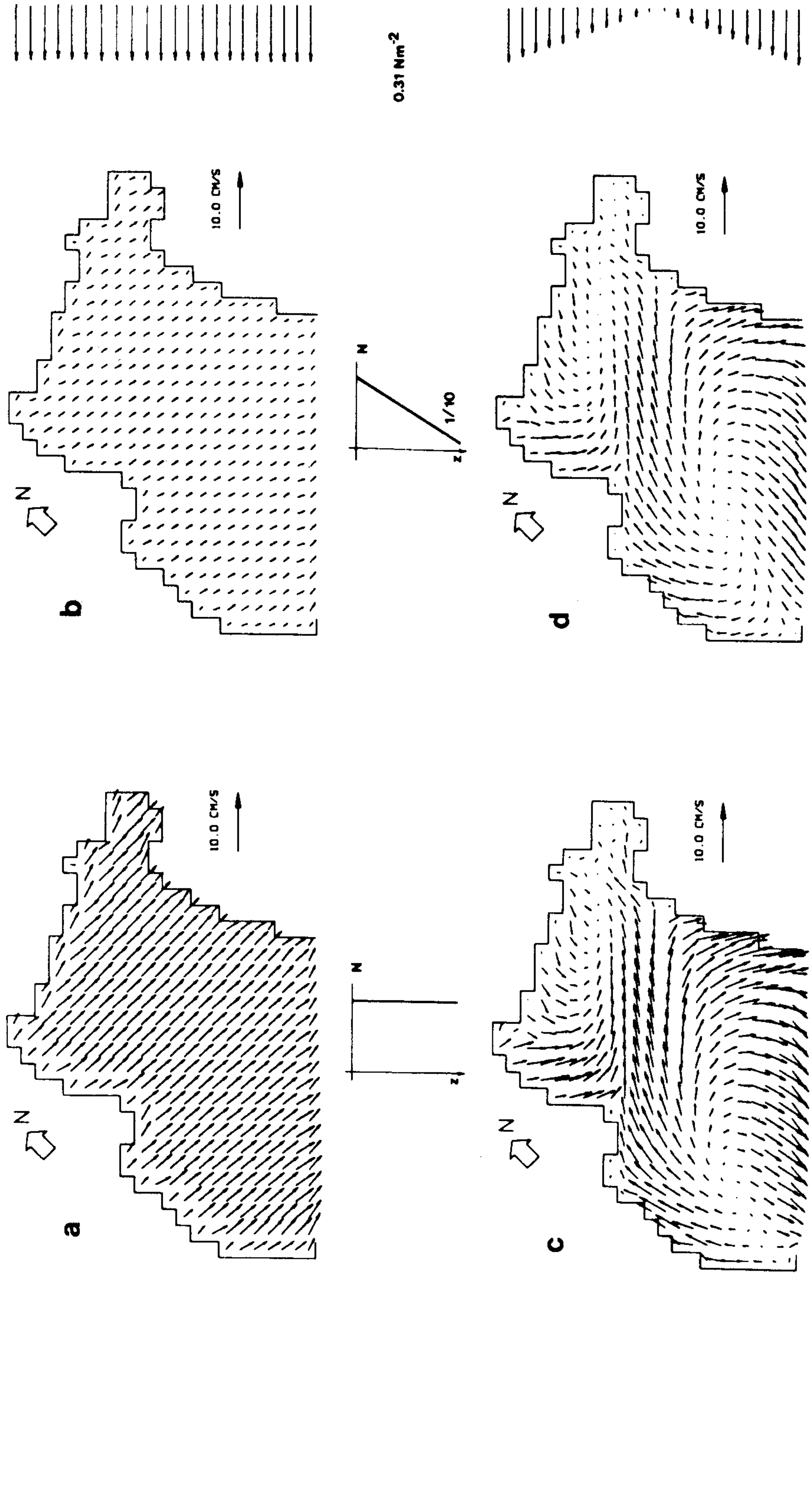

Figure 5. Bottom currents predicted by the model. (a) – (d) as in Figure 2.

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