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Walsh spectral resolution of earth's rotation and atmospheric circulation: a non sinusoidal perspective

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Lambeck and Hopgood (1981, 1982) have made a comparative study of spectral characteristics of the angular momentum of atmospheric circulation and fluctuations in the Length-of-Day (LOD) data. The Fourier analysis of these data reveals a good correlation between the two processes, including a large spread of power in the range of a 20- to 70-month period. We re-examine here the above two time series by using the Walsh spectral analysis, which seems to be more appropriate for series with sharp peaks and reversals. The comparative study shows that in addition to 12-, 6- and 3-month seasonal terms, the present work discovers statistically significant (90% confidence interval) periods of (i) 26 months (hitherto weakly resolved) associated with a quasibiennial term, and (ii) 40±4 months in both the data. The resolution of the spread components removes meteorological noise uncertainity from astronomical observations of LOD in the study of the excitation mechanism of solid earth.

Razlučivanje Zemljine rotacije i atmosferske cirkulacije Walshovim spektrima: nesinusoidalna perspektiva

Lambeck i Hopgood (1981, 1982) usporedili su spektralne karakteristike podataka o kutnom momentu atmosferske cirkulacije i fluktuacijama duljine dana. Fourierova analiza ovih podataka ukazuje na dobru korelaciju između spomenuta dva procesa, uključujući široki spektralni maksimum na periodima od 20 do 70 mjeseci. U ovom radu se navedena dva vremenska niza analiziraju primjenom Walshovih spektara, koji su — kako izgleda — pogodni za razmatranje nizova s oštrim maksimumima i obratima. Rezultati pokazuju da pored 12-, 6- i 3-mjesečnih oscilacija, nizovi sadrže i statistički signifikantne (na nivou 90%) oscilacije perioda (i) 26 mjeseci (do sada gotovo neprepoznate), te (ii) 40±4 mjeseca. Rezolucija ovih komponenata otklanja meteorološki uvjetovanu nesigurnost iz astronomskih opažanja duljine dana.

1. Introduction

The astronomical observation of the Length-Of-Day (LOD) provides a measure of the variability of the total angular momentum of the atmosphere superimposed upon other longer term fluctuations, possibly of non-meteorological origin. The study of the nature of the longer-term excitation mechanism is quite interesting, as it is generally believed to be related to either the processes of the core-mantle interactions or some other solid earth excitation functions. However, due to the dominant contribution of the zonal wind circulation component to the LOD fluctuations, covering the frequency range from about 0.2 cycles per year to about 2 cycles/week and possibly higher terms (Lambeck, 1980; Hide, 1977), the exact nature of the response from longer-term fluctuations has not been well understood.

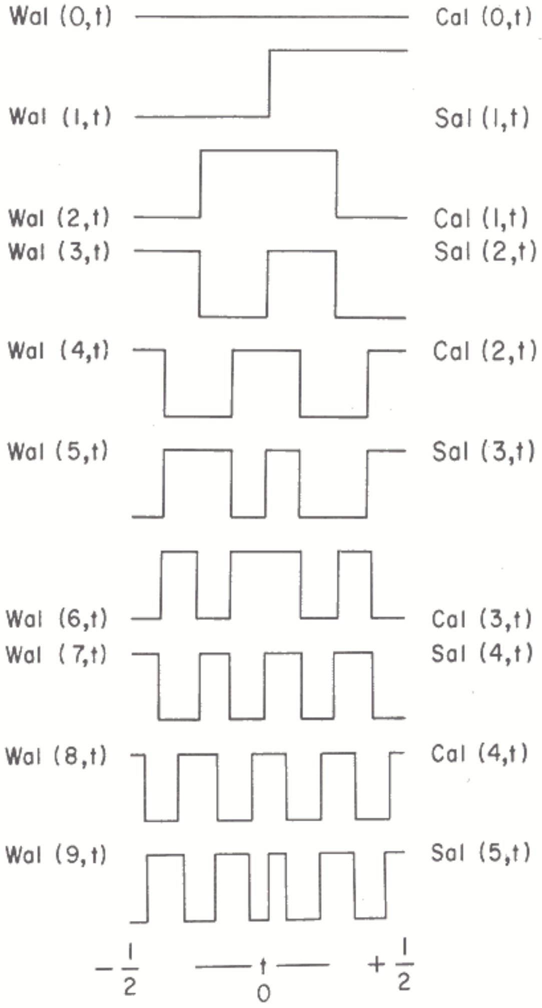

Figure 1. Example of Wal, Sal and Cal functions.

To examine the contributions of the zonal wind and to make the proper correction of the LOD time series, Lambeck and Hopgood (1981, 1982) made a comparative Fourier spectral analysis of these time series. The analysis showed (i) the power spread in the period range of 20—70 months, (ii) the indicated poor resolution of a quasibiennial (26 month) term reported by Reed et al. (1961) and observed by many other Fourier analyses. The spread in power introduces an uncertainty in making an appropriate correction for isolating the zonal wind contribution from the above astronomical observations of the LOD. As noted by Lambeck and Hopgood (1981) the reason for the spread was due to an erratic and non-sinusoidal behaviour of the quasi-biennial variations. Along with the several difficulties that are involved in understanding the excitation mechanism, the presence of meteorological noise and its fairly unknown spectral characteristics pose an additional problem. Thus, it becomes imperative to re-examine these data using a non sinusoidal perspective.

The angular momentum associated with the zonal wind near the tropopause can be characterised by a sequence of almost rectangular functions in which changes from easterly momentum to westerly momentum and vice-versa occur in a relatively short time interval between which the amplitude remains constant (Lambeck and Hopgood, 1981; Figures 1a, b). The Walsh function resembles more closely the signal characteristics and physical behaviour of these discontinuous processes. The LOD fluctuations record also exhibits an almost rectangular perturbations associated with much longer excitation functions. Negi and Tiwari (1983, 1984) and Tiwari (1987a, b) have successfully applied the Walsh spectral technique to the binary telegraphic wave form of geomagnetic reversals and non-sinusoidal palaeomagnetic/palaeoclimatic time series. It is reasonable to expect that the Walsh technique would be somewhat more appropriate to resolve the spectral peaks than the sinusoidal based techniques. We re-examine here the two time series analysed by Lambeck and Hopgood (1981, 1982) and resolve the spread components. The spectral comparisons will provide information about the maximum contribution of meteorological noise to the LOD record. It may be noted that we have deliberately chosen the record of Lambeck and Hopgood (1981, 1982) for the present analysis with a view to maintain comparative status.

2. Walsh transform and power spectra

The theory of Walsh transform is given in detail in the book of Beauchamp (1975) and discussed in detail elsewhere (Tiwari, 1985, 1987b). Brief descriptions of the Walsh technique are given here. The first eight Walsh functions are shown in Figure 1. Any geological time series having discrete time functions $\alpha_0, \alpha_1, \alpha_2, \ldots$ (in Fig. 1) can be expressed as the sum of Walsh functions so that:

$$\alpha_i = \sum_{n=0}^{N-1} \beta(\mathrm{n}) \,\mathrm{Wal}(n, \mathrm{i}/N—1/2) \qquad (1)$$

and because of the completness and orthogonality of the Walsh functions, the equation (1) can be written as a finite Walsh transform:

$$\beta(n) = \frac{1}{N} \sum_{n=0}^{N-1} \alpha_i \mathrm{Wal}(n, \mathrm{i/N}—1/2). \qquad (2)$$

A further notation, to classify the Walsh functions in terms of their odd and even symmetry, has been introduced. Accordingly, the above equation can be written as:

$$\begin{aligned} \beta_c(K) &= \sum_{i=0}^{N-1} \alpha_i \mathrm{Cal}(K, i/N - 1/2), \\ \beta_s(K) &= \sum_{i=0}^{N-1} \alpha_i \mathrm{Sal}(K, i/N - 1/2). \end{aligned} \tag{3}$$

The square of Cal and Sal function coefficients of the same frequency are added in an analogous manner as used in the Fourier analysis so that:

$$\begin{aligned} &A(0) = \beta_c^2(0), \\ &A(K) = \beta_c^2(K) + \beta_s^2(K), \quad K = 1, 2, \dots, (N/2) - 1, \\ &A(N/2) = \beta_s^2(N/2), \end{aligned} \tag{4}$$

where β_c and β_s are Cal and Sal coefficients of the Walsh tansforms.

3. Walsh transform analysis of earth's rotation and atmospheric circulations time series

The time series of atmospheric circulation and LOD are reproduced in Figures 2a, b and 4a, b. The fast Walsh transform technique developed by Beauchamp (1975) is used here to compute the transform and power spectra.

Prior to discussing our result, it would be worth mentioning the comparative feature of Fourier and Walsh domain data windowing. It is known that an appropriate representation of abrupt/discontinuous waveform is not possible in the Fourier domain, as these functions exhibit oscillatory behaviour at discontinuities referred to as »Gibb's phenomena«. In practice, the abrupt truncation of data, such as occur at the start and end of the record, is inevitable. The Fourier analysis introduces leaking side lobes which can be the most serious source of error. The most common method of reducing such a leakage is to smooth these discontinuities by applying a split cosine bell window to the whole record. Fortunately, Walsh domain analysis is superior, having no Gibb's phenomena, as Walsh functions are themselves discontinuous. However, in order to remove the erratic behaviour in the spectrum and to obtain a stable spectral estimate, a simple method of smoothing is required. The choice of spectral window depends on the requirements of resolutions, stability and smoothness of the resultant spectrum. In the present investigations, we have applied a three point Hanning window (.25, .5, .25,) for the purpose of smoothing the spectral estimates. The resulting Walsh power spectra of Figures 2a, b are shown in Figures 3a, b. It is apparent that Figures 3a, b exhibit pronounced peaks centred around 40, 12, 10, 6 and 3 months. In addition to that, the spectra of zonal wind also show a relatively less pronounced peak at 22 months. The corresponding LOD spectra, however, do not appear, perhaps due to the dominance of seasonal term, significant power near 22 months. It is appropriate to perform a statistical significance test to examine the validity of various spectral peaks. To do this, we apply Fisher's test, which determines the probability that the largest amplitudes in a normalised form with respect to the data are the result of randomness of the data. Accordingly, we evaluated the probability distribution functions for all the above spectral components. The broken lines in all figures indicate noise level above which all the peaks are significant at P=0.1. Figures 5a, b show the Walsh power spectra of Figures 4a, b for a 22-year long LOD and zonal wind time series. The zonal wind spectrum (Figure 5b) apparently exhibits dominant peaks near 44, 26, 12, 10 and 6 months with remarkable resolutions. The Walsh power spectrum of the LOD record (Figure 5a), however, exhibits a large power towards the low sequency range indicating

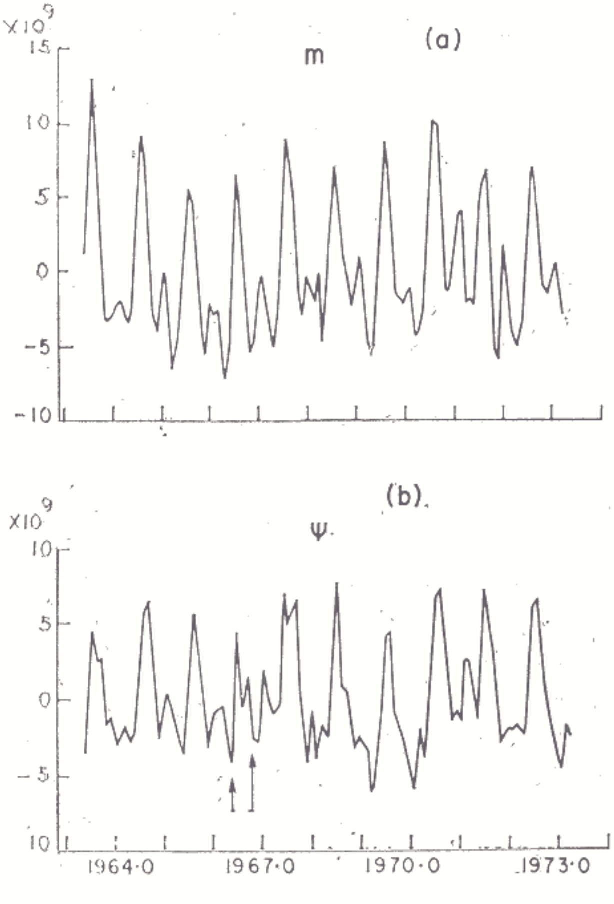

Figure 2. The time series of monthly values of the proportional change in length of day (m) and of the zonal wind excitation function (ψ) from 1963 May to 1973 April. The interval between the arrows indicates the period when the high latitude southern hemispheric coverage was inadequate (after Lambeck and Hopgood, 1981).

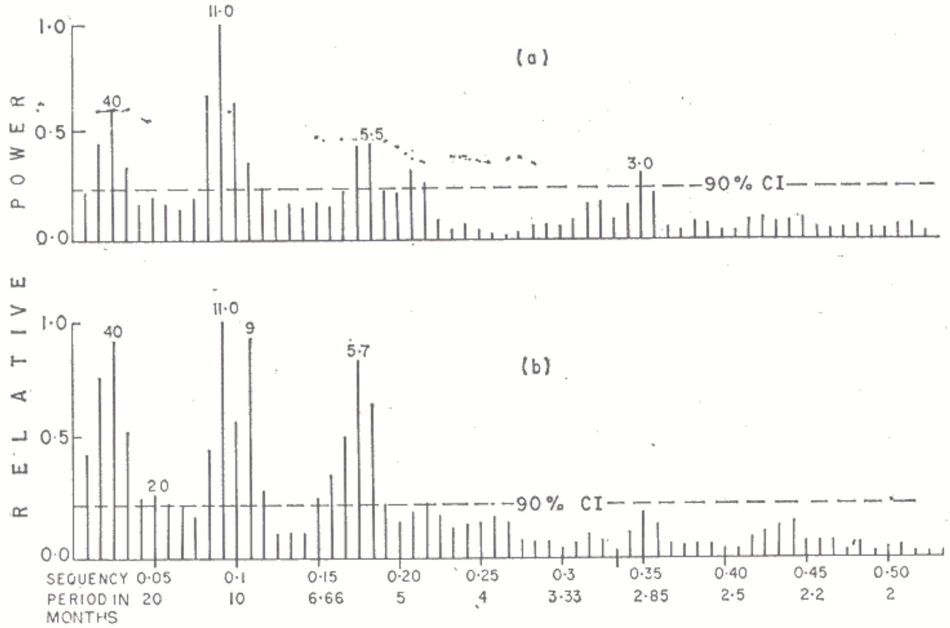

Figure 3. Walsh power spectra of (a) LOD and (b) atmospheric circulation time series as shown in Figure 2 for the period of 1963—1973.

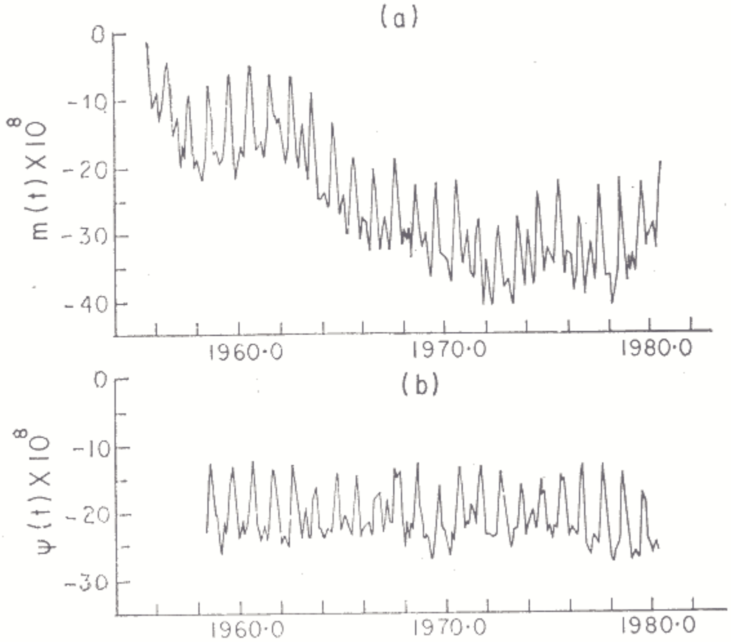

Figure 4. (a) Astronomical observations of the proportional change in the length of day m (t) since the introduction of the atomic time in mid-1955.
(b) Zonal wind excitation function $\psi(t)$ for 22 yrs period from 1958 May to 1980 April.

the dominant role of the long term excitation function. The apparent peaks are at 24, 12, 6, 10.4 and 5.8 months. Figure 6a shows the 22-year record of high-pass filtered time series LOD. The characteristic of these filters is that they posses high frequency signals but attenuate low frequency signals. The detailed features of such filters are discussed by Bath (1974). The corresponding Walsh power spectrum is shown in Figure 6b which reveals, in addition to a seasonal and quasi-biennial term, a relatively less pronounced, but significant peak at 36 months. The results are in close agreement with the zonal wind spectrum (Figure 5b). The spectral peaks corresponding to periods of 12 and 6 months could well be associated with seasonal terms as pointed out by Lambeck and Hopgood (1981). Other peaks in

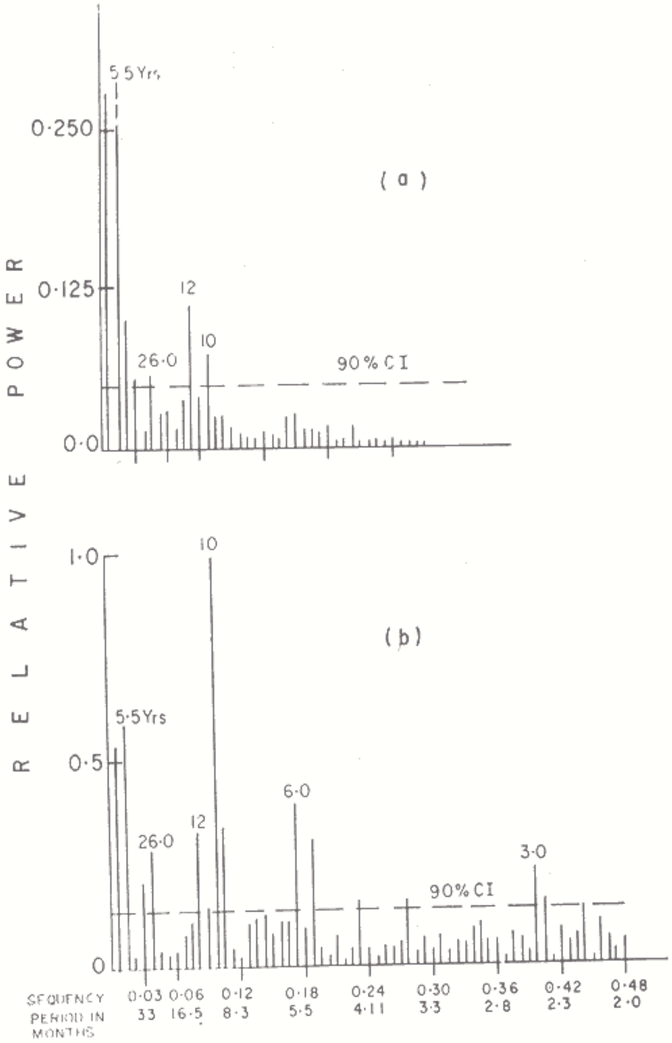

Figure 5. Walsh power spectra of (a) earth's rotation and (b) zonal wind circulation. For the period 1958—1980.

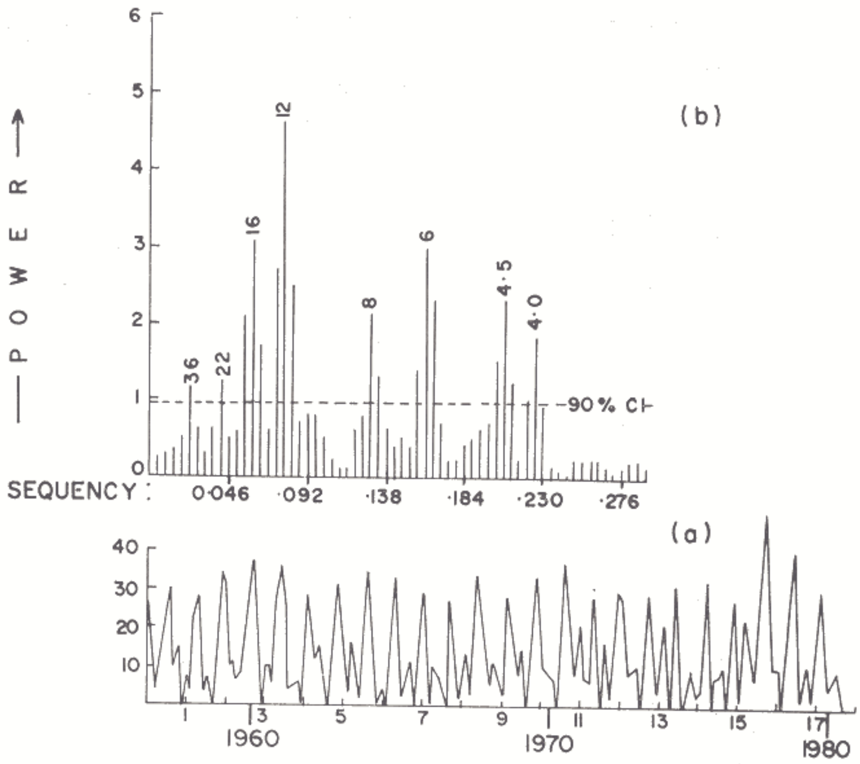

Figure 6 (a) LOD time series after removing longer trend.
(b) Walsh spectrum.

Table 1 (a) RESULTS FOR 10 YEAR PERIOD

Zonal wind circulation periodicities (in months)		Earth's rotation periodicities (in months)	
Lambeck and Hopgood (F.T.A.)	This paper (Walsh spectra)	Lambeck and Hopgood (F.T.A.)	This paper (Walsh spectra)
Spread of power, 20—50 months	40	Spread of power, 20—50 months	40
———	20	12	12
12	12	6	6
6			
3	3	3	3

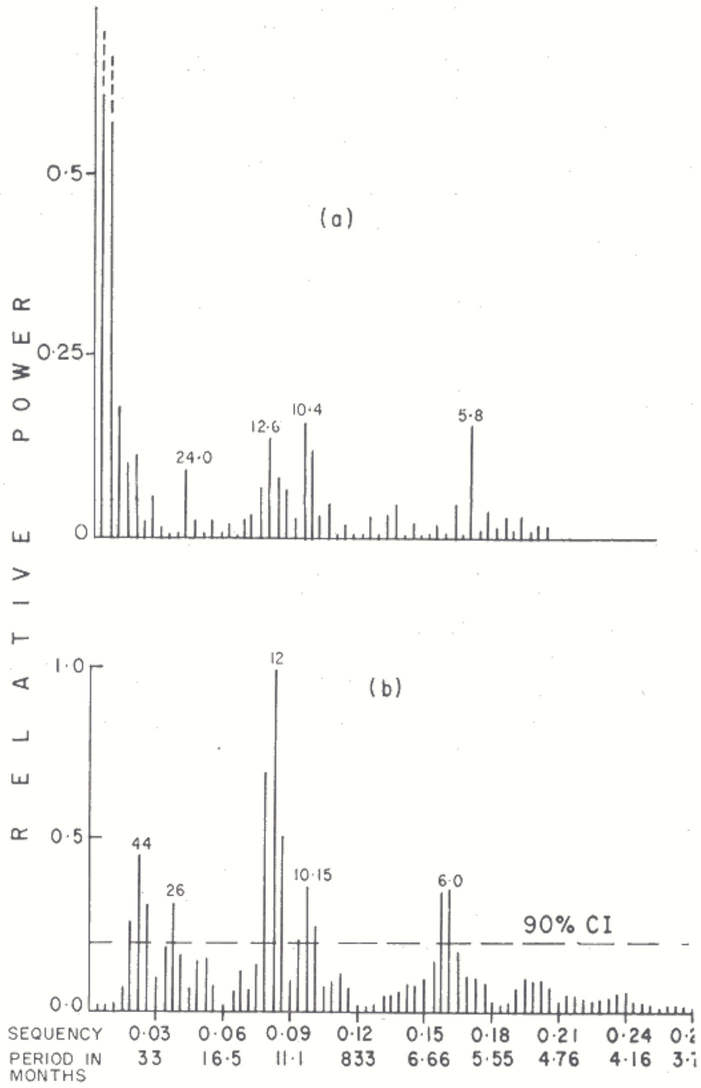

Figure 7. Walsh power spectra of earth's rotation data
(a) for the period 1958—1969
(b) 1969—1980.

a 24±2-month-period range may be related to a quasi-biennial term (Reed et al., 1961). The higher order periodicities (40±4 months) in the zonal wind time series indicate the maximum limit of the meteorological contribution to the LOD fluctuations and will have interesting implications for correcting the LOD period.

Figures 7a, b show the Walsh power spectra of the LOD record after dividing it into two equal sub-series. This helps to examine the stability of different peaks in both parts. Obviously, the results further confirm the stability of the quasi-biennial term of 26 months and other seasonal terms as noted earlier. There is also

Table 1 (b) RESULTS FOR 22 YEAR RECORD

Zonal wind circulation periodicities (in months)		Earth's rotation periodicities (in months)	
Lambeck and Hopgood (F.T.A.)	This paper (Walsh spectra)	Lambeck and Hopgood (F.T.A.)	This paper (Walsh spectra)
Spread of power in range of 25—70 months	44	Spread of power 25—70 months	66
20 (Poor resolution)	26	20 (Poor resolution)	26
12	12	12	12
9 (Poor resolution)	9	9 (Poor resolution)	9
6	6	6	6

a peak corresponding to 5.5 years. Although there are several indications suggesting the existence of such peaks in the LOD record (Currie, 1980) and geomagnetic spectrum (Currie, 1968, and Jin and Thomas, 1977), we can not assign significant physical weight to this peak, due to the short time scale. The final results are summarised and compared with the results of Lambeck and Hopgood (1981, 1982) in Table 1. Indeed, the comparison (Table 1) clearly points out the relative advantage of the Walsh spectral analysis in adequately resolving the spectral components from such time series.

4. Conclusions

From the above analysis and foregoing discussions, it is clear that the Walsh transform analysis reveals a relatively clearer picture of erratic and non-sinusoidal zonal wind circulation and LOD time series. The main results can be summarized as follows:

1. The analysis resolves a dominant peak approximately in the period range of 40±4 months, which may have significant implications in isolating the meteorological contributions from the LOD time series.
2. The analysis confirms the existence of a 26-month periodicity, both in the zonal wind and LOD time series, which was weakly resolved in the previous investigations.

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